Bankruptcy Risk Premium and the Valuation of Stock Options

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Abstract

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JEL Classification: G12, G13, G33

Key Words: stock option; option pricing; bankruptcy risk premium; term structure of bankruptcy probability

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1. Introduction

Corporate bankruptcy is undoubtedly an important issue in economics and finance, for both academics and practitioners alike, since bankruptcy events may have devastating impacts on the economy and society. According to the risk-return tradeoff, equity holders have the potential to profit from capital gains, while they may lose the value of their shares when a bankruptcy event occurs. The risk of corporate bankruptcy therefore plays a crucial pricing factor for individual stocks as well as their derivatives. In view of this, this study develops an innovative option pricing model on individual stocks subject to bankruptcy risk under an equilibrium framework and analyzes the corresponding risk-return nexus. Broadly speaking, this study links three important branches of financial literature: asset pricing, credit risk, and derivatives pricing.

In the option pricing theory, Black and Scholes (1973) provide the most influential foundation, offering a simple but revolutionary approach to the valuation of option contracts. They use the assumption of continuous trading - that is, investors can hold the underlying asset and its call options to form a continuously riskless portfolio. Based on the principle of no-arbitrage opportunity, the constructed portfolio should earn risk-free returns, and thus the well-known Black-Scholes option pricing formula can be deduced. Rubinstein (1976), on the other hand, derives the option pricing formula by considering a representative investor whose goal is to maximize her expected utility in a discrete-time economy. Although the equilibrium option price derived by Rubinstein (1976) is equivalent to the Black-Scholes formula, the discrete-time economy appears to be more realistic than the continuous trading assumption. However, both Black and Scholes (1973) and Rubinstein (1976) do not consider bankruptcy risk.
Under the discrete-time economy, Câmara, Popova, and Simkins (2012) incorporate bankruptcy risk into the valuation of stock options, adding one parameter to Rubinstein’s model to capture the bankruptcy probability. Similar to Samuelson’s (1973) jump-to-default (JTD) model, Câmara et al. (2012) assume that if the firm goes bankrupt, then the shareholders will lose all claims on the firm and the stock price will fall to zero. Thus, the equilibrium option pricing formula derived by Câmara et al. (2012) is equivalent to the perfect ruin model of Merton (1976), making it a simple approach to link stock option valuation and bankruptcy risk.

Although Câmara et al. (2012) incorporate bankruptcy risk into the valuation of stock options in a simple way, this traditional JTD model lacks some economic insights. First, the bankruptcy probability is an exogenous constant unrelated to the firm’s future conditions. In contrast, Merton’s (1974) structural credit risk model defines a default event as when the asset value is less than the debt level, implying that the probability of default is endogenously triggered by the future state of the firm. Second, the risk-return relationship implied by Câmara et al. (2012) shows that there is no bankruptcy risk premium, which is counterintuitive, since risk-averse investors require compensations for bearing the non-diversifiable corporate default risk. Third, the term structure of bankruptcy probability (TSBP) in the JTD model is inflexible, in which the hazard rate is irrelevant to the maturity date. However, even though two firms have the same 1-year credit rating, they may have different short-term credit risk profiles due to different debt structures and cash flows.

To remedy the deficiencies discussed above, this study develops an option pricing model on individual stocks subject to bankruptcy risk under Rubinstein’s (1976) discrete-time economy. Similar to Câmara et al. (2012), this study adopts the reduced-form credit risk model, assuming that the stock price may jump to zero with a positive probability. Unlike Câmara et al. (2012), the probability of a firm’s
bankruptcy is stochastic, negatively related to its future stock prices.\textsuperscript{1} Since the bankruptcy event is driven by the firm’s future conditions, the proposed model echoes the structural concept of Merton (1974). Consequently, the total risk premium on a stock can be decomposed into two parts: the traditional equity risk premium and the bankruptcy risk premium, which describes the excess returns for bearing market risk and the risk of bankruptcy, respectively. In addition, TSBP is flexible in our model. The relationship between default intensity and maturity date can be decreasing, increasing, or hump-shaped, depending on the credit risk-profile of the firm.

There is a controversial debate about expected stock returns and credit risk of a firm in the asset pricing literature. From the perspective of risk theory, risk-averse investors require compensations to bear the non-diversifiable credit risk.\textsuperscript{2} However, Dichev (1998), Griffin and Lemmon (2002), and Campbell, Hilscher, and Szilagyi (2008), among others, document a counterintuitively negative relation between expected stock returns and credit risk measures. To conciliate this debate, Friewals, Wagner, and Zechner (2014) use Merton’s (1974) structural model to show that the market price of risk depends on both the physical and the risk-neutral probabilities of default, whereas previous studies use only one of them and subsequently observe incomplete conclusions. Consistent with Friewals et al. (2014), the introduced bankruptcy risk premium depends on both the physical and the risk-neutral bankruptcy probabilities. In addition, the bankruptcy risk premium is driven by the dependency between bankruptcy probability and future stock prices and is affected by

\textsuperscript{1} Linetsky (2006), Carr and Linetsky (2006), and Carr and Wu (2010) adopt similar concepts to capture the negative relation and to derive option prices, but none of them provide economic mechanisms and equilibrium conditions for asset pricing. Câmara, Popova, and Simkins (2014) employ a negative relation under the equilibrium framework, but they do not explore the bankruptcy risk premium and the term structure of bankruptcy probability.

\textsuperscript{2} Chava and Purnanandam (2010), Garlappi, Shu, and Yan (2008), Vassalou and Xing (2004), among others, offer empirical evidence for the positive relation between expected stock returns and the measures of credit risk.
stock volatility.

The most common proxy of credit risk premium in the literature is the ratio of the risk-neutral default intensity to the physical default intensity. Using U.S. corporate bond prices and average historical default frequencies by credit rating, Driessen (2005) estimates the ratio to be about two, which means that credit risk is positively priced. Similar results are documented by Berndt, Douglas, Duffie, Ferguson, and Schranz (2005), where they focus on the time-varying features of the ratio using credit default swaps (CDS) data and Moody’s KMV expected default frequency. Similarly, Das and Sundaram (2007) use the ratio of risk-neutral probability to real-world probability as a metric of the credit risk premium, finding that the ratio is on average greater than one, which echoes the finding of Driessen (2005). Although the ratio of intensity (probability) is a simple metric to compare the credit risk under both measures, the linkage to the risk-taking compensation is not clear. In contrast, this study provides an economic mechanism to interpret the additional risk premium for credit events.

There is another potential illogical issue concerning probability ratios or intensity ratios. On the one hand, the ratio of two probabilities is sensitive when both bankruptcy probabilities are small (e.g., less than 0.1%) and the corresponding credit risk premium may be significant even if the bankruptcy risk is negligible. On the other hand, when both bankruptcy probabilities are extremely high (e.g., greater than 90%), the ratio of two probabilities must be close to one, subject to the upper bound of a probability. This example suggests that the required rate of return to hold such a dangerous stock is relatively small. However, the bankruptcy risk premium proposed in this study increases monotonically in the bankruptcy probability, which is consistent with economic intuition.

After constructing the theory of bankruptcy risk premium, we analyze the impacts of the term structure of bankruptcy probability. As documented in Das and
Hanouna (2009), the short-run likelihood of default is high for high credit risk firms. In addition, Duan, Sun, and Wang (2012) find that the term structure of Lehman Brothers’ default probabilities corresponds strongly to its bankruptcy event. The forward default probability of Lehman Brothers exhibits an increasing pattern during the normal period, whereas it becomes a decreasing curve three months prior to its bankruptcy. Motivated by the fact that firms with high- and low-bankruptcy risk possess disparate term structures of bankruptcy probability, we specify a short-run factor to distinguish the risk profile of the underlying stock.

This study offers the closed-form solution for the stock option prices. The pricing formula is associated with the magnitude of bankruptcy risk premium and the term structure of the risk-neutral bankruptcy probability, offering a channel to extract forward-looking information of bankruptcy expectations from the derivatives market. Easley, O’Hara, and Srinivas (1998) argue that the option market is an alternative trading venue for informed traders, which means that private information may be reflected in the option trading. Cremers, Diressen, Maenhout, and Weinbaum (2008) and Cao, Yu, and Zhong (2010) note that option-implied volatility is a crucial determinant of corporate bond credit spreads and CDS spreads, respectively. Taylor, Tzeng, and Widdicks (2014) find that bankruptcy probabilities extracted from option prices of six banks are high during the financial crisis. Based on these results, the market prices of stock options may contain useful information about corporate bankruptcy.

Using the market prices of stock options and returns for 23 bankruptcy firms, our empirical results show that the relative option trading volume is abnormally high three months before the bankruptcy event. At the same time, the estimated bankruptcy risk premium surges in the six months before the bankruptcy event, supporting bankruptcy expectations contained in the options market. In addition, our empirical results show
that specifying a short-run credit risk factor into the term structure of bankruptcy probability can fit the market implied-volatility better than the traditional jump-to-default model does. The short-run factor is statistically significant prior to bankruptcy events, which echoes the findings of Das and Hanouna (2009) and Duan et al. (2012) and supports the theory proposed herein.

The remainder of this study proceeds as follows. Section 2 develops the model set-up, derives the option pricing formula, and introduces the bankruptcy risk premium. Section 3 shows the numerical analyses. Section 4 provides the empirical discussions. The conclusions are finally presented in Section 5.

2. The Theory

2.1 The Economic Framework

This study considers the pure exchange economy of Rubinstein (1976) where there is a representative investor who aims to maximize her expected utility of consumption when she makes her consumption and investment decisions. The utility function $U$ is assumed to be the power-utility specification:

$$U(C_t) = \beta \frac{C_t^{1-\gamma}}{1-\gamma},$$ (1)

where $\beta \leq 1$ captures the degree of patience, $\gamma > 0$ is the coefficient of proportional risk aversion, and $C_t$ is the aggregate consumption at date $t$. As shown in Cochrane (2001), the Euler equation implies that the current price of the underlying asset $A_t$, in equilibrium, is given by:

$$A_t = \mathbb{E}_t^P \left[ \left( \frac{U'(C_{t+\tau})}{U'(C_t)} \right) A_{t+\tau} \right] = \mathbb{E}_t^P \left[ \beta^\tau \left( \frac{C_{t+\tau}}{C_t} \right)^{-\gamma} A_{t+\tau} \right],$$ (2)

where $\mathbb{E}_t^P [\cdot]$ is the expectation conditional on information available at date $t$ under the actual probability measure $\mathbb{P}$ ($\mathbb{P}$-measure, hereafter), $\tau > 0$ represents the time to maturity of an option contract, and the marginal rate of substitution $\frac{U'(C_{t+\tau})}{U'(C_t)}$ is...
called the *pricing kernel*. Under this economic framework, this study considers three financial assets: a riskless zero coupon bond \( B_t \), an individual stock \( S_t \) subject to bankruptcy risk, and a European-style call option \( \text{Call}_t \) on the individual stock.

### 2.2 Aggregate Consumption and Stock Price

For the aggregate consumption process, this study follows Rubinstein (1976) and Câmara et al. (2012) to assume that \( C_{t+\tau} = C_t \epsilon_{c,\tau} \), where \( \epsilon_{c,\tau} \) is a lognormal random variable with mean \((\mu_c - \sigma_c^2/2)\tau\) and variance \(\sigma_c^2\tau\) under the \( \mathbb{P} \)-measure. This assumption implies that the aggregate consumption is a geometric random walk, and the pricing kernel is lognormally distributed.

For the stock price process, this study considers the bankruptcy risk in which the underlying firm may go bankrupt with a positive probability. If the firm goes bankrupt before the maturity date, then this study follows Câmara et al. (2012) to assume that the stock price will fall to zero, which means that the shareholders lose all their claims against the firm. On the contrary, if the firm does not go bankrupt until the maturity date, then this study follows Rubinstein (1976) and Câmara et al. (2012) to assume that the stock price and aggregate consumption are bivariate lognormally distributed. Consequently, the stock price process consists of two parts: the probability of bankruptcy and its distribution conditional on survival.

In notation, the stock price has the form:

\[
S_{t+\tau} = S_t \epsilon_{s,\tau} \mathbf{1}_{\{t^* > t+\tau\}}, \tag{3}
\]

where \( \epsilon_{s,\tau} \) is a lognormal random variable with mean \((\mu_s - \sigma_s^2/2)\tau\) and variance \(\sigma_s^2\tau\) under the \( \mathbb{P} \)-measure, and the covariance between \( \ln \epsilon_{c,\tau} \) and \( \ln \epsilon_{s,\tau} \) is \(\sigma_{cs}\). The notations \( \mathbf{1}_{\{\cdot\}} \) and \( t^* \) respectively denote the indicator function and the bankruptcy date, determining whether the underlying firm survives at maturity. The bankruptcy date \( t^* \) is a random variable. If the bankruptcy date \( t^* \) is earlier than the
maturity date $t + \tau$, then the indicator function returns to zero such that the stock price also goes to zero - that is, the indicator function captures the survival status, with probability $\Pr(t^* > t + \tau)$.

2.3 Expected Bankruptcy Probability

The bankruptcy probability $\Pr(t^* \leq t + \tau)$ in this study is a random variable driven by the same random source of the stock price. More specifically, this study allows a negative relation between the bankruptcy probability and the future states of the stock price, which means capturing the spirit of the structural credit risk model of Merton (1974). The bankruptcy probability is assumed to have the form:

$$\Pr(t^* \leq t + \tau) = \frac{\delta(\tau)}{S_t} - \frac{\alpha(\tau)}{e^{-\alpha(\tau) \left( \mu_s - (1 + \alpha(\tau)) \sigma_s^2 / 2 \right) \tau}}.$$  

(4)

where $\delta(\tau)$ captures the force of default, and $\alpha(\tau)$ captures the dependency between bankruptcy probability and future stock prices. In particular, this model reduces to that in Câmara et al. (2012) when $\alpha(\tau) \equiv 0$. Note that $\delta(\tau)$ and $\alpha(\tau)$ are functions of $\tau$, and we determine their functional forms by the properties of their expected probabilities.

The expected bankruptcy probability under the $\mathbb{P}$-measure, $PD_{t,t+\tau}$, can be derived as:

$$PD_{t,t+\tau} = \mathbb{E}_t^{\mathbb{P}}[\Pr(t^* \leq t + \tau)] = \delta(\tau) S_t^{-\alpha(\tau)} e^{-\alpha(\tau) \left( \mu_s - (1 + \alpha(\tau)) \sigma_s^2 / 2 \right) \tau}.$$  

(5)

Assume that model parameters $\{\sigma_s^2, \delta, \alpha\}$ are the same under the $\mathbb{P}$-measure and the risk-neutral measure ($\mathbb{Q}$-measure, hereafter), and the difference between the growth rate of return conditional on survival under two measures is $\mu_s - \mu_s^Q = \gamma \sigma_{cs}$, where $\gamma \sigma_{cs}$ is the traditional equity risk premium as shown in Rubinstein (1976) and Câmara et al. (2012). In other words, we do not impose an additional risk premium into the growth rate of return conditional on survival, since this study emphasizes the
risk premium contributed by the randomness of the bankruptcy probability. Therefore, the corresponding expected bankruptcy probability under the $\mathbb{Q}$-measure, $PD_{t,t+\tau}^\mathbb{Q}$, can be derived as:

$$PD_{t,t+\tau}^\mathbb{Q} = \mathbb{E}_t^\mathbb{Q}[\Pr(t^* \leq t + \tau)] = \delta(\tau)S_t^{-\alpha(\tau)} e^{-\alpha(\tau)(\mu_0^\mathbb{Q} -(1+\alpha(\tau))\sigma_s^2\tau)},$$

(6)

where $\mathbb{E}_t^\mathbb{Q}[\cdot]$ is the expected value conditional on information available at date $t$ under the $\mathbb{Q}$-measure. Hence, (5) and (6) imply that:

$$PD_{t,t+\tau}^\mathbb{Q} = PD_{t,t+\tau}e^{\alpha(\tau)\gamma\sigma_cs\tau}.$$ 

(7)

According to the property of a probability, $PD_{t,t+\tau}$ and $PD_{t,t+\tau}^\mathbb{Q}$ should be strictly increasing in $\tau$, with initial values equal to zero and upper limits equal to one. Without loss of generality, this study assumes: $\alpha(\tau) = \alpha/\tau$ and $\lim_{\tau \to \infty} \delta(\tau) = \delta$, where $\alpha$ and $\delta$ are constants. These assumptions imply that:

$$PD_{t,t+\tau}^\mathbb{Q} = PD_{t,t+\tau}e^{\alpha\gamma\sigma_cs},$$

(8)

which echoes the common proxy of the credit risk premium in the literature. Driessen (2005) and Berndt et al. (2008) define the proxy of the default risk premium as the ratio of the risk-neutral default intensity to the actual default intensity, and Das and Sundaram (2007) use the proxy as the ratio of the risk-neutral probability to the actual probability. Thus, the probability ratio implied herein is simultaneously driven by the traditional equity risk premium $\gamma\sigma_cs$ and the dependency between bankruptcy probability and future stock prices $\alpha$.

### 2.4 Asset Pricing

This implies that the limits of $PD_{t,t+\tau}$ and $PD_{t,t+\tau}^\mathbb{Q}$ are $\delta e^{-\alpha(\mu_0^\mathbb{Q} - \sigma_s^2\tau)/2}$ and $\delta e^{-\alpha(\mu - \gamma\sigma cs - \sigma_s^2\tau)/2}$, respectively, which are restricted to be less than one.

The condition (7) implies the relation between two default intensities as:

$$e^{-\lambda^\mathbb{Q}\tau} = 1 - (1 - e^{-\lambda\tau})e^{\alpha\gamma\sigma_cs}.$$ 

Using the Taylor expansion, we obtain that $\lambda^\mathbb{Q} \approx \lambda e^{\alpha\gamma\sigma_cs}$. 

11
In this subsection, we provide the consumption capital asset pricing model in closed-form. First, the riskless zero coupon bond $B_t$ pays one dollar at maturity date $t + \tau$ with current value $e^{-r\tau}$, where $r$ is the continuously compounded interest rate. The Euler equation implies that the bond price in equilibrium is given by $E_t^F[\beta^e c^{-\gamma}]$. Using the moment generating function of the normal distribution as shown in Câmara et al. (2012), the continuously compounded interest rate, in equilibrium, is given by:

$$r = -\ln(\beta) + \gamma \left( \mu_c - \frac{\sigma_c^2}{2} \right) - \frac{1}{2} \gamma^2 \sigma_c^2. \quad (9)$$

The equilibrium interest rate is the same as that of Rubinstein (1976) and Câmara et al. (2012) due to the same assumption of the aggregate consumption process.

We next note that the stock price in equilibrium satisfies $S_t = E_t^F[\beta^c e^{-\gamma} S_{t+\tau}]$, which implies that:

$$\mu_s - \gamma \sigma_{cs} = r - \frac{1}{\tau} \ln \left( 1 - e^{-\alpha \sigma_s^2} P Q_{t+\tau} \right). \quad (10)$$

The equilibrium condition (10) nests the models in the literature. For example, if $P Q_{t+\tau} = 0$, then it reduces to that in Rubinstein (1976): $\mu_s = r + \gamma \sigma_{cs}$, indicating that the expected rate of return $\mu_s$ consists of the risk-free rate $r$ and the equity risk premium $\gamma \sigma_{cs}$. If $\alpha = 0$ and $P Q_{t+\tau} = 1 - (1 - \delta)^\tau$, then it reduces to that in Câmara et al. (2012): $\mu_s + \ln(1 - \delta) = r + \gamma \sigma_{cs}$, indicating that the expected rate of return is diminished due to the possibility of bankruptcy.

When the stock option price has strike price $K$ and time to maturity $\tau$ is $E_t^F[\beta^c e^{-\gamma} (S_{t+\tau} - K)^+]$, this yields the following option pricing formula:

$$Call_t = e^{-r\tau} \left[ S_t e^{(\mu_s - \gamma \sigma_{cs})\tau} \Pi_1 - K \Pi_2 \right], \quad (11)$$

where

$$\Pi_1 = \left[ \Phi(d_+) - e^{-\alpha \sigma_s^2} P Q_{t+\tau} \Phi(q_+) \right], \quad (12)$$

$$\Pi_2 = \left[ \Phi(d_-) - P Q_{t+\tau} \Phi(q_-) \right]. \quad (13)$$
\[ d_\pm = \frac{\ln\left(\frac{S_t}{K}\right) + \left(\mu_s - \gamma \sigma_{cs} \pm \frac{\sigma_s^2 \tau}{2}\right)}{\sigma_s \sqrt{\tau}}, \quad (14) \]

and \( q_\pm = d_\pm - \alpha \sigma_s / \sqrt{\tau}. \) In particular, \( \Phi(\cdot) \) denotes the cumulative density function of the standard normal distribution. We leave the proof in Appendix.

We note that the option pricing formula (11) is preference-free after incorporating the equilibrium condition (10). This pricing formula (11) is equivalent to the option pricing formula of Black and Scholes (1973) and Rubinstein (1976) when \( PD_{t,t+\tau}^Q = 0 \) and is equivalent to the jump-to-default model of Câmarã et al. (2012) as well as the ruin option pricing model of Merton (1976) when \( \alpha = 0 \) and \( PD_{t,t+\tau}^Q = e^{-\lambda \tau}. \) The corresponding put option price can be derived from the put-call-parity: \( Put_t = Call_t + Ke^{-r\tau} - S_t. \)

2.5 Bankruptcy Risk Premium

The equity risk premium measures the expected excess return over the risk-free return, which can be decomposed into two parts: the traditional equity risk premium and the bankruptcy risk premium. The former is the risk premium capturing the difference in expected growth rates of returns between \( \mathbb{P} \)- and \( \mathbb{Q} \)-measures conditional on the firm’s survival, which is a traditional term in the literature. The latter, however, captures the difference in expected bankruptcy probabilities between \( \mathbb{P} \)- and \( \mathbb{Q} \)-measures.

Equating the equilibrium condition (10) with the expected bankruptcy probability (7), the expected excess return can be deduced as:

\[ \mu_s + \frac{1}{\tau} \ln(1 - PD_{t,t+\tau}) - r = \frac{\gamma \sigma_{cs}}{\text{traditional risk premium}} + \frac{\text{BRP}}{\text{bankruptcy risk premium}}, \quad (15) \]

where
BRP = $\frac{-1}{\tau} \ln \left( 1 - b \frac{PD_{t,t+\tau}}{1-PD_{t,t+\tau}} \right)$, \hspace{1cm} (16)

and

$$b = \left( e^{\alpha(\gamma \sigma_{cs} - \sigma_s^2)} - 1 \right).$$ \hspace{1cm} (17)

Note that the BRP deduced by this study is determined by the level of expected bankruptcy probability $PD_{t,t+\tau}$ (or $PD_{t,t+\tau}^Q$), the equity risk premium conditional on survival $\gamma \sigma_{cs}$, the dependency between bankruptcy probability and future stock prices $\alpha$, and the stock volatility $\sigma_s$.

There are three special cases where BRP is zero. First, when $PD_{t,t+\tau} = 0$ or $PD_{t,t+\tau}^Q = 0$, there is no risk of bankruptcy, implying no additional risk premium. Interestingly, even if the expected bankruptcy probability is non-zero, BRP remains zero when $\alpha = 0$. This special case implies that bankruptcy risk premium can be driven by the randomness of bankruptcy probability, which explains why there is no bankruptcy risk premium in the JTD model of Câmara et al. (2012). Third, when $\gamma \sigma_{cs} = \sigma_s^2$, BRP is also zero regardless of the bankruptcy probability. To investigate the sign of BRP, we rewrite the traditional equity risk premium $\gamma \sigma_{cs}$ as the proportion of stock variance, $\theta \sigma_s^2$, which is a common expression in the literature such as by Heston and Nandi (2000) and Christoffersen, Jacobs, Ornthanalai, and Wang (2008), among others. The notation $\theta$ captures the market price of risk conditional on survival.\(^5\) Thus, BRP is positive when $\theta > 1$. It means that the sign of BRP is determined by the sign of the market price of risk.

### 2.6 Term Structure of Bankruptcy Probability

The term structure of bankruptcy probability (TSBP) is defined as the cumulative

\(^5\) The notation for the market price of risk in Heston and Nandi (2000) and Christoffersen et al. (2008) is $\lambda$ rather than $\theta$, while the notation $\lambda$ in the credit risk literature usually represents default intensity.
probability of each future maturity, which is a non-decreasing curve with an upper bound of one that describes the relationship between bankruptcy probabilities and maturity date. In the traditional jump-to-default model of Câmara et al. (2012), the survival probability is assumed to be exponentially distributed - that is, the bankruptcy probabilities are \( PD_{t,t+\tau} = 1 - e^{-\lambda \tau} \) and \( PD^Q_{t,t+\tau} = 1 - e^{-\lambda^Q \tau} \), where \( \lambda \) and \( \lambda^Q \) are the default intensities under the physical and the risk-neutral measures, respectively.\(^6\) In order to explore the short-run and long-run credit factors, this study proposes a general TSBP, assuming that the distribution of survival probability is a mixture of exponential and Weibull distributions.

The Weibull distribution can be treated as a generalized exponential distribution and is widely used in statistical problems in the actuarial literature.\(^7\) The corresponding bankruptcy probability has the form: \( PD_{t,t+\tau} = 1 - e^{-\lambda \kappa \tau} \) and \( PD^Q_{t,t+\tau} = 1 - e^{-\lambda^Q \kappa \tau} \), where the shape parameter \( \kappa \) determines the pattern of the default intensity. The default intensity is decreasing (increasing) in \( \tau \) for \( \kappa < 1 \) (\( \kappa > 1 \)), meaning that the bankruptcy probability is driven by the short-run (long-run) credit factor. Hence, a combination of multiple Weibull distributions may be the ideal way to generate the TSBPs.

Without loss of generality, this study adopts the following term structure:

\[
PD_{t,t+\tau} = 1 - e^{-[\lambda_E(S_t,\alpha)\tau + \lambda_W(S_t,\alpha)\tau^\kappa]}, \tag{18}
\]

\[
PD^Q_{t,t+\tau} = 1 - e^{-[\lambda^Q_E(S_t,\alpha)\tau - \lambda^Q_W(S_t,\alpha)\tau^\kappa]}, \tag{19}
\]

where \( \lambda_E(S_t,\alpha) \) and \( \lambda^Q_E(S_t,\alpha) \) capture the traditional default intensity in the exponentially distributed survival probability, and \( \lambda_W(S_t,\alpha) \) and \( \lambda^Q_W(S_t,\alpha) \) capture the short-run (long-run) credit risk factor in the Weibull distributed survival

\(^6\) In the model of Câmara et al. (2012), \( \lambda = \lambda^Q \).

\(^7\) See, for example, Jasiulewicz (1997), Ma and Ma (2013), and Valdez, Vadiveloo, and Dias (2014).
probability when $\kappa < 1$ ($\kappa > 1$). In particular, when $\lambda_W(S_t, \alpha) = 0$ and $\lambda^Q_W(S_t, \alpha) = 0$, this model degrades to the exponential distribution, or the traditional case. When $\lambda_E(S_t, \alpha) = 0$ and $\lambda^Q_E(S_t, \alpha) = 0$, this model degrades to the Weibull distribution. Note that the default intensities in (18) and (19) are functions of stock price and dependency relationship between bankruptcy probability and future stock prices. To echo the impacts of state-dependence addressed in Section 2.3, we assume that $\lambda_E(S_t, \alpha) = \lambda_E S_t^{-\alpha}$, $\lambda^Q_E(S_t, \alpha) = \lambda^Q_E S_t^{-\alpha}$, $\lambda_W(S_t, \alpha) = \lambda_W S_t^{-\alpha}$, and $\lambda^Q_W(S_t, \alpha) = \lambda^Q_W S_t^{-\alpha}$, where $\lambda_E$, $\lambda^Q_E$, $\lambda_W$, and $\lambda^Q_W$ are constants.

3. Numerical Analysis

3.1 Bankruptcy Risk Premium

Figure 1 illustrates the relation between BRP and the risk-neutral expected bankruptcy probability, with parameters $\tau \in \{30/365, 1\}$, $\gamma \sigma_{cs} \in \{0.1, 0.2\}$, $\sigma_s^2 = 0.09$, and $\alpha \in \{0.5, 1.0, 1.5\}$ in the range of $PD^Q_{t,t+\tau} \in [0,1]$. In general, the patterns for all panels are quite similar, while the BRP levels in the right panels ($\gamma \sigma_{cs} = 0.2$) are higher than the BRP levels in the left panels ($\gamma \sigma_{cs} = 0.1$). In addition, as shown in each panel, BRP is positively related to the dependency between bankruptcy probability and future stock prices. These relations can be deduced from equations (16) and (17), ceteris paribus, as BRP is positively related to $\gamma \sigma_{cs}$ and $\alpha$.

Different from the intensity ratio adopted by Driessen (2005) and Berndt et al. (2005) or the probability ratio employed by Das and Sundaram (2007), Figure 1 shows that BRP increases monotonically in $PD^Q_{t,t+\tau}$. While high risk premiums accompanied by high bankruptcy risk are intuitive, the intensity ratio or the probability ratio does not satisfy this property. In addition, the concave upward curve
shows that the marginal BRP is positive. For high (low) bankruptcy risk firms, *ceteris paribus*, the required rate of return for bearing an additional unit of expected bankruptcy probability is high (low). However, the curvatures of the intensity ratio and the probability ratio are not clear.

Compared with the traditional risk premium $\gamma \sigma_{cS}$, the BRP level is relatively small in Panels A and C. The traditional risk premium is 0.1, and BRP reaches 0.1 only when the expected bankruptcy probability approaches one. However, the BRP level becomes significant in Panels B and D. The traditional risk premium is 0.2, and BRP in Panels B and D may reach 0.5 and 0.7, respectively. Since the BRP level may exceed the traditional equity risk premium and then dominate the total risk premium, we provide an alternative analysis in Figure 2.

Given the total equity risk premium, Figure 2 shows the relationship between the traditional risk premium and the risk-neutral expected bankruptcy probability, with parameters $\tau \in \{30/365, 1\}$, $\sigma_s^2 = 0.09$, and $\alpha \in \{0.5, 1.0, 1.5\}$ in the range of $PD_{t,t+\tau}^Q \in [0,1]$. The total equity risk premium is 0.1 in Panels A and C and 0.2 in Panels B and D, respectively. As a result, the traditional risk premium decreases monotonically in $PD_{t,t+\tau}^Q$. This finding supports the importance of BRP, and the neglect of BRP may mislead a conclusion that firms with high bankruptcy risk have a lower return potential, which is known as the credit risk premium puzzle.

<Figure 2 is inserted about here>

### 3.2 Term Structure of Bankruptcy Probability

This section analyzes the term structure of bankruptcy probability (TSBP). Figure 3 depicts the model-implied TSBP within one year, conditional on the same 1-year bankruptcy probability. The annualized bankruptcy probabilities are 18.08% and
63.95%, respectively, for the two sample periods, as estimated by Câmara et al. (2014) using the market prices of Bear Stearns’ stock options. The first sample period, as shown in Panels A and B, is from March 3 to March 12, 2008, and the second sample period, as shown in Panels C and D, is from March 13 to March 20, 2008.

In Panels A and C we show the TSBPs implied by the Weibull distribution with shape parameters $\kappa$ are 0.25, 0.50, 1.50, and 2.00, respectively, marked by W1, W2, W3, and W4. In Panels B and D we demonstrate the TSBP implied by the mixture of exponential and Weibull distributions with $\kappa=0.25$, marked by EW1, as well as the TSBP implied by the unsystematic bankruptcy risk model of Câmara et al. (2014), marked by UBR. The benchmark is the TSBP implied by the traditional jump-to-default model, marked by JTD.

Panels A and C of Figure 3 show that the term structure generated by W1 and W2 are higher than those generated by JTD for all maturity dates. On the contrary, the term structures generated by W3 and W4 are lower than those generated by JTD for all maturity dates. Conditional on the same 1-year bankruptcy probability, both panels show that the TSBP of the Weibull distribution with $\kappa<1$ ($\kappa>1$) increases rapidly (slowly) when $\tau$ is small, which in turn captures the relatively short-run (long-run) credit factor. For example, the 1-month bankruptcy probability of JTD in the first period is 1.65%, whereas those generated by W1, W2, W3, and W4 are 10.16%, 5.59%, 0.48%, and 0.14%, respectively. In fact, for any two Weibull distributions $W_i$

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8 For any $\kappa \neq 0$, we have $(\lambda_{E}^{\sigma}, \lambda_{W}^{\sigma}) = (0, 0.1994)$ and $(0, 1.0203)$ when $PD_{t+1}^{\sigma} = 18.08\%$ and 63.95\%, respectively. On the contrary, for $\kappa = 0$, we have $(\lambda_{E}^{\sigma}, \lambda_{W}^{\sigma}) = (0.1994, 0)$ and $(1.0203, 0)$, or the exponential distribution for the traditional jump-to-default model.

9 More specifically, for the mixture of exponential and Weibull distributions, we assume $(\lambda_{E}^{\sigma}, \lambda_{W}^{\sigma}) = (0.0997, 0.0997)$ and $(0.5102, 0.5102)$ when $PD_{t+1}^{\sigma} = 18.08\%$ and 63.95\%, respectively.
and $W_j$ with parameters $\kappa_i$ and $\kappa_j$, the TSBP generated by $W_i$ is greater than the TSBP generated by $W_j$ for all maturity dates within one year if and only if $\kappa_i < \kappa_j$.

Panels B and D of Figure 3, however, show that when $\tau$ is small enough, the TSBP of EW1 is greater than the TSBP of W2, but with $\tau$ increasing, this relation is completely opposite. In this example, the mixture distribution EW1 is exactly the average of JTD and W1. In fact, our model also allows unequal weights, thus offering flexibility to capture the short-run risk and the long-run credit risk simultaneously. We will present more discussions in Section 4 by calibrating the distribution parameters from empirical data.

Although Câmara et al. (2014) claim that their UBR model can generate a high bankruptcy probability before the Bear Stearns’ collapse (the second period), Panel D of Figure 3 shows that the UBR model in fact captures the long-run factor. The 1-year bankruptcy probability of UBR is 63.95%, whereas the 1-month bankruptcy probability is only 3.64%. However, the corresponding 1-month bankruptcy probabilities generated by JTD, W2, and EW1 are 8.15%, 25.25%, and 27.14%, respectively. According to the findings of Das and Hanouna (2009) and Duan et al. (2012), the cumulative default probability is dominated by short-run risk factors. In addition, Duan et al. (2012) find that TSBP is strongly related to the bankruptcy event. Therefore, the bankruptcy probability of such a high bankruptcy risk in Bear Stearns should be driven primarily by the short-run factor, indicating the deficiency of the UBR model.

4. Empirical Analysis

4.1 Stock Options Data

The stock options data for all trading days between January 4, 1996 and August 31, 2016 are from OptionMetrics. As the purpose of this study is to explore the
option-implied bankruptcy information before bankruptcy events, we focus on those bankruptcy firms that have active option trades before their bankruptcy events. Limited by the fact that option-listed firms are usually large firms, this study selects 23 bankruptcy firms and adopts the sample period three years prior to the event date. This study omits those options with fewer than five days and more than 91 days to maturity, because it is less convincing to assume that bankruptcy information is reflected in long-term contracts. In addition, all observations with zero trading volume are discarded due to the liquidity concern. Finally, the summary statistics of selected bankruptcy firms are shown in Table 1.

Although the options on individual stocks are American-style contracts, the implied volatility provided by OptionMetrics is estimated using a binomial tree method that takes into account the early exercise premium. Therefore, it is appropriate to convert the implied volatility to the European option price using the Black-Scholes formula. This study takes implied volatility quotes rather than closing prices and applies them to estimate parameters of our pricing formulae (11) to (14).

The option pricing formula (11) assumes no dividends. Following Christoffersen, Heston, and Jacobs (2013), for each day and each option, we use the risk-free rate to discount the dividend realized from the current underlying value during the expiration of the option. When using (11) to calculate the option prices, we take this adjusted stock price to get rid of the dividend concern. This study follows Christoffersen, Jacobs, and Ornthanalai (2012) to employ the daily time-series of 3-month Treasury bills as the proxy for the risk-free rate, whereas the term structure of the risk-free rate is calculated for each derivative contract through the interpolation of zero curve surfaces obtained from the OptionMetrics database to fit the contract maturity.
Chakravarty, Gulen, and Mayhew (2004) and Pan and Poteshman (2006) find that option trading volume contains useful information about future prices of the underlying assets. Therefore, we investigate the average of relative option trading volumes before bankruptcy events in Figure 4. The bankruptcy date is denoted as $t^*$, and we examine the weekly trading volumes starting from $t^* - 3$, three years before bankruptcy. The relative trading volume of firm $i$ at week $t$ is defined as $RelVOL_{i,t} = \left(\frac{VOL_{i,t}}{MedVOL_i}\right) - 1$, where $VOL_{i,t}$ is the trading volume of firm $i$ at week $t$ and $MedVOL_i$ is the median of weekly trading volume between day $t^* - 3$ and day $t^*$. Finally, we depict the average of $RelVOL_{i,t}$ for each week $t$. As a result, the trading volume in the options market is abnormally high about three months before the bankruptcy event, implying that option prices may contain useful information about corporate bankruptcies.

<Figure 4 is inserted about here>

### 4.2 Bankruptcy Probability Data

The proxy of actual bankruptcy probability is obtained from the available Credit Research Initiative (CRI) database provided by the Risk Management Institute, National University of Singapore.\(^{10}\) The probability of default at the end of each month can be directly downloaded from the RMI-CRI website (http://rmicri.org), whereas we obtain the daily data from the CRI team, which covers 14,248 U.S. firms during the period from December 3, 1990 to December 30, 2013. For each calendar date, the CRI database offers the default probability for seven maturities: 1-month, 3-month, 6-month, 12-month, 24-month, 36-month, and 60-month. However, considering that option contracts are usually less than one year, this study does not

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\(^{10}\) Source: National University of Singapore, Risk Management Institute, CRI database. Available at: http://rmicri.org [Accessed 6 8 2014].
discuss the maturities longer than one year. Finally, we utilize the cubic spline to generate the continuous and smooth term structure curves.

Figure 5 shows the TSBP within one year. Solid lines show the empirical term structure provided by the CRI database, and the dashed line shows the JTD model-implied term structure by fitting the 1-year bankruptcy probability. Panel A of Figure 5 shows the average term structure for all firm-date observations, where the JTD model fits the empirical pattern quite well. This figure indicates that the overall empirical survival probability approximates to the exponential distribution.

<Figure 5 is inserted about here>

The term structure, however, becomes mis-specified for the subsample results in Panels B, C, and D of Figure 5. Panels B and C show the term structure for firms with the top-20% highest PD and the bottom-20% lowest PD, respectively. Panel D shows the term structure of the 23 bankruptcy firms listed in Table 1 during the period one year before the bankruptcy events. As a result, the term structures of those firms with high bankruptcy risk (Panel B) and facing bankruptcy (Panel D) increase rapidly in the short run, indicating a short-run factor in the bankruptcy probability. On the contrary, the term structure of those firms with low bankruptcy risk (Panel C) is mainly driven by a long-run factor. Hence, Figure 5 supports the model set-up of this study, specifying the mixture of exponential and Weibull distributions to identify the short-run and long-run credit risk factors.

4.3 Empirical Methodology

To estimate the risk-neutral parameters from option prices, minimizing the sum of squared error is a common approach in the literature. For example, Bakshi, Cao, and Chen (1997) and Câmara et al. (2012, 2014) calibrate model parameters on a daily basis by minimizing the sum of squared option-pricing errors. Minimizing the same
objective function, Christoffersen and Jacobs (2004) calibrate model parameters by using more than one day of option prices. On the other hand, Christoffersen, Heston, and Jacobs (2009) minimize the sum of squared vega-adjusted option-pricing errors on an annual basis for the calibration. The vega-adjusted error is an approximation for the implied volatility error, which measures the time value of the option. Since the intrinsic value is model-free, fitting implied volatility can avoid the effect of intrinsic value levels.

As the purpose of this study is to extract forward-looking bankruptcy information contained in the stock options market, the first approach adopted in this study is to estimate the risk-neutral parameters \( \Theta = \{ \sigma_s^2, \lambda_E^Q, \lambda_W^Q, \kappa, \alpha \} \) from option prices for each firm-week. There is a trade-off between the number of observations and the updated information. Weekly estimates can, on the one hand, alleviate the shortage of option data, because firms with high bankruptcy risk are usually small firms whose option contracts are relatively illiquid and may have no observation on some trading days.\(^{11}\) On the other hand, compared with a long estimation period, estimating model parameters on a weekly basis may be helpful for updating market expectation. For example, if informed traders reflect their expectations to Lehman’s stock options in July 2008 and we adopt option observations since January 1996, then the estimation must capture the overall view, but miss the most recent and the more important information.

Instead of minimizing the vega-adjusted option-pricing errors directly, this study follows Christoffersen, Feunou, Jacobs, and Meddahi (2014) to estimate the risk-neutral parameters by maximizing the option error likelihood:

\[
\ln L^0 = -\frac{N}{2} \ln(2\pi \sigma^2_e) - \frac{1}{2} \sum_{n=1}^{N} e_n^2 / \sigma^2_e. \tag{20}
\]

\(^{11}\) If the number of observations is less than five, then we remove the firm-week.
where \( N \) is the total number of options in the sample; \( e_n = \frac{C_n(\Theta) - C_n^{mkt}}{\text{Vega}_n} \) is the vega-weighted option error, which is assumed to be normally distributed with zero mean and variance \( \sigma^2 \); \( C_n(\Theta) \) presents the theoretical value of the \( n \)th call option in the firm-week; \( C_n^{mkt} \) presents the corresponding market option price; and \( \text{Vega}_n \) is the Black-Scholes sensitivity of the market option price with respect to volatility. In particular, Christoffersen et al. (2014) show that \( \sigma^2 = \frac{1}{N} \sum_{n=1}^{N} e_n^2 \). The estimation is implemented for each firm-week, and we will report the summary statistics of the estimated parameters.

Although estimating parameters from option prices may extract forward-looking information contained in the derivatives market, the estimated parameters may not be consistent with their time-series characteristics. In view of this, Christoffersen et al. (2012, 2013, 2014) jointly estimate the model parameters from options and returns by maximizing the weighted likelihoods of options and returns. Considering that the observations of daily returns and option contracts may be quite different, the weighted likelihood adopted by Christoffersen et al. (2012) ensures that the joint estimates are not dominated by one or the other - that is, they solve the optimization:

\[
\max_{\Theta^P, \Theta} \frac{T+N}{2} \ln L^R + \frac{T+N}{2} \ln L^O,
\]

where \( \Theta^P = \{ \sigma^2_S, \lambda_E, \lambda_W, \kappa, \alpha, \theta \} \) denotes the \( \mathbb{P} \)-measure parameters, \( T \) is the number of trading dates for each estimation, and \( \ln L^R \) is the return log-likelihood:

\[
\ln L^R = \sum_{t=1}^{T-1} \ln(f_t(R_{t+1})) = \sum_{t=1}^{T-1} \ln(f_t(R_{t+1}|t^* > t + 1) \times \Pr(t^* > t + 1)),
\]

where \( R_t = \ln(S_{t+1}/S_t) \) is the stock log-return at day \( t \), and \( f_t \) is the conditional density of returns.

Similar to Christoffersen et al. (2012, 2013, 2014), the second approach of this study jointly estimates model parameters from stock returns and option prices to
ensure time-series consistency. However, this study adopts a rolling estimation on a weekly updated basis. In other words, the estimation is implemented for each firm-week, where the estimation period for returns is one year before the week. In addition, the return likelihood (22) may raise the sample selection issue for a jump-to-default model. When the historical returns can be observed, it is clear that the underlying firm is surviving during this period for sure. Consequently, the probability $\Pr(t^* > t + 1)$ in (22) essentially leads the corner solution with zero bankruptcy probability. The intuition is that, if the observed historical returns are all small variations, then the maximum likelihood of a jump model is exactly the model with no jumps. Therefore, directly applying the return likelihood of Christoffersen et al. (2013) may not be an appropriate method for the bankruptcy topic. In view of the sample selection concern, this study maximizes the log-likelihood conditional on the firm’s survival. - That is:

$$
\sum_{t=1}^{T-1} \ln(f_t(R_{t+1}|t^* > t + 1)) = -\frac{N}{2} \ln(2\pi\sigma_s^2) - \sum_{t=1}^{T-1} \left\{ \frac{(R_{t+1} - (\theta - 0.5)\sigma_s^2)^2}{2\sigma_s^2} \right\},
$$

which is independent of bankruptcy parameters. This approach can also be treated as a modification of Bakshi et al. (1997) and Câmara et al. (2012, 2014) to incorporate the time-series consistency of stock variance, $\sigma_s^2$.

### 4.4 Empirical Results

#### 4.4.1 Bankruptcy Risk Premium

Table 2 reports the mean of the estimated parameters for the JTD model of Câmara et al. (2012) and the state-dependent jump-to-default model (11), denoted by SDD. The bankruptcy probability is assumed to be $PD_{lt+\tau} = 1 - e^{-\frac{\theta S_{lt}}{\sigma_s^2 t}}$, which emphasizes the influences of bankruptcy risk premium (BRP) without raising the issue of the term structure of bankruptcy probability. Panels A and B show the estimates generated by
maximizing the log-likelihood of option \( \ln L^O \) and the joint log-likelihood of returns and options (21), respectively. As the SDD model uses one more parameter than the JTD model does, we examine four constrained models to rule out the benefits of the number of parameters. SDD-1, SDD-2, SDD-3, and SDD-4 are the SDD models with constraints \( \alpha=0.1, 0.2, 0.5, \) and 0.8, respectively. Standard errors are shown in parentheses, and the asterisk indicates constrained parameters.

<Table 2 is inserted about here>

Table 2 shows that the estimated \( \lambda^Q_E \) is increasing in the dependency between bankruptcy probability and future stock prices, \( \alpha \). For example, the estimated \( \lambda^Q_E \) in Panel A are 0.130, 0.239, 0.324, 6.952, and 16.529 when \( \alpha \) are 0, 0.1, 0.2, 0.5, and 0.8, respectively. In fact, the level of \( \lambda^Q_E \) is driven by the corresponding stock price. However, both Panels A and B show that the log-likelihoods of options are not monotonically increasing in \( \alpha \). For example, the log-likelihood of options is 72.747 at \( \alpha=0 \), which increases to 73.368 at \( \alpha=0.5 \) and then decreases to 73.359 at \( \alpha=0.8 \). This humped shape implies that there exists an optimal choice of \( \alpha \), and the last column of Table 2 shows that the optimal \( \alpha \) is on average 0.634 and significantly differs from zero.

Panel B of Table 2 shows that the estimated BRP is increasing in \( \alpha \) and can be as large as 0.183 when \( \alpha = 0.8 \). For the SDD models, BRP is on average 0.083 and significantly differs from zero. In Figure 6 we depict the average of estimated BRP for the 23 bankruptcy firms three years before their bankruptcy events. Consequently, Figure 6 shows that BRP may be negative during the normal period, while BRP becomes significant before the bankruptcy events. However, the BRP of the JTD model is zero, where the bankruptcy risk is not priced.

<Figure 6 is inserted about here>
4.4.2 Term Structure of Bankruptcy Probability

Table 3 reports the mean of the estimated parameters for the JTD model of Câmara et al. (2012) and the state-dependent jump-to-default model (11), denoted by SDD. For both JTD and SDD models, we compare three survival distributions for both models to investigate the term structure of bankruptcy probability: the exponential distribution (Exponential), the Weibull distribution (Weibull), and a mixture of exponential and Weibull distributions (Mixture). The survival distribution of Mixture has the form $1 - PD_{tt+	au}^{Q} = e^{-(\lambda_{E}^{Q}t+\lambda_{W}^{Q}t^{\kappa})/S_{t}^{Q}}$, and Exponential and Weibull can be treated as constrained models with $\lambda_{W}^{Q}=0$ and $\lambda_{E}^{Q}=0$, respectively. In addition, we impose a constraint $\alpha=0.6$ on the SDD model with Mixture, which is close to the average of optimal $\alpha$ in Table 2. This constraint ensures that the maximum number of parameters in this study is five.

<Table 3 is inserted about here>

Both Panels A and B of Table 3 show that the log-likelihoods of the Weibull model are higher than the log-likelihoods of the Exponential model. For the JTD model, the joint log-likelihood of Exponential is 578.826, and the joint log-likelihood of Weibull is 596.562. For the SDD model, the joint log-likelihood of Exponential is 582.827, and the joint log-likelihood of Weibull is 598.966. Our untabulated results show that all the differences of log-likelihood between Exponential and Weibull models are statistically significant.\(^{12}\) In addition, the estimated $\kappa$ for all Weibull models are significantly smaller than one, with a mean of about 0.85. These results show evidence that specifying a short-run credit risk factor can fit the market implied-volatility better than the traditional jump-to-default model does.

\(^{12}\) The t-statistics of the difference for the JTD model in Panels A and B are 22.86 and 35.70, respectively. The t-statistics of the difference for the SDD model in Panels A and B are 31.93 and 38.92, respectively.
Compared with the JTD and SDD models, Table 3 shows that all the log-likelihoods of the SDD models are higher than those of the corresponding JTD models. The estimated $\alpha$ for all SDD models are significantly smaller than zero, with a mean of about 0.6 and consistent with the findings of Table 2. Our untabulated results show that all the differences of log-likelihood between the JTD and SDD models are statistically significant. These results echo the findings in Table 2 that incorporating BRP can improve the implied-volatility fits, albeit the magnitudes of improvements made by $\alpha$ are less significant than those made by $\kappa$.

Figure 7 shows the mean and the median of estimated short-run/long-run credit risk factor ($\kappa$) in Panels A and B, respectively, for the 23 selected bankruptcy firms during the sample period three years prior to the event date. The term structure of bankruptcy probability is defined by (19), which is estimated by the state-dependent jump-to-default model (11) with $\lambda_w^Q = 0$ and $\alpha = 0$ for each firm-week. The mean and the median of the estimated $\kappa$ are all less than one, and those during the sample period six months prior to the event date are significantly less than one. These results are consistent with the findings of Das and Hanouna (2009) and Duan et al. (2012) that TSBP is strongly related to the bankruptcy event and the bankruptcy probability is primarily dominated by short-run risk factors.

5. Concluding Remarks

This study develops an innovative option pricing model on individual stocks subject to bankruptcy risk. Under the equilibrium framework, we decompose the total risk premium of a stock into two parts: the traditional equity risk premium and the

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13 The t-statistics of the differences in the Exponential model in Panels A and B are 8.23 and 14.14, respectively. The t-statistics of the differences in the Weibull model in Panels A and B are 21.49 and 30.35, respectively.
bankruptcy risk premium, which describe the excess return for bearing market risk and the risk of bankruptcy, respectively. Subsequently, we offer the closed-form solution for the option prices that is associated with the magnitude of bankruptcy risk premium and the term structure of risk-neutral bankruptcy probability, offering a channel to extract forward-looking expectations of bankruptcy risk premium and the short-run credit risk from the derivatives market. Using the market prices of stock options and returns for 23 bankruptcy firms during the period from three year before the bankruptcy events, our empirical results show that: (1) the relative option trading volume is abnormally high three months before the bankruptcy event; (2) bankruptcy risk premium surges before the bankruptcy event; and (3) specifying a short-run credit risk factor into the term structure of bankruptcy probability can fit the market implied-volatility better than the traditional jump-to-default model does.
Appendix. Proof of the Equilibrium Stock Return and Stock Options Price

For the sake of simplicity, we define the notations $X = \ln \epsilon_{c, \tau}$ and $Y = \ln \epsilon_{s, \tau}$ such that $X$ and $Y$ are jointly normally distributed with correlation coefficient $\rho = \frac{\sigma_{cs}}{\sigma_c \sigma_s}$

The mean and variance of $X$ are denoted by $\mu_X = \left( \mu_c - \frac{\sigma_c^2}{2} \right) \tau$ and $\nu_X = \sigma_c^2 \tau$, respectively, and those of $Y$ are denoted by $\mu_Y = \left( \mu_s - \frac{\sigma_s^2}{2} \right) \tau$ and $\nu_Y = \sigma_s^2 \tau$. The joint probability density function of $X$ and $Y$ is:

$$f_{XY}(x, y) = \frac{1}{2\pi \sqrt{\nu_X \nu_Y (1-\rho^2)}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[ \frac{x-\mu_X}{\sqrt{\nu_X}} \right]^2 - 2\rho \frac{x-\mu_X}{\sqrt{\nu_X}} \frac{y-\mu_Y}{\sqrt{\nu_Y}} \right\}.$$  \hspace{1cm} (A1)

where $\overline{X} \equiv \frac{x-\mu_X}{\sqrt{\nu_X}}$ and $\overline{Y} \equiv \frac{y-\mu_Y}{\sqrt{\nu_Y}}$. For any positive real-value $z$, we define the function $M_z(a, b)$ as:

$$M_z(a, b) \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{ax+by} f_{XY}(x, y) dx dy. \hspace{1cm} (A2)$$

Using the moment generating function of the normal distribution, (A2) can be derived as:

$$e^{a\mu_X+b\mu_Y+\frac{a^2\nu_X}{2}+\frac{b^2\nu_Y}{2}+ab\sqrt{\nu_X\nu_Y}\rho} \times \Phi \left( \frac{\mu_X-\ln z}{\sqrt{\nu_Y}} + b\sqrt{\nu_Y} + a\sqrt{\nu_X}\rho \right). \hspace{1cm} (A3)$$

In addition, we define $M_0(a, b)$ as the limit case

$$\lim_{z \to 0^+} M_z(a, b) = e^{am_x+bm_y+\frac{a^2\nu_x}{2}+\frac{b^2\nu_y}{2}+ab\sqrt{\nu_x\nu_y}\rho}. \hspace{1cm} \text{Hence, the expected bankruptcy probability under the physical measure, Equation (5), can be derived as:}$$

$$\mathbb{E}_t^{\mathbb{P}} \left[ \frac{\delta(t)}{S_{t+\tau}} \right] = \frac{\delta(t)}{S_{t+\tau}} \mathbb{E}_t^{\mathbb{P}} \left[ h_t \right]. \hspace{1cm} (A4)$$

Using similar arguments, the equilibrium interest rate is $e^{-\gamma \tau} = \mathbb{E}_t^{\mathbb{P}} \left[ \beta_t^\tau \epsilon_{c, \tau}^{-\gamma} \right] = \beta_t^\tau M_0(-\gamma, 0)$, which means that Equation (9) holds. Assuming that $\delta(t) = \delta$ and $\alpha(t) = \alpha/\tau$, the stock price in equilibrium satisfies:

$$1 = \mathbb{E}_t^{\mathbb{P}} \left[ \beta_t^\tau \left( \frac{c_{t+\tau}}{c_t} \right)^{-\gamma} \left( \frac{S_{t+\tau}}{S_t} \right) 1_{\{t+\tau \geq s_t \}} \right] = \mathbb{E}_t^{\mathbb{P}} \left[ \beta_t^\tau e^{-\gamma Y} \left( 1 - \frac{\delta}{S_t} e^{-\gamma Y / \tau} \right) \right]. \hspace{1cm} (A5)$$

which can be expressed as:
\begin{align*}
\beta^\top M_0(-\gamma, 1) & - \frac{\delta}{S_t^{\alpha/\tau}} \beta^\top M_0(-\gamma, 1 - \alpha/\tau). 
\end{align*}

(A6)

Combining (A4)-(A6) and (6), we obtain the relation:

\begin{align*}
1 &= e^{(\mu_s - \gamma\sigma_{cs} - \tau)\tau} \left(1 - \frac{\delta}{S_t^{\alpha/\tau}} e^{-(\mu_s - \gamma\sigma_{cs})\tau + \left(1 - \frac{\alpha}{\tau}\right)\frac{\sigma_s^2}{2}}\right) \\
&= e^{(\mu_s - \gamma\sigma_{cs} - \tau)\tau} \left(1 - e^{-\alpha\sigma_s^2 PD_{t,t+\tau}^Q}\right),
\end{align*}

(A7)

which is equivalent to the condition (10):

\begin{align*}
\mu_s - \gamma\sigma_{cs} &= r - \frac{1}{\tau} \ln \left(1 - e^{-\alpha\sigma_s^2 PD_{t,t+\tau}^Q}\right).
\end{align*}

(A8)

Finally, the call option price (11) can be derived by:

\begin{align*}
S_t \mathbb{E}_t^P \left[\beta^\top e^{-\gamma X_1} 1_{\{Y > \ln(K/S_t)\}}\right] - K \mathbb{E}_t^P \left[\beta^\top e^{-\gamma X_1} 1_{\{Y > \ln(K/S_t)\}}\right].
\end{align*}

(A9)

The first expectation of (A9) can be expressed as:

\begin{align*}
\beta^\top M_{K/S_t}(-\gamma, 1) - \frac{\delta}{S_t^{\alpha/\tau}} \beta^\top M_{K/S_t}(-\gamma, 1 - \alpha/\tau),
\end{align*}

(A10)

and the second expectation of (A9) can be expressed as:

\begin{align*}
\beta^\top M_{K/S_t}(-\gamma, 0) - \frac{\delta}{S_t^{\alpha/\tau}} \beta^\top M_{K/S_t}(-\gamma, -\alpha/\tau).
\end{align*}

(A11)

Combining (A9)-(A11), (6), and (9), the call option price becomes:

\begin{align*}
S_t e^{-r\tau} \left[e^{(\mu_s - \gamma\sigma_{cs})\tau} \Phi(d_+) - e^{-\alpha\sigma_s^2 PD_{t,t+\tau}^Q} \Phi(q_+)\right] \\
- Ke^{-r\tau} \left[\Phi(d_-) - PD_{t,t+\tau}^Q \Phi(q_-)\right],
\end{align*}

(A12)

where

\begin{align*}
d_\pm &= \frac{\ln(S_t/K) + (\mu_s - \gamma\sigma_{cs} \pm \frac{\sigma_s^2}{2}) \tau}{\sigma_s \sqrt{\tau}}, 
\end{align*}

(A13)

and \( q_\pm = d_\pm - \sigma_s \sqrt{\tau}. \)
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Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous.


Figure 1 Bankruptcy Risk Premium and Expected Bankruptcy Probability

This figure shows the relation between bankruptcy risk premium (BRP) and the expected bankruptcy probability under the risk-neutral measure \( PD^Q_{t,t+\tau} \) in the range of \( PD^Q_{t,t+\tau} \in [0,1] \). The maturity date \( \tau=30/365 \) in Panels A and B, and \( \tau=1 \) in Panels C and D, respectively. The traditional risk premium \( \gamma \sigma_s \) is 0.1 in Panels A and C and 0.2 in Panels B and D, respectively. Other parameters are \( \sigma_s^2=0.09 \) and \( \alpha \in \{0.5, 1.0, 1.5\} \).
Figure 2 Traditional Risk Premium and Expected Bankruptcy Probability
Given the total equity risk premium, this figure shows the relation between traditional risk
premium (RP) and the risk-neutral expected bankruptcy probability \( PD_{t影音}^{\Omega} \) in the range of
\( PD_{t影音}^{\Omega} \in [0,1] \). The total risk premium is 0.1 in Panels A and C and 0.2 in Panels B and D,
respectively. The maturity date \( \tau = 30 \) in Panels A and C and \( \tau = 1 \) in Panels C and D,
respectively. Other parameters are \( \sigma^2 = 0.09 \) and \( \alpha \in \{0.5, 1.0, 1.5\} \).
Figure 3 Term Structure of Bankruptcy Probability

This figure shows the model-implied term structure of risk-neutral bankruptcy probability within one year, conditional on the same 1-year bankruptcy probability. The bankruptcy probability is estimated by Câmara et al. (2014) by using the market price of Bear Stearns’ stock options. The first sample period, as shown in Panels A and B, is from March 3 to March 12, 2008, and the second sample period, as shown in Panels C and D, is from March 13 to March 20, 2008. JTD shows the term structure implied by the jump-to-default model. W1, W2, W3, and W4 show the term structure implied by the Weibull distribution with shape parameter χ=0.25, 0.5, 1.5, and 2.0, respectively. EW1 shows the term structure implied by the mixture of exponential and Weibull distributions with χ=0.25, which is the average of JTD and W1. UBR shows the term structure implied by the unsystematic bankruptcy risk model of Câmara et al. (2014).
Figure 4. Average Relative Trading Volume

This figure shows the average relative trading volumes for 23 selected bankruptcy firms (listed in Table 1). The bankruptcy date is denoted as $t^*$, and we examine the weekly trading volumes starting from $t^* - 3$, three years before bankruptcy. The relative trading volume of firm $i$ at week $j$ is defined as $\text{RelVol}_{i,j} = (\text{Vol}_{i,j}/\text{MedVol}_{i}) - 1$, where $\text{Vol}_{i,j}$ is the trading volume of firm $i$ at week $j$ and $\text{MedVol}_{i}$ is the median of weekly trading volume between $t^* - 3$ and $t^*$. Finally, this figure depicts the average of $\text{RelVol}_{i,j}$ for each week $j$. 
Figure 5. Empirical Term Structure of Bankruptcy Probability

This figure shows the term structure of bankruptcy probability within one year. Solid lines show the empirical term structure provided by the CRI database, and the dashed line shows the jump-to-default model-implied term structure by fitting the 1-year bankruptcy probability (PD). Panel A shows the average term structure for all firm-date observations, Panels B and C show the term structure for the firms with the top-20% highest PD and the bottom-20% lowest PD, respectively. Panel D shows the term structure of the 23 bankruptcy firms listed in Table 1 during the period one year before their bankruptcy events.
Figure 6. Estimated Bankruptcy Risk Premium

This figure shows the average of estimated bankruptcy risk premium for 23 selected bankruptcy firms (listed in Table 1). The bankruptcy risk premium is defined by (16), which is estimated by the state-dependent jump-to-default model (11) with $\lambda_W^{(0)} = 0$ and $\kappa = 0$ for each firm-week. The bankruptcy date is denoted as $t^*$, and this figure shows the estimated bankruptcy risk premium starting from $t^* - 3$, three years before bankruptcy.
Panel A. Mean Level of Estimated $\kappa$

Panel B. Median Level of Estimated $\kappa$

Figure 7. Estimated Short-run/Long-run Credit Risk Factor

This figure shows the mean and the median of estimated short-run/long-run credit risk factor ($\kappa$) in Panels A and B, respectively, for 23 selected bankruptcy firms (listed in Table 1). The term structure of bankruptcy probability is defined by (19), which is estimated by the state-dependent jump-to-default model (11) with $\lambda^Q_W = 0$ and $\alpha = 0$ for each firm-week. The bankruptcy date is denoted as $t^*$, and this figure shows the estimated bankruptcy risk premium starting from $t^* - 3$, three years before bankruptcy.
Table 1 Selected Bankruptcy Firms

This table reports on the 23 bankruptcy firms selected in this study. SECID and CRIID represent the identification of the OptionMetrics and the Credit Research Initiative (CRI) databases, respectively. SIC denotes the four-digit SIC code. Event Date indicates the date of each firm’s bankruptcy. IV and VOL show the averages of implied volatility and trading volume, respectively.

<table>
<thead>
<tr>
<th>SECID</th>
<th>CRIID</th>
<th>Company Name</th>
<th>SIC</th>
<th>Event Date</th>
<th>IV</th>
<th>VOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>101057</td>
<td>28512</td>
<td>Adelphia Communications Corp.</td>
<td>4841</td>
<td>2002/06/25</td>
<td>0.704</td>
<td>149.970</td>
</tr>
<tr>
<td>101311</td>
<td>27022</td>
<td>Ambac Financial Group, Inc.</td>
<td>6351</td>
<td>2010/11/01</td>
<td>1.310</td>
<td>379.096</td>
</tr>
<tr>
<td>101397</td>
<td>27035</td>
<td>American International Group, Inc.</td>
<td>6331</td>
<td>2008/09/16</td>
<td>0.398</td>
<td>528.693</td>
</tr>
<tr>
<td>102061</td>
<td>27116</td>
<td>The Bear Stearns Companies, Inc.</td>
<td>6211</td>
<td>2008/05/30</td>
<td>0.407</td>
<td>271.526</td>
</tr>
<tr>
<td>102281</td>
<td>38314</td>
<td>Borders Group, Inc.</td>
<td>5942</td>
<td>2011/02/16</td>
<td>0.880</td>
<td>75.595</td>
</tr>
<tr>
<td>102636</td>
<td>39345</td>
<td>Calpine Corporation</td>
<td>4911</td>
<td>2005/12/20</td>
<td>0.770</td>
<td>539.189</td>
</tr>
<tr>
<td>103165</td>
<td>33084</td>
<td>Colonial Bancgroup, Inc.</td>
<td>6022</td>
<td>2009/08/25</td>
<td>0.642</td>
<td>95.776</td>
</tr>
<tr>
<td>103675</td>
<td>27326</td>
<td>Dana Holding Corporation</td>
<td>3714</td>
<td>2006/03/03</td>
<td>0.479</td>
<td>127.692</td>
</tr>
<tr>
<td>103757</td>
<td>27336</td>
<td>Delta Air Lines, Inc.</td>
<td>4512</td>
<td>2005/09/14</td>
<td>0.904</td>
<td>385.181</td>
</tr>
<tr>
<td>104118</td>
<td>27377</td>
<td>Eastman Kodak Company</td>
<td>3861</td>
<td>2012/01/19</td>
<td>0.746</td>
<td>344.889</td>
</tr>
<tr>
<td>104355</td>
<td>27401</td>
<td>Enron Corporation</td>
<td>1311</td>
<td>2001/12/02</td>
<td>0.519</td>
<td>172.279</td>
</tr>
<tr>
<td>105171</td>
<td>33315</td>
<td>General Growth Properties, Inc.</td>
<td>6798</td>
<td>2009/04/16</td>
<td>0.536</td>
<td>90.717</td>
</tr>
<tr>
<td>105175</td>
<td>27494</td>
<td>General Motor Company</td>
<td>3711</td>
<td>2009/06/01</td>
<td>0.705</td>
<td>727.432</td>
</tr>
<tr>
<td>106078</td>
<td>27306</td>
<td>IndyMac Bancorp Inc</td>
<td>6035</td>
<td>2008/07/31</td>
<td>0.758</td>
<td>201.924</td>
</tr>
<tr>
<td>106893</td>
<td>35160</td>
<td>Lehman Brothers Holdings Inc.</td>
<td>6162</td>
<td>2008/09/15</td>
<td>0.532</td>
<td>516.579</td>
</tr>
<tr>
<td>108083</td>
<td>42436</td>
<td>New Century Financial Corporation</td>
<td>6798</td>
<td>2007/04/02</td>
<td>0.449</td>
<td>168.378</td>
</tr>
<tr>
<td>108609</td>
<td>27929</td>
<td>Pacific Gas and Electric Company</td>
<td>4911</td>
<td>2001/04/06</td>
<td>0.777</td>
<td>93.817</td>
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<tr>
<td>111286</td>
<td>28247</td>
<td>United Airlines, Inc.</td>
<td>4512</td>
<td>2002/12/09</td>
<td>0.688</td>
<td>102.896</td>
</tr>
<tr>
<td>111448</td>
<td>28289</td>
<td>US Airways</td>
<td>4512</td>
<td>2002/08/11</td>
<td>0.743</td>
<td>69.913</td>
</tr>
<tr>
<td>111884</td>
<td>32291</td>
<td>Washington Mutual, Inc.</td>
<td>6022</td>
<td>2008/09/26</td>
<td>0.523</td>
<td>438.723</td>
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<tr>
<td>112129</td>
<td>31434</td>
<td>WorldCom Inc.</td>
<td>4813</td>
<td>2002/07/21</td>
<td>0.568</td>
<td>308.509</td>
</tr>
<tr>
<td>115929</td>
<td>45586</td>
<td>CIT group Inc.</td>
<td>6159</td>
<td>2009/11/01</td>
<td>0.825</td>
<td>184.042</td>
</tr>
<tr>
<td>133879</td>
<td>43639</td>
<td>MF Global Holdings Ltd.</td>
<td>6211</td>
<td>2011/10/31</td>
<td>0.642</td>
<td>76.462</td>
</tr>
</tbody>
</table>

Average   0.618  360.253
This table reports the mean of the estimated parameters for the jump-to-default model (JTD) of Câmara et al. (2012) and the state-dependent jump-to-default model (11), denoted by SDD. The bankruptcy probability is assumed to be \( PD_{t,t+\tau}^Q = 1 - e^{-\lambda_{Q}^{E}/\tau} \). The sample covers 23 bankruptcy firms listed in Table 1 during the period from 3 years before the bankruptcy date. For each firm-week, the estimates are generated by maximizing the log-likelihood of options in Panel A, and the estimates are generated by maximizing the joint log-likelihood of returns and options (21) in Panel B, where the estimation period for returns is one year before the week. SDD-1, SDD-2, SDD-3, and SDD-4 are the SDD models with constraints \( \alpha = 0.1, 0.2, 0.5, \) and 0.8, respectively. Lik presents the joint log-likelihood, and BRP presents the estimated bankruptcy risk premium defined as (16). Standard errors are shown in parentheses, and the asterisk indicates constrained parameters.

### Panel A. Maximum likelihood estimates of selected models on stock options

<table>
<thead>
<tr>
<th>JTD</th>
<th>SDD-1</th>
<th>SDD-2</th>
<th>SDD-3</th>
<th>SDD-4</th>
<th>SDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_s^2 )</td>
<td>0.407</td>
<td>0.405</td>
<td>0.406</td>
<td>0.415</td>
<td>0.420</td>
</tr>
<tr>
<td>(0.0071)</td>
<td>(0.0070)</td>
<td>(0.0071)</td>
<td>(0.0072)</td>
<td>(0.0073)</td>
<td>(0.0071)</td>
</tr>
<tr>
<td>( \lambda_{Q}^{E} )</td>
<td>0.130</td>
<td>0.239</td>
<td>0.324</td>
<td>6.952</td>
<td>16.529</td>
</tr>
<tr>
<td>(0.0023)</td>
<td>(0.0042)</td>
<td>(0.0056)</td>
<td>(0.1210)</td>
<td>(0.2876)</td>
<td>(0.3118)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0*</td>
<td>0.1*</td>
<td>0.2*</td>
<td>0.5*</td>
<td>0.8*</td>
</tr>
<tr>
<td>(0.0027)</td>
<td>(0.0047)</td>
<td>(0.0063)</td>
<td>(0.0518)</td>
<td>(0.3330)</td>
<td>(0.2236)</td>
</tr>
<tr>
<td>ln ( L^Q )</td>
<td>72.747</td>
<td>73.259</td>
<td>73.301</td>
<td>73.368</td>
<td>73.359</td>
</tr>
<tr>
<td>(1.2660)</td>
<td>(1.2749)</td>
<td>(1.2758)</td>
<td>(1.2749)</td>
<td>(1.2766)</td>
<td>(1.2810)</td>
</tr>
</tbody>
</table>

### Panel B. Maximum likelihood estimates of selected models on stock options and returns

<table>
<thead>
<tr>
<th>JTD</th>
<th>SDD-1</th>
<th>SDD-2</th>
<th>SDD-3</th>
<th>SDD-4</th>
<th>SDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_s^2 )</td>
<td>0.377</td>
<td>0.375</td>
<td>0.377</td>
<td>0.388</td>
<td>0.397</td>
</tr>
<tr>
<td>(0.0066)</td>
<td>(0.0065)</td>
<td>(0.0066)</td>
<td>(0.0068)</td>
<td>(0.0069)</td>
<td>(0.0066)</td>
</tr>
<tr>
<td>( \lambda_{Q}^{E} )</td>
<td>0.156</td>
<td>0.270</td>
<td>0.361</td>
<td>2.976</td>
<td>19.134</td>
</tr>
<tr>
<td>(0.0027)</td>
<td>(0.0047)</td>
<td>(0.0063)</td>
<td>(0.0518)</td>
<td>(0.3330)</td>
<td>(0.2236)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0*</td>
<td>0.1*</td>
<td>0.2*</td>
<td>0.5*</td>
<td>0.8*</td>
</tr>
<tr>
<td>(0.0027)</td>
<td>(0.0024)</td>
<td>(0.0022)</td>
<td>(0.0024)</td>
<td>(0.0024)</td>
<td>(0.0024)</td>
</tr>
<tr>
<td>( \theta )</td>
<td>1.258</td>
<td>1.289</td>
<td>1.289</td>
<td>1.288</td>
<td>1.286</td>
</tr>
<tr>
<td>(0.0219)</td>
<td>(0.0224)</td>
<td>(0.0224)</td>
<td>(0.0224)</td>
<td>(0.0224)</td>
<td>(0.0224)</td>
</tr>
<tr>
<td>Lik</td>
<td>578.826</td>
<td>580.916</td>
<td>581.082</td>
<td>581.320</td>
<td>581.299</td>
</tr>
<tr>
<td>(10.0730)</td>
<td>(10.1094)</td>
<td>(10.1138)</td>
<td>(10.1225)</td>
<td>(10.1160)</td>
<td>(10.1426)</td>
</tr>
<tr>
<td>BRP</td>
<td>0</td>
<td>0.009</td>
<td>0.020</td>
<td>0.069</td>
<td>0.138</td>
</tr>
<tr>
<td>(0.0002)</td>
<td>(0.0004)</td>
<td>(0.0012)</td>
<td>(0.0024)</td>
<td>(0.0014)</td>
<td></td>
</tr>
</tbody>
</table>
Table 3 Parameter Estimates and the Term Structure of Bankruptcy Probability

This table reports the mean of the estimated parameters for the jump-to-default model (JTD) of Câmara et al. (2012) and the state-dependent jump-to-default model (11), denoted by SDD. The survival probability is a mixture of exponential and Weibull distributions, $1 - PD_{t,t+\tau}^Q = e^{-(\lambda_0^E+\lambda_0^W)\tau / S_t^\tau}$. The sample covers 23 bankruptcy firms listed in Table 1 during the period from 3 years before the bankruptcy date. For each firm-week, the estimates are generated by maximizing the log-likelihood of options in Panel A, and the estimates are generated by maximizing the joint log-likelihood of returns and options (21) in Panel B, where the estimation period for returns is one year before the week. Exponential and Weibull show the models with $\lambda_0^W = 0$ and $\lambda_0^E = 0$, respectively. Lik presents the joint log-likelihood. Standard errors are shown in parentheses, and the asterisk indicates constrained parameters.

<table>
<thead>
<tr>
<th>Panel A. Maximum likelihood estimates of selected models on stock options</th>
<th>JTD</th>
<th>SDD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exponential</td>
<td>Weibull</td>
</tr>
<tr>
<td>$\sigma_s^2$</td>
<td>0.407</td>
<td>0.388</td>
</tr>
<tr>
<td>(0.0071)</td>
<td>(0.0068)</td>
<td>(0.0067)</td>
</tr>
<tr>
<td>$\lambda_0^E$</td>
<td>0.130</td>
<td>0*</td>
</tr>
<tr>
<td>(0.0023)</td>
<td>(0.0007)</td>
<td>(0.3118)</td>
</tr>
<tr>
<td>$\lambda_0^W$</td>
<td>0*</td>
<td>0.345</td>
</tr>
<tr>
<td>(0.0060)</td>
<td>(0.0058)</td>
<td>(0.1797)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.856</td>
<td>0.810</td>
</tr>
<tr>
<td>(0.0149)</td>
<td>(0.0141)</td>
<td>(0.0149)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0*</td>
<td>0*</td>
</tr>
<tr>
<td></td>
<td>(0.0110)</td>
<td>(0.0116)</td>
</tr>
</tbody>
</table>

| Panel B. Maximum likelihood estimates of selected models on stock options and returns | |
|-----------------------------|-----|-----|
|                             | Exponential | Weibull | Mixture | Exponential | Weibull | Mixture |
| $\sigma_s^2$               | 0.377 | 0.364 | 0.364 | 0.378 | 0.368 | 0.377 |
| (0.0066)                   | (0.0063) | (0.0063) | (0.0066) | (0.0064) | (0.0066) |
| $\lambda_0^E$             | 0.156 | 0* | 0.045 | 12.848 | 0* | 6.371 |
| (0.0027)                   | (0.0008) | (0.2236) | (0.1109) |
| $\lambda_0^W$             | 0* | 0.362 | 0.343 | 0* | 5.452 | 12.455 |
| (0.0063)                   | (0.0060) | (0.0949) | (0.2167) |
| $\kappa$                  | 0.843 | 0.762 | 0.832 | 0.749 |
| (0.0147)                   | (0.0133) | (0.0145) | (0.0130) |
| $\alpha$                  | 0* | 0* | 0* | 0.610 | 0.634 | 0.6* |
|                             | (0.0106) | (0.0110) | (0.0110) |
| $\theta$                  | 1.258 | 1.301 | 1.301 | 1.286 | 1.298 | 1.299 |
| (0.0219)                   | (0.0226) | (0.0226) | (0.0224) | (0.0226) | (0.0226) |
| Lik                        | 578.826 | 596.562 | 597.071 | 582.827 | 598.966 | 597.684 |
| (10.0730)                  | (10.3817) | (10.3905) | (10.1426) | (10.4235) | (10.4012) |