CONDITIONING INFORMATION AND IDIOSYNCRATIC VOLATILITY PUZZLE

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Abstract. We argue that the idiosyncratic volatility (IVOL) puzzle documented in Ang et al. (2006) can partly be attributed to estimation bias that occurs when estimating systematic risk using information not in investor’s information set. We first analytically prove there exists a conditioning bias in IVOL estimates when β and IVOL are estimated using contemporaneous daily return data. To mitigate this estimation bias, we suggest using conditional factor model as in Avramov and Chordia (2006) to estimate the systematic risk and IVOL. Empirically, we find that using Q-theory based firm characteristics as the conditioning variables, such as size, book-to-market and operation leverage, the conditionally estimated IVOL does not command a negative return premium and the associated benchmark-adjusted abnormal return is also much smaller than that in the original IVOL puzzle.

JEL Classification: C13, G12, G14.

Key words: Idiosyncratic volatility, conditional factor models, conditioning bias, firm characteristics

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I. Introduction

In finance literature, risk and return trade-off is the fundamental paradigm. If investors hold well-diversified portfolios, only systematic risk should matter. Bearing idiosyncratic risk should not be compensated with higher expected return. However, in an influential work, Ang et al. (2006) find strong negative correlation between idiosyncratic volatility (IVOL) and expected stock return at the firm level. Ang et al. (2009) provide further international evidence supporting the negative relation. Up to now, many follow-up papers aim to explain this puzzling result. However, Hou and Loh (2016) find that most of the existing explanations can only account for roughly half of the puzzle leaving a substantial part unexplained.

In this paper, we argue that the IVOL puzzle may not be as robust or economically large as been documented in the existing literature, due to conditioning biases incurred in the IVOL estimation. In fact, there are potentially two sources of conditioning biases associated with the IVOL puzzle. The first conditioning bias arises from how the IVOL is estimated. Starting from Ang et al. (2006, 2009) and later in Bali and Cakici (2008) among many others, the IVOL at month $t$ is estimated along with $\beta$ in a one-step linear regression using daily realized stock return and contemporaneous risk factor returns. Boguth et al. (2011) show that such contemporaneous rolling regression estimation leads to conditioning biases in the slope and intercept estimates if there exists a nonlinear relationship between stock and factor returns. We extend this notion and further derive that such biased estimates of betas and alphas give arise to a conditioning bias in the estimated IVOL. Intuitively, the estimated IVOL using contemporaneously estimated betas is not an accurate proxy for the idiosyncratic risk perceived from investor’s perspective, because it uses information that does not belong to the investor’s information set at month $t$. Moreover, we show in simulations that the conditioning bias is associated with a negative premium in the cross-sectional stock returns, which tends to strengthen the IVOL puzzle. To solve this over-conditioning issue, one could use lagged beta estimate to compute current IVOL. But as Lewellen and Nagel (2006) point out that true beta is time varying and lagged beta may not be the best proxy for the current beta. Following Avramov and Chordia (2006) and Boguth...
et al. (2011), we estimate the betas and $IVOL$ in a conditional factor model framework, where
firm specific characteristics serve as state variables that are scaled into the beta estimation. We further show that the conditional model provides a better way to reduce the bias in the estimated $IVOL$. Using this approach, we find large decrease in return spread between high and low $IVOL$ portfolio (42% in CAPM and 35% in Fama-French 3 factor model) compared with the portfolio sorted based on the realized $IVOL$ estimated from the contemporaneous regressions. Empirically, we find that firm specific information such as size, book-to-market and operation leverage, can help explain the puzzle. If correct state variables are used, then the economic importance of IVOL puzzle is not as large as previously thought.

The second bias largely follows insights of Jagannathan and Wang (1996) and Lewellen and Nagel (2006) that incorrectly evaluating a conditional model with a misspecified unconditional model will cause a bias in the alpha estimation. In most of previous studies, $IVOL$ portfolios are constructed using daily return data and updated monthly, which implicitly assume that the conditional factor model holds. When evaluating risk-adjusted abnormal returns of the zero-investment portfolios sorted by $IVOL$, however, an unconditional time-series regression is used. This inconsistence leads to a downward bias in the estimated abnormal return that is proportional to the concavity of the payoff curve (Lewellen and Nagel, 2006). In other words, the true alpha of the $IVOL$ spread portfolio may also be evaluated with errors. Following the component beta approach (Boguth et al., 2011) to calculate the sorted portfolio beta, we compute the abnormal return and such adjustment in beta significantly reduce the abnormal returns (43% in CAPM and 52% in Fama-French 3-factor model).

Prior research try to solve the $IVOL$ puzzle from different perspectives. Fu (2009) also questions using the simple second moment of realized daily idiosyncratic return as the estimated $IVOL$. But he takes a quite different approach from ours. He proposes to use estimated volatility from EGARCH model as the “expected” $IVOL$ and find the new measure is positively correlated with expected return. Bali, Cakici, and Whitelaw (2011) show that the negative relation between the $IVOL$ and expected future stock return is no longer significant after controlling for the
maximum daily return (MAX) of the previous month. However, given its high correlation (over 0.8) with idiosyncratic volatility, people generally think of MAX as an alternative measure of IVOL. Rachwalski and Wen (2016) justify that in the long run the correlation between idiosyncratic risk and expected return should be positive. However, they leave the short run negative correlation unexplained. Stambaugh, Yu, and Yuan (2015) treat the IVOL as the proxy of arbitrage risk for individual stocks. Due to the short sale constraints, overpriced stocks with high arbitrage risk could not be easily arbitraged, which generates a negative expected return in the future. Herskovic et al. (2016) document commonalities among IVOL sorted portfolios and build an equilibrium model that links the common idiosyncratic volatility (CIV) to idiosyncratic labor income risk. However, the value-weighted return spread between the quintile 5 (high) and quintile 1 (low) IVOL portfolio remains significantly positive even after controlling for the CIV factor.

Different from the previous explanations, this paper tries to understand the IVOL puzzle from the estimation and conditioning information perspective. We first analytically derive the bias in the estimated IVOL and show explicitly that both the conditioning bias in beta estimation (first moment effect) will lead to bias in the IVOL estimation (second moment effect). In a calibrated model, we show that the payoff nonlinearity and conditioning bias jointly generate a negative premium. Empirically, we find that sorting based on the biased IVOL estimates tends to overstate the return spread between the high and low IVOL quintiles. Moreover, we show that the large abnormal return of the zero-investment portfolio is partly due to the estimation bias of unconditionally evaluating a conditional factor model.

The rest of the paper is organized as follows. Section 2 analyzes the estimation biases associated with the idiosyncratic volatility estimation and portfolio performance evaluation. We quantitatively evaluate the magnitude of the conditioning bias and its cross-sectional asset pricing implication in a calibrated model. In section 3, we discuss the conditional model adopted in this paper and present the main empirical results. We revisit the idiosyncratic volatility puzzle by comparing the performance of the quintile portfolios sorted based on the contemporaneous
and conditional model idiosyncratic volatility. Section 4 provides various robustness analyses of the factors used in our study. We examine the IVOL calculated with respect to different time horizons and conditional models with different state variables. Using bootstrap method proposed in [Fama and French (2010)], we find that the pattern of idiosyncratic volatility portfolios generated from a useless factor model is significantly different from the pattern we observed from the data both statistically and economically. Section 5 concludes.

II. CONDITIONING BIAS IN STOCK IDIOSYNCRATIC VOLATILITY


The conditional model with a single market factor is used as our benchmark. We assume that the excess return $R_t$ follows the conditional factor model

$$R_t = \alpha_t + \beta_t R_{Mt} + \varepsilon_t,$$

(1)

where $\alpha_t$, $\beta_t$ and $\varepsilon_t$ represent the conditional abnormal return, conditional beta, and idiosyncratic return, respectively. If $\alpha_t = 0$ then the conditional CAPM holds. Let $\{F_t\}_{t=1}^{\infty}$ denote the investor’s information set, and we assume that it is identical to the true information set with respect to which the the conditional beta is defined, i.e., $\beta_t = \text{Cov}(R_t, R_{Mt}|F_{t-1})/\text{Var}(R_{Mt}|F_{t-1})$. Econometricians replace $\alpha_t$ and $\beta_t$ with the estimated counterpart $\hat{\alpha}_t$ and $\hat{\beta}_t$ and work with the model

$$R_t = \hat{\alpha}_t + \hat{\beta}_t R_{Mt} + \hat{\varepsilon}_t,$$

(2)

where $\text{Var}_{t-1}(\hat{\varepsilon}_t)$ denotes an estimator of IVOL. $\text{Var}_{t-1}(\hat{\varepsilon}_t)$ closely relates to the estimated risk exposure $\hat{\beta}_t$ because $\hat{\varepsilon}_t$ is the residual in Eq. (2).

From investor’s perspective, the risk exposure $\beta_t$ should be measured using only the ex-ante information, in this way, the significant profit may be seen as the evidence of financial market anomaly. However, scholars tend to estimate beta using information that does not belong to

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1The estimators $\hat{\alpha}_t$ and $\hat{\beta}_t$ may suffer from the error-in-variable (EIV) bias that will affect the result of cross-sectional regression ([Shanken, 1992]), but the bias that is being addressed in this paper mainly relates to the conditioning information that the econometrician relies on.
the investors’ information set to see if the anomalies can generate profit ex post. Lewellen and Nagel (2006) propose to use the contemporaneous realized beta as the proxy for the conditional beta, which is widely adopted in the empirical studies. We argue, however, if the payoff curve is non-linear, this approach would cause a bias in the IVOL estimation.

To demonstrate this, we use a similar decomposition method as in Boguth et al. (2011). Let \( \hat{\beta}_t \) denote an empirical estimator of \( \beta_t \), which may or may not be a precise proxy depending on whether correct information is used for the estimation. We denote the difference between \( \hat{\beta}_t \) and \( \beta_t \) as \( \varepsilon_{\beta_t} \), i.e., \( \varepsilon_{\beta_t} = \hat{\beta}_t - \beta_t \). If the econometrician uses correctly the investor’s information set \( F_{t-1} \), then \( E(\varepsilon_{\beta_t}) = 0 \).

Similarly, the realized market return \( R_{Mt} \) can be decomposed as the sum of predictable (with respect to the investor information set \( F_{t-1} \)) return \( \overline{R}_{Mt} = E(R_{Mt}|F_{t-1}) \) and residual \( \varepsilon_{Mt} \), such that

\[
R_{Mt} = \overline{R}_{Mt} + \varepsilon_{Mt}. \tag{3}
\]

It can be shown that the estimator \( \hat{\alpha}_t \) generally deviates from the true \( \alpha_t \) with an expected bias\(^2\)

\[
E(\hat{\alpha}_t - \alpha_t|F_{t-1}) = -Cov(\varepsilon_{\beta_t}, \varepsilon_{Mt}|F_{t-1}). \tag{4}
\]

This bias is over-conditioning bias because this term is a covariance of two variables that does not belong to the investor information set. We further assume \( \hat{\alpha}_t = \alpha_t + u_{\alpha t} \), where \( u_{\alpha t} \) represents the sampling error. It can be shown that the biased first moment estimation in \( \hat{\alpha}_t \) and \( \hat{\beta}_t \) will generate a bias in the estimated idiosyncratic volatility.

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\(^2\)In Boguth et al. (2011), the true information set \( \Omega_t \), with respect to which \( \beta_t \) is defined, is allowed to be different from the investor’s information set \( F_t \). They show that the bias in conditional alpha consists of two components,

\[
E(\hat{\alpha}_t - \alpha_t|F_{t-1}) = -(\bar{\beta}_t - \beta_t)\overline{R}_{Mt} - Cov(\varepsilon_{\beta_t}, \varepsilon_{Mt}|F_{t-1}).
\]

\( \overline{\beta}_t - \beta_t \) in the first term is called under-conditioning bias as it depends on the difference between the true conditional beta and its time \( t \) estimates. This bias raises because the investor information set \( F_{t-1} \) is always a subset of the true information set \( \Omega_{t-1} \). Boguth et al. (2011) show that the under-conditioning bias can be corrected using the contemporaneous rolling window regression method proposed in Lewellen and Nagel (2006).
Proposition II.1. If the difference terms \( \varepsilon_{\beta t} \) and \( \varepsilon_{Mt} \) are conditionally correlated, the estimated idiosyncratic volatility \( \text{Var}_{t-1}(\hat{\varepsilon}_t) \) is biased from the true idiosyncratic volatility \( \text{Var}_{t-1}(\varepsilon_t) \),

\[
\text{Var}_{t-1}(\hat{\varepsilon}_t) - \text{Var}_{t-1}(\varepsilon_t) = (1 - 2\theta_1)\sigma^2_{\alpha t} - 2\theta_3(\text{Cov}(\varepsilon_{\beta t}, \varepsilon_{Mt}|F_{t-1}))^2 + 2\overline{R}_{Mt}\text{Cov}(\varepsilon_{\beta t}, \varepsilon_{Mt}|F_{t-1}) \]
\[ +(\overline{R}_{Mt}^2 - 2\theta_2\overline{R}_{Mt})\text{Var}(\varepsilon_{\beta t}|F_{t-1}) + (1 - 2\theta_3)\text{Var}(\varepsilon_{\beta t}\varepsilon_{Mt}|F_{t-1}), \tag{5}\]

where the parameter \( \theta_1, \theta_2 \) and \( \theta_3 \) describes the sensitivity between errors.

The first term \((1 - 2\theta_1)\sigma^2_{\alpha t}\) represents the conditional sampling error in which \( \sigma^2_{\alpha t} \) is the conditional variance of the sampling error \( u_{\alpha t} \). All the remaining terms in Eq.(5) relate to the over-conditioning bias because they are covariances of \( \varepsilon_{\beta t}, \varepsilon_{Mt} \) or their squares. In short, Eq.(5) shows that the estimated IVOL consists of sampling error and the over-conditioning bias. This finding suggests that sorting on the estimated IVOL in the existing literature may suffer from estimation bias.

In the following sections, we calibrate a conditional CAPM with payoff non-linearity in the realized returns and show that the the conditioning bias and payoff non-linearity in the realized returns can jointly generate a pattern of negative premium in the cross-sectional stock returns.


We present a simple model of a time-varying risk premium and volatility, along with beta dynamics and payoff nonlinearities in individual stock returns. Following [Brandt and Kang (2004)] and [Boguth et al. (2011)], among many others, we assume that for each month \( \tau \), the conditional mean and variance of the market return is governed by the realization of two state variables \( \{X_{\tau}, Y_{\tau}\}_{\tau=0}^{\infty} \) at the end of month \( \tau - 1 \),

\[
\overline{R}_{Mt} = \mathbb{E}(R_{Mt}|F_{\tau-1}) = \overline{R}_M \exp[\lambda_M X_{\tau-1} - \lambda_M^2/2], \tag{6}\]
\[
\sigma^2_{Mt} = \text{Var}(R_{Mt}|F_{\tau-1}) = \overline{\sigma}_M^2 \exp[\lambda_\sigma Y_{\tau-1} - \lambda_\sigma^2/2]. \tag{7}\]

The state variables follow stationary AR(1) processes:

\[
X_{\tau} = \varphi_x X_{\tau-1} + \sigma_x \varepsilon_{x\tau}, \quad Y_{\tau} = \varphi_y Y_{\tau-1} + \sigma_y \varepsilon_{y\tau}, \tag{8}\]
where the innovations $\varepsilon_{x\tau}$ and $\varepsilon_{y\tau}$ are bivariate normal with correlation $\rho_\varepsilon$. We set $\sigma_x = (1 - \varphi_x^2)^{1/2}$ and $\sigma_y = (1 - \varphi_y^2)^{1/2}$ to ensure the unconditional variances of $X$ and $Y$ are equal to 1. Under this setting, the conditional mean and variance of the market return follow a bivariate lognormal process and $\bar{R}_M$ and $\sigma_M^2$ represent the unconditional expectation of conditional mean and variance of the market return.

In a given month $\tau$, each trading day is labeled as $\tau(t)$ where $t = 1, 2, ..., T$. The realized daily market return is simply

$$R_{M\tau(t)} = \bar{R}_{M\tau} + \sigma_{M\tau} \varepsilon_{M\tau(t)}, \quad \varepsilon_{M\tau(t)} \sim N(0, 1).$$

The stock excess return $R_t$ satisfies the conditional model,

$$\bar{R}_{\tau(t)} = \alpha_{\tau} + \beta_{\tau} \bar{R}_{M\tau},$$

and based on the empirical observations of asymmetry in betas in market upside and downside (Ang and Chen, 2002, Ang, Chen, and Xing, 2006), we follow Boguth et al. (2011) and assume the existence of payoff non-linearity in the realized stock excess return,

$$R_{\tau(t)} = \alpha_{\tau} + \beta_{\tau} R_{M\tau(t)} - \Delta_\beta \sigma_{M\tau} \left( \varepsilon_{M\tau(t)}^4 - \mathbb{E} \left( \varepsilon_{M\tau(t)}^4 \right) \right) + \varepsilon_{\tau(t)}, \quad \varepsilon_{\tau(t)} \sim N(0, \sigma_{\tau}^2), \quad (9)$$

where the innovation $\varepsilon_{\tau(t)}$ is independent from $\varepsilon_{M\tau(t)}$. We choose this specification to better match the magnitude of upside and downside betas from the true data. $\beta_{\tau}$ in Eq.(9) is stock’s conditional beta because $\text{Cov}(R_{M\tau(t)}, R_{\tau(t)}|F_{\tau(t)-1})/\sigma_{M\tau}^2 = \beta_{\tau} - \Delta_\beta \text{Cov}_{t-1} \left( \varepsilon_{M\tau(t)}^4, \varepsilon_{M\tau(t)} \right) = \beta_{\tau}$. The parameter $\Delta_\beta$ in Eq.(9) is different from the $\Delta_\beta_t$ defined in the Eq.(5). In fact for a given stock, $\Delta_\beta$ is a fixed parameter that determines the degree of payoff nonlinearity. If $\Delta_\beta = 0$, the realized stock return is a linear function of the realized market return. The realized individual stock return is concave in realized market return if $\Delta_\beta > 0$, and convex if otherwise. When $\Delta_\beta \neq 0$, estimation based on the realized stock returns in the market up- and downturns will yield up- and downside betas that are not equal to the true conditional beta $\beta_{\tau}$. 
For the beta dynamics, we assume the conditional beta $\beta_\tau$ is a linear function of the state variables $X_{\tau-1}$ and $Y_{\tau-1}$,

$$\beta_\tau = \bar{\beta} + b_x X_{\tau-1} + b_y Y_{\tau-1}. \quad (10)$$

To build the connection between the conditioning bias and cross-sectional stock returns, we make a simple assumption that the curvature parameter $\Delta\beta$ in Eq.(9) for a stock increases in its magnitude with $\bar{\beta}$, the unconditional mean of the conditional beta in Eq.(10). Under this assumption, the conditioning bias associated with the contemporaneous estimation will generate a negative premium in the cross-section.

A simple diagram helps to illustrate the intuition behind. Fig.1 shows a world of four states, among which two are labeled as “good” (state ‘G’) and two are labeled as “bad” (state ‘B’). In two good states, the return on stock and market both exceed their means and in two bad states, both returns are below their means. For simplicity, we assume that the cross-section has two stocks A and B, with $\beta_A > \beta_B$. We also assume that the realization of states cannot be predicted using investor’s ex-ante information thus the unconditional CAPM holds. In the figure, the solid lines passing through the origin show the true return generating processes for both stocks. Possible return realizations are marked in solid dots and dashed lines connect the return pairs show the payoff nonlinearity. For any possible realization of returns, contemporaneous estimation using the realized return pairs will incorrectly obtain a zero $IVOL$, which underestimates the true $IVOL$ in either market up- or downturn. The true $IVOL$ in state ‘G’ and ‘B’ are equal to the length of the vertical dashed line from the realized returns to the solid unconditional return generating lines. The figure shows that if the payoff exhibits stronger non-linearity, contemporaneous estimation causes greater conditioning bias (underestimate the true $IVOL$). Meanwhile, the greater payoff non-linearity implies larger stock beta (larger future return). Combining two channels together, the conditioning bias from contemporaneous will generate a negative premium. In section III, we try to identify the related firm characteristics that proxy for different economic states.

We conduct two sets of simulations. First, we show that the contemporaneous estimation using the realized return pairs will cause a conditioning bias in the estimated $IVOL$. Second, we show that this bias along with payoff non-linearity is able to generate a negative premium pattern in the cross-sectional stock returns. In both simulations, we set number of trading days $T = 21$ for each calendar month $\tau$.

The first simulation focuses on the monthly conditioning bias for a given stock. We simulate a total of $10^7$ months for each specification of the calibrated model in section II.2. In each specification, the stock $IVOL$ is estimated using two methods and their results are compared directly. One is the contemporaneous estimation commonly used in current $IVOL$ literature. Econometricians use the realized daily stock and market returns in a given calendar month $\tau$ to estimate the market model

$$R_{\tau(t)} = \alpha_{\tau} + \beta_{\tau} R_{M_{\tau(t)}} + \eta_{\tau(t)}$$

in the usual manner. Once obtained the estimated residual $\hat{\eta}_{\tau(t)}$, the estimated month $\tau$ivol $\hat{IVOL}_{Ctmp} = \frac{\sum_{t=1}^{T} \eta_{\tau(t)}^2}{T-1}$. Alternatively, the econometrician can incorporate the observed state variables and estimate a scaled factor model, as in Shanken (1990), Lettau and Ludvigson (2001), and many others. That is, the econometrician substitute Eq. (10) into the monthly market model and estimate

$$R_{\tau} = \alpha + (\tilde{\beta} + \tilde{b}_x X_{\tau-1} + \tilde{b}_y Y_{\tau-1}) \cdot R_{M_{\tau}} + \epsilon_{\tau}$$

unconditionally using the whole time series of the realized monthly market and stock returns. At the daily level, the conditional beta for month $\tau$ is simply $\hat{\beta}_{\tau} = \hat{\beta} + \hat{b}_x X_{\tau-1} + \hat{b}_y Y_{\tau-1}$. Then we substitute the $\hat{\beta}_{\tau(t)}$ into the daily market model

$$R_{\tau(t)} = \alpha_{\tau} + \hat{\beta}_{\tau} R_{M_{\tau(t)}} + \xi_{\tau(t)},$$

(12)
and the estimated ivol $\hat{IVOL}_{\tau}^{\text{Cond}} = \frac{1}{T-1} \sum_{t=1}^{T} \hat{\xi}_{\tau(t)}^2$. The estimator proposed in Eq. (11) is able to significantly reduce both conditioning biases. The unconditional regression on the whole time series would not generate any over-conditioning bias (Boguth et al., 2011) as $\varepsilon_{\beta t} = 0$.

In the calibration exercise, we set the true abnormal return $\alpha_t = 0$. $\bar{\beta}$, the unconditional mean of $\beta_{\tau}$, is normalized to be 1. We assume $R_M = 0.03\%$ and $\sigma_M = 1\%$ to match the unconditional market return and volatility in the real data. Since the $IVOL$ of individual stock is the main interest of this paper, we assume $\sigma_{\tau} = 3\sigma_{M\tau}$ to match the magnitude of the average stock return volatility.

Table 1 presents the mean and standard deviation of the bias in the estimated $IVOL$ for the representative stock for both the contemporaneous and conditional $IVOL$ estimators. Different parameter combinations are examined in the calibrated model. In all specifications, we set $\lambda_M = 0.9$ and $\lambda_{\sigma} = 1.2$, which according to Brandt and Kang (2004), are able to match the observed conditional market mean and volatility. The $IVOL$ biases are defined as the difference between the estimated and the realized $IVOL$, i.e., the standard error of the realized innovations.

We examine in total 6 model specifications. In case (1), the model is simply an unconditional CAPM since $b_x = b_y = 0$ turns off the conditioning channel. Moreover, there is no non-linearity in the payoff, e.g., $\Delta_{\beta} = 0$. The contemporaneous $IVOL$ on average yields a bias of $-0.07\%$. The negative bias is not due to estimation with improper information set. In fact, this bias corresponds to the sampling error term $(1 - 2\theta_1)\sigma_{\alpha t}^2$ in Eq. (5). Mechanically, the negative bias raises because the OLS is designed to achieve the global minimum of the sum of residual squares. We will show in the next set of simulation that only the conditioning bias generated by the payoff nonlinearity has the asset pricing implication in the cross-sectional stock returns.
relative betas of ±1. **Boguth et al. (2011)** show that the contemporaneous alpha estimation is biased when payoff nonlinearity exists and the estimation is conducted with respect to improper information set. We further provide numerical evidence showing that the biased slope will result in a biased residual variance due to over-conditioning, which is consistent with the derivation in Eq.(5). In the simulation, we find that the over-conditioning corresponds to an average monthly bias of −0.02%. Meanwhile, the standard deviation of the IVOL bias increases significantly to 0.18% per month, which suggests that the investor who forms the IVOL portfolios based on the contemporaneous estimation is facing a substantial error-in-variable problem. On the other hand, the conditional estimation consistently produce better estimation results. The average estimation is unbiased with extreme low dispersion. In case (3), we allow for conditioning but close the nonlinearity channel. We find that the contemporaneous IVOL estimation is biased downward (a monthly average of -0.07%) and more volatile (a monthly standard deviation of 0.12%) while the IVOL estimated from the conditional model is unbiased and more precise. In case (4), (5) and (6), we keep both the conditioning channel and payoff nonlinearity while change the correlation between the state variables X and Y. According to Eq.(5) the contemporaneous IVOL estimation will contain the sampling error, under- and over-conditioning biases. The simulations show that the average conditioning bias in the contemporaneous IVOL estimation is about −0.02% per month (with a monthly standard deviation of around 0.18-0.19%). The estimated IVOL obtained from the scaled factor model, which is free from conditioning biases, provides the correct IVOL estimation in all three specifications.

In the second simulation, we examine cross-sectional asset pricing implications of the conditioning bias using the calibrated model. The cross section has in total 100 stocks, with unconditional betas $\bar{\beta}$ ranging from 0.6 to 2.6, increasing at a step size of 0.02. We set $b_x$ and $b_y$ in Eq.(10) equal to 0 for all stocks, thus the unconditional CAPM holds in the simulation. We estimate stock monthly IVOLs contemporaneously using the realized return pairs and compare the results with payoff non-linearity to those without. In the case where the non-linearities exist, the parameters $\Delta \beta$ and $\bar{\beta}$ follow a simple linear relation $\Delta \beta = (\bar{\beta} - 0.5) \cdot 0.091$. Under this setting,
small beta stocks exhibit less payoff non-linearity and thus smaller up- and downside beta spread whereas the opposite is true for large beta stocks. For example, for the stock with $\beta = 0.6$, we have $\beta^- - \beta^+ = 0.7 - 0.5 = 0.2$, but this spread increases dramatically to 4.2 for the stock with $\beta = 2.6$. Moreover, we assume in Eq.(9) that $\sigma_\tau = \sigma_{M_\tau}$ for all stocks. This assumption may appear to be too restrictive since the cross-sectional IVOL dispersion is then determined solely by the degree of non-linearity in stocks’ realized payoffs. We argue, however, this assumption avoid introducing other channel between IVOL and the cross sectional stock returns, which helps us identify the link between the conditioning bias of the IVOL estimation and future stock returns. We set $\rho_\varepsilon = 0.5$ and all rest parameters follow the previous setting.

Table 2 presents the time series average of the quantile statistics of the stock cross section and the premium of the conditioning bias from the Fama and MacBeth (1973) regression. Estimations are based on a simulated return panel of 1000 months with 21 monthly trading days. To mitigate the concern of sampling randomness, all quantities reported are the average outcome from 100 simulated panels. In case (1) we assume no non-linearities in stock payoffs by letting $\Delta_\beta = 0$ for all stocks. Once the non-linearity channel is turned off, the stock IVOLs are solely determined by the innovation $\varepsilon_\tau(t)$. Therefore we obtain almost identical IVOLs among all stocks in the cross section. The simulation shows that although the contemporaneous estimation generally underestimates stock IVOLs, the bias dispersion is too small to generate a significant premium (average $\gamma$ and its $t$-stats are 0.026 and 0.044, respectively) in the cross section if payoff non-linearity does not exist. In case (2), we allow for non-linearities in stock payoffs while keep all other settings unchanged to examine the cross sectional effect of the conditioning bias. Simulation results suggest that the non-linearity in stock payoffs enlarge not only the magnitude but also the dispersion of stock IVOLs in the cross section. Moreover, we find that the bias of the IVOL estimated contemporaneously decreases with stock beta in the cross section, meaning that the conditioning bias decreases with the payoff non-linearity. Meanwhile, the cross-sectional spread
in conditioning bias is able to generate a negative premium that is significant both statistically and economically (average $\gamma$ is $-2.048$ while the average $t$-stats is equal to $-2.94$).

The negative conditioning bias premium cannot fully account for the IVOL puzzle documented in the existing literature. In fact, the beta-non-linearity channel alone will incorrectly imply a positive premium between the current IVOL and future return. Moreover, when comparing the quantile statistics of stock IVOLs in case (2) of our simulation with the real data, we find that the IVOL produced by the simulation is much smaller. The average monthly IVOL of the real data is around 3% (as shown in Table 3), which is more than twice as large as that reported in Table 2 (1.12% per month). This result suggests that the payoff non-linearity, if exists, accounts for a small portion of the IVOL in the real data. Both reasons explain why the conditioning bias is largely ignored in the IVOL literature. However, our results show that conditioning biases that arise from the contemporaneous estimation can enhance the negative premium observed in the cross-sectional stock returns and thus should not be ignored. On the other hand, the simulation results indicate that there is room for additional channels that link the IVOL and cross-sectional stock returns. For example, one possible modeling scheme for an empiricist is to impose structures in $\sigma_t$ in the Eq.(9). An important question we left for future research is whether there exists a channel between IVOLs and future stock returns that can fully explain the cross-sectional negative premium observed in the real data, after conditioning biases are properly taken into account.


The EIV problem that rose from the contemporaneous IVOL estimation can largely be corrected by the scaled factor estimation. However, there may still exist another bias in the performance measurement after the IVOL-sorted portfolio is formed. Once the portfolio is formed, the conditional beta for each component stock is also known. Therefore at the portfolio level, the conditional beta of the portfolio is simply an weighted average of component beta. Lewellen and Nagel (2006) show that, if a conditional model is evaluated unconditionally, the alpha estimation
is biased

\[
\alpha^U - \mathbb{E}(\alpha_t) = \left(1 + \frac{R_M^2}{\sigma_M^2}\right) \text{Cov}(\beta_t, R_Mt) - \left(\frac{R_M}{\sigma_M^2}\right) \text{Cov}(\beta_t, R_M^2),
\]

(13)

where the right terms can be rewritten as the sum of biases related to the market timing and volatility timing. Boguth et al. (2011) analyze the abnormal return of the return spread between the winner and loser portfolio and find incorporating the portfolio ex-ante beta can reduce the abnormal return of the momentum strategy by 20%-40%. Similarly, the true risk of IVOL portfolios may be over-estimated if the portfolio ex-ante beta fails to be taken into account. We will show later that the alpha of the zero-investment IVOL portfolio is not as large as reported in the existing literature if the component betas are incorporated at the portfolio level.

III. Empirical Results

III.1. Conditional Model with Firm Fundamental Information.

The dynamic CAPM presented in section II.3 assumes that the conditional beta of individual stock is determined by two state variables \(X\) and \(Y\), which also govern the dynamics of the conditional market return and volatility. But more generally, firm betas can be a function of firm specific variables, such as firm characteristics.

Firm fundamental characteristics are well known to affect stock returns. Karolyi (1992) notes that incorporating the information of firm industry help estimate stock betas. Fama and French (1992) suggest that firm size and book-to-market may proxy for some risk factor. Daniel and Titman (1997) argue that firm characteristics help explain the cross-sectional stock returns. Gomes, Kogan, and Zhang (2003) build a general equilibrium model which links the conditional beta to firm size and book-to-market ratio. In their model, the firm size proxies for the firm’s systematic risk that related to its growth opportunity and the book-to-market ratio captures the risk of firm’s existing projects. The risk premia in their model is driven by the changing in business cycle conditions, which also motivates the inclusion of conditioning variables that reflects the state of the economy. Carlson, Fisher, and Giammarino (2004) show that firm investment
decision will affect the stock returns. Value firms are more vulnerable during recession because of higher operating leverage. Zhang (2005) develops a competitive equilibrium model in a product economy where the costly reversibility of capital makes the investment of value firms deviates from their optimal choice during the recession. Petkova and Zhang (2005) find some empirical evidence that support this theory. Motivated by these theoretical results, Avramov and Chordia (2006) directly model a firm’s conditional beta as a function of firm size, book-to-market ratio, a macroeconomic variable and their interaction. They find the conditional model help explain size and value anomalies in the cross sectional stock returns. Livdan, Sapriza, and Zhang (2009) extend the investment-based asset pricing model to incorporate firm’s financial constraint. They find that more constrained firms are less likely to flexibly finance desired investments, which prevents them from smoothing the dividend streams when facing negative shocks. Cosemans et al. (2016) adopt a similar econometric framework as Avramov and Chordia (2006). In their paper, the conditional beta is estimated with a shrinkage estimator that combines the information of both the contemporaneous realized beta and a beta estimation obtained using conditional model. They find the shrinkage beta has many desired properties that can be directly applied in the field such as portfolio management.


We adopt a similar econometric model as Avramov and Chordia (2006), in which the conditional beta is a function of firm characteristics, a variable that describes the state of macroeconomy and their interaction, i.e.,

$$\beta_{jt} = \beta_{j0} + \beta_{j1}Z_{t-1} + \beta'_{j2}X_{j,t-1} + \beta'_{j3}X_{j,t-1}Z_{t-1},$$  \hspace{1cm} (14)$$

where the variable $Z_{t-1}$ represents the lagged state variable, and $X_{j,t-1}$ is an $n$-vector of lagged firm characteristics. This specification allows a time-varying relation between the conditional beta and firm characteristics over the business cycle, as supported by empirical findings in the existing literature, e.g., Petkova and Zhang (2005).
Following Jagannathan and Wang (1996), we use the default spread as the state variable of the macroeconomy. Other commonly used state variables, such as the term spread, dividend yield, and Treasury bill rate are also examined. The results are robust to the choice of state variable. Although many variables are related to beta dynamics\(^4\), we include the firm size, book-to-market ratio, and operation leverage in the firm characteristics vector \(X_{jt}\). The main results are largely unchanged if different state variables or characteristics vectors are chosen. Following Avramov and Chordia (2006) and Cosemans et al. (2016), all characteristics are standardized by subtracting the cross-sectional mean and dividing by the cross-sectional standard deviation every month to remove any time trend in their cross-sectional average. In the empirical analysis, all firm characteristics are in the form of logarithm and all truncated at 0.005 fractile and 0.995 fractile to eliminate the effect of extreme values. As many coefficients are involved in Eq.(14), we require at least 36 monthly observations for a stock to be included in the analysis.

Following the spirit of simulations in section II.3, we first use the whole time series of monthly stock return to estimate the coefficients \(\beta_j \equiv [\beta_{j0}, \beta'_{j1}, \beta'_{j2}]'\) in Eq.(14). That is, we estimate the following model

\[
R_{j\tau} = \alpha_j + (\beta_{j0} + \beta_{j1}Z_{\tau-1} + \beta'_{j2}X_{j\tau-1} + \beta'_{j3}X_{j\tau-1}Z_{\tau-1})R_{Mt} + \epsilon_{j\tau}.
\]  

At each month \(\tau\), the estimated conditional beta \(\hat{\beta}_{j\tau}\) is given by

\[
\hat{\beta}_{j\tau} = \hat{\beta}_{j0} + \hat{\beta}_{j1}Z_{\tau-1} + \hat{\beta}'_{j2}X_{j\tau-1} + \hat{\beta}'_{j3}X_{j\tau-1}Z_{\tau-1}.
\]

We then substitute Eq.(15) into the daily market model (12) and estimate the monthly \(IVOL\) under the usual manner. As shown in section II.3 the conditional estimation approach helps mitigate the conditioning biases in the \(IVOL\) estimation. In the empirical analysis, we examine the \(IVOL\) defined with respect to the CAPM model and the Fama-French 3 factor model.

\(^4\)Example includes financial leverage (Livdan, Sapriza, and Zhang 2009) and momentum (Grundy and Martin 2001).
For each model, the results include both contemporaneous and conditional $IVOL$ for direct comparison.

III.3. Data.

Stock return, volume and related data are from July 1964 until December 2015 on Center for Research in Securities Prices (CRSP). The data includes all common stocks (with share codes of 10 or 11) listed on NYSE, AMEX and NASDAQ. The trading volume in NASDAQ is adjusted according to Gao and Ritter (2010). Firm size at each month $\tau$ is measured using the market value of equity (in million dollars) at the end of month $\tau$. The book value of equity, book value of total assets are from COMPSTAT. Book value of equity is supplemented by the hand-collected book value data from Kenneth French’s website. The book to market ratio is calculated by the book value of equity (assumed to be available six months after the fiscal year end) divided by current market capitalization. Operating leverage is the ratio of the percentage change in operating income before depreciation to the percentage change in sales. All firm characteristics are in the form of logarithm and all truncated at 0.005 fractile and 0.995 fractile to eliminate the effect of extreme values. Following Jegadeesh and Titman (1993), the past cumulative return of each stock in month $\tau$ is measured from month $\tau$-6 to month $\tau$-1. Following Acharya and Pedersen (2005), we calculate illiquidity as the normalized Amihud (2002) Ratio.

The commonly used indicator of the macroeconomy are used in the empirical analyses. Following Goyal and Welch (2008), we examine default spread, term spread, dividend yield and treasury bill rates.


Ang et al. (2009) report that the past contemporaneous $IVOL$ can help explain the cross-sectional dispersion of current stock excess returns. In this section, we want to examine whether this conclusion still holds if the $IVOL$ estimated via the conditional model, which possesses a smaller conditioning bias according to the simulation, is used as the dependent variable. We

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5We are grateful to Kenneth French for making the data available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

6We are grateful to Amit Goyal for making the predictor data available at http://www.hec.unil.ch/agoyal/
apply the two-stage Fama and MacBeth (1973) regression by first regressing the cross-sectional firm excess returns onto the lagged \( IVOL \)

\[
R_{it+1} = \alpha + \gamma \cdot IVOL_{it} + \varepsilon_{it+1},
\]  

and then testing whether the time series average of the coefficient on the lagged \( IVOL \) measure is significantly different from zero.

Table 3 reports results of the Fama-MacBeth regression for both \( IVOL \) estimations. The left (right) panel reports the results for the \( IVOL \) that is defined with respect to the CAPM (Fama-French 3-factor model). In the table, \( IVOL^{ctmp} (IVOL^{cond}) \) represents the \( IVOL \) estimated using the contemporaneous realized beta (conditional beta). The estimated \( IVOL \) has a mean and standard deviation of about the same magnitude in both models. Moreover, the time series average of the cross sectional correlation (not shown in the table) between the contemporaneous and conditional \( IVOL \) estimates is 0.986 (for CAPM) and 0.954 (for FF3). Although \( IVOL^{ctmp} \) and \( IVOL^{cond} \) share some large co-movement, their cross-sectional asset pricing implications are quite different. When the \( IVOL \) is estimated using the contemporaneous realized beta, the corresponding \( \gamma \) is significant with negative premium, which is consistent with the results in Ang et al. (2009) and many others. When the conditional \( IVOL \) is used as the dependent variable in Eq.(16), however, the point estimates of \( \gamma \) decrease in magnitude and no longer statistically significant. The \( t \)-statistics decreases from \(-2.16 \) to \(-0.56 \) in CAPM and from \(-2.10 \) to \(-0.18 \) in Fama-French 3 factor model. The results suggest that when the \( IVOL \) is estimated using a conditional beta, the negative premium puzzle disappear in the cross-sectional regression.

III.5. Sorting.

We also examine the performance of the zero-investment long-short \( IVOL \) sorted portfolio. The ivol puzzle is first documented in portfolio sorting under the value-weighted scheme (Ang et al.).
Later research, such as Bali and Cakici (2008), show that the puzzle is more prominent when portfolio component stocks are weighted by their market capitalization. In this part, we mainly focus on the return difference among value-weighted ivol portfolios.

At the beginning of each month, stocks are sorted into five value-weighted quintile portfolios based on the estimated ivol in the last month. Portfolios are held for one month before rebalancing. Previous literature suggests that the quintile 1 (low ivol) portfolio on average earns higher return than the quintile 5 (high ivol) portfolio. Moreover, the return spread cannot be explained by standard risk factors.

We construct the zero-investment portfolio by simultaneously long quintile 5 and short quintile 1 ivol portfolio that are sorted by both ivol estimates. Table 4 presents the raw and abnormal returns for ivol estimated with respect to CAPM and Fama-French 3-factor model. While the raw return of the long-short strategy is simply the return spread between the quintile 5 and 1 portfolio for both ivol estimations, the abnormal return are calculated differently for the contemporaneous and conditional ivol. The abnormal return for the contemporaneous ivol is estimated unconditionally by regressing the time series of raw return $R_{5-1,\tau}$ on the realized contemporaneous factor, i.e., the contemporaneous abnormal return $\alpha^u$ for CAPM is obtained by estimating

$$R_{5-1,\tau} = \alpha^u + \beta_{5-1}^u R_{M\tau} + \nu_{\tau}.$$  \hfill (17)

However, when forming portfolio, the conditional beta for individual stock is already known. As shown by Lewellen and Nagel (2006), ignoring the conditioning information will result in a bias in the alpha estimation. Instead, Boguth et al. (2011) provide an alternative way of alpha estimation by incorporating the information of the component beta into the performance evaluation. When the ivol is estimated with respect to CAPM, Boguth et al. first estimate the portfolio beta by taking the value-weighted average of its component beta. Then the conditional beta of the long-short portfolio is simply $\hat{\beta}_{5-1,\tau} = \hat{\beta}_{5,\tau} - \hat{\beta}_{1,\tau}$. The abnormal return for the
strategy is given by the $\alpha^c$ in

$$R_{5-1,\tau} = \alpha^c + (\phi_0 + \phi_1 \hat{\beta}_{5-1,\tau})R_{M,\tau} + u_{\tau}. \quad (18)$$

When the ivol is defined with respect to CAPM, we find a significant difference in both raw and abnormal return between low and high ivol portfolios. When the ivol is estimated using the contemporaneous realized beta, the monthly return on low ivol portfolio on average exceeds that of high ivol portfolio by 0.80%. The monthly abnormal return from Eq.(17) is around $-1.18\%$. Both numbers are close in magnitude to those reported in the literature. However, if the ivol is estimated using the conditional model (12), the monthly raw return spread decreases in absolute value by 42% from 0.80% to 0.46%. The $t$-statistic changes from $-2.77$ to $-1.59$, which is no longer statistically significant. For the monthly abnormal return, the coefficient $\phi_1$ of the interaction term $\hat{\beta}_{5-1,\tau}R_{M,\tau}$ is statistically significant and the sign of coefficient $\phi_0$ flips, which means that the conditional component beta do help describe the time series of portfolio return and all information in the unconditional model has been absorbed. The point estimate of absolute value of $\alpha$ also exhibits a significant drop of around 43% from 1.18% to 0.67%. For ivol with respect to the Fama-French model, we find similar results. The average monthly absolute raw return spread drop in absolute value by 24% from 0.71% to 0.54% and the absolute value of abnormal return drops by 47% from 1.18% to 0.62%. These results suggest that the original ivol puzzle documented in the literature may not be as economically significant as it was perceived.

III.6. **Cross-sectional Asset Pricing Implications of the Conditioning Bias.**

The simulation results in Section II.3 suggest that the ivol estimated using the contemporaneous realized stock and market returns may contain conditioning bias that may generate a cross-sectional negative premium. In this section, we want to examine whether this result holds in the real data.

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7We find a larger decrease if other state variables $Z_{\tau-1}$ is used in Eq.(14), details will be provided in the section of robustness analysis.
Both contemporaneous and conditional ivol measures can be decomposed as the true ivol plus estimation biases. Because of the flexibility in the choice of conditioning variable and longer time series used in the estimation, the ivol estimated using the conditional model yields greater accuracy (with smaller conditioning bias). Let the $IVOL^{\text{cond}}$ proxy the combination of true ivol and the sampling error, then the difference between the contemporaneous and the conditional ivol can proxy the stock’s conditioning bias,

$$IVOL_{i,\tau}^{\text{ctmp}} - IVOL_{i,\tau}^{\text{cond}} = \text{bias}_{i,\tau}.$$  

If the channel between the conditioning bias and cross-sectional stock returns is correct, the difference term $\text{bias}_{i,\tau}$ should bear a negative premium. We run a cross-sectional Fama and MacBeth (1973) regression between stock excess returns and lagged bias term, along with a vector of factor loadings and firm characteristics,

$$R_{i,\tau+1} = \alpha + \gamma \cdot \text{bias}_{i,\tau} + \beta_i' \lambda + \epsilon_{i,\tau+1}.$$  

We examine various specifications that cover the most commonly used combinations of factors and characteristics in the literature. We are interested in the coefficient $\gamma$ on the lagged conditioning bias, which should be significant negative if our hypothesis is correct.

The Panel A. of Table 5 reports the results of the Fama and MacBeth (1973) regressions in Eq.(20) for ivols estimated with respect to CAPM. The univariate regression in specification (1) show that the lagged bias commands a significant negative premium in the cross-sectional stock returns. The results suggest that a 0.01 decrease in the lagged bias will cause a 106 basis points increase in expected stock excess returns. In specification (2), we show that the conditioning bias premium still exist after controlling for the market betas. The point estimation of coefficient on the bias term drop in magnitude, suggesting that the premium associated with the bias can
partly be explained by the market factor. However, a 0.01 increase in the lagged conditioning bias will still cause a 40 basis points decrease in expected stock excess returns. In specification (3) we incorporate both market betas and firm characteristics, such as the size, book-to-market, momentum, and the normalized [Amihud (2002) illiquidity measure proposed by Acharya and Pedersen (2005). Although the point estimation of $\gamma$ drops in magnitude to around -0.20%, which means the conditioning bias can also be explained partly by firm characteristics, the result still suggests that $\gamma$ is significant both economically and statistically. Huang et al. (2010) argue that the ivol puzzle is caused by omitting the return reversal in Eq.(20), they show that the ivol premium is no longer significant once the previous month’s stock return is included. In specification (4), we incorporate the lagged stock return as the extra characteristic variable in Eq.(20). The point estimates of the bias premium (-0.27%) are almost identical to the previous specifications, and still statistically significant. Bali, Cakici, and Whitelaw (2011) argue that ivol is essentially a proxy for the stock maximum daily return. The negative premium is due to investor’s lottery preference on stock selection. They show that the (contemporaneous) ivol premium is no longer significant once the lagged maximum daily return is included in the regression. In specification (5), we follow [Bali, Cakici, and Whitelaw (2011) by including the lagged max return in Eq.(20). The magnitude of point estimate (-0.29%) and the statistical significance (-2.92 in $t$-stats) of the bias premium almost remains the same as in the previous specifications. In other words, the lagged max return variable cannot rule out the lagged conditioning bias.

We also evaluate the cross-sectional asset pricing implications of the conditioning bias using portfolio sorting. In Panel B. of Table 5 we present results from both single and double sorting. In the single sorting, we sort all stocks into 5 quintile portfolios under the equal-weighted scheme each month based on the magnitude of conditioning bias. We then hold the portfolio for one month before reform. The single sorting results show that the average monthly return spread between the quintile 5 and 1 portfolios is -0.6% (about -7.2% per annum), which is statistically significant. Moreover, this return spread cannot be explained by the market factor, with a monthly average abnormal return around -0.28% (with a $t$-stat of -2.62). We further evaluate
the relation between the contemporaneous ivol and the conditioning bias using double sorting. At the end of each month, we first sort all stocks into 5 portfolios based on their conditioning biases, then we further sort the stocks within each bias quintile into 5 quintiles based on their contemporaneously estimated ivol. The portfolio is formed under the value-weighted scheme and is rebalanced at monthly frequency. We report the return spread between the quintile 5 (high contemporaneous ivol) and 1 (low contemporaneous ivol) portfolio within each conditioning bias quintile, along with the average return spread and abnormal return among all bias quintiles.

Economically, the double sorting exercise examines whether the negative premium of the contemporaneous ivol would survive after controlling for the conditioning bias. In bias quintile 1, the monthly return spread of the ivol quintiles is just -0.15% and statistically insignificant. In bias quintile 4 and 5, the monthly ivol return spreads are around -0.5%, only marginally significant from a statistical perspective. The average monthly return spread between the high and low ivol quintiles is -0.54%, with a t-stat of -2.05, indicating a smaller negative premium compared with the single sorting results (-0.8% per month) reported in Table 4. All the results show that conditioning biases that associated with the contemporaneous estimation contribute to the ivol puzzle, the true ivol puzzle may not be as large if conditioning biases have been taken into account.

IV. Robustness Analyses

In this section, we examine various specifications to check the robustness of our findings.

IV.1. Ivol of Different Time Horizons.

The contemporaneous ivol can be estimated using a longer period. Ang et al. (2009) examine the cross-sectional results of the contemporaneous ivol estimated from 3- and 6-month rolling window and show that the negative premium puzzle still exists. We replicate their finding using the value weighted sorting in Table 6.

[Insert Table 6 about here]
We estimate the contemporaneous ivol with a 3- and 6-month rolling window with respect to the CAPM and then sort all stocks into five quintile portfolios based on the lagged ivol estimates. We find a significant negative monthly return spread of around 0.80% in both cases. Moreover, the return spread cannot be explained by the market factor. We then use the conditional scaled factor framework to estimate the ex ante beta and the ivol. The sorting results based on the conditional ivol, which should be less affected by the conditioning bias, indicate that the monthly return spread drop significantly to 0.27% (0.01%) for the 3-month (6-month) ivol, both of which are statistically insignificant. Furthermore, when correctly incorporate the conditional beta in the abnormal return evaluation, we find the abnormal return is no longer statistically significant (0.45% for 3-month ivol, and 0.32% for the 6-month ivol).

IV.2. Results with Different State Variables.

The previous results can be driven by the choice of state variable. To eliminate this possibility, we examine various state variables that are widely used in the literature such as the term spread, dividend yield and the Treasury bill rate. We estimate the monthly conditional beta in Eq.(14) with different state variables $Z_{t-1}$ then calculate the ivol accordingly. The same sorting exercise is conducted and the results are presented in Table 7.

The monthly return spreads between the low and high ivol portfolio are comparable among different state variables. When the ivol is estimated with respect to the CAPM, the monthly return spreads are between 0.47% to 0.56% and the estimations are marginally significant at 10% at most. The abnormal return estimates show similar pattern, compared to the $\alpha$ of around 1.19% in the contemporaneous estimation, the abnormal return in the conditional model is around 0.56% to 0.59%, suggesting a decline of about 48%. Similar results are also found in the Fama-French model. The monthly return spread is between 0.46% to 0.55% for three state variables, and the abnormal return estimation also exhibits a significant decrease to 0.57% to 0.65%.

In previous sections we have shown that comparing to the contemporaneous model, the alphas of the long-short ivol strategy in the conditional model decrease by roughly 50%, though the point estimates are still significant both economically and statistically. In this section we would like to further check whether such 50% alpha reduction is statistically significant. In other words, we want to test the null hypothesis \( H_0 : \alpha_{5-1}^{\text{cond}} = \hat{\alpha}_{5-1}^{\text{ctmp}} \). Further, we want to check whether the variables we used in Eq.(14) truly contain information about stock betas. Due to the more complicated beta structure in our conditional beta model, it may raise concerns that any random variable with certain variation could possibly generate similar reduction in alphas. To mitigate this concern, we generate “useless” (purely random) conditioning variables that has the same mean and standard deviation as the real macro and firm characteristics variables and test whether the alpha of the ivol strategy in the simulated factor model is equal to the alpha observed under the conditional model, i.e., we test the null hypothesis \( H_0 : \alpha_{5-1}^{\text{sim}} = \hat{\alpha}_{5-1}^{\text{ctmp}} \).

To implement both hypothesis tests, following Fama and French (2010), we use cross-sectional bootstrap approach to construct confidence intervals for the test statistic, the \( t \)-statistic of \( \hat{\alpha} \). According to Kosowski et al. (2006) and Fama and French (2010), \( t_\alpha \) is a pivotal statistic in which the sampling error in each bootstrap sample has been taken into account. Hence this approach can provide more reliable inferences about the abnormal returns. The entire bootstrapped distribution percentiles are reported in Table 8. Specifically, for Panel A, we bootstrap each time one cross-section from original sample of contemporaneous ivol quintiles with replacement and then construct the bootstrapped distribution of \( t_{\hat{\alpha}^{\text{ctmp}}} \) based on the resampled data. As we can see from the results, the \( t \)-statistic of \( \hat{\alpha}^{\text{cond}} \), the abnormal return for ivol spread portfolio based on conditional model, is -2.399, which indicates that the 50% reduction is statistically significant at the 1% level. For Panel B, we randomly generate conditioning variables and simulate the data. The state variable \( Z_\tau \) is generated using a normal distribution with the same mean and variance

[Insert Table 8 about here]
as in the observed default premium and the firm characteristics are simply random numbers drawn from the standard normal distribution. To test the hypothesis $H_0 : \alpha_{5-1}^{\text{sim}} = \alpha_{5-1}^{\text{ctmp}}$, we calculate the t-statistic of the $\alpha$ obtained from the sample with simulated factors. The point estimate (t-statistic) of $\hat{\alpha}_{5-1}^{\text{sim}}$ is $-1.09 \ (-4.51)$, which are close to the abnormal return between the contemporaneous ivol quintiles in the Table 4. After pivoting the $\alpha^{\text{sim}}$, we have the pivotal t-statistic is equal to -0.40. Based on the bootstrapped distribution, the t-statistic is significant at conventional levels of significance, indicating that the conditioning variables we use indeed contain information about stock betas.

V. Conclusion

The ivol puzzle has drawn serious research interests and has been extensively studied for a decade. Quite a few papers have proposed different approaches to solve this puzzle, this paper provides yet another novel explanation focusing on the conditioning bias in the estimation procedure. We show analytically that estimating ivol using contemporaneous daily return regression incurs conditioning bias. Moreover, we provide numerical evidence that the conditioning bias and the payoff nonlinearity can jointly create a negative premium in the future stock return. We further argue that if the true factor pricing model is conditional, then the abnormal returns should also be evaluated under the conditional factor model framework. Empirically, we show that after correcting the conditioning bias using ex-ante beta, the value-weighted return difference between the top and bottom quintile ivol sorted portfolios decreases about 43%. When evaluating the spread portfolio return using conditional model, we find that the conditional alpha is reduced by almost one half. The reduction is significant both statistically and economically. Separately, we find direct empirical evidence that the bias incurs a significant negative risk premium. Our findings are robust to the horizon of the ivol estimation and the choice of the state variables used in the conditional factor pricing model.

\footnote{not reported in the table.}


Proof of proposition II.1. Here we give a proof of Proposition II.1 in a more general case following the model setup in [Boguth et al. (2011)], where the investor’s information set $F$ is allowed to be different from the true information set $\Omega$ that determines the conditional beta.

The proposition straightforwardly follows if we let $F = \Omega$.

Let $\{\Omega_t\}_{t=1}^\infty$ denote the information set with respect to which the conditional beta is defined, i.e.,

$$\beta_t = \frac{\text{Cov}(R_t, R_{Mt}|\Omega_{t-1})}{\text{Var}(R_{Mt}|\Omega_{t-1})}.$$

Let $\{F_t\}_{t=1}^\infty$ denote the investor information set, where $F_{t-1} \subseteq \Omega_{t-1}$, then the true idiosyncratic volatility with respect to the investor is $\text{Var}_{t-1}(\epsilon_t) = \mathbb{E}(\epsilon_t^2|F_{t-1})$.

Consider the conditional model in [Boguth et al. (2011)]

$$R_t = \alpha_t + \beta_t R_{Mt} + \epsilon_t.$$

Suppose the investor use the estimators $\hat{\alpha}_t$ and $\hat{\beta}_t$ to estimate the unknown coefficients $\alpha_t$ and $\beta_t$, then the model being evaluated is

$$R_t = \hat{\alpha}_t + \hat{\beta}_t R_{Mt} + \hat{\epsilon}_t.$$

Similar to [Boguth et al. (2011)], non-linear payoff in realized asset returns will cause a conditioning bias in idiosyncratic volatility estimation. Specifically, the bias is given by

$$\text{Var}_{t-1}(\hat{\epsilon}_t) - \text{Var}_{t-1}(\epsilon_t) = \mathbb{E}(\epsilon_t^2 - \hat{\epsilon}_t^2|F_{t-1}) = \mathbb{E}\left( (\alpha_t - \hat{\alpha}_t + (\beta_t - \hat{\beta}_t)R_{Mt} + \epsilon_t)^2 - \epsilon_t^2 |F_{t-1} \right)$$

$$= \mathbb{E} \left( (\alpha_t - \hat{\alpha}_t)^2 + (\beta_t - \hat{\beta}_t)^2 R_{Mt}^2 + 2(\alpha_t - \hat{\alpha}_t)(\beta_t - \hat{\beta}_t)R_{Mt} |F_{t-1} \right)$$

$$+ 2\mathbb{E} \left( (\alpha_t - \hat{\alpha}_t + (\beta_t - \hat{\beta}_t)R_{Mt})\epsilon_t |F_{t-1} \right). \quad (21)$$

Simplifying the Eq. (21) gives

$$\text{Var}_{t-1}(\hat{\epsilon}_t) - \text{Var}_{t-1}(\epsilon_t) = \mathbb{E} \left( (\alpha_t - \hat{\alpha}_t)^2 |F_{t-1} \right) + \mathbb{E}(\hat{\beta}_t^2 |F_{t-1})\mathbb{E}(R_{Mt}^2 |F_{t-1}) - 2\beta_t \mathbb{E}(\hat{\beta}_t |F_{t-1}) \mathbb{E}(R_{Mt}^2 |F_{t-1}).$$
\[ + \beta_t^2 \mathbb{E}(R_{Mt}^2 | \mathcal{F}_{t-1}) + \text{Cov}(\hat{\beta}_t^2, R_{Mt}^2 | \mathcal{F}_{t-1}) - 2 \beta_t \text{Cov}(\hat{\beta}_t, R_{Mt}^2 | \mathcal{F}_{t-1}) + 2 \mathbb{E}(\hat{\alpha}_t \hat{\beta}_t | \mathcal{F}_{t-1}) \mathbb{E}(R_{Mt} | \mathcal{F}_{t-1}) \]

\[-2 \beta_t \mathbb{E}(\hat{\alpha}_t | \mathcal{F}_{t-1}) \mathbb{E}(R_{Mt} | \mathcal{F}_{t-1}) - 2 \alpha_t \mathbb{E}(\hat{\beta}_t) \mathbb{E}(R_{Mt} | \mathcal{F}_{t-1}) + 2 \alpha_t \beta_t \mathbb{E}(R_{Mt} | \mathcal{F}_{t-1}) \]

\[+ 2 \mathbb{Cov}(\hat{\alpha}_t \hat{\beta}_t, R_{Mt} | \mathcal{F}_{t-1}) - 2 \beta_t \mathbb{Cov}(\hat{\alpha}_t, R_{Mt} | \mathcal{F}_{t-1}) - 2 \alpha_t \mathbb{Cov}(\hat{\beta}_t, R_{Mt} | \mathcal{F}_{t-1}) \]

\[+ 2 \mathbb{E}(\beta_t R_{Mt} \Lambda_t | \mathcal{F}_{t-1}) + 2 \mathbb{E}((\alpha_t - \hat{\alpha}_t) \Lambda_t | \mathcal{F}_{t-1}) \].

\[ (22) \]

Let \( R_{Mt}, \beta_t \) and \( \alpha_t \) be predictable part of \( R_{Mt} \), \( \beta_t \) and \( \alpha_t \) with respect to investors’ information set \( \mathcal{F}_{t-1} \), i.e.,

\[ \mathbb{R}_{Mt} = \mathbb{E}(R_{Mt} | \mathcal{F}_{t-1}), \quad \beta_t = \mathbb{E}(\hat{\beta}_t | \mathcal{F}_{t-1}), \quad \alpha_t = \mathbb{E}(\hat{\alpha}_t | \mathcal{F}_{t-1}). \]

Then

\[ R_{Mt} = \mathbb{R}_{Mt} + \varepsilon_{Mt}, \quad \hat{\beta}_t = \beta_t + \varepsilon_{\beta t}, \quad \hat{\alpha}_t = \alpha_t + u_{\alpha t}, \]

where \( \varepsilon_{Mt} \), \( \varepsilon_{\beta t} \) and \( u_{\alpha t} \) denote unpredictable market return, over-conditioning bias in beta and sampling error in alpha, respectively. We assume

\[ \mathbb{E}(\varepsilon_{Mt} u_{\alpha t} | \mathcal{F}_{t-1}) = \mathbb{E}(\varepsilon_{\beta t} u_{\alpha t} | \mathcal{F}_{t-1}) = \mathbb{E}(\varepsilon_{Mt} \varepsilon_{\beta t} | \mathcal{F}_{t-1}) = 0. \]

According to Boguth et al. [2011], the \( \alpha \) bias is given by

\[ \mathbb{E}(\hat{\alpha}_t - \alpha_t | \mathcal{F}_{t-1}) = (\beta_t - \beta_t) \mathbb{R}_{Mt} - \text{Cov}(\varepsilon_{\beta t}, \varepsilon_{Mt} | \mathcal{F}_{t-1}) \]

Therefore,

\[ \mathbb{E}((\hat{\alpha}_t - \alpha_t)^2 | \mathcal{F}_{t-1}) = \mathbb{E}((\beta_t - \beta_t) \mathbb{R}_{Mt} - \text{Cov}(\varepsilon_{\beta t}, \varepsilon_{Mt} | \mathcal{F}_{t-1}) + u_{\alpha t})^2 | \mathcal{F}_{t-1}) \]

\[ = \sigma_{\alpha t}^2 + (\beta_t - \beta_t)^2 \mathbb{R}_{Mt}^2 - 2(\beta_t - \beta_t) \mathbb{R}_{Mt} \text{Cov}(\varepsilon_{\beta t}, \varepsilon_{Mt} | \mathcal{F}_{t-1}) + (\text{Cov}(\varepsilon_{\beta t}, \varepsilon_{Mt} | \mathcal{F}_{t-1}))^2. \] 

The cross product term is equal to 0 because of the law of iterated expectation. Similarly,

\[ \mathbb{E}(\beta_t^2 | \mathcal{F}_{t-1}) = \beta_t^2 + \mathbb{V} \text{ar}(\varepsilon_{\beta t} | \mathcal{F}_{t-1}), \quad \mathbb{E}(\hat{\beta}_t | \mathcal{F}_{t-1}) = \mathbb{R}_{t}, \quad \mathbb{E}(R_{Mt}^2 | \mathcal{F}_{t-1}) = \mathbb{R}_{Mt}^2 + \sigma_{Mt}^2. \] 

\[ (24) \]

\[ \text{Cov}(\beta_t^2, R_{Mt}^2 | \mathcal{F}_{t-1}) = \text{Cov}(\beta_t^2, \varepsilon_{Mt}^2 | \mathcal{F}_{t-1}) + 2 \beta_t \text{Cov}(\varepsilon_{\beta t}, \varepsilon_{Mt}^2 | \mathcal{F}_{t-1}) + 2 \mathbb{R}_{Mt} \text{Cov}(\beta_t^2, \varepsilon_{Mt} | \mathcal{F}_{t-1}) + 4 \beta_t \mathbb{R}_{Mt} \text{Cov}(\varepsilon_{\beta t}, \varepsilon_{Mt} | \mathcal{F}_{t-1}), \] 

\[ (25) \]

\[ \text{Cov}(\hat{\beta}_t, R_{Mt}^2 | \mathcal{F}_{t-1}) = \text{Cov}(\hat{\beta}_t, \varepsilon_{Mt}^2 | \mathcal{F}_{t-1}) + 2 \mathbb{R}_{Mt} \text{Cov}(\hat{\beta}_t, \varepsilon_{Mt} | \mathcal{F}_{t-1}), \quad \text{Cov}(\hat{\alpha}_t \hat{\beta}_t, R_{Mt} | \mathcal{F}_{t-1}) = \mathbb{R}_{t} \text{Cov}(\varepsilon_{\beta t}, \varepsilon_{Mt} | \mathcal{F}_{t-1}). \] 

\[ (26) \]
Each of the last two terms in Eq. (22) can be written as the product of $\varepsilon_t$ and the sum of over- and under-conditioning terms, e.g.,

\[
E_{t-1} \left( (\alpha_t - \hat{\alpha}_t) \varepsilon_t | F_{t-1} \right) = E_{t-1} \left( (\alpha_t - \bar{\alpha}_t - u_{at}) \varepsilon_t | F_{t-1} \right) = -E_{t-1} (u_{at} \varepsilon_t | F_{t-1})
\]

\[
E_{t-1} \left( (\beta_t - \hat{\beta}_t) R_M \varepsilon_t | F_{t-1} \right) = E_{t-1} \left( (\beta_t - \bar{\beta}_t - \varepsilon_{\beta t}) (\bar{R}_M + \varepsilon_M) \varepsilon_t | F_{t-1} \right)
\]

\[
= -E_{t-1} (\varepsilon_{\beta t} (\bar{R}_M + \varepsilon_M) \varepsilon_t | F_{t-1}),
\]\n
The last equality in Eq. (28) follows the fact that the conditional covariance between the innovation in market return $\varepsilon_M$ and the error $\varepsilon_t$ is 0. However, $\varepsilon_{\beta t}$ and $\varepsilon_t$ are conditional dependent.

To further simplify Eq. (27) and (28), we follow Jagannathan and Wang (1996) to project $\varepsilon_t$ on to $u_{at}$, $\varepsilon_{\beta t}$ and $\varepsilon_{\beta t} \varepsilon_M$.

\[
\varepsilon_t = \theta_1 u_{at} + \varepsilon_{\alpha t}, \quad \varepsilon_t = \theta_2 \varepsilon_{\beta t} + \varepsilon_{\beta t}, \quad \varepsilon_t = \theta_3 \varepsilon_{\beta t} \varepsilon_M + \varepsilon_{\beta M t},
\]

where the parameter $\theta_1$, $\theta_2$ and $\theta_3$ describes the sensitivity between errors. The conditional expectation of terms that include the orthogonal error $\varepsilon_{\alpha t}$, $\varepsilon_{\beta t}$ and $\varepsilon_{\beta M t}$ are simply 0. Thus Eq. (27) and (28) can be rewritten as

\[
E_{t-1} \left( (\alpha_t - \hat{\alpha}_t) \varepsilon_t | F_{t-1} \right) = -\theta_1 \sigma_{\alpha t}^2,
\]

\[
E_{t-1} \left( (\beta_t - \hat{\beta}_t) R_M \varepsilon_t | F_{t-1} \right) = -\theta_2 \bar{R}_M \text{Var}(\varepsilon_{\beta t} | F_{t-1}) - \theta_3 \text{Var}(\varepsilon_{\beta t} \varepsilon_M | F_{t-1})
\]

\[
-\theta_3 (\text{Cov}(\varepsilon_{\beta t}, \varepsilon_M | F_{t-1}))^2.
\]\n
Substitute Eq. (23), (24), (25), (26), (30) and (31) into Eq. (22) and we will have the bias term

\[
\text{Var}_{t-1}(\hat{\varepsilon}_t) - \text{Var}_{t-1}(\varepsilon_t) = (1 - 2\theta_1) \sigma_{\alpha t}^2 + (\Delta \beta_t)^2 \sigma_M^2 + 2\Delta \beta_t \bar{R}_M \text{Cov}(\varepsilon_{\beta t}, \varepsilon_M | F_{t-1}) + 2\Delta \beta_t \text{Cov}(\varepsilon_{\beta t}, \varepsilon_M^2 | F_{t-1})
\]

\[
-2\theta_3 (\text{Cov}(\varepsilon_{\beta t}, \varepsilon_M | F_{t-1}))^2 + 2 \bar{R}_M \text{Cov}(\varepsilon_{\beta t}, \varepsilon_M | F_{t-1})
\]

\[
+(\bar{R}_M^2 - 2\theta_2 \bar{R}_M) \text{Var}(\varepsilon_{\beta t} | F_{t-1}) + (1 - 2\theta_3) \text{Var}(\varepsilon_{\beta t} \varepsilon_M | F_{t-1}).
\]
where $\Delta \beta_t \equiv \bar{\beta}_t - \beta_t$ represents the under-conditioning bias in conditional beta estimates. Eq. (32) shows that the bias in estimated idiosyncratic volatility consists of the sampling error ($\sigma^2_{\alpha t}$), under-conditioning bias ($\sigma^2_{\Delta \beta t}$) and over-conditioning bias (all remaining terms, which include some interaction terms).

Specifically, $\Delta \beta_t \equiv 0$ when $F = \Omega$, then we have the results provided in proposition II.1.
Figure 1. Conditioning Bias and Cross-Sectional Stock Returns in a Four-State Example

Note: This figure plots the realized stock returns against the market return to illustrate the channel between the conditional bias and cross-sectional stock returns in a four-state example. The stock return generating processes follow an unconditional CAPM, which are represented by the solid lines passing through the origin. Stock A (red) has a larger unconditional beta than the stock B (blue). The true ivol for each stock in a given month is plotted by the vertical dashed line from the realization of stock returns (solid black dots) on the contemporaneous estimation lines (dashed lines) to the unconditional solid lines. The figure suggests that stronger payoff non-linearity in realized stock return generates larger negative conditioning bias but higher future stock returns.
Table 1. Bias in the idiosyncratic volatility measurement

<table>
<thead>
<tr>
<th>Case</th>
<th>Market parameters</th>
<th>Stock parameters</th>
<th>Daily relative betas</th>
<th>Performance measure</th>
<th>E(Bias)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda_M$</td>
<td>$\lambda_\sigma$</td>
<td>$\rho_v$</td>
<td>$b_x$</td>
<td>$b_y$</td>
</tr>
<tr>
<td>(1)</td>
<td>No conditioning, no nonlinearities</td>
<td>0.30</td>
<td>1.20</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(2)</td>
<td>Nonlinearities only</td>
<td>0.30</td>
<td>1.20</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.01</td>
<td>-0.01</td>
<td>0.09</td>
<td>1.00</td>
</tr>
<tr>
<td>(3)</td>
<td>Conditioning, no nonlinearities</td>
<td>0.30</td>
<td>1.20</td>
<td>0.00</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.50</td>
<td>-0.50</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(4)</td>
<td>Conditioning, nonlinearities, $\rho_v = 0$</td>
<td>0.30</td>
<td>1.20</td>
<td>0.00</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.51</td>
<td>-0.50</td>
<td>0.09</td>
<td>1.00</td>
</tr>
<tr>
<td>(5)</td>
<td>Conditioning, nonlinearities, $\rho_v &gt; 0$</td>
<td>0.30</td>
<td>1.20</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.51</td>
<td>-0.52</td>
<td>0.09</td>
<td>1.00</td>
</tr>
<tr>
<td>(6)</td>
<td>Conditioning, nonlinearities, $\rho_v &lt; 0$</td>
<td>0.30</td>
<td>1.20</td>
<td>0.00</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.50</td>
<td>-0.47</td>
<td>0.09</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: This table reports the mean and standard deviations of the biases in ivol estimation (in %) from the calibrated conditional CAPM with parameter $\lambda_M = 0.03\%$, $\sigma_M = 1\%$, $\sigma_t = \sigma_M$ and $\beta = 1$. The persistence of the conditioning variables $X$ and $Y$ is $\phi_X = \phi_Y = 0.9$. The definition of relative betas is following Ang, Chen, and Xing (2006), in which relative $\beta^- = \beta^- - \beta_t$ and relative $\beta^+ = \beta^+ - \beta_t$. Estimations are based on the simulated returns of $10^7$ months with 21 monthly trading days. The ivol are measured using the contemporaneous realized beta (contemp) and the monthly scaled multi-factor model (conditional). Biases are defined as the difference between the estimated ivol and realized ivol, i.e., the standard error of the realized innovations.
Table 2. Conditioning Bias and the Cross-sectional Stock Returns

<table>
<thead>
<tr>
<th>Case</th>
<th>Quantile statistics</th>
<th>Fama-MacBeth Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%</td>
<td>25%</td>
</tr>
<tr>
<td>(1) No nonlinearities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VOL</td>
<td>1.244</td>
<td>1.447</td>
</tr>
<tr>
<td>IVOL</td>
<td>0.780</td>
<td>0.783</td>
</tr>
<tr>
<td>IVOL_{c tmp}</td>
<td>0.760</td>
<td>0.763</td>
</tr>
<tr>
<td>Bias</td>
<td>-0.019</td>
<td>-0.020</td>
</tr>
<tr>
<td>(2) Nonlinearities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VOL</td>
<td>1.389</td>
<td>1.677</td>
</tr>
<tr>
<td>IVOL</td>
<td>0.883</td>
<td>0.952</td>
</tr>
<tr>
<td>IVOL_{c tmp}</td>
<td>0.853</td>
<td>0.905</td>
</tr>
<tr>
<td>Bias</td>
<td>-0.030</td>
<td>-0.046</td>
</tr>
</tbody>
</table>

Note: This table presents the cross-sectional asset pricing implication of the conditioning bias using the simulated model in section II. We report the time series average of the quantile statistics of the stock cross section and the premium of the conditioning bias from the Fama and MacBeth (1973) cross-sectional regression. The cross-sectional quantile statistics include stocks’ total volatility, ivol, the contemporaneous estimated ivol and the estimation bias (in %). The model is calibrated with parameter \( R_M = 0.03\%, \sigma = 1\%, \sigma_t = \sigma_M \). We set \( \lambda_M = 0.3, \lambda_\sigma = 1.2 \) and \( \rho_{\epsilon} = 0.5 \), following Boguth et al. (2011). The persistence of the conditioning variables \( X \) and \( Y \) is \( \phi_x = \phi_y = 0.9 \). Estimations are based on a simulated return panel of 1000 months with 21 monthly trading days. To mitigate the concern of sampling randomness, each quantity in this table is the average outcome of 100 such simulated panels. ‘***’, ‘**’, and ‘*’ stand for statistical significance at 1%, 5%, and 10% level.

Table 3. Idiosyncratic volatility and expected stock returns

<table>
<thead>
<tr>
<th></th>
<th>CAPM</th>
<th>FF3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ctmp</td>
<td>Cond</td>
</tr>
<tr>
<td>IVOL_c tmp</td>
<td>-0.0891**</td>
<td>(-2.16)</td>
</tr>
<tr>
<td>IVOL_cond</td>
<td>-0.0250</td>
<td>(-0.56)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.0017***</td>
<td>0.8426***</td>
</tr>
<tr>
<td>Mean(%)</td>
<td>2.9152</td>
<td>2.9631</td>
</tr>
<tr>
<td>Stdev(%)</td>
<td>2.4243</td>
<td>2.1607</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.019</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Note: This table reports Fama and MacBeth (1973) regression (Eq.(16)) of different ivol estimations. The idiosyncratic volatility is defined with respect to CAPM and Fama-French 3-factor model. \( IVOL\_c tmp (IVOL\_cond) \) represents the ivol estimated using the contemporaneous realized beta (conditional beta). In the conditional beta estimation (Eq.(14)), we choose the default spread as the indicator of the state of the macroeconomy, and the firm characteristics vector contains firm size, book-to-market, and operation leverage. The mean and standard deviation are calculated as the time series average of cross-sectional mean and standard deviation of the estimated ivol (both in %). ‘***’, ‘**’, and ‘*’ stand for statistical significance at 1%, 5%, and 10% level. The sample period spans from July 1964 to December 2015.
Table 4. Portfolios Sorted by the Estimated Ivol

<table>
<thead>
<tr>
<th></th>
<th>CAPM</th>
<th></th>
<th>FF3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ctmp</td>
<td></td>
<td>Cond</td>
</tr>
<tr>
<td>$R_{Mt}$</td>
<td>0.7853***</td>
<td>-0.2094***</td>
<td>0.4046***</td>
</tr>
<tr>
<td></td>
<td>(13.95)</td>
<td>(-2.69)</td>
<td>(9.67)</td>
</tr>
<tr>
<td>$\beta_t R_{Mt}$</td>
<td>-1.3030***</td>
<td>-1.1302***</td>
<td>0.0624</td>
</tr>
<tr>
<td></td>
<td>(-16.35)</td>
<td>(-20.99)</td>
<td>(0.77)</td>
</tr>
<tr>
<td>$SMB_t$</td>
<td>1.4199***</td>
<td>0.0624</td>
<td>-0.9441***</td>
</tr>
<tr>
<td></td>
<td>(24.19)</td>
<td>(0.77)</td>
<td></td>
</tr>
<tr>
<td>$s_t SMB_t$</td>
<td>-0.2307***</td>
<td>0.0882*</td>
<td>0.0882*</td>
</tr>
<tr>
<td></td>
<td>(-3.62)</td>
<td>(1.94)</td>
<td></td>
</tr>
<tr>
<td>$HML_t$</td>
<td>-1.2043***</td>
<td>-1.2043***</td>
<td>0.0882*</td>
</tr>
<tr>
<td></td>
<td>(-21.80)</td>
<td>(-21.80)</td>
<td>(1.94)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.8001***</td>
<td>-1.1812***</td>
<td>-0.5425*</td>
</tr>
<tr>
<td></td>
<td>(-2.77)</td>
<td>(-4.66)</td>
<td>(-1.92)</td>
</tr>
</tbody>
</table>

Note: This table reports the return spread and abnormal return (in %) of the zero-investment long-short ivol sorted portfolios. In each month, all stocks are sorted into five value-weighted quintile portfolios based on the lagged ivol estimation. The zero-investment portfolio is formed by simultaneously short the quintile 1 (low ivol) and long the quintile 5 (high ivol) portfolio. $R_{5-1}$ and $\alpha_{5-1}$ represent the return spread and abnormal return on the long-short portfolio in the following month, respectively. The idiosyncratic volatility is defined with respect to CAPM and Fama-French 3-factor model. ‘Ctmp’ (‘Cond’) represents the ivol estimated using the contemporaneous realized beta (conditional beta). In the conditional model, the strategy abnormal return is estimated following Eq.(18). ‘***’, ‘**’, and ‘*’ stand for statistical significance at 1%, 5%, and 10% level. The sample period spans from July 1964 to December 2015.
Table 5. Conditioning bias and expected stock returns

<table>
<thead>
<tr>
<th></th>
<th>Panel A. Fama-MacBeth Cross-sectional Regression</th>
<th>Panel B. Portfolio Sorting</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β(MKT)</td>
<td>RΔt</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>β(MKT)</td>
<td>0.7625***</td>
<td>0.7888***</td>
</tr>
<tr>
<td></td>
<td>(4.36)</td>
<td>(4.66)</td>
</tr>
<tr>
<td>Bias</td>
<td>-1.0565***</td>
<td>-0.4027***</td>
</tr>
<tr>
<td></td>
<td>(-5.23)</td>
<td>(-3.01)</td>
</tr>
<tr>
<td>Size</td>
<td>-0.0821**</td>
<td>-0.0703**</td>
</tr>
<tr>
<td></td>
<td>(-2.30)</td>
<td>(-2.06)</td>
</tr>
<tr>
<td>Book-to-market</td>
<td>0.3706***</td>
<td>0.2669***</td>
</tr>
<tr>
<td></td>
<td>(7.16)</td>
<td>(5.46)</td>
</tr>
<tr>
<td>Momentum</td>
<td>0.0106***</td>
<td>0.0089***</td>
</tr>
<tr>
<td></td>
<td>(7.76)</td>
<td>(6.55)</td>
</tr>
<tr>
<td>Illiquidity</td>
<td>0.0389***</td>
<td>0.0372***</td>
</tr>
<tr>
<td></td>
<td>(2.64)</td>
<td>(2.55)</td>
</tr>
<tr>
<td>Lagged return</td>
<td>-0.0519***</td>
<td>-0.0519***</td>
</tr>
<tr>
<td></td>
<td>(-15.37)</td>
<td>(-15.37)</td>
</tr>
<tr>
<td>Lagged max</td>
<td>-0.0850***</td>
<td>-0.0850***</td>
</tr>
<tr>
<td></td>
<td>(-13.35)</td>
<td>(-13.35)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.6384***</td>
<td>-0.0892</td>
</tr>
<tr>
<td></td>
<td>(3.08)</td>
<td>(-0.96)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.010</td>
<td>0.073</td>
</tr>
</tbody>
</table>

Note: This table reports the cross-sectional asset pricing implications of the conditioning bias with regression and sorting methods. The conditioning bias in the estimated ivol is calculated using Eq. (21), in which the ivol is defined with respect to CAPM. In Panel A we report Fama and MacBeth (1973) regression (Eq. (20)) of stock excess returns and lagged conditioning bias (‘Bias’) in the estimated ivol, along with different factor loadings and lagged firm specifications. ‘Size’ is the log market capitalization of the firm at the beginning of the month. ‘Book-to-market’ is the book-to-market ratio available six months prior. ‘Momentum’ is defined as the stock’s cumulative return over the previous six months. ‘Illiquidity’ is defined as the normalized Amihud (2002) Ratio following Acharya and Pedersen (2005). ‘Lagged return’ is the lagged monthly stock excess return as in Huang et al. (2010). ‘Lagged max’ is stock’s maximum daily return in the last month as in Bali, Cakici, and Whitelaw (2011). ‘***’, ‘**’, and ‘*’ stand for statistical significance at 1%, 5%, and 10% level. The sample period spans from July 1964 to December 2015. In Panel B we show the single and double sorting results of the conditioning bias. In single sorting, we sort all stocks into 5 quintile portfolios every month based on the conditioning bias. In the double sorting, we first sort all stocks into 5 quintile portfolios every month then within each bias quintile we further sort stocks into 5 quintiles based on their contemporaneous estimated ivol. All portfolios are held for one month before reform.
### Table 6. Portfolios Sorted by the Estimated 3- and 6-month Ivol

<table>
<thead>
<tr>
<th></th>
<th>3 month</th>
<th></th>
<th>6 month</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ctmp</td>
<td>Cond</td>
<td>Ctmp</td>
<td>Cond</td>
</tr>
<tr>
<td>$R_{5-1}$</td>
<td></td>
<td></td>
<td>$R_{5-1}$</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{5-1}$</td>
<td></td>
<td></td>
<td>$\alpha_{5-1}$</td>
<td></td>
</tr>
<tr>
<td>$R_{Mt}$</td>
<td>0.8303***</td>
<td>-0.2775***</td>
<td>0.8562***</td>
<td>-0.1633*</td>
</tr>
<tr>
<td></td>
<td>(13.17)</td>
<td>(-3.04)</td>
<td>(12.90)</td>
<td>(-1.84)</td>
</tr>
<tr>
<td>$\beta_t R_{Mt}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_t R_{Mt}$</td>
<td>-1.4446***</td>
<td>(-15.19)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.7907***</td>
<td>-1.1936***</td>
<td>-0.2676</td>
<td>-0.4496*</td>
</tr>
<tr>
<td></td>
<td>(-2.48)</td>
<td>(-4.20)</td>
<td>(-0.84)</td>
<td>(-1.86)</td>
</tr>
</tbody>
</table>

*Note:* This table reports the return spread and abnormal return (in %) of the zero-investment long-short ivol sorted portfolios. In each month, all stocks are sorted into five value-weighted quintile portfolios based on the lagged ivol estimation. The zero-investment portfolio is formed by simultaneously short the quintile 1 (low ivol) and long the quintile 5 (high ivol) portfolio. $R_{5-1}$ and $\alpha_{5-1}$ represent the return spread and abnormal return on the long-short portfolio in the following month, respectively. The idiosyncratic volatility is defined with respect to CAPM and Fama-French 3-factor model. ‘Ctmp’ (‘Cond’) represents the ivol estimated using the contemporaneous realized beta (conditional beta). In the conditional model, the strategy abnormal return is estimated following Eq.(18). ‘***’, ‘**’, and ‘*’ stand for statistical significance at 1%, 5%, and 10% level. The sample period spans from July 1964 to December 2015.
Table 7. Sorting with Different State Variables

<table>
<thead>
<tr>
<th></th>
<th>CAPM Term spread</th>
<th>CAPM Dividend Yield</th>
<th>CAPM T-bill Rate</th>
<th>FF3 Term spread</th>
<th>FF3 Dividend Yield</th>
<th>FF3 T-bill Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_{\alpha} )</td>
<td>( \alpha )</td>
<td>( R_{\alpha} )</td>
<td>( \alpha )</td>
<td>( R_{\alpha} )</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>( R_{Mt} )</td>
<td>-0.1896**</td>
<td>-0.1669*</td>
<td>-0.1980**</td>
<td>-0.0648*</td>
<td>-0.0540</td>
<td>-0.0315</td>
</tr>
<tr>
<td></td>
<td>(-2.22)</td>
<td>(-2.15)</td>
<td>(-2.35)</td>
<td>(-1.74)</td>
<td>(-1.58)</td>
<td>(-0.88)</td>
</tr>
<tr>
<td>( \beta_t R_{Mt} )</td>
<td>-1.3437***</td>
<td>-1.2603***</td>
<td>-1.3201***</td>
<td>-1.1623***</td>
<td>-1.0975***</td>
<td>-1.0506***</td>
</tr>
<tr>
<td></td>
<td>(-14.12)</td>
<td>(-16.40)</td>
<td>(-14.49)</td>
<td>(-19.13)</td>
<td>(-21.29)</td>
<td>(-18.22)</td>
</tr>
<tr>
<td>( s_t SMB_t )</td>
<td>-0.1476</td>
<td>-0.0960</td>
<td>-0.1334</td>
<td>-0.1476</td>
<td>-0.0960</td>
<td>-0.1334</td>
</tr>
<tr>
<td></td>
<td>(-1.58)</td>
<td>(-0.97)</td>
<td>(-1.46)</td>
<td>(-1.58)</td>
<td>(-0.97)</td>
<td>(-1.46)</td>
</tr>
<tr>
<td>( HML_t )</td>
<td>0.0225</td>
<td>0.0517</td>
<td>0.0800*</td>
<td>0.0225</td>
<td>0.0517</td>
<td>0.0800*</td>
</tr>
<tr>
<td></td>
<td>(0.48)</td>
<td>(1.12)</td>
<td>(1.67)</td>
<td>(0.48)</td>
<td>(1.12)</td>
<td>(1.67)</td>
</tr>
<tr>
<td>( h_t HML_t )</td>
<td>-1.1592***</td>
<td>-1.0911***</td>
<td>-1.2292***</td>
<td>-1.1592***</td>
<td>-1.0911***</td>
<td>-1.2292***</td>
</tr>
<tr>
<td></td>
<td>(-19.74)</td>
<td>(-18.77)</td>
<td>(-22.06)</td>
<td>(-19.74)</td>
<td>(-18.77)</td>
<td>(-22.06)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.5643*</td>
<td>-0.5867***</td>
<td>-0.4721</td>
<td>-0.5705***</td>
<td>-0.5103*</td>
<td>-0.5552***</td>
</tr>
<tr>
<td></td>
<td>(-1.95)</td>
<td>(-2.65)</td>
<td>(-1.58)</td>
<td>(-2.62)</td>
<td>(-1.76)</td>
<td>(-2.53)</td>
</tr>
</tbody>
</table>

Note: This table reports the return spread and abnormal return (in %) of the zero-investment long-short ivol sorted portfolios when different state variables \( Z_{t-1} \) are used in the beta estimation (Eq. 14). In each month, all stocks are sorted into five value-weighted quintile portfolios based on the lagged ivol estimation. The zero-investment portfolio is formed by simultaneously short the quintile 1 (low ivol) and long the quintile 5 (high ivol) portfolio. \( R_{\alpha} \) and \( \alpha \) represent the return spread and abnormal return on the long-short portfolio in the following month, respectively. The idiosyncratic volatility is defined with respect to CAPM and Fama-French 3-factor model. The strategy abnormal return is estimated following Eq. 18. ‘***’, ‘**’, and ‘*’ stand for statistical significance at 1%, 5%, and 10% level. The sample period spans from July 1964 to December 2015.
<table>
<thead>
<tr>
<th>Hypo A. $H_0: \alpha^{cond} = \hat{\alpha}_{ctmp}$</th>
<th>Hypo B. $H_0: \alpha^{sim} = \hat{\alpha}_{ctmp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{\hat{\alpha}^{cond}}$</td>
<td>$-2.3988709^{***}$</td>
</tr>
<tr>
<td>1%</td>
<td>-2.2505225</td>
</tr>
<tr>
<td>2%</td>
<td>-2.0011197</td>
</tr>
<tr>
<td>3%</td>
<td>-1.8447664</td>
</tr>
<tr>
<td>4%</td>
<td>-1.7324272</td>
</tr>
<tr>
<td>5%</td>
<td>-1.6219774</td>
</tr>
<tr>
<td>10%</td>
<td>-1.2300824</td>
</tr>
<tr>
<td>20%</td>
<td>-0.81441104</td>
</tr>
<tr>
<td>30%</td>
<td>-0.51503474</td>
</tr>
<tr>
<td>40%</td>
<td>-0.23787734</td>
</tr>
<tr>
<td>50%</td>
<td>0.005964</td>
</tr>
<tr>
<td>60%</td>
<td>0.26418182</td>
</tr>
<tr>
<td>70%</td>
<td>0.53074953</td>
</tr>
<tr>
<td>80%</td>
<td>0.842724</td>
</tr>
<tr>
<td>90%</td>
<td>1.254925</td>
</tr>
<tr>
<td>95%</td>
<td>1.6179385</td>
</tr>
<tr>
<td>96%</td>
<td>1.7261329</td>
</tr>
<tr>
<td>97%</td>
<td>1.8484701</td>
</tr>
<tr>
<td>98%</td>
<td>2.0177319</td>
</tr>
<tr>
<td>99%</td>
<td>2.2900499</td>
</tr>
</tbody>
</table>

Note: This table reports values of $t_\alpha$ at selected percentiles of the distribution of $t_\alpha$ estimates for the zero-investment long-short ivol portfolios. The percentiles are obtained from the empirical distribution of $t_\alpha$ constructed using 10,000 simulation bootstrap samples. The bootstrap algorithm generally follows Fama and French (2010) by repeated sampling the whole cross-section of ivol quintiles with replacement. For comparison, we also report the $t_\alpha$ of the conditional model and the simulated factor model. '***', '**', and '*' stand for statistical significance at 1%, 5%, and 10% level.