Dealer Trading at the Fix

Abstract

This paper examines a model of dealer trading at the London 4 pm “fix,” a major foreign exchange market benchmark. Dealer misconduct, long suspected because of the extreme returns and quick retracements common at 4 pm, was confirmed in 2015 with guilty pleas from major banks. Our model clarifies the dealers’ incentives and strategies, explains why price dynamics appear unchanged despite recent reforms, and provides insights relevant to benchmark design. Collusion is profitable because it shuts down a form of free riding that involves shifting trades across time. Identifying collusion based on fix-price volatility and retracements would be difficult because these dynamics could also reflect front-running or banging the close, neither of which is illegal in currency markets. Collusion has the relatively subtle implication that the pre-fix price path becomes more convex. A bootstrap test for convexity applied to seven major exchange rates vs. the US dollar does not reject the hypothesis that collusion took hold around 2008.

Key words: London Fix, foreign exchange, collusion, front-running, information sharing, banging the close, convexity

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Dealer Trading at the Fix

Extreme returns and quick retracements have long been common at a key foreign exchange market benchmark, the London 4 pm Fix (see Figure 1). Widespread suspicions that these price dynamics reflected dealer misconduct gained support in 2013 Bloomberg article alleging that major dealers were colluding in private chatrooms. Legal and regulatory investigations ensued, and by 2015 the major dealing banks had pled guilty to market manipulation and paid fines and settlements in excess of $11 billion.¹ In recent years related misconduct has come to light in other markets including non-deliverable forwards in Asia; gold, other precious metals, interest rate derivatives, and Treasury securities.² Evidently, it is important to identify superior fix calculation methodologies, but unfortunately the literature can provide little guidance. Price dynamics around fixes are “not well accounted for in existing microstructure models” (Melvin and Prins, 2011, p. 1), and represent “a challenge to [existing] theories of trading behavior” (Evans, 2015, p. 4).

This paper develops a model of dealer trading at a fix that clarifies the economics of collusion, information sharing, excess trading, and banging-the-close. Banging the close involves pushing large trades through the market all at once with the intent of causing a big price move (Comerton-Forde and Putniņš, 2011). Excess trading involves a dealer taking proprietary positions in parallel with his customers. Like front running, which is illegal in most markets (though not in foreign exchange), excess trading moves the price adversely for customers. The results of our analysis will be of interest to academics and to at least three practitioner communities: authorities investigating reported misconduct; bank compliance officers; and regulators evaluating fix calculation methodologies.

The profits associated with a dealer’s fix trading arise from to unique feature of fill-at-fix orders: there is a substantial time delay between the time at which the dealer learns the quantity to trade and the time at which the customer’s price is set (Comerton-Forde and Putniņš, 2011). In consequence,  

¹ For the guilty pleas, see Department of Justice (2015). For fines see CFTC (2015a, 2015b); U.S. Department of Justice (2015); FCA (2014a); Bray et al. (2014). For class-action settlements see Raymond (2015).
² See Armstrong (2013) For Asian NDFs; Harvey (2014) and FCA (2014b) for gold; McLaughlin and Schoenberg (2015) for other precious metals; Leising and van Voris (2014) for interest rate derivatives; and Stempel (2015) for Treasury securities.
dealers profit as the inventory accumulated early in the pre-fix interval appreciates when continued trading in the same direction creates a price trend at the end of the pre-fix interval.

If fix orders were uncorrelated, dealers would ignore one another and maximize profits by trading equal amounts in each period before the fix. In reality, fix orders will be positively correlated because of their shared origin in economy-wide forces. In consequence, each fix dealer will effectively free-ride on the other fix dealers by shifting doing most of his trading early in the pre-fix interval, counting on the other dealers to sustain the trend through the moment at which the fix is calculated. Naturally enough, free-riding reduces average dealer profits. If dealers share information about their customer orders free-riding intensifies, further reducing dealer profits. Collusion is more profitable than either competition or information sharing because it shuts down free riding.

According to the model, high exchange rate volatility should be expected at the fix for a simple and neutral reason: many orders must be executed within a short time frame. Retracements should likewise be expected, but not for a neutral reason. Excess trading is profitable in all three competitive settings and the retracements arise when the dealers’ proprietary positions are unwound after the fix. To explain banging-the-close and the strictly convex pre-fix price path apparent since 2007 (Figure 1) we enhance the model’s realism in two dimensions. First, a trade’s price impact rises with order flow, consistent with the limit-order structure of most interdealer trading. Second, dealers are risk averse.

The model provides a cogent explanation for the stability of fix-price dynamics despite recent reforms (Saks-McLeod, 2015). The underlying incentives of fix trading, according to the model, were essentially unchanged by the extension of the fix-price calculation window from one to five minutes. There were more significant implications of the other major reform: requiring fix dealers to process fix orders via automated trading algorithms distributing customer fix trades over the entire pre-fix interval. This eliminated dealers as strategic agents but it may have enabled other participants to adopt trading strategies that sustain excess volatility and retracements. In particular, non-dealers could use the price trend immediately following 3:45 as a signal of the dealers’ net fix orders. If so, they would optimally adopt the model’s strategy for a dealer under information sharing with zero net fix orders from his own customers. This strategy, which is to open a speculative position immediately after 3:45 and liquidate it partly before and partly after the fix, would evidently generate excess volatility and retracements. It is
notable that SmartFix, a new software package marketed to active traders, is designed to facilitate this strategy among non-dealers (Albinus, 2016).

The model also reveals that volatility and retracements at the fix may represent a sustainable equilibrium in an efficient market. Fix dealers, their customers, and other dealers are all connected in a web of strategic complementarities in which each agent’s behavior enhances the incentives for the others’ behavior. To illustrate, as more customers place fix orders fix volatility rises, which increases the incentive for dealers and others to take speculative positions at the fix, which increases fix volatility and the incentive for customers to place fix orders.

The model suggests that it would be difficult to make a convincing case for collusion based exclusively on price data. Heightened volatility before and after the fix, for example, is consistent with dealer behaviors that would arise in the absence of collusion, such as excess trading. Changes in volatility could reflect changes in many features of the trading environment rather than changes in dealer strategies. The model does highlight one dimension of price dynamics that is influenced by collusion and is less likely to be influenced by changes in the trading environment: the convexity of the pre-fix price path. We develop a measure of convexity and calculate it using high-frequency dealer quotes on month-end dates for the period 1996 through 2013 for seven major currencies vis-à-vis the US dollar: EUR, JPY, GBP, CHF, CAD, NZD, and DKK. Results of this analysis support the hypothesis that convexity increased around 2008 when fix dealers reportedly began their collusive activities.

Our model could help explain dramatic price dynamics and evidence of misconduct that have emerged around benchmarks in other OTC markets. In precious metals, for example, "[t]he behaviour patterns in precious metals were somewhat similar to the behaviour patterns in foreign exchange" (Mark Branson, director of the Swiss financial authority, quoted in Harvey, 2014; Slater and Jones, 2014).

Research on fix trading per se has just begun and is therefore limited to working papers, so far as we are aware. Saakvitne (2016) examines a model of benchmark trading in which dealers know nothing about other dealers so information sharing and collusion do not arise. Evans (2016) concludes that non-collusive fix dealers will wait to manage fix orders until after the fix price has been calculated. This implies that dealers forego the profits associated with the price impact of order flow, which raises the possibility that the model does not capture all the relevant incentives. Onur and Reiffen (2016) develop
a model of benchmark trading specific to futures markets that focuses on settlement rules and the distribution of trading between floor and electronic markets.

Fix prices and closing prices have much in common, and concerns about closing prices are nothing new.³ A tendency for U.S. equity prices to rise at the end of the day was documented by the mid-1980s (Wood et al., 1985; Harris, 1986) and a number of explanations have been suggested that involve misconduct. Carhart et al. (2002) suggest that equity fund managers may have intended to inflate quarter-end mutual fund values; Kumar and Seppi (1992) suggest equity prices may be manipulated to influence derivative prices benchmarked to the close; and Hillion and Suominen (2004) suggest that equity brokers may have intended to inflate their apparent skill. These ideas seem unlikely to be important for price dynamics at the London fix, however, given that the chatroom conversations included no fund managers, the dealers did not focus on derivatives trading, and forex trades are handled on a principal basis rather than an agency basis.

Cushing and Madhavan (2000), who examine price dynamics at the close for Russell 1000 stocks in the late 1990s, suggest that high volatility and post-close retracements were unrelated to misconduct and instead reflect the common tendency among market makers to temporarily lower (raise) prices when they hold excess (insufficient) inventory. This phenomenon is well documented for equity markets (Hendershott and Menkveld, 2014), but existing studies find little or no evidence of such price shading among forex dealers (e.g., Osler et al., 2011). This absence may reflect a simple cost-benefit analysis: price shading communicates information about a dealer’s position that other dealers can exploit, which is costly, and the benefits in forex are limited because the interdealer market is fast and inexpensive. Anecdotal reports indicate that price shading has become more common in forex as the market has become more concentrated. We view price pressures as complementary to the misconduct that emerges from our model, the relative contribution of which is an empirical question.

The rest of this paper has seven sections. Section I outlines the baseline model. Section II analyzes that model when dealers trade independently. Section III examines that model when dealers share information and when they collude. Section IV examines the model when price impact is sensitive to

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³ Cordi et al. (2016) provide a brief survey.
order flow and when dealers are risk averse. Section V tests the model’s prediction that convexity under collusion. Section VI discusses whether volatility and retracements should be competed away and considers the model’s policy implications. Section VII concludes.

I. The Model

This section outlines the model and highlights how key assumptions reflect the actual structure of trading at the WM/Reuters London 4 pm fix while incorporating critical insights from microstructure.

Agents: Customer fix orders are managed by $N+1 < \infty$ identical OTC dealers who trade in the interdealer market with each other and with a fringe of atomistic dealers. We assume a finite number of competitors because the market share of the top four foreign exchange (forex) dealing banks exceeds 50% (Euromoney, 2013). Fix trading is yet more concentrated because small and regional banks generally pass their fix orders on to the dominant dealers.

Customer Fix Orders: Before trading begins each dealer receives a random set of customer fix orders. We focus on representative dealer $d$, whose net fix order, $F_d$, includes a component shared by all other dealers, $\Phi$, and a dealer-specific deviation, $\eta_d$: $F_d = \Phi + \eta_d$. The terms $\Phi$ and $\eta_d$ are i.i.d. and mutually uncorrelated with mean zero and variances $\sigma^2_\Phi > 0$ and $\sigma^2_{\eta_d} > 0$, respectively. The correlation in fix orders is denoted $0 < \rho = \frac{\sigma^2_{\Phi}}{\sigma^2_\Phi + \sigma^2_{\eta_d}} < 1$. Without loss of generality we assume that dealer $d$’s customers are buyers, $F_d > 0$, so dealer $d$ himself is a buyer in the interdealer market.

The model takes customer fix orders as exogenous but their origin in reality is well understood. Cochrane (2015) points out that international equity funds worth $9$ trillion are benchmarked to the MSCI indexes and another $2$ trillion are benchmarked to the Citi World Government Bond Index, and all of these indexes are marked to market with the WM/Reuters Closing Spot Rates. These institutions have a high incentive to avoid tracking risk, which they can achieve by trading exactly at the fix price. Fix trading is concentrated at month end, when firms set aside employee retirement income and

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4 Note: This paper provides a positive rather than a normative analysis of dealer behavior around OTC fixes. The paper does not advocate prohibited behaviors.

5 We use male pronouns throughout because almost all forex interdealer traders are male.
international investing institutions adjust their currency hedge positions. Melvin and Prins (2015) provide evidence that month-end portfolio hedging influences trading and prices at the London 4 pm fix.

**Time:** The pre-fix trading interval has two trading periods, periods 1 and 2, during which representative dealer \(d\) trades quantities \(D_{1d}\) and \(D_{2d}\) in the interdealer market at prices \(P_1\) and \(P_2\), respectively. Orders to trade at the London 4 pm fix must be received by 3:45 pm, which suggests that periods 1 and 2 could, in reality, be roughly seven minutes long. It is also possible, however, that dealers wait to trade until just before the fix price is calculated, in which case periods 1 and 2 could be a minute or less. After the fix is calculated dealers trade with each other one more time, at interdealer price \(P_3\).

**Inventory Management:** The model follows the literature in assuming that each dealer’s inventory is at its target level when fix orders arrive and is restored to that level by the end of period 3. In general, foreign exchange dealers target a zero inventory position, but we leave desired inventory unspecified. This inventory constraint implies:

\[
X_d = D_{1d} + D_{2d} - F_d .
\]

(1)

**The Fix:** The fix price is set equal to the period-2 interdealer price, \(P_F = P_2\), and dealer \(d\) trades \(F_d\) with his customers at that price. Due to the generality of this structure, our analysis should be relevant to fixing prices calculated with a variety of methodologies. Indeed, foreign exchange dealers appear to have behaved similarly at both the London 4 pm fix and the ECB fix, which occurs at 2:15 European Central time, though the methodologies for determining these fixing prices are distinct. Until December, 2014, the WM/Reuters 4 pm fix used (roughly) the median of traded interdealer prices sampled once per second over the 60-second interval centered on the hour. The ECB fix is set according to a central bank “concertation procedure,” the details of which are not published.

**Dealer Objectives:** In the baseline model dealers are risk-neutral profit maximizers; Section IV investigates the implications of dealer risk aversion. The analysis does not incorporate potential costs to dealers or their employers of violating laws, regulations, or bank policies.

Dealer \(d\)’s revenues comprise \(P_F F_d\) from selling to customers at the fix plus \(P_3 X_{1d}\) from liquidating any proprietary position after the fix. His costs come from purchasing inventory in periods 1

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6 International asset managers typically hedge substantial portion of their currency exposure – most commonly 50% – via forward contracts. Forward contracts are structured as a spot trade combined with a swap so they directly affect spot prices.
and 2: $P_1D_{1d} + P_2D_{2d}$. Interest expense is irrelevant because fix trading occurs intraday. We abstract from the cost of bank equity capital following the literature. Dealer $d$'s profits, $\pi_d$, are:

$$
\pi_d = P_1F_d + P_3X_{1d} - P_1D_{1d} + P_2D_{2d}.
$$

(2)

Our analysis relies heavily on the following decomposition of profits:

$$
\pi_d = D_{1d}(P_2 - P_1) + X_d(P_3 - P_2).
$$

(3)

The first term on the right represents profits or losses on dealer $d$'s period-1 inventory purchase as the price moves over period 2. The second term on the right captures gains or losses incurred upon liquidating any proprietary position. In equilibrium the first term is always positive in expectation and the second term is always negative in expectation. In other words, in equilibrium the profitability of fix trading derives entirely from the interaction between early inventory accumulation and later returns.

**Price Generating Process:** When executing fix trades, dealers trade against each other and against the atomistic fringe. Following closely-related research (e.g., Bertsimas and Lo, 1998; Cushing and Madhavan, 2000) we initially assume that fix trades have a linear contemporaneous price impact proportional to $\theta > 0$.\(^7\) (Section IV examines the model when price impact responds positively to instantaneous trading volume.) Returns are also driven by the arrival of public information and other factors orthogonal to the fix. These shocks, denoted $\varepsilon_t$, are i.i.d. with zero mean and variance $\sigma^2 > 0$:

$$
P_t - P_{t-1} = \theta\left(D_{td} + \sum_N D_{md} + \varepsilon_t\right), \quad t = \{1,2,3\}.
$$

(4)

Equation (4) shows that the price follows a random walk, $P_t - P_{t-1} = \theta\varepsilon_t$, outside of the fix trading interval, with one-period return variance $\theta\sigma^2$. Figure 1 shows unambiguously that fix trades have a permanent price impact, consistent with Equation (4) and with empirical research showing that order flow has a permanent price impact in all major asset classes.\(^8\)

\(^7\) The model abstracts from questions of order choice – meaning the choice between making and taking liquidity – because influence of order flow on financial prices arises whether informed agents make or take liquidity. If a dealer takes liquidity the price necessarily moves by the bid-ask spread. If he makes liquidity by placing a limit order the price will move in the same direction in expectation because liquidity on the bid (ask) side reduces the likelihood of a price decline (rise).

Equation (4) is well-grounded theoretically as well as empirically. A permanent price impact of order flow is implied by models with finite elasticity of demand, in which price changes elicit shifts in order flow. In forex this endogenous response of order flow to price can come from risk-averse speculators (Evans and Lyons, 2002) or from firms engaged in international trade (Osler, 2006). A permanent price impact of order flow is also implied by models of asymmetric information (Kyle, 1985; Glosten and Milgrom, 1985). Though “inside information” as defined in equity markets is rare in forex, asymmetric information is nonetheless an integral feature of the market. Private information about exchange rates arises, for example, when hedge funds attempt to anticipate macro news including shifts in central bank policies (Rime et al., 2010). Within the interdealer market larger banks are generally better informed than smaller banks (Bjønnes et al., 2009).

II. Front-running, Free-riding, Excess Volatility, and Retracements

We first examine dealer strategies and fix-price dynamics under independent trading. Representative dealer $d$ must analyze later trading decisions before making his initial trading decision. His final trading decision occurs in period 2, not period 3, because in period 3 he must restore inventory to its initial level (Equation (1)).

A. The Period-2 Decision

Before delving into details we streamline the analysis with a change of variables. Let $\alpha_i$ represent the share of dealer $d$’s net fix order that he trades in period 1, $\alpha_i = \frac{D_{1d}}{F_d}$, and $\hat{\alpha}_i$ represent the average of the corresponding fraction for all other dealers: $\hat{\alpha}_i = \frac{\sum_N D_{1n}}{\sum_N F_n}$. In period 2 dealer $d$ takes $\alpha_i$ and $\hat{\alpha}_i$ as given. He automatically purchases $(1-\alpha_i)F_d$, which ensures he has sufficient inventory to service his customer fix orders. He also chooses $X_d$, his proprietary trading, to maximize expected profits. Combining Equations (1), (3), and (4) gives this optimization problem:

$$\text{Max}_{X_d} E_{2d} \{ \pi_d \} = \alpha_i F_d \theta \left( \left[ (1-\alpha_i)F_d + (1-\hat{\alpha}_i)E_{2d} \left\{ \sum_N F_n \right\} \right] \right) + (\alpha_i F_d - X_d) \theta (X_d + E_{2d} \left\{ \sum_N X_n \right\} ) . \quad (5)$$
The first-order condition below shows that \( X_d \) depends on the (conditional) expected proprietary trading of other dealers, \( E_{2d} \left( \sum_{N} X_n \right) \):

\[
X_d = \frac{1}{2} \left[ \alpha_1 F_d - E_{2d} \left( \sum_{N} X_n \right) \right].
\]  

As shown in the Appendix, which derives the rational expectations solution for \( X_d \), dealer \( d \)'s proprietary trading is the fraction \( 0 < q < 1 \) of his period-1 trade:

\[
X_d = \frac{\alpha_1}{2 + \rho N} F_d = q \alpha_1 F_d, \quad 0 < q \equiv \frac{1}{2 + \rho N} < \frac{1}{2}.
\]  

B. The Period-1 Decision

Having identified the functional form for his period-2 decision, dealer \( d \) chooses his period-1 trade by solving the following optimization problem:

\[
\text{Max} \quad E_{1d} \{ x_d \} = \alpha_1 F_d \theta \left[ F_d \left( 1 - \alpha_1 \right) + (1 - \hat{\alpha}_1) \rho NF_d \right] + \alpha_1 F_d \left[ (1 - q) \theta \left[ \hat{q} \alpha_1 F_d + E_{1d} \left( \sum_{N} X_n \right) \right] \right].
\]  

The profit-maximizing value of \( \alpha_1 \) depends on the other dealers' expected behavior as captured by \( \hat{\alpha}_1 \) and \( \hat{q} \equiv E_{2d} \left( \sum_{N} X_n / \sum_{N} F_n \right) \):

\[
\alpha_1 = \frac{(1 + \rho N) - \hat{\alpha}_1 \rho N[1 - \hat{q}(1 - \hat{q})]}{2[1 - q(1 - q)]}.
\]

Dealer symmetry implies that \( \alpha_1 = \hat{\alpha}_1 \) and \( q = \hat{q} \) in market equilibrium, closing the model. Lemma 1 describes equilibrium trading shares using \( \text{Cov}(F_d, \sum_{N+1} F_n) = \sigma_F^2 \equiv (N + 1)\sigma_a^2 + \sigma_q^2 \):

**Lemma 1**: In competitive equilibrium \( (N \geq 1) \) with positively-but-imperfectly correlated fix orders \( (1 > \rho > 0) \),

a. A dealer's trades are proportional to his own fix orders in every period:

\[
D_{1d} = \frac{(2 + \rho N)(1 + \rho N)}{(2 + \rho N)^2 - (1 + \rho N)} F_d \equiv \alpha_1 F_d, \quad \frac{2}{3} < \alpha_1 < 1.
\]  

\[
D_{2d} = \frac{(2 + \rho N)}{(2 + \rho N)^2 - (1 + \rho N)} F_d \equiv \alpha_2 F_d, \quad 0 < \alpha_2 < \frac{2}{3}.
\]
\[ X_d = -D_{3d} = \frac{\alpha_1}{2 + \rho N} = \frac{(1 + \rho N)}{(2 + \rho N)^2 - (1 + \rho N)} \quad 0 < x < \frac{1}{3}. \]

b. Each fix dealer's trades decline over time in absolute magnitude: \( \alpha_1 > \alpha_2 > x \).

c. Expected per-dealer profits are positive: \( E_0 \{ \bar{\pi}^{\text{Indep}} \} = \theta \sigma_f^2 \) \( x > 0 \).

d. As competition intensifies, trading shifts from period 2 towards period 1, proprietary trading declines in absolute magnitude: \( \bar{\partial} \alpha_1 / \partial N > 0 \), \( \bar{\partial} \alpha_2 / \partial N < 0 \), \( \bar{\partial} x / \partial N < 0 \) (see Appendix).

Profit-maximizing fix trades have three critical properties: they are distributed across both pre-fix periods ("distributed trading"), they are concentrated in period 1 ("free riding"), and they exceed customer fix orders ("proprietary" or "excess" trading). Each property is best understood by comparing the equilibrium when fix orders are correlated and dealers act strategically (\( 1 > \rho > 0 \)) to the equilibrium when orders are uncorrelated (\( \rho = 0 \)) and dealers do not act strategically.

**Distributed Trading:** With uncorrelated orders (\( \rho = 0 \)) dealer \( d \) would conclude that he must trade in both periods based on the profit decomposition, Equation (3). Without a period-1 purchase, his period-2 purchase would bring a price rise but he would have no inventory to gain value. Without a period-2 purchase, he would have an inventory position but he could expect no period-2 appreciation. The profit-maximizing strategy with \( \rho = 0 \) is to trade equal amounts in both periods so \( \alpha_2^{N=0} / \alpha_1^{N=0} = 1 \).

**Free-riding:** In reality, fix orders should be correlated across dealers because they are driven by economy-wide forces. With positive-but-imperfect correlation of fix orders (\( 1 > \rho > 0 \)), dealers still trade in both periods but they shift some trading from period 2 to period 1. Every dealer expects the other dealers to trade in his same direction in period 2 given the positive correlation among fix orders:

\[ E_{1d} \{ \sum_N F_n \} = \rho NF_d. \]

Each dealer therefore expects an appreciation in his period-1 inventory even if he himself does not trade in period 2. Of course, if the other dealers apply the same logic they will all skip trading in period 2, the expected period-2 return will be zero, and fix profits will be zero for all dealers. In a rational expectations equilibrium each dealer shifts some but not all of his period-2 trading to period 1 relative to the equilibrium with \( \rho = 0 \). This reduces the ratio of period-2 trading to period-1 trading, \( 0 < \alpha_2 / \alpha_1 = 1/(1 + \rho N) < 1 \).
This trade shifting represents the dealers’ attempt to free-ride on each other. Free-riding, which can be measured by the extent to which $\alpha_2/\alpha_1$ falls short of 1, rises with $N$ for two reasons. First, the foregone profits are distributed equally across all dealers; with higher $N$ each dealer expects to bear less of those costs. Second, the benefits of free riding, in terms of the anticipated period-2 price trend if the dealer abstains from trading, are rising in $N$.

**Excess Trading:** Profit-maximizing dealers accumulate more inventory than required to fulfil their customer fix orders: $x_d \equiv X_d / F_d > 0$. Excess trading is profitable in expectation due to unique incentives at the fix. For normal OTC trades, prices and quantities are agreed simultaneously and the dealer’s subsequent inventory management trades cannot influence the customer’s price. The dealers’ incentive on normal trades is to minimize price impact (Bertsimas and Lo, 1998); proprietary trading would raise the dealer’s costs with no offsetting benefit. At the fix, by contrast, the price is set after the dealer and customer agree on the quantity. The dealer can enhance his revenues from fix trades, $F_d P_F$, by trading for his own account and increasing the price move prior to the fix. The profit-maximizing amount of excess trading is not infinite, however, because those trades bring an offsetting loss when they are liquidated in period 3.

If fix orders are uncorrelated ($\rho = 0$), excess trading is exactly $1/3$ of customer fix orders:

$$ (\alpha_1^{N=0} + \alpha_2^{N=0} - 1) F_d \equiv x^{N=0} F_d = F_d / 3. $$  \hspace{1cm} (11)

Because fix orders are correlated, a dealer’s own order is informative about the orders of other dealers, so dealers will rationally expect other dealers to trade in excess in their same direction. This increases the expected period-3 retracement and reduces the anticipated profitability of excess trading, so such trading shrinks as a share of customer fix orders: $x^{\rho>0} < 1/3$.

Proposition 1 summarizes key features of equilibrium when dealers trade independently:

**Proposition 1:** Under competitive fix trading ($N \geq 1$) and with positively but imperfectly correlated fix orders ($1 > \rho > 0$), equilibrium dealer trading exhibits three key features:

1. **Distributed trading:** Pre-fix inventory accumulation occurs in both periods 1 and 2.
2. **Free riding:** Dealers shift pre-fix inventory accumulation from period 2 to period 1 relative to the equilibrium with uncorrelated orders ($\rho=0$).
c. **Excess trading: Every fix dealer accumulates more inventory before the fix than required to service his customer fix orders.**

The prediction that dealers take proprietary positions at the fix is consistent with dealer comments. CFTC transcripts of chatroom comments report one dealer telling another “haha I [sic] sold a lot up there and over sold by 100” (CFTC, 2015c). The original Bloomberg story indicated that dealers viewed excess trading as a professional responsibility: “Three [forex dealers] said that when they received a large [fix] order they would adjust their own positions knowing that their client’s trade could move the market. If they didn’t do so, they said, they risked losing money for their banks” (Vaughan et al., 2013).

Excess at the London fix was observed to be a common feature of dealer trading strategies in the Bank of England’s investigation (Grabiner, 2014):

> Traders increased the volume traded by them at the fix in the desired direction in excess of the volume necessary to manage the risk associated with the firm’s fix position. Traders have referred to this process as “overbuying” or “overselling” (p. 11).

Excess trading was also reported at the silver fix by the Swiss financial authority (Harvey, 2014).

Front-running is generally illegal because the dealer’s own trade moves the price adversely for his customers. Front-running is not illegal in currency markets, however, in part because foreign exchange dealing has no worldwide equivalent of the SEC or FSA to set and enforce regulations. In addition, currencies are neither securities nor financial instruments so they are not covered by major financial regulations (e.g., MIFID).

Practitioners acknowledge the absence of regulation in forex: “As for front-running, ... it’s simply impossible to regulate. Consequently there is a general perception that front-running is not illegal, and any abuse in this regard can only be discouraged on reputational grounds” (Kaminska, 2013). Regulators have naturally attempted to exploit the leverage associated with reputation. In 2011 the Bank of England coordinated a statement signed by all major foreign exchange banks that says, in part:

> The handling of customer orders requires standards that strive for best execution for the customer in accordance with such orders subject to market conditions. In particular, caution should be taken so that customers’ interests are not exploited when financial intermediaries trade for their own accounts. ... Manipulative practices by banks with each other or with clients constitute unacceptable trading behaviour (Bank of England, 2011).
The Bank for International Settlements is currently coordinating a worldwide effort to design an FX Global Code, a set of very-specific guidelines intended to “promote the integrity and effective functioning of the wholesale foreign exchange market” (Bank for International Settlements, 2016).

C. Fix Price Dynamics When Dealers Trade Independently

This model predicts high pre-fix volatility, consistent with Evans’ (2015) finding that “across all time periods and currency pairs changes in rates before and after the Fix are regularly of a size rarely seen in normal trading activity” (p. 44). The model also predicts partial post-fix trend reversals, consistent with Evans’ finding that “pre- and post-Fix rate changes also display a strong degree of negative autocorrelation that is not found elsewhere during normal forex trading” (p. 44).

Volatility Before the Fix, $\Sigma$, will be measured as the variance of returns from $P_0$ to $P_F = P_2$:

$$\Sigma^{\text{Indep}} = E\left( (P_F - P_0)^2 \right) = \theta^2 E\left( \sum_{n=1}^{N+1} F_n (1 + x) + \varepsilon_1 + \varepsilon_2 \right)^2$$

$$= 2\theta^2 \sigma_x^2 + \theta^2 (N+1) \sigma_F^2 + \theta^2 (N+1) \sigma_F^2 x(2+x).$$

Two-period volatility at normal times would be $2\theta^2 \sigma_x^2$, the first term on the right in Equation (12). Volatility at the fix naturally exceeds volatility during normal times due to the high concentration of customer orders executed within a short time frame. This is captured by $\theta^2 (N+1) \sigma_F^2$, the second term on the right. By implication high fix volatility need not reflect dealer misconduct. Nonetheless, the third term on the right shows that fix volatility can be intensified by excess trading. We refer to the portion of volatility associated with excess trading as “excess volatility,” in the sense that it would not arise in the absence of strategic trading.

Post-fix Retracements: Return autocorrelation around the fix, denoted $\Lambda$, will be measured as the coefficient from a regression of post-fix returns, $P_3 - P_F$, on pre-fix returns, $P_F - P_0$:

$$\Lambda^{\text{Indep}} = \frac{-\theta^2 M \sigma_F^2 (1 + x)x}{\Sigma} = \frac{-(N+1) \sigma_F^2 (1 + x)x}{2\sigma_x^2 + (N+1) \sigma_F^2 + (N+1) \sigma_F^2 x(2+x)} < 0. \quad (13)$$

Equation (13) shows that post-fix retracements in the model are due entirely to excess trading: $\Lambda^{\text{Indep}} = 0$ if $x = 0$. 
Cushing and Madhavan (2000) suggest that retracements at the NASDAQ close had a different source in the late 1990s: an inventory effect known as “price pressures.” Price pressures emerge when dealers with excess (insufficient) inventory lower (raise) the price, hoping to attract trades that rectify the imbalance (Hendershott and Menkveld, 2014). Price pressures are not explicitly incorporated in this model in the interests of transparency, but the excess trading and price pressure hypotheses are not mutually exclusive and there’s every reason to assume both are relevant: The comments of dealers and regulators indicate that excess trading was common at the London fix; price pressures have been documented for many markets and, as noted by Hendershott and Menkveld (2014), they should apply in virtually every financial market: “the fundamental economic forces that generate price pressure and intermediaries’ inventory risk exist in all markets on which investor trading needs are not perfectly synchronized.”

Convexity: The model has implications for the acceleration or deceleration of the pre-fix price path. We refer to an accelerating path as convex and measure convexity, denoted $\Pi$, as the ratio of the expected period-2 return to the expected period-1 return for a given set of fix orders:

$$0 \leq \Pi^{\text{Indep}} \equiv \frac{E(P_2 - P_1 | \sum_{n=1}^{N+1} F_n)}{E(P_2 - P_1 | \sum_{n=1}^{N+1} F_n)} = \frac{\theta \alpha_2 \sum_{n=1}^{N+1} F_n}{\theta \alpha_1 \sum_{n=1}^{N+1} F_n} = \frac{\alpha_2}{\alpha_1} = \frac{1}{1 + \rho N} < 1. \quad (14)$$

The pre-fix price path is strictly convex (concave) if $\Pi>1$ ($\Pi<1$), so Equation (14) implies that the pre-fix path under independent trading is necessarily concave in the baseline model.

Proposition 2 summarizes fix price dynamics when fix dealers trade independently:

**Proposition 2:** Under competitive fix trading ($N \geq 1$) with positively-but-imperfectly correlated orders ($1 > \rho > 0$),

a. Return volatility before the fix will be higher than normal. This is due to the customer fix orders per se and to the dealers’ proprietary trading

b. The dealers’ proprietary trading will cause partial retracements of the pre-fix trend

c. The pre-fix price path is concave due to free riding.
III. Information Sharing and Collusion

This section examines market equilibrium when dealers share confidential information about customer orders or collude outright.

A. Information Sharing

Sharing information about customer orders with another dealer is considered unethical because it puts the customer at risk of manipulation. Forex dealers know this because bank compliance officers regularly tell them so\(^9\). Nonetheless, conversation transcripts available to the public show that forex dealers were accustomed to sharing information about customer fix orders in private chat rooms (FCA, 2014a-g). To model such behavior we assume that each fix dealer gives accurate information to the other fix dealers about his customer net fix order once all orders arrive (i.e., shortly after 3:45 pm for the London fix). Each dealer still undertakes a two-stage analysis to determine his own trades: He first identifies how his period-2 trades will depend on his period-1 trade and then chooses his period-1 trade.

In equilibrium under information sharing dealer \(d\)’s trading is influenced as much by the average customer order, \(\bar{F} = (F_d + \sum_{n} F_n) / (N + 1)\), as by his own customer order, \(F_d\):

Lemma 2: If dealers share information about customer fix orders, dealer \(d\) trades exclusively for his own account in periods 1 and 3 and accumulates inventory for customers only in period 2. His trading, as a function of his own and the average fix order, will be:

\[D_{1d} = \left( \frac{(1+N)(2+N)}{(2+N)^2 - (1+N)} \right) \bar{F} = \delta_1 \bar{F}, \quad \frac{2}{3} < \delta_1 < 1, \quad (15a)\]

\[D_{2d} = F_d - D_{1d} + X_d = \left( \frac{(1+N)^2}{(2+N)^2 - (1+N)} \right) \bar{F} = F_d - \delta_2 \bar{F}, \quad 0 < \delta_2 < \frac{2}{3}, \quad (15b)\]

\[X_d = -D_{3d} = \left( \frac{1+N}{(2+N)^2 - (1+N)} \right) \bar{F} = \chi \bar{F}, \quad 0 < \chi < \frac{1}{3}. \quad (15c)\]

---

\(^9\) The Bank of England’s Non-Investment Products Code (2011), signed by all major dealing banks, forcefully discourages information sharing:

Confidentiality is essential for the preservation of a reputable and efficient market place. ... Principals or brokers should not, without explicit permission, disclose or discuss, or apply pressure on others to disclose or discuss, any information relating to specific deals which have been transacted, or are in the process of being arranged ... All relevant personnel should be made aware of, and observe, this fundamental principle.
Equations (15a) through (15c) show that fix dealers who share information, like dealers who trade independently, trade in both pre-fix periods, free ride on each other, and trade in excess of their customer orders. Indeed, the excess trading can now be exactly identified as front-running because the dealer only trades for his customers in period 2. His period-1 and period-3 trades are determined entirely by the average fix order, which he uses as a signal of the direction of pre-fix returns.

In fact, free riding intensifies under information sharing: every dealer takes exactly the same proprietary position in period 1 and liquidates it at the same pace over periods 2 and 3. A dealer with no fix orders of his own, or with net fix orders in the opposite direction to the majority, nonetheless takes the same inventory position in period 1 as the other dealers. By contrast, recall that under independent trading every dealer trades a fraction of his own fix order in every period. If he has no fix order he does not trade; if his customers are buying when all the other customers are selling, he nonetheless buys.

In Period 2 under information sharing, dealers whose fix orders are against the majority nonetheless begin liquidating the proprietary position opened in period 1. These trades undermine the profits of dealers with majority-direction fix orders by moderating the period-2 price trend.

Proposition 3: If dealers share information about customer fix orders, fix trading is distributed across both pre-fix periods, dealers free-ride, and dealers undertake excess trading, as observed under independent trading. Under information-sharing, however,

a. Free-riding is more intense than under independent trading:

\[ 0 < \left( \frac{\alpha_2}{\alpha_1} \right)^{\text{InfoShare}} = \frac{1}{1 + \rho N} < \left( \frac{\alpha_2}{\alpha_1} \right)^{\text{Indep}} = \frac{1}{1 + \rho N} < 1 \quad , \tag{16a} \]

b. The additional free-riding undermines dealer profits relative to independent trading:

\[ 0 < E_0 \left\{ \bar{\pi}^{\text{Indep}} \right\} = \theta \sigma_F^2 \bar{\pi} \leq E_0 \left\{ \bar{\pi}^{\text{InfoShare}} \right\} = \theta \sigma_F^2 \left( \frac{1 + N}{(2 + N)^2 - (1 + N)} \right)^2 , \tag{16b} \]

c. On average, excess trading is lower than under independent trading:

\[ 0 < \chi^{\text{InfoShare}} = \left( \frac{1 + N}{(2 + N)^2 - (1 + N)} \right) \leq \chi^{\text{Indep}} = \left( \frac{1 + N}{(2 + \rho N)^2 - (1 + \rho N)} \right) \leq \frac{1}{3} . \tag{16c} \]

Proposition 4 summarizes how information sharing affects fix price dynamics:
Proposition 4: If dealers share information about customer fix orders there is more free riding and less excess trading than under independent trading, and in consequence

a. Volatility is less pronounced than under independent trading, though it still exceeds volatility during non-fix periods:
\[
\sum_{\text{InfoShare}} = E\left\{ (P_{F} - P_{0})^2 \right\} = \theta^2 2\sigma^2 + \theta^2 M\sigma^2(1 + \overline{x})^2 < \sum_{\text{Indep}},
\]

(17a)

b. Post-fix trend retracements are less pronounced than under independent trading:
\[
\Lambda_{\text{Indep}} < \Lambda_{\text{InfoShare}} = -\frac{M\sigma^2(1 + \overline{x})\overline{x}}{2\sigma^2 + M\sigma^2(1 + \overline{x})^2} < 0 , \quad \text{and}
\]

(17b)
c. Convexity of the average pre-fix price path is lower than under independent trading.

Discussion. The analysis above indicates that dealers are better off sharing zero information than sharing truthfully about their fix orders without active strategic collusion. This has direct parallels in the analysis of information sharing among oligopolistic firms. Clarke (1983), whose model of oligopoly is closest to ours, shows that in an oligopolistic market with stochastic elements the firms’ welfare is minimized if firms pool their information. Nonetheless, a key choice variable for fix dealers, trade timing, has no parallel in a traditional model of the firm. This could help explain a notable difference between our findings and those of traditional models: Competition among fix dealers is closest to Cournot competition, insofar as dealers choose quantities not prices, but traditional oligopoly models show that information sharing is beneficial to firms under Cournot competition and costly otherwise (Vives, 1990). The conclusion that information sharing is costly to dealers may be robust to the possibility that dealers do not fully reveal their information. Gal-Or (1985), which analyzes such a model, concludes that in Nash equilibrium firms still do best by revealing zero information. Clarke (1983) finds that firms do best if they not only share information but also collude on trading strategies. We examine that possibility next.

B. Collusion

Collusion violates antitrust laws, standard regulatory limits, and bank policies; the penalties can include jail time. One might therefore wonder why dealers who already share information might also collude. This section shows that collusion raises average dealer profits.

Collusive strategies could be arranged in a variety of different ways. At the London 4 pm fix, dealers in the self-described “cartel” often assigned a single dealer to control all customer fix trading for the
group (FCA, 2014c – 2014g). But the same collusive strategy could be executed with multiple dealers sharing trading responsibilities, so long as they can trust each other. For convenience we refer to a single dealer who has full control, and assume the other members of the cartel do not cheat on him. We also assume that all dealers join just one cartel. The outcome with \( K \) separate cartels is isomorphic to independent trading with \( N = K - 1 \).

The profit-maximizing collusive strategy depends entirely on total customer fix orders,

\[
F_{\text{tot}} \equiv \left( F_d + \sum_{n} F_n \right).
\]

Otherwise the strategy is identical to the strategy of a single dealer \( (N = 0) \) under independent trading. Lemma 3 summarizes this using \( D_1, D_2 \), and \( X \equiv x^{\text{Collude}} F_{\text{tot}} \) to denote pre-fix inventory accumulation and excess trading under collusion.

**Lemma 3:** In the baseline model a dealing cartel will trade \( 2/3 \) of total fix orders in period 1 and again in period 2 and the cartel’s excess trading will be \( 1/3 \) of total fix orders:

\[
\frac{D_1}{F_{\text{tot}}} = \frac{D_2}{F_{\text{tot}}} = \frac{2}{3}, \quad \frac{X}{F_{\text{tot}}} = x^{\text{Collude}} = \frac{1}{3}.
\]

Collusion is more profitable than information sharing because it shuts down free riding. As would be true if fix orders were uncorrelated, pre-fix inventory accumulation is evenly distributed between periods 1 and 2. By eliminating free riding, collusion brings higher per-dealer profits and encourages excess trading.

**Proposition 5:** When dealers collude, fix trading is still distributed across both pre-fix periods and dealers trade for their own account, as observed under independent trading and information sharing. Under collusion, however,

a. **Dealers cannot free-ride,**

b. **Expected profits are maximized**

\[
E_0 \left\{ \bar{\pi}^{\text{Collude}} \right\} = \theta \sigma_{\pi}^2 / 3 > E_0 \left\{ \bar{\pi}^{\text{Indep}} \right\} = \theta \sigma_{\pi}^2 x^{\text{Indep}} > E_0 \left\{ \bar{\pi}^{\text{InfoShare}} \right\} = \theta \sigma_{\pi}^2 \bar{x}^{\text{InfoShare}},
\]

c. **Excess trading is maximized:**

\[
x^{\text{Collude}} = \frac{1}{3} > x^{\text{Indep}} > x^{\text{InfoShare}}.
\]

Collusive fix profits emerge for reasons unrelated to traditional monopoly power because collusive fix dealers do not agree on a specific price. They agree instead on strategy for the amount and timing of a trade sequence intended to influence the price. Collusion at the fix does share some
notable features with a different microeconomic phenomenon, shrouding. Introduced by Gabaix and Laibson (2006), shrouding applies to producers who advertise a low price for a headline product while hiding the high price of an add-on. Manufacturers of home printers, for example, advertise low prices for the printers while hiding the high price of ink, which is actually the biggest cost in home printing. The headline product for liquidity provision is immediacy. For normal-sized forex trades (up to roughly $25 million) immediacy is the entire product and its price is the bid-ask spread. Larger trades are handled on a best-efforts basis, however, so dealers are normally compensated for both immediacy and their skill at minimizing price impact and spreads are wider. Dealers typically charged a zero bid-ask spread on fix orders, even though they’re typically quite large, in effect setting a low price on their headline product. The banks’ guilty pleas of 2015 suggest that they could charge low spreads on such trades because they earned profits on the side by maximizing the orders’ price impact and trading for their own account. Information about these additional costs was unavailable to customers: The dealers hid their information sharing and collusion by operating in private chat rooms and their actual trades were not observable because most customers have no access to the forex interdealer market.

Collusion has clear implications for price dynamics at the fix:

**Proposition 6: In equilibrium under collusion**

a. *Pre-fix volatility is higher than under independent trading or information sharing:*

\[ \Sigma_{\text{Collude}} = 2\theta^2 \sigma^2 + \theta^2 M \sigma^2 (1 + \chi_{\text{Collude}})^2 > \Sigma_{\text{Indep}} > \Sigma_{\text{InfoShare}}, \]  

(20a)

b. *Post-fix reversals are more pronounced than under independent trading or information sharing:*

\[ \Lambda_{\text{Collude}} = -\frac{(N + 1)\sigma^2 \chi_{\text{Share}} (1 + \chi_{\text{Share}})}{2\sigma^2 + (N + 1)\sigma^2 (1 + \chi_{\text{Share}})^2} < \Lambda_{\text{Indep}} < \Lambda_{\text{InfoShare}} < 0, \]  

(20b)

c. *Convexity is higher than under independent trading or information sharing:*

\[ 0 < \Pi_{\text{InfoShare}} = \frac{1}{1 + N} < \Pi_{\text{Indep}} = \frac{1}{1 + \rho N} < \Pi_{\text{Collude}} = 1. \]  

(20c)
In sum, information sharing and collusion have opposing effects on per-dealer profits, excess trading, pre-fix volatility, post-fix retracements, free riding, and convexity of the pre-fix price path. This is consistent Clarke’s (1983) findings for oligopolistic firms in a stochastic environment.

**Dynamic Collusion:** The possibility of cheating by cartel members, which has so far been ignored, cannot reasonably be ruled out given the dealing banks’ admission that dealers violated bank ethical standards and anti-trust laws. Indeed, given the strong incentives for dealers to cheat on each other identified by the model, fix trading was isomorphic to a prisoner’s dilemma and cheating should perhaps be expected. The cooperative strategy was colluding as outlined above; non-cooperative strategies would have included lying about one’s fix orders and trading those orders independently, or trading for one’s own account separately from the cartel and liquidating those trades earlier than the rest of the cartel.

As a repeated game fix trading can be analyzed in terms of dynamic collusion (e.g., Bishop, 1960; Stigler, 1964). If demand and supply functions are known with certainty, equilibrium cartel behavior would be determined by the fact that cheating can be identified unambiguously. In forex, however, dealers face many sources of uncertainty and signals of cheating are inherently noisy. Green and Porter (1984) and Abreu et al. (1986) show that Bertrand competitors facing such uncertainty can rationally adopt both carrots and sticks: They cooperate if and only if the price remains within a certain range; otherwise they perceive a high likelihood of cheating among their peers, retaliate for a finite number of rounds, and then revert to collusion. In fix trading the trigger for retaliation could, by analogy, have been an observed price path that was inconsistent with the other dealers’ expected path under collusion. The price rise might begin or end earlier than expected, for example. Retaliation itself need not take place within the context of fix trading and could take many forms. A suspicious dealer could quote slower or wider prices in direct trading, accommodate smaller amounts, or engage in social exclusion.

Given randomness in the price process, Green and Porter (1984) and Abreu et al. (1986) predict an irregular cycle of cheating, retaliation, and renewed cooperation which could generate irregularities in fix-price dynamics. Empirical research on cartels does not entirely support this theoretical perspective, however. Cartels typically survive for at least a few years – many last
beyond ten years – and price wars are less frequent and less intense than theory predicts (Levenstein and Suslow, 2006). The empirical research also highlights conditions under which collusion tends to thrive, at least two of which were met by forex dealing at the fix. First, members of a successful cartel will make compensation responsive to market conditions, because this allows them to avoid disagreement over how to adjust collusive rents over time (Levenstein and Suslow, 2011). In the fix cartel, the dominant dealer for the day – a role that rotated according to the relative magnitude of each dealer’s fix orders – earned the day’s entire gains, and those were determined by that day’s net fix order, that day’s market conditions, and his own skill. Second, members of a successful cartel will typically apply a high discount rate to the future (Levenstein and Suslow, 2016). In the fix cartel, dealers were secure within their respective banks and interest rates were generally low.

IV. Banging the Close and Convexity

The baseline model has explained high volatility and retracements at the fix and clarified important dealer trading strategies including front-running, free riding, and collusion. However, the model consistently implies that dealers make half or more of their trades early in the pre-fix period, which means it does not capture banging the close or convexity. Banging the close, a practice often ascribed to fix dealers by the media, involves “concentrating orders in the moments before and during the 60-second window” surrounding the calculation of the London fix (Vaughan, Finch, and Choudhury, 2013). If dealers bang the close the pre-fix price path should be convex, consistent with the pattern observed in forex since the mid-2000s (Figure 1).

This section examines two natural extensions of the model that produce banging the close and strict convexity of the pre-fix price path: sensitivity of price impact to order flow and dealer risk aversion.

A. Quantity-sensitive Price Impact

Cushing and Madhavan (2000) provide evidence that order flow has a stronger impact on NASDAQ prices just prior to the close than at other times. As discussed in Comerton-Forde and Putniņš (2011), this could reflect the dealers’ intentional efforts to maximize it. Traders in a limit-order market know they can minimize price impact by spreading many smaller transactions out over time, allowing depth at
the best quotes to be replenished between each trade (Bertsimas and Lo, 1998). By symmetry, traders in
a limit-order market know they can maximize price impact by trading a large amount all at once,
exhausting available depth at the best quote and many price levels beyond. These observations imply
that instantaneous price impact is sensitive to order flow.

To capture the dealers’ influence over price impact we assume that $\theta$ is an increasing function of
total order flow, $Q_t = \sum_{N+1} D_{nt} + \varepsilon_i : \theta = \theta(Q_t), \theta(Q_t) > 0$. For tractability we also assume constant elasticity:
$\theta(Q_t)Q_d / \theta(Q_t) = e > 0$; with this assumption the current model becomes a straightforward generalization
of the baseline model, for which $e=0$.

The sequence of analysis once again begins with representative dealer $d$ analyzing his period-2
choice of excess trading, $X_d$, as a function of period-1 variables and then choosing his period-1 trade.
Under independent trading each dealer must estimate the other dealers’ orders and trades. Under
information sharing the other dealers’ orders are known but not their trading strategies. Under collusion
one dealer controls all orders and trading. The derivation is provided in the Appendix, along with
expressions for trading as a function of fix orders. Lemmas 4 through 6 summarize dealer trading in our
three competitive settings. Note that $D_{2d} = F_d - D1d + X_d$ under independent trading and information
sharing; likewise $D_2 = F_{1d} - D1 + X$ under collusion.

Lemma 4: If price rises with order flow with constant elasticity $e>0$, dealers trade independently
($N\geq1$), and fix orders are positively-but-imperfectly correlated ($1>\rho>0$), a dealer’s trades will be the
following shares of his own fix orders:

\begin{align*}
\text{a. } D_{1d} &= \frac{(2+e+\rho N)(1+\rho N)}{(2+e+\rho N)^2-(1+e)(1+\rho N)} \frac{F_d}{\theta_3} = \alpha_1 F_d, \\
\text{b. } X_d &= -D_{3d} = \frac{(1+e)(1+\rho N)}{(2+e+\rho N)^2-(1+e)(1+\rho N)} \frac{F_d}{\theta_3} = xF_d.
\end{align*}

Lemma 5: When price impact is sensitive to order flow with constant elasticity $e>0$ and dealers share
information, every dealer will trade exclusively for his own account in periods 1 and 3; in period 2
every dealer accumulates inventory required to service his own customers and partially liquidates the
inventory acquired in period 1.
Lemma 6: When price impact is sensitive to order flow with constant elasticity $e > 0$ and dealers collude, trades will be the following fractions of total fix orders every period:

1. $D_1 = \frac{(2 + e)}{(2 + e)^2 - (1 + e)} F_{tot} \equiv \alpha_1 F_{tot}$,  
   \hspace{2cm} (23a)

2. $X = -D_3 = \frac{(1 + e)}{(2 + e)^2 - (1 + e)} F_{tot} \equiv \chi F_{tot}$,  
   \hspace{2cm} (23b)

These solutions are highly non-linear and cannot be expressed in closed form: the trading shares for each period depend on the ratio of expected price impacts, $\theta_2/\theta_3$, which depends non-linearly on the trading shares. Nonetheless, comparative statics reveal that a rise in the sensitivity of price impact to order flow, $e$, brings an increase in excess trading and a shift of trading from period 1 to period 2 unless parameters are extreme (specifically $N < e$).

A pair of thought experiments clarifies these shifts. Assume the market start in equilibrium with $e = 0$ and $e$ rises slightly. A dealer considers shifting a unit of trading from period 1 to period 2. With $e = 0$ and a constant price impact, the proportionately larger period-2 return would increase the appreciation of period-1 inventory and these gains would exactly offsetting the loss from the inventory’s smaller size. With $e > 0$, however, the period-2 price impact will increase as well as the amount traded in that period, so the period-2 return will rise more than with $e = 0$ and the benefits of the shift exceed the costs.

Now the dealer considers adding a unit of excess trading in period 2, to be liquidated in period 3. With $e = 0$ and constant price impact, the benefits of a larger period-2 return would exactly offset the loss from liquidating the extra trading in period 3. With $e > 0$ the period-2 price impact rises relative to the period-3 price impact, so the gains exceed the costs.
Given the non-linearity of this equilibrium we use simulations to examine its properties. Equilibrium is determined by just three exogenous parameters: $N$, $\rho$, and $e$. Figure 2A plots $\alpha_2 / \alpha_1$ for representative values of $e$ and $N$ with $\rho=0.5$. Additional solutions, not reported, show that the equilibrium is not highly sensitive to $\rho$. Period-2 trading can be quite high relative to period-1 trading, consistent with banging the close, and the expected price path can be strictly convex. These occur if dealers collude ($N=0$) or if competition is limited in the other settings, meaning specifically if $\rho N < e$ when dealers trade independently or if $N < e$ when dealers share information.

When price impact is sensitive to order flow, volatility and retracements become more pronounced according to the simulations. Volatility, though not retracements, is also affected by shifts in the distribution of pre-fix trading between periods 1 and 2: it rises (falls) if trading becomes less (more) evenly distributed. The variance-based measured used heretofore now involves third and fourth moments of all distributions, so to assess the effect of dealer strategies on volatility we rely on a measure of “relative volatility,” $V$:

$$V = \frac{\mathbb{E}_0\left\{\theta(\cdot)\left[\alpha_1 \sum M F_n + \varepsilon_1\right] + \theta(\cdot)\left[\alpha_2 \sum M F_n + \varepsilon_2\right] \right\} \mathbb{E}_0\left\{\sum M F_n\right\}}{\mathbb{E}_0\left\{\theta(\cdot)\left[\sum M F_n / 2 + \varepsilon_1\right] + \theta(\cdot)\left[\sum M F_n / 2 + \varepsilon_2\right] \right\} \mathbb{E}_0\left\{\sum M F_n\right\}}.$$  \hspace{1cm} (24)

Relative volatility is the ratio of (a) the price change under equilibrium strategic behavior to (b) the price change in the absence of strategic or manipulative behavior, meaning when dealers trade only the amount of their own fix orders divided evenly across pre-fix periods. When price impact is fixed ($e=0$) $V_{\text{Indep}}^{e=0} = 1 + x_{e=0}$. When price impact responds to order flow ($e>0$) $V$ has no closed-form solution but a Taylor series expansion allows us to approximate it as $V_{\text{Indep}}^{e>0} \approx 2^e \left[\alpha_1^{1+e} + \alpha_2^{1+e}\right]$. Figure 2B, which plots $V$ for representative values of $e$ and $N$ with $\rho=0.5$, shows that it is increasing with $e$.

### B. Risk Aversion

Our second extension of the model incorporates dealer risk aversion. One might reasonably wonder whether forex dealers actually care about risk, but media interviews suggest that they do. The article that broke the news of manipulation (Vaughan, Finch, and Choudhury, 2013), for example, notes:
“Dealers colluded with counterparts to boost chances of moving the rates, said two of the people, who worked in the industry for a total of more than 20 years” (italics added). Reporting shortly thereafter, Kaminska (2013) corroborated that dealers were concerned about risk, indicating specifically the risks that are built into our model:

“[Manipulative strategies] could still backfire if another dealer with a larger position bets in the other direction or if market-moving news breaks during the 60-second window, [a current dealer] said. A former dealer characterized it as a risky strategy that he only attempted when he had a high degree of knowledge of other banks’ positions and a particularly large client order. Typically, that would need to exceed 200 million euros to have a chance of moving the rate, two of the traders estimated” (italics added).

We examine the model when fix dealers have mean-variance utility with risk aversion $\gamma/2$:

$$Max \ E(\pi_d) - \frac{\gamma}{2} Var(\pi_d).$$  \hspace{1cm} (25)$$

As before, we begin by analyzing representative dealer $d$ who trades independently and who analyzes period 2 before period 1. His expected profits, conditional on period-2 information, remain as shown in Equation (5). The conditional variance of those profits is determined by the non-fix trade shocks, $\varepsilon_2$ and $\varepsilon_3$; by dealer $d$’s error in forecasting other dealers’ fix orders, $q = \sum_{n} F_n - E_{2d}[\sum_{n} F_n]$; and by his error in forecasting other dealers’ excess trading, $\mu = \sum_{n} X_n - E_{2d}[\sum_{n} X_n]$. These risk terms have variance $\sigma^2_\varepsilon > 0$ and $\sigma^2_\mu > 0$, respectively, and covariance $Cov(\varepsilon, \mu)$. The equilibrium trading strategy is derived in the Appendix. Lemma 6 reports that equilibrium trading shares depend on two risk terms,

$$R_x \equiv \gamma \theta (\sigma^2_\varepsilon + \sigma^2_\mu) \quad \text{and} \quad R_d \equiv \gamma \theta [2 \sigma^2_\varepsilon + \sigma^2_\mu + (1 - \tilde{\alpha})Cov(\mu, \varepsilon)].$$

**Lemma 7:** Risk-averse dealers in competitive equilibrium with positively-but-imperfectly correlated fix orders ($1 > \rho > 0$) will trade the following shares of their own customer fix order:

- a. $$D_{id} = \frac{(1 + \rho N)}{(2 + \rho N)[1 - q(1 - q)] + \gamma \theta V_1(\pi_d)} F_d \equiv \alpha_1 F_d, \quad \frac{1}{2} < \alpha_1 < 1, \hspace{1cm} (26a)$$

- b. $$X_d = \frac{1 + R_d}{2 + R_x + \rho N} \alpha_1 F_d \equiv q \alpha_1 F_d \equiv x F_d, \quad 0 < x < \alpha_1 < 1. \hspace{1cm} (26b)$$

This solution is highly non-linear; period-1 trading as a share of fix orders, $\alpha_1$, depends on the risk terms $R_x$ and $R_d$ as well as on $q$, the ratio between excess trading and period-1 trading; these, in turn, depend
non-linearly on each other as well as $\alpha_i$. Equilibrium trading shares cannot be expressed in closed form and comparative statics are inconclusive. Simulations for a wide variety of parameters, presented in Figures 3A and 3B, confirm that trading shifts towards the end of the pre-fix interval as risk aversion increases, consistent with banging the close. Period-2 trading exceeds period-1 trading, and the price path is strictly convex, for many parameter combinations. Similar shifts accompany a rise in any of the market’s underlying risk variables ($\sigma_\theta^2$, $\sigma_\gamma^2$, and $\sigma_\varepsilon^2$) or an increase in $N$. The latter effect occurs because rising $N$ amplifies the variance of total fix order flow.

When dealers share information about customer fix orders the only source of risk is non-fix trading noise, $\varepsilon$, because the dealers no longer face risks associated with predicting the orders of other dealers and $\sigma_\theta^2 = \sigma_\gamma^2 = \text{Cov}(\theta, \mu) = 0$. The variance of profits for the dealer’s period-2 decision becomes $\theta^2 \sigma_\varepsilon^2 (X_d^2 + D_{1d}^2)$. Equilibrium trading shares do not involve recursive non-linearities and can be expressed in closed form.

**Lemma 8:** Risk-averse dealers who share information will trade the following amounts:

a. $D_{1d} = \left( \frac{(1 + N)(2 + N + \theta \gamma \sigma_\varepsilon^2)}{(2 + N + \theta \gamma \sigma_\varepsilon^2)^2 - (1 + N)} \right) \bar{F} \equiv \bar{\delta}_1 \bar{F}, \quad 0 < \delta_1 < 1 \quad (27a)$

b. $X_d = -D_{3d} = \left( \frac{(1 + N)}{(2 + N + \theta \gamma \sigma_\varepsilon^2)^2 - (1 + N)} \right) \bar{F} \equiv \chi \bar{F}, \quad 0 < \chi < 1 \quad (27b)$

Under information sharing, risk-averse dealers trade less in period 1 and more in period 2 than risk-neutral dealers: $\partial \delta_1 / \partial \gamma < 0$, $\partial \delta_2 / \partial \gamma < 0$. If $N(3 + 2N) < 2\theta \theta \sigma_\varepsilon^2$, trading will be concentrated in period 2, consistent with banging the close, and the price path will be strictly convex. Intuitively, dealers are more inclined to postpone trading and bang the close if they are highly risk averse, if risk is high, or if competition is limited. Note that the relation between convexity and $N$ changes sign between independent trading and information sharing. Under information sharing convexity falls as $N$ rises because fix dealers are not certain about each others’ orders and free riding rises with competition.

When dealers collude the only source of risk remains non-fix trading noise, $\varepsilon$ and equilibrium trading strategies can again be expressed in closed form.
**Lemma 9:** Risk-averse dealers who collude will trade less in period 1 than period 2; the pre-fix price path will be strictly convex; convexity is rising in risk aversion.

\[
\text{a. } D_1 = \left( \frac{(2 + \theta \gamma \sigma_x^2)}{(2 + \theta \gamma \sigma_x^2)^2 - 1} \right) F_{\text{tot}} = \alpha_1 F_{\text{tot}}, \quad 0 < \alpha_1 < 1,
\]

\[
\text{b. } X = \left( \frac{1}{(2 + \theta \gamma \sigma_x^2)^2 - 1} \right) F_{\text{tot}} = x F_{\text{tot}}, \quad 0 < x < \frac{1}{3},
\]

\[
\text{c. } D_1 > D_2 > X, \quad \frac{\partial \alpha_1}{\partial \gamma} < 0, \quad \frac{\partial \alpha_2}{\partial \gamma} > 0.
\]

If price impact is sensitive to order flow or if dealers are risk-averse the model predicts both banging the close and convexity of the pre-fix price path under collusion. Banging the close can also be optimal when dealers trade independently if competition is limited.

**V. Identifying Collusion from Fix-Price Dynamics**

Our analysis of fix trading has implications for an important practical question: Could one extract reliable evidence for collusion from data on currency trades and prices? Could collusion be indicated, for example, by high volatility before the fix and/or pronounced retracements after the fix? The paper’s findings are not encouraging in this regard. With respect to volatility, the model implies that volatility just prior to the fix will inevitably arise given the concentration of customer orders that must be executed within a short time frame. Volatility could be even more pronounced if dealers engage in excess trading or bang the close but these are not illegal in currency markets and would occur regardless of whether dealers collude.\(^\text{10}\) Retracements, similarly, could reflect an entirely neutral force, specifically price shading by non-fix dealers whose inventories have shifted in response to trading by fix dealers (Hendershott and Menkveld, 2014), or a force that is considered unethical but legal, excess trading.

Collusion is no easier to identify based on changes over time in volatility or retracements because these could reflect changes in factors other than collusion including: the variance of non-fix order flow, \(\sigma_x^2\); the variance of aggregate fix orders \((N+1)\sigma_F^2\); and the correlation of fix orders across dealers, \(\rho\).

---

\(^{10}\) Evans (2016) suggests that heightened volatility should arise after the fix rather than before, but this can only occur if dealers forego the profit incentives identified in Comerton-Forde and Putninš (2008) and natural profit opportunities associated with slippage or, more precisely, with the well-documented response of order flow to dealer trading.
and the number of fix dealers, \(N+1\). Some of these factors – most notably \(\sigma_2^2\) – are known to vary over time but difficult to measure, so it would not be possible to identify whether changes in volatility or retracements reflected the changes in collusion or other factors.

According to the model, convexity of the pre-fix price path might have more potential for identifying periods of collusion, though it probably could not be definitive. Convexity is influenced by just two factors beyond the competitive environment – the number of dealers and the correlation of orders across dealers – both of which are likely to be fairly stable over time. Every version of the model considered above implies a more convex price path under collusion than under independent trading or information sharing. Regulators claim, and dealing banks have admitted, that there was collusion among dealers beginning around 2007-2008. The model therefore predicts that pre-fix price dynamics should display higher convexity after 2007-2008 than before. Convexity has not traditionally been the focus of statistical analysis, so we develop a measure of convexity and test whether it changed around 2008 for seven major currencies vis-à-vis the U.S. dollar: EUR, GBP, JPY, CHF, CAD, NZD, and DKK.

**Data:** Our data comprise tick-by-tick OTC quotes among interbank dealers from the Reuters Dealing platform. They begin in February 1996 for JPY, GBP, CHF, CAD, NZD, and DKK and in January, 1999 for EUR. Data for all currencies end in December 2013 but for CHF we drop all observations after October, 2011, when the Swiss National Bank began active intervention in support of a floor on the exchange rate vis-à-vis the euro. Following Melvin and Prins (2015) we focus exclusively on end-of-month trading days. We use mid-quotes sampled at the end of every minute and rely on minute-by-minute signed returns: 

\[
(p_{t,m} - p_{t,m-1})I_t
\]

where \(p_{t,m}\) is the log price at the end of minute \(m\) on day \(t\) and \(I_t\) is an indicator variable, \(I_t=1\) (-1) if the exchange rate rises (falls) between 3:45 and 4:00 on day \(t\).

**Measuring Convexity:** We measure of convexity in two steps, as illustrated in Figure 4A. We first take the difference between (a) the area under the actual average the pre-fix price path and (b) the corresponding area if the path had linearly connected the same beginning and ending price level. We standardize this by dividing it by the area under a linear path, (b).

Figure 4B plots convexity of the average month-end price path from the beginning of the sample through December, 2002, and through the end of every subsequent year in the sample. Early in the
sample convexity was negative for many currencies, consistent with the concave price path that might emerge as a result of free riding. By the end of the sample, by contrast, convexity was positive for all currencies, consistent with the hypothesis that collusion in the later years of the sample period curtailed free riding and encouraged banging-the-close, thereby raising convexity.

**Bootstrap Methodology:** Our null hypothesis is that convexity is unchanged between the early part of the sample, when collusion is not widely alleged, and the later part of the sample, when collusion is admitted. We evaluate this hypothesis by bootstrapping the statistical distribution of convexity in the early period for each currency (Efron, 1979; 1982), partitioning each sample after December, 2007. The bootstrap involves 72,000 simulated average price series designed to match the underlying distribution of returns before collusion apparently took hold.

The simulated series capture both the autoregressive structure of returns and the statistical distribution of volatility in the years before 2008. They are created by sampling with replacement from price shocks and empirical volatilities derived as follows:

- We capture the autoregressive structure by estimating an AR(1) using one-minute returns during the pre-fix interval (3:45 to 4:00 pm) on end-month dates through December 2007:
  \[ p_{t,m+1} - p_{t,m} = \beta_0 + \beta_1 (p_{t,m} - p_{t,m-1}) + \zeta_{t,m+1}. \]
- We capture the distribution of volatility by estimating a GARCH(1,1) using all available 15-minute returns on all trading days through December 2007.\(^{11}\) We then select the fitted standard deviations of returns for the 15-minute fix intervals for each end-month day \( t \), \( \kappa_t \), which serve as the empirical volatilities that are sampled to create the simulated series.
- The estimated fix-interval volatilities are used to standardize the AR(1) return residuals:
  \[ \chi_{t,m} = \frac{\zeta_{t,m}}{\kappa_t}. \]
- We also identify all returns from 3:44 to 3:45 for end-month days through December 2007.

A given simulated series is created as follows:

1. Sample with replacement one of the 3:45 returns, \( \hat{r}_{3:45} \). Sample with replacement one of the fix-interval standard deviations, \( \hat{\kappa} \). Sample with replacement one of the standardized return shocks, \( \hat{\chi}_1 \). Multiply all of the latter by \( \hat{\kappa} \) to create a return shocks, \( \hat{\zeta}_1 = \hat{\kappa}\hat{\chi}_1 \).
2. Generate the 3:46 value of the simulated series as follows:
   \[ r_{3:46} = \hat{\beta}_0 + \hat{\beta}_1 \hat{r}_{3:45} + \hat{\zeta}_1 \]
   \[ P_{3:46} = 100 + r_{3:46}. \]

\(^{11}\) Trading days were defined to exclude weekends and major holidays.
4. To generate the next element of this simulated series, repeat step 1 to create \( \hat{\chi}_2 = \hat{\chi}_2 \).

5. Generate the 3:47 value of the simulated series as follows:

\[
\begin{align*}
    r_{3,47} &= \hat{\beta}_0 + \hat{\beta}_1 r_{3,46} + \hat{\epsilon}_1 \\
    P_{3,47} &= P_{3,46} + r_{3,47}
\end{align*}
\]

6. Repeat steps 4 and 5 thirteen more times to create \( P_{3,48}, P_{3,49}, \ldots, P_{400} \).

Steps 1 through 6 are then repeated 71 times to create a total of 72 simulated series, where 72 is the number of end-month series between January 2008 and December 2013. (For CHF we repeat the process only 46 times because the post-2007 sample ends in October, 2011, when the Swiss National Bank put a floor under the EUR-CHF exchange rate.) The average of these 72 simulated series serves as a single simulated average price path. We then create 999 more sets of 72 simulated series, taking the average of each set. Finally, we calculate the convexity of each simulated average path. The 1,000 simulated convexity values should capture the distribution of convexity under the null.

The 1,000 simulated convexity values should faithfully represent the distribution of the average pre-fix price path in the pre-collusion observations of February, 1996 through December, 2007. We identify the marginal significance of the difference between actual convexity and its simulated distribution under the null by finding the rank of actual convexity for the post-2007 period relative to the 1,000 simulated convexity levels and dividing by 1,000.

**Results:** Table 1 presents observed pre-2008 and post-2007 convexity values and the marginal significance levels for post-2007 convexity values. Convexity is higher in the later period for all currencies. Before examining the bootstrap tests we first take a much simpler approach by testing the null hypothesis that convexity has an equal chance of being higher or lower in the later period. Under this null, the number of currencies for which convexity rises has a binomial distribution with \( n = 7 \) and \( p = 0.5 \). The likelihood of observing a rise in convexity for seven out of seven currencies is just 0.0078, which implies a rejection of the null.

This test is only valid if convexity values are independent across currency pairs. Independence could well characterize the convexity of high-frequency price paths even though daily exchange-rate dynamics, per se, are not independent. We examine the validity of the independence assumption by considering cross-currency correlations in the convexity in the month-end fix paths. Of the 21 bilateral
correlations among our seven currencies, only eight are positive; the overall average correlation is just -0.00453. On this basis we infer that independence is a reasonable assumption.

For most individual currencies the bootstrap supports the hypothesis that convexity was higher after than before December, 2007. The test is significant at the 5% level for five of the seven currencies, and at the 7% and 14% levels for the remaining two currencies. These results are consistent with the common view that collusion began in these markets around 2008.

VI. Discussion

Before closing we step back to examine whether fix-price dynamics are consistent with an efficient market and whether the model has insights relevant to regulating the fix.

A. Price Dynamics and Market Efficiency

Evans (2016) raises the possibility that high pre-fix volatility and predictable retracements are inconsistent with market efficiency. Strong and predictable price dynamics often disappear in the long run, and it is commonly assumed that this will occur inevitably in an efficient market (Lo, 2004). Predictable price patterns may not be competed away, however, if the behaviors generating them are strategic complements, meaning each agent’s behavior raises profits or reduces risk for other agents (Bulow et al., 1985).

Our model suggests that the behaviors generating fix price dynamics are multi-dimensional strategic complements. Customers are strategic complements to each other: fix orders from one customer intensify pre-fix volatility and make it more important for other customers to place orders. The customers are also strategic complements with the dealers: customer orders motivate dealers to trade in excess and collude, which increases fix volatility and the customers’ incentive to place fix orders. Even the small and mid-sized dealers may have contributed to these positive feedback loops: their decision to pass fix orders on to the big banks increases the big banks’ control over total fix order flow and with it the incentive to engage in excess trading; additional excess trading increases volatility, reinforces the smaller dealers’ incentive to step back and provides additional incentive for funds to place fix orders.

To identify whether fix-price dynamics are consistent with efficiency it is not sufficient to consider whether one could profit solely from their behavior immediately after the fix (Evans, 2015). As discussed
below, non-fix traders now have an incentive to trade on the fix by taking a position early in the pre-fix interval, and this speculative activity should magnify rather than eliminate existing patterns.

B. Fix Regulation

Since 2013 there have been notable changes in the calculation methodology for the London 4 pm fix and in dealer handling of fix orders. Nonetheless, market participants report that fix-price dynamics are qualitatively unchanged (Saks-McLeod, 2015). Our model provides an explanation for the stability of these dynamics and other insights regarding fix regulation.

The London 4pm fix is now calculated over five minutes rather than one minute, effectively increasing the sample for selecting the median price from 60 to 300 traded prices. The extended time interval should act like an increase in the variance of non-fix trading, \( \sigma_e^2 \), in the model. If dealers are risk averse this could reduce their incentive to misconduct and encourage them to shift trading towards the end of the fix period, as shown in Section IV. Otherwise this modification does not change the incentives that drive the behavior identified in this paper.

Most banks now require their dealers to process fix orders via automated algorithms that essentially eliminate dealer autonomy and spread the trades out over the pre-fix interval. This appears to preclude dealer misconduct, though curiously it might support dealer profits by eliminating free riding. More importantly, the predictability of trading by automated algorithms could allow non-dealers to replace dealers in driving fix-price dynamics. Dealer execution of fix orders now follows a predictable pattern, so non-dealers can interpret the return immediately following 3:45 as a signal of the upcoming price trend. If so, they would rationally behave like a zero-order dealer under information sharing: take a position in the dominant direction immediately after getting the signal and then liquidate that position partly before and partly after the fix. Indeed, an algorithm recently came on the market that facilitates exactly this strategy (Albinus, 2016). Like the dealer trading it replaces, this non-dealer trading would generate excess volatility and retracements.

The model provides little reason for optimism with respect to another proposal, the implementation of a clearing auction for fix orders. This could provide an equilibrium price for all fix buy and sell orders that can be matched off. However, it would provide no match for the fix order imbalance because fix
orders have zero price elasticity — customers have instructed their banks to trade a certain amount regardless of the market price. According to the model it is the imbalance that drives the striking fix-price dynamics. But if the goal is to incorporate non-fix trading into the execution of fix orders, the solution might look much like the market as it exists at present.

To moderate fix-price dynamics one could try to elicit non-fix orders to match the net fix order imbalance. The NASDAQ once attempted to do something along these lines by publishing market-on-close imbalances shortly before the close (Cushing and Madhavan, 2000). The model suggests that instead of dampening fix-price dynamics this could intensify them because rational non-dealers informed of the dealers’ net order imbalance will engage in excess trading. The NASDAQ dropped this approach and instituted a closing call in 2004.

It worth noting that fill-at-fix orders have negative externalities. They bring excess volatility which raises the risks faced by dealers and thus execution costs for end-users. They also introduce distortions in critical benchmark rates that reduce asset values for underlying investors and can compromise economic analysis. Long-established economic logic points to a specific remedy for negative externalities: calibrated disincentives for the behavior that generates them. Of course, any such policy would inevitably have side-effects that could mitigate or eliminate their value, so it would be impossible to say without a careful analysis whether the costs of a disincentive policy would outweigh the benefits.

VII. Summary

This paper examines dealer behavior at the London 4 pm fix in foreign exchange, a benchmark for index funds valued at $11 trillion (Cochrane, 2015). Funds that choose to trade at the fix must inform their dealers well in advance, providing these dealers with opportunities to engage in profitable misconduct. Observers long wondered whether such misconduct explained the high volatility and retracements around 4 pm. Investigations beginning in 2013 brought fines and guilty pleas from major banks (Department of Justice, 2015).

This paper examines a model of dealer trading at the fix in which dealers rationally choose misconduct including front-running, sharing confidential information about customer orders, banging the close, and outright collusion in the design and implementation of trading strategies. The model’s
assumptions about price formation and the structure of trading are based on core theories and evidence in microstructure. A critical finding is that high volatility before the fix will inevitably result from a neutral force: the magnitude of fill-at-fix orders that must be executed during a short time interval. The model also shows, however, that volatility may also rise, and retracements be introduced, because dealers trade for their own account in parallel with customer orders, a proprietary trading strategy akin to front-running.

Information sharing brings lower dealer profits and lower volatility; collusion has the opposite effects. Both sets of effects are related to free riding among dealers, a strategy in which dealers accelerate their pre-fix trading. When dealers share information about customer fix orders, free riding is encouraged because dealers have a more precise signal of the price trend. Collusion maximizes profits for the dealing community by shutting down free riding.

We show that dealers will delay trading if price impact is sensitive to order flow and if dealers are risk averse. With these extensions the model captures two additional phenomena associated with the London Fix. First, dealers will bang the close, meaning they will concentrate fix trading near the moment of fix calculation. Second, the price trend will accelerate on average as the fix approaches so the average price path will be convex.

We test the model by examining the convexity of the pre-fix price path before and after December 2007, around the time that collusive trading reportedly began in earnest. Our tick data cover seven major currencies vis-à-vis the US dollar: GBP, EUR, JPY, CHF, CAD, NZD, and DKK during 1996-2013. We develop a measure of convexity under the null that convexity remained constant after 2007 and evaluate the statistical significance of convexity from 2008-2013 using a bootstrap test. The results show that convexity of the pre-fix price path was higher during 2008-2013 than 1996-2007 for all seven currencies, as the model predicts under collusion.

The model can explain why excess volatility and price retracements continue to characterize exchange rate dynamics around 4 pm, despite recent reforms. Now that dealer trading at the fix is automated agents with no fix orders could use price movements immediately following 3:45 pm as a signal of dealer fix orders and speculate accordingly. These trades that by non-dealers can be expected to generate excess volatility and retracements. These price dynamics are unlikely to be traded away
because fix trading sets in motion a nexus of mutually reinforcing behaviors among dealers, customers, and other traders. Fix-price dynamics might be moderated if the incentives funds faced weaker incentives to avoid tracking risk.

Our findings could be relevant to benchmark prices in other OTC markets including precious metals. The findings are not likely to be relevant to manipulation of the LIBOR (Wheatley, 2012). Fix prices in foreign exchange and precious metals have limited relevance to a bank’s financial health. LIBOR, by contrast, affects the costs or returns for a significant share of bank assets and liabilities (Gandhi et al., 2016). Further, a bank’s LIBOR submission could influence the market’s perception of its financial health, and thus its funding costs (Abrantes-Metz et al., 2012), because individual-bank LIBOR submissions were immediately released to the public. The relevance of our analysis to LIBOR is also limited by differences in calculation methodology. LIBOR is based on banks’ reported borrowing costs while the forex and precious metals fixes are based on prices that should represent a market equilibrium.

Future research could extend the model to include other asset classes. Dealers who trade in options indexed to the fix, for example, might influence the fix to ensure that a given option matures in-the-money (e.g., Slater and Jones, 2014). Future research could also fruitfully endogenize the customers’ decision to place fix orders. Finally, future research could examine carefully the externalities associated with fix orders and whether adjusting the incentives to place fix orders could reduce excess volatility and the distortions of critical benchmarks.
References


39
Appendix: Solution Details

A.1. Rational expectations equilibrium under risk neutrality and independent trading

Dealer \( d \)'s period-2 expectation of the other dealers' proprietary trading, \( E_{2d}(\sum_{n} X_n) \), necessarily depends on his period-2 information set, \( \Omega_{2d} = \{F_d, D_{1d}, P_1 - P_0\} \). To identify the functional form of this expectation, assume that it is linear and apply the method of undermined coefficients:

\[
E_{2d}(\sum_{n} X_n) = A(P_1 - P_0) + BD_{1d} + CF_d.
\]  
(A.1)

The coefficients \( A, B, \) and \( C \) are identified from rationality constraints. The first is that dealers should expect their own proprietary trading, as a share of their net fix order, to be neither more nor less than the unconditional expected value of that share:

\[
E_{id}(X_d)/F_d = E_{id}\left(\sum_{n} X_n / \sum_{n} F_n\right).
\]  
(A.2)

This implies the following equality which can only be satisfied if \( A = 0 \):

\[
\frac{E_{id}(X_d)}{F_d} = \frac{\alpha_i}{2}\left[1 - B - \frac{A}{\theta}(1 + \rho N)\right] = \frac{\alpha_i}{2}\left[1 - B - \frac{A}{\theta}(1 + N)\right] = E_{id}\left(\sum_{n} X_n / \sum_{n} F_n\right).
\]  
(A.3)

A second rationality constraint is that dealers should not make predictable forecast errors, or:

\[
E_{id}(\Psi_d) = E_{id}\left(\sum_{n} X_n - E_{2d}\left(\sum_{n} X_n\right)\right) = 0.
\]

With \( A = 0 \), this implies:

\[
E_{id}(\Psi_d) = E_{id}\left(\frac{1 - B}{2} \sum_{n} D_{1n} - BD_{1d} - C \left(\sum_{n} F_n + 2F_d\right)\right) = 0.
\]  
(A.4)

This can be solved for \( B \) and \( C \) by considering (a) the model's symmetry, which implies that \( \alpha_{in} = \alpha_{im} \) for all \( n \), and (b) the structure of fix orders, which implies \( E_{id}(F_n) = \rho F_n \) where \( \rho \equiv \sigma_{d}^{2}/(\sigma_{d}^{2} + \sigma_{r}^{2}) \). Equation (A.4) becomes \( E_{id}(\Psi_d) = 0 = \alpha_i [B(2 + \rho N) - \rho N] - C(2 + \rho N) \) or

\[
C = \frac{\rho N}{2 + \rho N} - B.
\]  
(A.5)

Applying this to Equation (A.1) reveals that \( E_{2d}(\sum_{n} X_n) \) depends only on \( D_{1d} \):

\[
E_{2d}(\sum_{n} X_n) = \frac{\rho N}{2 + \rho N} D_{1d}.
\]  
(A.6)

Thus \( B = \rho N/(2 + \rho N) \) and \( C = 0 \). In combination with Equation (5), this implies:

\[
X_d = \frac{1}{2 + \rho N} D_{1d} = qD_{1d}.
\]  
(A.7)

Comparative statics:

\[
\frac{\partial \alpha_1}{\partial \rho N} = \frac{3 + 2\rho N}{Y^2} > 0, \text{ where } Y \equiv (2 + \rho N)^2 - (1 + \rho N).
\]  
(A.8)

\[
\frac{\partial \alpha_2}{\partial \rho N} = -\frac{\partial \alpha_1}{\partial \rho N} + \frac{\partial x}{\partial \rho N} < 0.
\]  
(A.9)

\[
\frac{\partial x}{\partial \rho N} = -\frac{\rho N(2 + \rho N)}{Y^2} < 0.
\]  
(A.10)
A.2 Rational expectations equilibrium under independent trading if dealers are risk neutral and price impact is sensitive to order flow.

Assume \( \theta = \theta(Q_t), \theta(Q_t) > 0, \theta(Q_t)Q_t/\theta(Q_t) \equiv e \), where \( Q_t \) represents order flow. Representative dealer \( d \)'s period-2 optimal excess trading is:

\[
X_d = D_{id} \left( \frac{1+e}{2+e} \right) \frac{\theta_3}{\theta_1} - E_{2d} \left\{ \sum_N X_n \right\} \frac{1}{2+e}.
\]  

(A.11)

Using the method of undetermined coefficients, under the two rationality constraints outlined in Section A.1, it can be shown that \( E_{2d} \left\{ \sum_N X_n \right\} = BD_{id} \) with \( B \equiv \rho N \left( \frac{1+e}{2+e+\rho N} \right) \frac{\theta_2}{\theta_3} \), (using \( \theta_t \equiv E_{id} \{ \theta(Q_t) \} \)), and thus \( X_d = \left( \frac{1+e}{2+e+\rho N} \right) \frac{\theta_3}{\theta_1} D_{id} \). The rest of the solution follows.

Comparative statics:

\[
\frac{\partial D_{id}}{\partial e} = \frac{(1+N)\left[ (1+N)^2 \frac{\theta_2}{\theta_3} - (2+e+N)^2 \right]}{Z^2}
\]

(A.12)

where \( Z \equiv (2+e+N)^2 - (1+N)(1+e) \frac{\theta_2}{\theta_3} \).

\[
\frac{\partial X_d}{\partial e} = \frac{(1+N)(2+e+N) \frac{\theta_2}{\theta_3} (N-e)}{Z^2}.
\]

(A.13)

A.3 Rational expectations equilibrium with risk aversion under independent trading

Dealer \( d \) evaluates expected profits and the variance of profits for the period-1 and period-2 trading decisions. Unexpected profits for the period-2 decision are:

\[
\pi_d = -E_{2d} \{ \pi_d \} = D_{id} \theta (1-\hat{\alpha}_1) (\vartheta + \varepsilon_2 + \varepsilon_3) + (D_{id} - X_d) \theta (\mu - \varepsilon_3).
\]

(A.14)

The variance of profits conditional on period-2 information is:

\[
Var_2 (\pi_d) = \theta^2 \left[ \hat{\alpha}_1^2 F_d \left[ 2 \sigma^2 + (1-\hat{\alpha}_1)^2 \sigma_{\varepsilon}^2 \right] + (\alpha_1 F_d - X_d)^2 (\sigma^2 + \sigma_{\varepsilon}^2) \right]
\]

\[
+ \theta^2 \left[ 2 \alpha_1 F_d (\alpha_1 F_d - X_d) \left( 1-\hat{\alpha}_1 \right) \text{Cov}(\mu, \vartheta) - \sigma^2 \right].
\]

(A.15)

This depends on the variance of non-fix trading shocks, \( \sigma^2 \) and on the variance and covariance of dealer \( d \)'s prediction errors, \( \sigma^2_\mu \) and \( \sigma^2_\vartheta \), and \( \text{Cov}(\mu, \vartheta) \), respectively, where

\[
\vartheta \equiv \sum_N F_n - E_{2d} \left\{ \sum_N F_n \right\} \quad \text{and} \quad \mu \equiv \sum_N X_n - E_{2d} \left\{ \sum_N X_n \right\}.
\]

The prediction-error properties are partially endogenous because they depend on the dealer’s proprietary trading. In equilibrium these errors, as well as their variances and covariance, depend on the three underlying sources of randomness: \( \phi, \eta, \) and \( \varepsilon \).

To identify utility-maximizing period-2 trading, dealer \( d \) applies Equations (4) and (29) to his overall optimization problem, Equation (27). The first-order condition for \( X_d \) implies:

\[
X_d = D_{id} \left( \frac{1+R_d}{2+R_d} \right) - E_{2d} \left\{ \sum_N X_n \right\} \frac{1}{2+R_d}.
\]

(A.16)

The new risk terms, \( R_d \) and \( R_\alpha \), are defined in the text. Dealer \( d \)'s proprietary trading is again linear in his period-1 trading and that the proportionality coefficient, \( q_1 \), now depends non-linearly on risk. To identify
\[ E_{2d} \left\{ \sum_{n \neq d} X_n \right\} \] we again assume agnostically that it is linear in the dealer’s information:
\[ E_{2d} \left\{ \sum_{n \neq d} X_n \right\} = A(P_1 - P_0) + BD_{1d} \] (this excludes \( F_d \) based on the results of Section II).\(^{12}\) We once again infer that \( A = 0 \) from the rational expectation constraint that a dealer expects his overtrading, as a share of his fix orders, to equal the unconditional average share of overtrading. \( B \) can once again be identified from the rational expectation constraint that the dealer’s period-2 expectation error should have expected value of zero conditional on period-1 information: \( B = \rho N(1 + R_d)/(2 + R_\gamma + \rho N) \).

Applying this to the Equation (A.16), above, gives the following solution for excess trading:
\[ X_d = \frac{1 + R_d}{2 + R_\gamma + \rho N} \alpha_d F_d = q \alpha_d F_d, \quad q = \frac{1 + R_d}{2 + R_\gamma + \rho N} \] (A.17)

Dealer \( d \) next identifies the variance of profits from the perspective of period 1:
\[ \text{Var}_t(\pi_d) = \sigma_\varepsilon^2 \left( 2 - q(1 - q) \right) + \sigma_\theta^2 (1 - \hat{\alpha}_1)^2 + (1 - q)^2 q^2 \hat{\alpha}_1^2 + 2(1 - \hat{\alpha}_1) (1 - q) q \hat{\alpha}_1 \] (A.18)

The period-1 trading strategy in Lemma 5 solves the dealer’s period-1 optimization problem:
\[ \text{Max}_{\alpha_1} \alpha_1 F_d \left\{ F_d (1 - \alpha_1) + \sum_N F_n (1 - \hat{\alpha}_1) + \alpha_1 F_d (1 - q) \theta \left[ q \alpha_1 F_d + \sum_N X_n \right] \right\} - \frac{\gamma}{2} (\alpha_1 F_d)^2 \theta^2 \text{Var}_t(\pi_d) \] (A.19)

Comparative statics:
\[ \frac{\partial \hat{\alpha}_1}{\partial \gamma} = - \frac{(1 + N) \left( 2 + N + \gamma \theta \sigma_\varepsilon^2 \right)^2 + (1 + N)}{W^2} < 0 \] (A.20)
\[ \frac{\partial \hat{\alpha}_2}{\partial \gamma} = - \frac{(1 + N) \left( 2 + N + \gamma \theta \sigma_\varepsilon^2 \right) (N + \gamma \theta \sigma_\varepsilon^2) + (1 + N)}{W^2} < 0 \] (A.21)
\[ \frac{\partial \alpha}{\partial \gamma} = - \frac{2(1 + N)(2 + N + \gamma \theta \sigma_\varepsilon^2)}{W^2} < 0 \] (A.22)

where \( W \equiv (2 + N + \gamma \theta \sigma_\varepsilon^2)^2 - (1 + N) \).

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\(^{12}\) The irrelevance of \( F_d \) is confirmed in unreported analysis.
Figure 1: Exchange Rate Dynamics Around the London 4 pm Fix
Mean price path from 60 minutes before to 60 minutes after the London 4 pm fix using tick-by-tick quotes from Reuters Dealing, and interbank trading platform, for EUR-USD, GBP-USD, USD-JPY, USD-CHF, CAD-USD, NZD-USD, and DKK-USD. The series begin in February 1, 1996, except EUR-USD, which begins January 1, 1999. All series end on December 31, 2013 except CHF-USD, which ends in October, 2011. All series are indexed to 100 at 3:45 pm. Declining prices have trends reversed for the average.
Figure 2: Volatility and convexity when price impact is sensitive to order flow. For all simulations $\rho=0.5$. When $N\geq1$ dealers trade independently; when $N=0$ dealers collude.

2A: Ratio of period-2 trading to period-1 trading. If the ratio exceeds unity the pre-fix price path is convex. If the ratio exceeds unity by a substantial margin dealers could be said to “bang the close.”

2B. Chart plots simulated values of relative volatility under independent trading when price impact is an increasing function of trading volume. Relative volatility is the ratio of volatility with strategic dealing to volatility if dealers engage in zero proprietary trading and distribute trades evenly across the pre-fix periods.
Figure 3: Banging the close under risk aversion
Charts provide the simulated ratio of period-2 trading to period-1 trading for risk-averse dealers trading independently ($N \geq 1$) or colluding ($N=0$). For all simulations $\rho = 0.5$ and the risk of non-fix trading is constant at $\sigma^2 = 1$. Charts end at $N=9$ to ensure differences at $N=0$ are readily apparent. With high risk aversion $\gamma \theta = 0.5$. With moderate risk aversion $\gamma \theta = 0.25$. Banging the close is consistent with solutions that lie above the horizontal line at unity.

3A. Banging the close under risk aversion with high fix-orders risk: $\sigma^2 = \sigma_\varepsilon^2 = 1$.

3B. Banging the close under risk aversion with low fix-orders risk: $\sigma^2 = \sigma_\varepsilon^2 = 0.1$.
Figure 4. Testing for a Change in Convexity

4A. Measuring convexity. We calculate (1) the area of the triangle ABC with (2) the area lying between segment AB and the actual average price path. Specifically we measure convexity as the ratio (2)/(1). The actual path shown below is the average end-of-month path for GBP-USD from January, 2008 through December, 2013.

4B. Convexity: Chart shows convexity of the average month-end price path over 3:45-4:00. Data for each observation span the beginning of the sample through the end of December in the year specified. One-minute returns calculated from tick-by-tick data from Reuters Dealing beginning January, 1996, through May, 2013 except EUR which begins January, 1999, and CHF which ends October, 2011.
Table 1. Convexity of pre-fix price path

Table shows convexity of average pre-fix price path for end-month days from January, 2008 through December, 2013 except for CHF, where the sample ends in October, 2011. Marginal significance is based on the bootstrapped distribution of convexity for the average pre-fix path in the earlier part of the sample period. Sample begins February 1996 for all currency pairs but EUR, which begins January, 1999. Data are tick-by-tick interdealer dealer OTC quotes from the Reuters Dealing platform for exchange rates vis-à-vis USD. ** indicates significance at the 5% level.

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