ABSTRACT

Recent banking regulation can harm bond market liquidity by motivating a shift to agency intermediation. In a model, theoretical market makers are made to satisfy balance-sheet constraints that are stylizations of the banking rules from Basel III and the U.S. Volcker Rule. The regulations cause market makers to reduce their intermediation by refusing principal positions and instead match clients on an agency basis. As a result, asset prices exhibit greater price impact and greater price pressure. However, the regulations can improve the bid-ask spread because they induce entry by new market makers. We conclude the costs of regulation can be assessed by examining principal positions and asset prices but not by examining bid-ask spreads.

Keywords: Market making, market microstructure, fixed income, liquidity, regulation, Basel III, Volcker rule, securities financing

JEL Classifications: G14, G20, L10
The banking regulations motivated by the financial crisis have generated controversy. Critics argue that the new rules, intended to control risk-taking at banks, have the unintended consequence of harming bond market liquidity.\textsuperscript{1} The concern is that regulation unduly constrains the balance sheet of dealers and leads them to limit their market making. The unintended cost could be large, as banks are still the primary market makers in bond markets (Bao et al. 2016), yet it has proven difficult to identify the costs in the data. Liquidity metrics do not show a universal increase or decrease during the regulatory period. Some metrics such as bid-ask spreads have even improved after the regulations, whereas some metrics such as price impact have worsened.\textsuperscript{2} It is not clear whether the unintended cost is large or even present.

In this paper, we provide a theoretical analysis of the regulation to explain why the liquidity data could appear mixed and, moreover, to show how regulation on the balance sheet can indeed affect market making. Our finding is that regulation in the style of the Basel III framework or the U.S. Volcker Rule motivates a shift toward an agency model of intermediation. In the agency model, a market maker tries to match buyers to sellers directly and without using its balance sheet. The move to agency can reduce liquidity because end investors require price concessions to motivate them to trade.

The model we present is intentionally simple. An industry of market makers interacts with two populations of investors, buyers and sellers, who are segmented and can transact only with market makers. There is only one period and one asset, no risk, no agency friction, and capital-structure irrelevance would hold if there were no regulation. One might not expect rich behaviour in such an environment, but we can generate striking results using two


\textsuperscript{2}For example, compare Adrian et al. (2015) to Dick-Nielsen and Rossi (2016).
frictions that are novel to market microstructure. First, there is explicit corporate finance. Market makers cannot hold “negative” quantities and must explicitly finance operations using debt, equity, and securities financing, an essential tool for bond dealing (Huh and Infante 2016). Second, there is industrial organization. Market makers compete for investor business as Cournot quantity competitors, a friction that is appropriate to bond markets, which are characterized by substantial market power (O’Hara et al. 2016).

These frictions create a novel divergence in two popular liquidity metrics, the bid-ask spread and the price impact. We find spreads derive from the industrial organization, whereas price impacts derive from the corporate finance. The divergence distinguishes the model within microstructure theory, in which liquidity metrics typically co-move. Moreover, the divergence is useful to explain the data, in which tight bid-ask spreads coexist with prices that are sensitive to sudden demands for immediacy. As the recent regulations are balance-sheet regulations and therefore constraints on corporate financing, we find they worsen price impacts. In contrast, we find bid-ask spreads can improve, insofar as the regulation creates profit opportunities that lead to market-maker entry.

Using the model, we can match findings from the growing empirical literature on liquidity after Basel III and the U.S. Volcker Rule. We find regulation causes market makers to reduce their capital commitment, as in Bessembinder et al. (2016), and to become reluctant to take bonds in inventory, as in Schultz (2017). The result is worse price impact, as in Dick-Nielsen and Rossi (2016) and Bao et al. (2016). However, the bid-ask spread is largely unchanged or even improved, as in Trebbi and Xiao (2015), Bessembinder et al. (2016), and Adrian et al. (2015), because there is entry of new market makers, as in Bao et al. (2016). In addition,
the model predicts an increase in “price pressure” (Hendershott and Menkveld 2014), which is not yet studied in the context of regulation.\footnote{Friewald and Nagler (2016) give a methodology to measure corporate-bond price pressure.}

In the results, we study three types of regulation: capital, funding liquidity, and position. The capital and liquidity regulations are stylizations of the Basel III framework. The capital regulation, representing the Basel III Leverage Ratio, is simply an upper bound on debt financing.\footnote{We do not study the Basel III Capital Adequacy Ratio because it reduces to a leverage ratio in a model with one asset.} The liquidity regulations, representing the Basel III Liquidity Coverage Ratio (LCR) and Net Stable Funding Ratio (NSFR), require possession of a high-quality asset in a proportion to repo and reverse-repo exposures. Last, the position regulation is a stylization of the U.S. Volcker Rule, and it limits the absolute value of the market maker’s position. Although each regulation imposes a different balance-sheet constraint, we find they lead to a common outcome. Market makers respond by rationing their balance sheet, matching buyers to sellers as an agent rather than acting as their principal counterparty. By working as an agent, market makers avoid principal positions, relaxing both capital and position limits and also avoiding asset purchases obliged by liquidity regulations.

The move to agency intermediation has an asset-pricing effect. If market makers dedicate less capital to absorbing imbalances in trade, instead prices must adjust to find an equilibrium. For example, if market makers choose to pass inventory to investor sellers, the asset price must rise to solicit interest from sellers, aggravating price pressure. Though we find a negative effect on asset prices, the bid-ask spread can often improve. Capital and position limits make capital more scarce, incentivizing the entry of new market makers who can provide a scarce resource. When more firms compete do less trading per firm, the bid-ask spread improves.
Regulation in our model brings only costs. We do not consider the macroprudential benefits of regulation because we focus on an unintended consequence rather than the intended consequence. It is possible to embed a classical banking friction in the model, such as bank runs, but the prudential benefits of regulation lack an interesting interaction with the costs of regulation to market making. We seek to understand whether regulation effects liquidity and to reconcile empirical findings. In addition, we study only a static model and not the dynamics of inventory control under regulation. Dynamics complicate our results without changing them; regardless of when a client arrives, a balance-sheet constraint always reduces the market maker’s willingness to act as the principal counterparty.

We conclude that regulation on the balance sheet does decrease market liquidity. However, the costs are unlikely be measurable in spread-based liquidity metrics, which are determined by industrial organization and not by the ease of bearing inventory. Instead, the costs can be measured in new inventory premia in asset prices, such as in price pressure. Empiricists could study liquidity under regulation further by examining price pressure in the post-regulatory period, which we predict should increase.

A. Literature

This paper addresses the developing empirical literature on the effects of regulation on liquidity. Although the current slate of evidence could appear to be mixed, as some liquidity metrics improve whereas others worsen, we find the current data can be explained in a single framework. Specifically, spread metrics since the reforms show no evidence of deterioration (Trebbi and Xiao 2015; Adrian et al. 2015) or even a mild improvement (Bessebinder et al. 2016). This corresponds to the model’s predictions when the leverage or position regulations bind. Both Dick-Nielsen and Rossi (2016) and Bao et al. (2016) exploit events
that create a need for immediate trading and find price impact is significantly worse in
the post-regulatory period. This corresponds to the model’s predictions on price impact,
particularly if the number of firms is fixed (no endogenous entry).

Evidence on dealer behaviour corroborates the model’s predictions. Bessembinder et al.
(2016), Schultz (2017) and Bao et al. (2016) find a deterioration in dealers’ balance-sheet com-
mitment and an increase in trading intermediated on an agency basis in the post-regulatory
period. They find dealers who are less affected by regulation step in to provide liquidity.
The findings are consistent with our model, which predicts that market makers shift toward
an agency basis of trading and that outside market makers enter. That being said, we do
not claim regulation is solely responsible for the observed move to agency in bond markets,
as bond-market electronification and new bond ETFs have also contributed.

Last, evidence from foreign exchange (FX) prices corroborates the model’s predictions
on price pressure. Pinnington and Shamloo (2016) and Du et al. (2016) find large and
persistent deviations in covered interest parity in FX markets, which they attribute to a
binding leverage ratio. Our results support their attribution, as the deviations from parity
are mostly negative in sign. This would result if borrowing is more difficult than lending, an
outcome of a binding Basel III capital constraint.

Although there are already stylized models of dealer markets, a new model is necessary
to understand balance-sheet regulation. The literature on dealers focuses on informational
frictions (Vives 2011; Dutta and Madhavan 1997; Biais 1993; Kyle 1989), inventory control
(Laux 1995; Ho and Stoll 1983), or combinations of the two (Liu and Wang 2016). It does
not treat corporate finance, and the topic of balance-sheet regulation demands a framework
that allows for corporate-finance effects within market microstructure. Existing work on
informational frictions is less applicable to bond markets, as most information relevant for
bond valuation is public (Fleming and Remolona 1999), and existing work on dynamics is less applicable for understanding regulation, as the regulation binds on the state of the balance sheet and not its evolution.

The model assumes securities financing is frictionless, and we do not otherwise explore the securities-financing market or its role. Huh and Infante (2016) do study the importance of repo transactions to dealer intermediation in some detail. Bottazzi et al. (2012) and Foley-Fisher et al. (2015) also study the role of securities financing, and regulators are aware it is the practical basis of bond market making (Fontaine et al. 2016). Our contribution relative to work on securities finance is to focus on regulation.

I. Model

We present a model of Cournot-competitive market makers. This model has three periods: the financing period \( t = 0 \), the trading period \( t = 1 \) and the liquidation period \( t = 2 \). Financial institutions must pay an upfront cost in order to become market makers. In order to finance their costs, market makers must issue either debt or equity and pay a market rate of return.

A. Assets

**Traded asset:** There exists an asset in unlimited supply with a certain value \( v \). At the end of the game, the asset liquidates to deliver \( v \) in cash.

**High-quality liquid asset (HQLA):** There also exists an HQLA in unlimited quantity. This HQLA can be obtained frictionlessly, in unlimited supply, at a normalized price of 1. At the end of the game, a unit of HQLA liquidates to deliver \( 1 + r_F \) in cash.
Repurchase agreements (repo) and reverse-repurchase agreements (reverse repo):

Last, there exist repo and reverse-repo agreements in unlimited quantity. A repo transaction is a cash loan at a rate $r_R$ that is collateralized by equivalent value of the asset.$^5$ A reverse-repo transaction in our context is a specific reverse repo (Duffie 1996), a loan of the model’s asset at a rate $r_R$ that is collateralized by equivalent value of cash.

B. Agents

There are three types of agent: investor buyers, investor sellers, and financial institutions. Investor buyers and sellers are interested in buying or selling the asset. They can transact only with market makers and not with one another. Financial institutions are companies that can issue debt and equity and that can invest in a market-making technology. In addition, they have access to the market for HQLA and the market for repo and reverse-repo agreements.

C. Investors

Investor sellers are infinitesimal and are arranged on a representative inverse demand curve. Market makers can purchase $B$ of the asset from sellers at a price:

$$P_B = v - l_B + B. \tag{1}$$

Similarly, investor buyers are also infinitesimal and arranged on a representative inverse demand curve. Market makers can sell $S$ of the asset to buyers at a price:

$$P_S = v + l_S - S. \tag{2}$$

$^5$We consider repos with no haircut.
The values $B$ and $S$ represent the total quantities that market makers choose to buy from $(B)$ and sell to $(S)$ the market, respectively. The values $l_B$ and $l_S$ are parameters for the strength of investor buyer demand and investor seller demand. We refer to these variables as investor demand to buy or investor demand to sell, or together simply as investor demand.

D. Financial institutions

There exist an infinite number of risk-neutral financial institutions, indexed $i$. Each financial institution can choose to pay an exogenous cost $c$ to invest in the market-making technology and become a market maker. This cost must be financed in $t = 0$ through either debt issuance $D_0$ or equity issuance $E_0$. The total rate of return required on the institution’s assets is exogenously set by the macroeconomy at $1 + r_A$. For simplicity, we assume all financial institutions choose an equal debt-equity ratio.

A market maker has a balance sheet and cannot take “negative” positions. It cannot spend cash it does not have or sell bonds it does not have. Instead, market makers must explicitly short either cash or bonds. If a market maker lacks funds to make desired purchases, then it must issue debt. Unless this additional debt is a repo, the market maker must also earn the return $1 + r_A$ on additional debt. Debt created via repo has its own rate, $r_R$. Any additional debt issued in period 1 is designated $D_1$.

Each financial institution solves three problems: (1) the entry problem in $t = 0$; (2) the corporate financing decision in $t = 0$; and (3) the market maker’s problem in $t = 1$. The firm’s entry problem is to choose whether to pay $D_0 + E_0 = c$ in order to become a market maker. Financial institution $i$ does so if it can earn a return $1 + r_A$ on its assets. The firm’s corporate financing problem is to choose initial values $D_0$ and $E_0$ in order to maximize the final value of the firm $D_2 + E_2$, given the results of market making. Last, the firm’s market-
making problem is to maximize profits from buying and selling to investors, given investor demands $l_B$ and $l_S$. Each institution $i$ that participates in market making realizes a profit of:

$$\pi_i = (v - P_B)b_i + (P_S - v)s_i - r_R v(b_i - s_i) - (1 + r_A)c. \tag{3}$$

E. Market structure and timing

At the beginning of the model at $t = 0$, all financial institutions observe the investor demands $l_B$ and $l_S$ and choose whether to pay the cost $c$ to become a market maker. The firms then choose to finance through some combination of debt $D_0$ and equity $E_0$.

After the institutions have completed their financing decisions, $t = 1$ begins. Each market maker selects a quantity $b_i$ to buy from the market and $s_i$ to sell to the market, given aggregate quantities of $B = \sum_i b_i$ and $S = \sum_i s_i$. The markets clear at prices $P_B$ and $P_S$.

Finally, at $t = 2$, the asset liquidates and returns cash to market makers who hold the asset. They also receive the cash returned to the assets lent to their repo counterparties, and they manufacture payments to reverse-repo counterparties on asset collateral. Then the market makers liquidate and distribute their profits $D_2 + E_2$ to their financiers.

II. Baseline equilibrium

In this section, we present the baseline, unregulated equilibrium. An equilibrium in the model consists of: (i) a solution to each market maker’s profit maximization problem given a number of firms $N$ and investor demands $l_B$ and $l_S$; (ii) a solution to each financial institution’s corporate finance problem; and (iii) an equilibrium number of entrants $N^*$ such
that each financial institution earns a return at least $1 + r_A$ on its assets but, were $N^* + 1$ institutions to enter, they would not.

**Theorem 1** (Existence of a Baseline Equilibrium):

(i) Given a number of entrants $N$, there exist unique liquidity supplies $b_i$ and $s_i$ for each market maker $i$, such that each market maker solves his optimization problem.

(ii) Given (i), there exist debt and equity values $D_0$ and $E_0$, such that each financial institution maximizes its firm value.

(iii) Given (i) and (ii), there exists a unique equilibrium number of entrants $N^*$ such that each financial institution earns a return of at least $1 + r_A$ on its assets. Instead, were $N^* + 1$ financial institution to enter, each would earn less than $1 + r_A$ on its assets.

A. Market-making decision

**Definition 1:** Let $\gamma = 2r_Rv$ be the cost of accessing the repo market.

Given a number of entrants $N$, each market maker chooses $b_i$ and $s_i$ to maximize:

$$\pi_i(N) = \left( -\frac{\gamma}{2} + l_B - \sum_i b_i \right) b_i + \left( \frac{\gamma}{2} + l_S - \sum_i s_i \right) s_i - (1 + r_A)c.$$  \hspace{1cm} (4)

The result of this unconstrained optimization problem yields the standard symmetric Cournot results, with liquidity supplies of:

$$b_i^*(N) = \frac{l_B - \frac{\gamma}{2}}{N + 1}$$  \hspace{1cm} (5)

$$s_i^*(N) = \frac{\frac{\gamma}{2} + l_S}{N + 1}.$$  \hspace{1cm} (6)
B. Corporate-financing decision

The baseline equilibrium for the corporate-financing decision follows from the standard
Modigliani-Miller results. For a financial institution trying to maximize its final value
$V = D_2 + E_2$, any initial combination of debt and equity is equally optimal (this will not be true
under regulation). The financial institution must finance $c = D_0 + E_0$ in order to become
a market maker. Throughout trading, the market maker has no requirement to issue any
long-term debt, but holds short-term debt equal to its repo position $D_1 = v(b_i - s_i)$, which
it clears following trading.

C. Market-maker entry decision

Given a number of entrants $N$ and optimal behaviour, the profit of each market maker is:

$$\pi_i^*(N) = \frac{(l_B - \frac{g}{2})^2 + (l_S + \frac{g}{2})^2}{(N + 1)^2} - (1 + r_A)c.$$  (7)

Given the expected profit, financial institutions choose whether to become market makers.
Institutions will continue to enter as long as $\pi_i > 0$. Since $\pi_i^*(N)$ is decreasing in $N$, there
is a single equilibrium number of entrants $N^*$ such that:

$$N^* \leq \sqrt{\frac{(l_B - \frac{g}{2})^2 + (l_S + \frac{g}{2})^2}{(1 + r_A)c}} - 1 < N^* + 1.$$  (8)

III. Basel III regulations on funding liquidity and capital

In this section, we subject the market maker to balance-sheet constraints styled after the
Basel III framework for liquidity and capital regulation. For liquidity regulation, we model
the Basel III Liquidity Coverage Ratio (LCR) as well as the Net Stable Funding Ratio (NSFR). The Basel III LCR asks institutions to hold sufficient assets deemed high-quality to cover all liabilities due in 30 days or less. We model the LCR as an obligation to hold quantities of HQLA in proportion to repo liabilities. The Basel III NSFR asks institutions to secure funding deemed stable at terms at least as long as the terms of certain investments. We model the NSFR as an obligation to possess additional long-term funding (i.e. not repo) in proportion to cash invested in reverse repo. The extra funding is used to purchase HQLA. As such, the two liquidity requirements are mirrored: the LCR obligates HQLA purchases for repo, and the NSFR for reverse repo.

For capital regulation, we model the Basel III capital regulations as a single leverage constraint. The two capital regulations in Basel III, the Leverage Ratio and the Capital Adequacy Ratio, are distinguished by their risk weighting, which in our context is immaterial because there is only one asset in the model. We model a leverage ratio as a numerical limit on the total value of the institution’s debt ($D_0 + D_1$).

Formally, we model the Basel III framework via three assumptions:

**Assumption 1:** Financial institutions that wish to engage in a repo transaction must hold HQLA. For a repo transaction of size $b_i - s_i$, they must hold $H_i = \alpha_L(b_i - s_i)v$. This HQLA earns a return of $H_i(1 + r_F)$ and costs the firm $H_i(1 + r_A)$.

**Assumption 2:** Financial institutions that wish to engage in a reverse-repo transaction must issue additional long-term debt (which they invest in HQLA). For a reverse-repo transaction of size $s_i - b_i$, they must hold $H_i = \alpha_N(s_i - b_i)v$. This HQLA earns a return of $H_i(1 + r_F)$ and costs the firm $H_i(1 + r_A)$.

**Assumption 3:** Financial institutions are subject to a leverage ratio constraint, represented by $\beta$. The maximum amount of debt an institution may hold is $\beta \geq \frac{D_0 + D_1}{D_0 + D_1 + E_0}$.
Theorem 2 establishes that the number of market makers and their choices can be determined in Cournot equilibrium.

**Theorem 2 (Existence of a constrained equilibrium):**

(i) Given a number of entrants $N$, there exist unique liquidity supplies $b_i$ and $s_i$ for each market maker $i$, such that each market maker solves his optimization problem.

(ii) Given (i), there exist debt and equity values $D_0$ and $E_0$, such that each financial institution maximizes its firm value.

(iii) Given (i) and (ii), there exists either a single unique equilibrium, or two equilibria for the number of entrants, denoted by $N^*$ and $N_B$. $N^*$ is the equilibrium number of entrants such that the leverage ratio does not bind and each financial institution earns a return of $1 + r_A$ on its assets. $N_B$ is the equilibrium number of entrants such that the leverage ratio does bind and each financial institution earns a return of $1 + r_A$ on its assets. In each case, were an additional firm to enter, each firm would earn less than $1 + r_A$ on its assets.

A. Market-making decision

For a given a number of entrants $N$, each market maker chooses $b_i$ and $s_i$ to solve the constrained maximization problem:

$$\max_{b_i,s_i} \pi_i(N) = \left(-r_R v + l_B - \sum_i b_i\right) b_i + \left(r_R v + l_S - \sum_i s_i\right) s_i$$

$$+ (1 + r_F) H_i - (1 + r_A)(c + H_i),$$

(9)

s.t. $v(b_i - s_i) \leq \Psi$,  

(10)
where $H_i$ is the amount of HQLA, equal to either $H_i = \alpha_L(b_i - s_i)v$, if the market maker accesses the repo market ($b_i > s_i$), or $H_i = \alpha_N(s_i - b_i)v$, if the market maker accesses the reverse-repo market ($s_i > b_i$); and $\Psi$ is an upper bound on the market maker’s debt $D_1$.

To express equilibrium behaviour, it is convenient to define two constants:

**Definition 2:** Let $\Gamma_L = 2v(r_R + \alpha_L(r_A - r_F))$ be the net cost of accessing the repo market in the presence of the LCR.

**Definition 3:** Let $\Gamma_N = 2v(r_R - \alpha_N(r_A - r_F))$ be the net benefit of accessing the reverse-repo market in the presence of the NSFR.

In equilibrium, the two liquidity regulations never bind at the same time, because the market maker cannot be both long and short in the asset. The LCR binds if the market maker uses repo to fund a net long position ($b_i > s_i$), whereas the NSFR binds if the market maker uses reverse repo to borrow assets for a net short position ($b_i < s_i$). Since repo and reverse repo are made costly by the liquidity regulations, the market maker avoids them if it is economical by taking no net position ($b_i = s_i$). Alternatively, if investor demand is sufficiently strong, it is more profitable for the market maker to use repo or reverse repo despite the regulatory cost. The binding regions are formalized in Proposition 1.

**Proposition 1** (Regions of effect of the liquidity constraints):

In equilibrium, for a given number of market makers $N$,

(i) if $l_B - l_S > \Gamma_L$, the LCR binds, and the market maker chooses $b_i > s_i$.

(ii) if $\Gamma_L > l_B - l_S > \gamma$, the LCR binds, and the market maker chooses $b_i = s_i$.

(iii) if $\gamma > l_B - l_S > \Gamma_N$, the NSFR binds, and the market maker chooses $b_i = s_i$.

(iv) if $\Gamma_N > l_B - l_S$, the NSFR binds, and the market maker chooses $b_i < s_i$. 

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As for capital regulation, the leverage ratio binds only if the market maker takes a sufficiently long position in the asset. This is because it is an upper bound on debt, which increases in repo but not in reverse repo, as repo is accounted as debt whereas reverse repo is accounted as an investment (under the two major accounting standards, GAAP and IFRS). The binding regions are formalized in Proposition 2.

**Proposition 2 (Region of effect of the leverage ratio):**

*In equilibrium, for a given number of market makers \( N \), the leverage ratio binds only if \( l_B - l_S > (N + 1)\Psi/v + \Gamma_L \). The market maker chooses \( b_i - s_i = \frac{\Psi}{v} \).

Since the regulations bind only in certain regions of the investor demand variables \( l_B \) and \( l_S \), the equilibrium choices of \( b_i \) and \( s_i \) are piecewise defined in the regions of \( l_B \) and \( l_S \). In the regulated equilibrium, liquidity supply for a given \( N \) market makers is:

\[
\begin{align*}
   b_i(N) &= \begin{cases} 
   \frac{l_B - l_S}{N + 1} & \text{if } l_B - l_S \leq \Gamma_N \\
   \frac{l_S + l_B}{2(N + 1)} & \text{if } \Gamma_N < l_B - l_S \leq \Gamma_L \\
   \frac{l_B - l_S}{N + 1} & \text{if } \Gamma_L < l_B - l_S \leq \frac{(N + 1)\Psi}{v} + \Gamma_L \\
   \frac{l_B + l_S}{2(N + 1)} + \frac{\Psi}{2v} & \text{if } \frac{(N + 1)\Psi}{v} + \Gamma_L < l_B - l_S \\
   \end{cases} 
\end{align*}
\]

(11)

\[
\begin{align*}
   s_i(N) &= \begin{cases} 
   \frac{\Gamma_N + l_S}{N + 1} & \text{if } l_B - l_S \leq \Gamma_N \\
   \frac{l_S + l_B}{2(N + 1)} & \text{if } \Gamma_N < l_B - l_S \leq \Gamma_L \\
   \frac{l_S + l_B}{N + 1} & \text{if } \Gamma_L < l_B - l_S \leq \frac{(N + 1)\Psi}{v} + \Gamma_L \\
   \frac{l_B + l_S}{2(N + 1)} - \frac{\Psi}{2v} & \text{if } \frac{(N + 1)\Psi}{v} + \Gamma_L < l_B - l_S. \\
   \end{cases} 
\end{align*}
\]

(12)

We find the market maker responds to regulation by rationing its balance sheet, taking smaller net positions (\( |b_i - s_i| \)) than in the baseline case. It even takes no net position
(b_i = s_i) if buyer and seller demand are relatively balanced in value, which is the case in the second segment of the functions. The market maker achieves the rationing by shifting toward an agency basis of trade, meaning it acts more as a broker between investors than as an ultimate counterparty. For example, in the case that seller demand is stronger (l_B > l_S), the market maker buys less from sellers than in the baseline equilibrium (b_i < b_i^*), yet it sells more inventory to buyers (s_i > s_i^*), meaning it is brokering a greater flow of assets from sellers to buyers than before. The regulated equilibrium is worse than the baseline, as some investors who would like to trade more of the asset are unable.

Next, we give the effect of regulation on prices. In the regulated equilibrium, the prices for a given N market makers are functions of equilibrium liquidity supply and hence are also defined piecewise. They are:

\[
P_B(N) = \begin{cases} 
    v - \frac{N\Gamma_N + l_B}{N+1} & \text{if } l_B - l_S \leq \Gamma_N \\
    v - \frac{(N+2)l_B - Nl_S}{2(N+1)} & \text{if } \Gamma_N < l_B - l_S \leq \Gamma_L \\
    v - \frac{N\Gamma_L + l_B}{N+1} & \text{if } \Gamma_L < l_B - l_S \leq \frac{(N+1)\Psi}{v} + \Gamma_L \\
    v - \frac{(N+2)l_B - Nl_S}{2(N+1)} + \frac{N\Psi}{2v} & \text{if } \frac{(N+1)\Psi}{v} + \Gamma_L < l_B - l_S 
\end{cases}
\]  

\(13\)

\[
P_S(N) = \begin{cases} 
    v + \frac{l_S - N\Gamma_N}{N+1} & \text{if } l_B - l_S \leq \Gamma_N \\
    v + \frac{(N+2)l_S - Nl_B}{2(N+1)} & \text{if } \Gamma_N < l_B - l_S \leq \Gamma_L \\
    v + \frac{l_S - N\Gamma_L}{N+1} & \text{if } \Gamma_L < l_B - l_S \leq \frac{(N+1)\Psi}{v} + \Gamma_L \\
    v + \frac{(N+2)l_S - Nl_B}{2(N+1)} + \frac{N\Psi}{2v} & \text{if } \frac{(N+1)\Psi}{v} + \Gamma_L < l_B - l_S. 
\end{cases}
\]  

\(14\)

The asset price responds to regulation by exhibiting price pressure (Hendershott and Menkveld 2014), a pricing dislocation due to the cost of holding inventory. The prices are
lower than baseline if the market maker takes a long position \((b_i > s_i)\) and higher than baseline if the market maker takes a short position \((b_i < s_i)\). The reason price pressure appears is, again, the shift to agency intermediation. In order for the market maker to limit its net position, it must pass some of its position to investors, but they require a price concession to accept the position. To sell excess inventory, the market maker must lower the price to solicit interest from buyers; to cover excess shorts, the market maker must raise the price to solicit interest from sellers.

The results illustrate how securities financing has a stabilizing role in bond markets. Using repo, a market maker can sell an asset to investor clients by borrowing it rather than procuring it outright, which would move the price. Borrowing has less impact on prices because it increases the effective supply of the asset, as a borrowed asset is used by two different investors at the same time. This is similar to the way that lendable deposits at banks increase the effective money supply. Prices are more stable under a principal market maker because it is adjusting the asset supply to investor demand, whereas an agency market maker can only adjust the price.

Last, we give the effect of regulation on the bid-ask spread. The outcome is simple. In a Cournot oligopoly model in two markets, the sole determinant of the bid-ask spread is the number of firms, as the bid-ask spread is generated via imperfect competition. As the number of firms \(N\) is fixed in this section of the paper, the spreads in the regulated and baseline equilibria are equal: \(P_S(N) - P_B(N) = P^*_S(N) - P^*_B(N)\).

Since the LCR and NSFR never bind at the same time, there are testable implications on liquidity outcomes conditional on the regulation. The implications summarize the results in the section.
Testable implication 1: Market makers limit their net position by trading more on an agency basis: When market makers are already long, they buy less and sell more; when already short, they buy more and sell less.

Testable implication 2: There is greater price pressure: Prices exhibit greater correlation with inventory position. By implication, there is greater price impact after large or unexpected purchases.

Testable implication 3: The bid-ask spread does not change if the composition of participants does not change: When $N$ is fixed, the bid-ask spread is constant.

Some of these implications have already been studied. Bessembinder et al. (2016) studies implication 1 and finds that dealers are committing less capital to market making in the post-regulatory period. Dick-Nielsen and Rossi (2016), and Bao et al. (2016) study implication 2 and find that price impacts after events necessitating sudden bond sales have gotten worse in the post-regulatory period. However, there is no study specifically on inventory pressure on the asset price post-regulation. Trebbi and Xiao (2015) studies implication 3 and finds a variety of metrics related to bid-ask spreads have not changed or have even improved, as do Adrian et al. (2015).

B. Corporate financing decision

There is a loss of capital-structure irrelevance due to the leverage constraint, which limits debt at a financial institution created by its use of repo and thus limits its maximum net position in the asset. As in the baseline case, the financial institution finances $c = D_0 + E_0$ in order to become a market maker and may finance additional debt using repo, with $D_1 = v(b_i - s_i)$. The loss of irrelevance is formalized in Proposition 3.
Proposition 3 (Loss of capital-structure irrelevance):

For a given number of market makers $N$,

(i) If there is a leverage ratio, Modigliani-Miller capital-structure irrelevance does not hold. Financial institutions are able to increase their value by choosing sufficient initial equity such that $D_0 \leq \frac{\beta E_0 - (1-\beta)\nu(b_i - s_i)}{1-\beta}$.

(ii) If there is no leverage ratio, even if there are liquidity regulations, Modigliani-Miller capital structure irrelevance holds.

In contrast to liquidity regulation, which always binds, the leverage ratio constraint is limited in its application. For a financial institution that optimally chooses a capital structure low in debt, the leverage ratio binds only when the market maker faces relatively heavy demand to sell. This results in another testable implication of the model.

Testable implication 4: The leverage ratio binds on market making only during periods of heavy selling pressure relative to the level of the firm’s debt financing.

The necessary level of seller demand that causes the leverage ratio to bind is strictly greater than the level that causes the LCR to bind, as the LCR binds on any long position. Supposing the financial institution chooses to finance its operations fully with equity, the maximum repo position is of size:

$$\Psi = \frac{\beta c}{(1 - \beta)(1 + \alpha_L)}.$$  \hspace{1cm} (15)

Also, liquidity regulation can oblige financial institutions to obtain additional funding to purchase HQLA, which aggravates the debt limit of the leverage ratio.

Proposition 4 (Liquidity regulation aggravates the binding of the leverage ratio):

For a given number of market makers $N$, the maximum additional debt a market maker can enter, $\Psi$, is lower under the LCR that it would be without the LCR.
C. Market-maker entry decision

For a given number of market makers \( N \), the profit function follows from the equilibrium liquidity supply functions and price functions. It is defined piecewise:

\[
\pi_i(N) = \begin{cases} 
\frac{(l_B - \frac{\Gamma_N}{2})^2 + (l_S + \frac{\Gamma_N}{2})^2}{(N+1)^2} - (1 + r_A)c & \text{if } l_B - l_S \leq \Gamma_N \\
\frac{(l_B + l_s)^2}{2(N+1)^2} - (1 + r_A)c & \text{if } \Gamma_N < l_B - l_S \leq \Gamma_L \\
\frac{(l_B - \frac{\Gamma_L}{2})^2 + (l_S + \frac{\Gamma_L}{2})^2}{(N+1)^2} - (1 + r_A)c & \text{if } \Gamma_L < l_B - l_S \leq \frac{(N+1)\Psi}{v} + \Gamma_L \\
\frac{(l_B + l_s)^2}{2(N+1)^2} + \frac{\Psi(l_B - l_s - \Gamma_L)}{2v} - \frac{N\Psi^2}{2v^2} - (1 + r_A)c & \text{if } \frac{(N+1)\Psi}{v} + \Gamma_L < l_B - l_S.
\end{cases}
\]  

(16)

The profit function determines the number of entrants in equilibrium. It would determine a unique number of entrants were \( N \) to be continuous (that is, if “half a firm” could exist), as there is a single \( N \) satisfying \( \pi_i(N) = 0 \). Under certain parameterizations there are two equilibrium values of discrete \( N \), which occur in the relative vicinity of \( \pi_i(N) = 0 \). Specifically, if seller demand is large but not too large, it is possible the leverage ratio binds for a smaller number of market makers but does not bind if the number of market makers is larger. In the figures in this paper, we parameterize the model such that multiple equilibria do not arise. In this case we denote the equilibrium number of firms under Basel III regulation by \( \hat{N} \), which is the number of entrants such that

\[
\pi_i(\hat{N}) \geq 0 > \pi_i(\hat{N} + 1).
\]  

(17)

A property of \( \hat{N} \) is that it is increasing in \( l_S \) and also in \( l_B \). In other words, the more business there is to be done with investors, the more financial institutions enter the market-making industry.
The equilibrium number of entrants decreases when liquidity regulation applies. Profits decline due to obligatory purchases of HQLA, so fewer financial institutions invest in market making. However, unlike the case of liquidity regulation, the effect of the leverage ratio on market maker is ambiguous. If the leverage ratio is sufficiently tight, entry by new market makers can increase. This is formalized in Proposition 5.

**Proposition 5** (The change in the number of firms):

*For given investor demands $l_B$ and $l_S$,*

(i) If market makers are bound only by liquidity regulation and not by the leverage ratio, fewer firms enter in equilibrium compared to the baseline case.

(ii) If the leverage ratio is sufficiently tight ($\beta$ low enough such that $\frac{\sqrt{1+r_A^A}}{c} > \frac{\beta}{1-\beta}$), then for sufficiently large seller demand relative to buyer demand, more firms enter in equilibrium compared to the baseline case.

Proposition 5 part (ii) provides conditions under which the leverage ratio stimulates entry. The reason the leverage regulation can stimulate entry is because it acts as an restriction on quantity output. If firms in quantity competition can coordinate to reduce output, it enhances their returns, and the leverage ratio acts as such a coordination device. Market makers cannot produce liquidity for buyers beyond a certain level. Still, in order to stimulate entry successfully, the ratio has to be sufficiently tight relative to the expense of being a market maker. If it is too expensive to be a market maker in terms of the entry cost $c$ (or if the asset value $v$ is sufficiently cheap that it is easy to finance), even a binding leverage ratio does not stimulate entry. The proposition leads to a testable implication.

**Testable implication 5:** If the leverage ratio is binding, the level $\beta$ is sufficiently low (as in Proposition 5 Part ii), and seller demand is sufficiently high, outside market makers enter the industry. Otherwise, entry will not increase.
D. Illustrated results

We illustrate the equilibrium behaviour of quantities and prices in Figure 1. The first panel graphs the liquidity supply functions \( b_i(\hat{N}) \) and \( s_i(\hat{N}) \), and the second panel graphs the prices \( P_B(\hat{N}) \) and \( P_S(\hat{N}) \). The panels graph the functions against the seller demand \( l_s \) for a fixed \( l_B \). In other words, the graphs vary demand for selling holding demand for buying constant. Accordingly, on the left side of the graphs, the market makers are short, and on the right side of the graph, the market makers are long.

FIGURE 1 ABOUT HERE

The graph of liquidity supply in Figure 1, Panel A, deviates from liquidity supply in the baseline model. In the baseline model, a change in seller demand changes only quantity bought \( b_i \) and not quantity sold \( s_i \). In contrast, in Figure 1, a change in seller demand does change quantity sold \( s_i \), in the three regions marked NSFR, LCR and leverage ratio (LR). The three regions correspond to those in Propositions 1 and 2. As discussed, the Basel III regulations induce the market maker to shift its trading strategy toward an agency basis, distributing assets from sellers to buyers rather than taking them on the balance sheet. Accordingly, in the marked regions, quantity sold rises parallel with quantity bought, showing the market maker is trading on agency.

Asset prices in Figure 1, Panel B, also deviate from results in the baseline model and in the same three regions. The deviations illustrate how asset prices exhibit price pressure from inventory. The asset price in the NSFR region deviates by the cost of holding HQLA required by the NSFR, which is proportional to \( \alpha_N \). Similarly, the asset price in the LCR region deviates by the cost of holding HQLA for the LCR, which is proportional to \( \alpha_L \). Last, the asset price has a monotonic negative relationship with seller demand when the
leverage ratio binds. In each of these regions, the market maker is operating as an agency intermediary and is adjusting the price to incentivize investors to trade with one another.

We give additional figures showing the partial effects of each of the regulations in isolation. Figures 3, 4 and 5 show the individual effects of each regulation on equilibrium liquidity supply, prices and spreads respectively. In each case, we compare the individual effect of each regulation to the baseline case in which no regulation is imposed.

**FIGURES 3, 4 AND 5 ABOUT HERE**

IV. Extension: Position limits and the Volcker Rule

In this section we cease to examine the Basel III capital and liquidity regulations and instead study a position-limit regulation styled on the Volcker Rule. The Volcker Rule, section eight of the 2012 Dodd-Frank Act in the United States, is a complex ban on proprietary trading at financial institutions with access to federal backstops. It obligates regulated institutions to satisfy an array of metrics, including a set of internal position limits for each trading desk. The position limits are the primary way the regulation binds dealers (Bao et al. 2016), so we choose to stylize this aspect of the Volcker Rule in isolation of the others. In effect, we assume the asset already satisfies the other metrics of the Volcker Rule. Formally, we impose Assumption 4 (and relax the other assumptions).

**Assumption 4:** Market makers have an exogenous position limit $\Delta$, such that their net position must be below this limit following trading: $|b_i - s_i| \leq \Delta$.

We derive equilibrium behaviour under a position limit on the absolute net position. Our finding is that it can be understood in the same way as the leverage ratio. Like the leverage ratio, a position limit is a cap on the long position of the market maker. In addition, the
position limit also is a cap on the size of its short position. Thus it can be thought of as a leverage ratio plus the mirror image of the leverage ratio.

A. Market-making decision

The market maker solves the constrained maximization problem given a number of market makers $N$,

$$
\max_{b_i, s_i} \pi_i(N) = \left(-r_Rv + l_B - \sum_i b_i\right) b_i + \left(r_Rv + l_S - \sum_i s_i\right) s_i - (1 + r_A)c,
$$

s.t. $|b_i - s_i| \leq \Delta$. \hspace{1cm} (18)

The position limit binds when an imbalance in investor demand would lead the market maker to take a position larger than the limit $\Delta$. Formally, it binds when $l_B - l_S$ is large relative to repo borrowing costs:

$$
|l_B - l_S - \gamma| < (N + 1)\Delta. \hspace{1cm} (19)
$$

Since the position limit binds only in certain regions of the investor demand variables $l_B$ and $l_S$, the equilibrium choices of $b_i$ and $s_i$ are piecewise defined in the relevant regions of $l_B$ and $l_S$. In equilibrium, liquidity supply given a number of market makers $N$ is:

$$
b_i(N) = \begin{cases} 
\frac{l_B + l_S}{2(N+1)} - \frac{\Delta}{2} & \text{if } l_B - l_S - \gamma < -(N + 1)\Delta \\
\frac{l_B - \gamma}{N+1} & \text{if } |l_B - l_S - \gamma| \leq (N + 1)\Delta \\
\frac{l_B + l_S}{2(N+1)} + \frac{\Delta}{2} & \text{if } l_B - l_S - \gamma > (N + 1)\Delta
\end{cases} \hspace{1cm} (20)
$$
The previous analysis of the binding leverage ratio applies here. In short, the market maker responds to a binding position limit by shifting its trading to a pure agency basis. Every security in excess of the position limit that it buys or sells must in turn be sold or bought with investors. If the position limit does not bind, the market maker supplies the same liquidity as in the baseline case.

Next, we give the position limit’s effect on prices. Under the limit, the prices for a given $N$ market makers are functions of equilibrium liquidity supply and hence are also defined piecewise:

$$s_i(N) = \begin{cases} \frac{l_B + l_S}{2(N+1)} + \frac{\Delta}{2} & \text{if } l_B - l_S - \gamma < -(N+1)\Delta \\ \frac{l_B + l_S}{N+1} & \text{if } |l_B - l_S - \gamma| \leq (N+1)\Delta \\ \frac{l_B + l_S}{2(N+1)} - \frac{\Delta}{2} & \text{if } l_B - l_S - \gamma > (N+1)\Delta. \end{cases}$$

(21)

The previous analysis of the binding leverage ratio applies here. In short, the market maker responds to a binding position limit by shifting its trading to a pure agency basis. Every security in excess of the position limit that it buys or sells must in turn be sold or bought with investors. If the position limit does not bind, the market maker supplies the same liquidity as in the baseline case.

Next, we give the position limit’s effect on prices. Under the limit, the prices for a given $N$ market makers are functions of equilibrium liquidity supply and hence are also defined piecewise:

$$P_B(N) = \begin{cases} v - \frac{(N+2)l_B - Nl_S}{2(N+1)} - \frac{N\cdot\Delta}{2} & \text{if } l_B - l_S - \gamma < -(N+1)\Delta \\ v - \frac{N^2 + l_B}{N+1} & \text{if } |l_B - l_S - \gamma| \leq (N+1)\Delta \\ v - \frac{(N+2)l_B - Nl_S}{2(N+1)} + \frac{N\cdot\Delta}{2} & \text{if } l_B - l_S - \gamma > (N+1)\Delta \end{cases}$$

(22)

$$P_S(N) = \begin{cases} v + \frac{(N+2)l_S - Nl_B}{2(N+1)} - \frac{N\cdot\Delta}{2v} & \text{if } l_B - l_S - \gamma < -(N+1)\Delta \\ v + \frac{l_S - N^2}{N+1} & \text{if } |l_B - l_S - \gamma| \leq (N+1)\Delta \\ v + \frac{(N+2)l_S - Nl_B}{2(N+1)} + \frac{N\cdot\Delta}{2v} & \text{if } l_B - l_S - \gamma > (N+1)\Delta. \end{cases}$$

(23)

Again, the previous analysis of the binding leverage ratio applies. In short, when a market maker shifts to a pure agency basis, it must unwind its positions by trading with investors. As buyers or sellers require a price concession to motivate trade, the asset price adjusts to
motivate them to trade. The equilibrium liquidity supply and asset prices lead to a further testable implication.

**Testable implication 6:** There is greater price pressure: Prices exhibit greater correlation with inventory position. By implication, there is greater price impact after large or unexpected purchases.

**B. Corporate financing decision**

Capital-structure irrelevance holds under the position limit. Unlike the leverage ratio, in which the constraint is on debt, firms cannot improve their position limit by financing using a greater share of equity. The irrelevance result of the baseline model is therefore unaffected.

**C. Market-maker entry decision**

For a given number of market makers $N$, the profit function follows from the equilibrium liquidity supply functions and price functions. It is defined piecewise:

$$
\pi_i(N) = \begin{cases} 
\frac{(l_B + l_S)^2}{2(N+1)^2} - \frac{\Delta(l_B - l_S - \gamma)}{2} - \frac{N\Delta^2}{2} - (1 + r_A)c & \text{if } l_B - l_S - \gamma < -(N + 1)\Delta \\
\frac{\left(l_B - \frac{\gamma}{2}\right)^2 + (l_S + \frac{\gamma}{2})^2}{(N+1)^2} - (1 + r_A)c & \text{if } |l_B - l_S - \gamma| \leq (N + 1)\Delta \\
\frac{(l_B + l_S)^2}{2(N+1)^2} + \frac{\Delta(l_B - l_S - \gamma)}{2} - \frac{N\Delta^2}{2} - (1 + r_A)c & \text{if } l_B - l_S - \gamma > (N + 1)\Delta 
\end{cases}
$$

Again, as with the leverage ratio, the equilibrium number of entrants can increase if the position limit is sufficiently binding. This obtains from Proposition 5 if $\Psi$ is replaced with $\Delta$, so we do not repeat the proof. Since the profit function in (24) is symmetric about $l_B - l_S$, the same proof holds for large $l_B$ instead of large $l_S$. Thus the Volcker Rule leads to more
entry regardless of whether there is greater buyer or greater seller pressure, if the position limit is sufficiently tight relative to the cost of market making.

**Testable implication 7:** *If the Volcker Rule is binding, the position limit is sufficiently small, and either seller or buyer demand is sufficiently large, outside market makers enter the industry. Otherwise, entry will not increase.*

### V. Conclusions

This paper studies how three types of regulation change the behaviour of market makers. The regulations we study are the Basel III standards on capital and funding liquidity and the U.S. Volcker Rule limits on position. Though the regulations are different, they all create a common incentive for market makers to trade on an agency basis: Market makers respond to regulation by matching buyers to sellers rather than acting as the ultimate counterparty. The regulations have a second and long-term consequence. If financial institutions are leverage-constrained or Volcker-constrained, outside market makers enter the market-making business to supply capital made that has been made scarce by regulation.

The outcome for market liquidity as measured by the bid-ask spread can be an improvement. The entry of new market makers intensifies competition, tightening the bid-ask spread. However, liquidity is less resilient to sudden demands to trade. The price impact of a large imbalance of trade is worse, particularly when we fix the number of market makers (no entry). The model can thus account for data that show both improving bid-ask spreads as well as worsening costs to immediacy (Trebbi and Xiao 2015; Bessembinder et al. 2016; Dick-Nielsen and Rossi 2016; Bao et al. 2016).

Last, the model affords two new predictions for future empirical work. First, inventory premia or “price pressure” should increase. If market makers relegate their trading to the
matching of buyers and sellers, rather than using their capital to absorb imbalances, it is prices that must adjust to equilibriate supply and demand. Second, the Basel III reforms should affect market makers’ long positions differently from short positions. Regulation that affects repo rather than reverse repo affects long positions rather than short positions, and vice versa.
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A Appendix

A. Baseline equilibrium

Proof of Theorem 1

Parts i, ii and iii

The proof of the baseline equilibrium is consistent with a standard problem of Cournot competition with endogenous entry. The market maker has a profit function denoted:

$$\pi_i = (-r_Rv + l_B - \sum_i b_i)b_i + (r_Rv + l_S - \sum_i s_i)s_i - (1 + r_A)c. \quad (A.1)$$

The problem is solved in the standard way, first by taking the first-order conditions with respect to $b_i$ and $s_i$ for all $i$. Second, the symmetry conditions of $b_i = b_j$ and $s_i = s_j$ are imposed for all $N$ entrants, giving equilibrium liquidity supplies of

$$b_i = \frac{l_B - r_Rv}{N + 1} \quad (A.2)$$

$$s_i = \frac{r_Rv + l_S}{N + 1}. \quad (A.3)$$

The prices at which the market maker buys and sells to the market are given by:

$$P_B = v - \frac{N^2 v^2}{N + 1} \quad (A.4)$$

$$P_S = v - \frac{(N^2 v^2 - l_S}{N + 1}. \quad (A.5)$$

The equilibrium liquidity supplies and prices can then be inserted into the profit functions. Through algebraic manipulation, the equilibrium profit is then given by:

$$\pi_i = \frac{1}{(N + 1)^2} \left( (l_B - r_Rv)^2 + (l_S + r_Rv)^2 \right) - (1 + r_A)c. \quad (A.6)$$

This profit function is independent of the market maker’s debt and equity choice, which therefore have no effect on the firm. The optimal combination of debt and equity is any $D_0, E_0 \geq 0$ such that $D_0 + E_0 = c$. In equilibrium, more firms will continue to enter until
they earn zero profit. Setting the profit equation equal to 0, and solving for \( N \), gives an equilibrium number of entrants equal to:

\[
\hat{N} = \sqrt{\frac{(l_B - r_Rv)^2 + (l_S + r_Rv)^2}{(1 + r_A)c}} - 1.
\] (A.7)

As the number of firms is discrete, \( N^* \) is the integer value directly below the preceding equation, such that \( N^* \leq \hat{N} < N^* + 1 \). As with the rest of the baseline equilibrium, the equilibrium number of entrants is consistent with the general properties of a Cournot equilibrium with endogenous entry.

### B. Constrained equilibrium

#### Proof of Theorem 2

**Part i**

This proof is in four subparts: (1) a market maker who accesses the reverse repo market by selling more units than he buys \((s_i > b_i)\); (2) a market maker who holds zero net inventory \((s_i = b_i)\); (3) a market maker who accesses the repo market and must hold HQLA \((s_i < b_i)\); and, (4) a market maker whose leverage ratio binds.

First, for a market maker who accesses the reverse repo market and must hold HQLA. In doing so, he maximizes his profit function while adding HQLA holds equal to \(\alpha_N v(s_i - b_i)\). These holdings pay a return of \(r_F\) and cost a rate of \(r_A\). By taking the first derivatives of this profit function, equilibrium supplies are shown to be:

\[
b_i = \frac{l_B - v(r_R - \alpha_N(r_A - r_F))}{N + 1}
\] (A.8)

\[
s_i = \frac{l_S + v(r_R - \alpha_N(r_A - r_F))}{N + 1}.
\] (A.9)

Given the equilibrium supplies, the optimal prices are given by:

\[
P_B = v - \frac{Nv(r_R - \alpha_N(r_A - r_F)) + l_B}{N + 1}
\] (A.10)

\[
P_S = v - \frac{Nv(r_R - \alpha_N(r_A - r_F)) - l_S}{N + 1}.
\] (A.11)

Substituting these into the profit functions gives a total profit of:
\[ \pi_1 = \left( l_B - \frac{\Gamma_N}{2} \right)^2 + \left( l_S + \frac{\Gamma_N}{2} \right)^2 \left( \frac{1}{(N+1)^2} \right) - (1 + r_A)c. \] (A.12)

Since this profit function relies on the market maker who performs a net sale, this profit function is viable for \( s_i > b_i \) which, in equilibrium is given by \( l_s - l_b \geq -\Gamma_N \).

Second, a market maker may wish to hold a positive inventory, but because of the cost of holding HQLA, holds a zero inventory instead. To hold a zero inventory, a market maker sets \( s_i = b_i \) and maximizes his initial profit function. This results in equilibrium supplies of:

\[ b_i = s_i = \frac{l_S + l_B}{2(N+1)}. \] (A.13)

Given this equilibrium supply, the market prices are:

\[ P_B = v - \frac{(N+2)l_B - Nl_S}{2(N+1)} \] (A.14)
\[ P_S = v + \frac{(N+2)l_S - Nl_B}{2(N+1)}. \] (A.15)

Substitution into the profit function yields a profit of:

\[ \pi_2 = \frac{(l_B + l_S)^2}{2(N+1)^2} - (1 + r_A)c. \] (A.16)

This profit function is viable for any parameter set. By simply evaluating the reduced form profit functions, it would appear that \( \pi_2 < \pi_1 \). However, for all \( l_S - l_B < -\Gamma_N \), a market maker would optimally choose to hold \( s_i < b_i \) and thus performs a strategy inconsistent with \( \pi_1 \).

Third, a market maker may wish to access the repo market and hold HQLA. In doing so, he maximizes his profit function while adding HQLA holds equal to \( \alpha_L v(b_i - s_i) \). These holdings pay a return of \( r_F \) and cost a rate of \( r_A \). By taking the first derivatives of this profit function, equilibrium supplies are shown to be:

\[ b_i = \frac{l_B - v(r_R + \alpha_L(r_A - r_F))}{N+1} \] (A.17)
\[ s_i = \frac{l_S + v(r_R + \alpha_L(r_A - r_F))}{N+1}. \] (A.18)
Given the equilibrium supplies, the optimal prices are given by:

\[ P_B = v - \frac{Nv(r_R + \alpha_L(r_A - r_F)) + l_B}{N + 1} \]  
\[ P_S = v - \frac{Nv(r_R + \alpha_L(r_A - r_F)) - l_S}{N + 1}. \]  

Substituting these into the profit functions gives a total profit of:

\[ \pi_3 = \frac{(l_B - \frac{\Gamma_L}{2})^2 + (l_S + \frac{\Gamma_L}{2})^2}{(N + 1)^2} - (1 + r_A)c. \]  

As the market maker holds HQLA only when he wishes to buy more than he sells, this strategy is not viable for any parameter set where the market maker wishes to supply \( s_i > b_i \). In the baseline case, this occurs when \( l_S - l_B \geq -2r_Rv \), and thus the spaces spanned by \( \pi_3 \) and \( \pi_1 \) are mutually exclusive.

As above, by simply evaluating the reduced form \( \pi_2 \) and \( \pi_3 \) it would appear that \( \pi_3 \geq \pi_2 \). However, for all \( l_S - l_B > -\Gamma_L \), the market maker would wish to hold \( s_i > b_i \) which is inconsistent with the strategy in \( \pi_3 \). Thus, in equilibrium, the spaces over which these three functions are optimal are mutually exclusive.

Finally, consider the case where the leverage ratio binds. This occurs when:

\[ v(1 + \alpha_L)(b_i - s_i) \geq \Psi. \]  

Substituting the equilibrium values \( b_i, s_i \) from the case with the liquidity ratio alone, algebraic manipulation can show that the leverage constraint binds when:

\[ l_S - l_B < -\frac{(N + 1)\Psi}{v} - \Gamma_L. \]  

In the case where the market maker holds HQLA and the leverage ratio binds, substituting the new position limit of \( s_i = b_i - \frac{\Psi}{v} \) into the market maker's initial profit function gives a new profit function of:

\[ \pi_i = \left( -\frac{\Gamma_L}{2} + l_B - \sum_i b_i \right) b_i + \left( \frac{\Gamma_L}{2} + l_S - \sum_i b_i + \sum_i \frac{\Psi}{v} \right) (b_i - \frac{\Psi}{v}) - (1 + r_A)c. \]
Taking the first-order condition gives equilibrium liquidity supply values of:

\[ b_i = \frac{l_B + l_S}{2(N + 1)} + \frac{\Psi}{2v} \]  \hspace{1cm} (A.25) \\
\[ s_i = \frac{l_B + l_S}{2(N + 1)} - \frac{\Psi}{2v} \]  \hspace{1cm} (A.26)

Given this equilibrium supply, the market prices are:

\[ P_B = v - \frac{(N + 2)l_B - Nl_S}{2(N + 1)} + \frac{N \cdot \Psi}{2v} \]  \hspace{1cm} (A.27) \\
\[ P_S = v + \frac{(N + 2)l_S - Nl_B}{2(N + 1)} + \frac{N \cdot \Psi}{2v} \]  \hspace{1cm} (A.28)

**Part ii**

Given debt \( D_0 \) and equity \( E_0 \) from the initial issuance, the maximum value of repo exposure is equal to:

\[ v(1 + \alpha_L)(b_i - s_i) = \Psi = \frac{\beta E_0 - (1 - \beta)D_0}{1 - \beta}. \]  \hspace{1cm} (A.29)

Profit is decreasing when repo exposure is restricted and thus the market maker optimally chooses an initial debt level that prevents the repo constraint from binding. This is given by:

\[ D_0 \leq \frac{\beta E_0 - (1 - \beta)(1 + \alpha_L)v(b_i - s_i)}{1 - \beta}. \]  \hspace{1cm} (A.30)

As a result of the additional leverage required to purchase HQLA, the corner case worsens and the maximum size of the repo transaction is:

\[ \Psi = v(b_i - s_i) = \frac{\beta c}{(1 - \beta)(1 + \alpha_L)}. \]  \hspace{1cm} (A.31)

Thus, as in the case with the leverage constraint alone, in equilibrium the market maker’s optimal solution is to select any value \( D_0 \geq 0 \) such that Equation A.30 holds or \( D_0 = 0 \) as a corner solution.

**Part iii**
The profitability of the market maker depends on whether his leverage constraint binds. When the leverage constraint does not bind, his profit is identical to the liquidity ratio cases $\pi_1$, $\pi_2$ and $\pi_3$, as shown in Proof of Theorem 2, Part i. When the leverage ratio does bind, substitution of the supply functions from Proof of Theorem 2, Part i results in a profit function of:

$$\pi_{B2} = \frac{(l_B + l_s)^2}{2(N + 1)^2} + \frac{\Psi (l_b - l_s - \Gamma_L)}{2v} - \frac{N \Psi^2}{2v^2} - (1 + r_A)c.$$  \hspace{1cm} (A.32)

The combination of these equations yields a total profit of:

$$\pi = \mathbb{1} (l_s - l_B \geq -\Gamma_N) \cdot \pi_1$$

$$+ \mathbb{1} (-\Gamma_N > l_S - l_B \geq -\Gamma_L) \cdot \pi_2$$

$$+ \mathbb{1} \left( -\Gamma_L > l_s - l_B \geq -\frac{(N + 1) \Psi}{v} - \Gamma_L \right) \cdot \pi_3$$

$$+ \mathbb{1} \left( -\frac{(N + 1) \Psi}{v} - \Gamma_L > l_s - l_B \right) \cdot \pi_{B2}. \hspace{1cm} (A.33)$$

In the case where the leverage ratio does not bind, the number of entrants $N^*$ is determined by inverting the relevant profit function. If $\hat{N}$ is the value which solves the inverted profit function, then the equilibrium number of entrants is $N^* \leq \hat{N} < N^* + 1$.

When the leverage constraint does bind, there exists $N^*_B > 0$, which solves:

$$(1 + r_A)c = \frac{(l_B + l_s)^2}{2(N^*_B + 1)^2} + \frac{\Psi (l_b - l_s - \Gamma_L)}{2v} - \frac{N^*_B \Psi^2}{2v^2}. \hspace{1cm} (A.34)$$

As before, the number of entrants $N_B$ is such that $N_B \leq N^*_B < N_B + 1$.

While the equilibrium supply functions and prices are unique given the number of entrants, the number of entrants is not necessarily unique itself. When the market maker’s leverage constraint does not bind, the number of entrants in equilibrium is identical to that in the baseline model. The number of entrants is unique if, for the values $N^*$ and $N_B$ above, the market maker’s leverage constraint either binds when $N$ is replaced by both $N^*$ and $N_B$, or does not bind when $N$ is replaced by both $N^*$ and $N_B$. Consider the case where either $N^*$ or $N_B$ firms enter, and:
\[ l_B - l_S \leq \frac{(N^* + 1)\Psi}{v} + \Gamma_L \]  
\[ \leq \frac{(N_B + 1)\Psi}{v} + \Gamma_L. \]  

(A.35)

If \( N^* \) firms enter, leverage constraint does not bind. Through the profit function in the baseline model each firm earns zero profit in expectation and, thus, this is an equilibrium. If \( N_B \) firms enter, the leverage constraint does not bind. Since \( N_B < N^* \), substitution of the baseline liquidity supply into the profit function gives a profit greater than zero. Thus, more firms could have entered and this is not an equilibrium.

Alternatively, there is a unique equilibrium where the leverage constraint binds and \( N_B \) firms enter if:

\[ l_B - l_S > \frac{(N^* + 1)\Psi}{v} + \Gamma_L \]  
\[ > \frac{(N_B + 1)\Psi}{v} + \Gamma_L. \]  

(A.36)

The uniqueness of this equilibrium is symmetric to the one above, with one alteration. If \( N^* \) firms enter, each earns less than zero profit and defaults, and if \( N_B \) firms enter, each earns zero profit. Thus, in this case, there is a unique number of entrants \( N_B \).

Finally, there exist two equilibria if, given equilibrium entrants \( N^* \), market makers earn zero profit and the leverage constraint does not bind, but given entrants \( N_B \), market makers also earn zero profit and the leverage constraint does bind. This occurs, given values \( N^* \) and \( N_B \) defined above, if:

\[ N_B < \frac{v}{\Psi}(l_B - l_S - \Gamma_L) - 1 \leq N^*. \]  

(A.37)

Thus, given the conditions above, there exists either: (1) a unique equilibrium number of entrants \( N^* \) or \( N_B \), or (2) two equilibria number of entrants \( N^* \) or \( N_B \).

**Proof of Proposition 1**

As shown in Theorem 2, Part i, when the market maker buys more than he sells \((s_i > b_i)\), he pays the costs of holding HQLA and his optimality decisions are changed from the baseline.
model. In the baseline equilibrium, \( s_i < b_i \) when \( l_S - l_B < -2r_Rv \). Thus, substituting for \( \gamma \), when \( l_S - l_B \geq -\gamma \), the LCR has no effect on the market maker’s decision. From Theorem 2, Part i, the market maker begins holding zero inventory at this point and continues to do so until \( l_B \) increases such that \( \Gamma_L < l_B - l_S \).

These results are reversed in the case of the NSFR.

**Proof of Proposition 2**
The leverage ratio affects the market maker if he wishes to take on more additional debt than his maximum allowable \( \Psi \), such that \( \Psi < v(b_i - s_i) \).

By substituting the equilibrium value of \( \Psi \) from Proof of Theorem 2, Part ii, and the liquidity supplies from Proof of Theorem 2, Part i, it can be shown that the market maker is affected by the leverage ratio only when:

\[
l_B - l_S > \frac{(N + 1)\Psi}{v} + \Gamma_L.
\]

(A.38)

**Proof of Proposition 3**
Proof follows from Proof of Theorem 2, Part ii. As the market maker is able to increase outputs by selecting a capital structure with \( D_0 \leq \frac{\beta E_0 - (1-\beta)v b_i}{1-\beta} \), then he is able to increase his firm’s value through capital structure. This is driven by the assumption that the market maker is able to finance his subsequent market making costs only through additional debt (the repo market), rather than additional equity issuances.

Thus, Modigliani-Miller capital structure irrelevance does not hold under these assumptions.

**Proof of Proposition 4**
Consider the case of the LCR. A market maker holds HQLA or holds zero inventory when \( b_i \geq s_i \), which in equilibrium occurs when \( l_S - l_B \geq -\gamma \). Consider the equilibrium value \( \pi \) from Proof of Theorem 1, Part iii and the equilibrium values \( \pi_2 \) and \( \pi_3 \) from Proof of Theorem 2, Part iii. First, we evaluate the conditions \( \pi \geq \pi_2 \) and \( \pi \geq \pi_3 \). Algebraic manipulation shows that, were \( \pi \) viable, these conditions would hold over the entire space where \( l_S - l_B < -\gamma \) also holds. However, since \( \pi \) is not viable, the market maker must choose either of the strategies resulting in \( \pi_2 \) or \( \pi_3 \). Thus, if the market maker is affected by the liquidity ratio, his profit is less than the profit he would earn in the baseline model given the same number of entrants.
Given that \( \pi \geq \pi_2 \) and \( \pi \geq \pi_3 \) over the space where \( l_S - l_B < -\gamma \), then \( \Pi \geq \Pi_2 \) and \( \Pi \geq \Pi_3 \), where \( \Pi_i = (\pi_i + (1 + r_A)c) \cdot (N + 1)^2 \). Thus, \( N^* \) from Theorem 2, Part iii is strictly lower than \( N^* \) from Theorem 1, Part iii, in any case where \( l_S - l_B < -\gamma \). The results for the NSFR are symmetric.

**Proof of Proposition 5**

The decrease in participants under liquidity regulation follows directly from the profit functions present in Theorem 2, Part iii.

In order to give conditions under which leverage regulation can increase the number of equilibrium entrants, it is sufficient to show the profit function in the regulated equilibrium is positive for the equilibrium number of firms in the unregulated equilibrium under those conditions. This would imply that the market makers under regulation would earn positive returns above \( r_A \), and thus too few market makers have entered to satisfy the entry condition.

The maximizing continuous-valued \( N \) in the unregulated baseline case is \( \arg \max_N \pi^*_i(N) \) where \( \pi^*_i(N) \) is given in (7). Inserting it as the argument to the profit function under regulation assuming the leverage constraint binds, segment four of \( \pi_i(N) \) as given in (??), the profit is:

\[
(p_B - l_S - \Gamma_L) \frac{\Psi}{2v} - \frac{l_B - l_S - \Gamma_L}{(l_B - \frac{\gamma}{2})^2 + (l_S + \frac{\gamma}{2})^2} \frac{1}{2} (1 + r_A)c \left( \frac{(l_B - \frac{\gamma}{2})^2 + (l_S + \frac{\gamma}{2})^2}{(1 + r_A)c} - 1 \right) \frac{\Psi^2}{2v}. 
\]

(A.39)

The Taylor expansion of (A.39), dropping all terms below order one in \( l_B \), is

\[
\left( \frac{\Psi}{2v} - \frac{\Psi^2}{2v^2} \frac{1}{\sqrt{1 + r_A}c} \right) (l_B - l^*_B). 
\]

(A.40)

So the limit of (A.39) as \( l_B \to \infty \) goes to positive infinity if

\[
\sqrt{1 + r_A}c > \frac{\Psi}{v}, 
\]

(A.41)

or, substituting for \( \Psi \), the equilibrium number of entrants increases for a debt-equity ratio \( \beta \) if:

\[
\frac{\sqrt{1 + r_A}c}{v} > \frac{\beta}{1 - \beta}. 
\]

(A.42)
Figure 1 illustrates the quantity supplied by market makers given varying degrees of sell pressure $l_B$. The buying pressure $l_S$ is fixed, so the chart shows net selling pressure. The top panel shows the base case. More seller interest causes market makers to buy more, and in the absence of regulation, it does not lead them to sell more. Liquidity regulation adds costs when market makers rely on repo transactions to fund inventory or source assets. The NSFR creates additional costs to reverse repo and causes market makers to trade on agency, which can be seen in the region marked NSFR, as the buy and sell lines are parallel (and overlap). The LCR creates additional costs to repo and similarly causes market makers to trade on agency, which can be seen in the region marked NSFR, as the buy and sell lines are parallel (and overlap). Finally, a leverage ratio or position limit restricts the total size of market makers’ positions, forcing trading on agency, which can be seen in the regions marked LR and Volcker, as the buy and sell lines are parallel. The leverage ratio only binds on long positions, whereas the position limit binds on both long and short positions.
Figure 2
Bid and ask prices, base case and regulated case

(a) Base case

(b) Regulated case

Figure 2 illustrates the equilibrium bid and ask prices given varying degrees of sell pressure $l_B$. The buying pressure $l_S$ is fixed, so the chart shows net selling pressure. The top panel shows the base case. Higher or lower seller interest does not lead to visible movement in the prices. Liquidity regulation adds costs when market makers rely on repo transactions to fund inventory or source assets. Prices rise or fall when market makers trade on agency in order to incentivize the opposite side of the market to supply liquidity.
Figure 3
Liquidity supply under each regulation

Figure 3 illustrates the quantity supplied by a market maker, given varying degrees of sell pressure $l_B$. The buying pressure $l_S$ is assumed to be constant. In the baseline model, the quantity bought by the market maker increases linearly with buy pressure, while the quantity sold remains unaffected. The introduction of liquidity constraints, the LCR and the NSFR, introduce kinks into the liquidity supply functions. One kink represents the point where the market maker maintains zero net inventory in order to avoid holding HQLA, while the second kink represents the point where he begins holding HQLA. The introduction of a leverage ratio introduces a single kink in the supply function, at the point where the leverage ratio binds. At this point, the market maker begins selling a unit for every unit bought.
Figure 4
Bid and ask prices under each regulation

Figure 4 illustrates the prices at which market makers buy and sell the asset, given varying degrees of sell pressure $l_B$. The buying pressure $l_B$ is assumed to be constant. In the baseline model, prices remain at approximately the value of the asset $v$, as more market makers enter when sell pressure increases. The introduction of liquidity constraints, the LCR and NSFR, introduces kinks into the price functions. When market makers must hold HQLA, their cost of marking markets in the asset changes. In the case of the LCR, it becomes cheaper for them to sell the asset than to buy it, shifting the price downward. In the case of the NSFR, it becomes cheaper for them to buy the asset than to sell it, shifting the price upwards. The introduction of a leverage ratio also introduces a kink into the pricing function at the point where the leverage ratio binds. At this point, market makers are unable to buy as much as the baseline case and must sell in excess, reducing the price.
Figure 5 illustrates the bid-ask spread, given varying degrees of sell pressure $l_B$. The buying pressure $l_S$ is assumed to be constant. In the baseline case, spreads slowly narrow as sell pressure increases. The entry of new market makers narrows spreads quicker than the increase in sell pressure increases them. With the introduction of liquidity constraints, spreads widen slightly, due to the cost of holding HQLA. The introduction of a leverage ratio introduces a kink in the spreads. When the leverage ratio binds, market makers are restricted in their ability to increase their amount bought and must increase their amount sold. Unlike other regulations, the effect of the leverage ratio on spreads in ambiguous. In the illustrated case, new market makers enter at a faster rate, decreasing spreads rapidly. Alternatively, in the unillustrated case, the entry of new market makers may slow, increasing spreads.