Uncertainty, Financial Frictions and Investment Sensitivity to Cash Flow

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Abstract

This paper introduces a new state variable, uncertainty over business prospects, into the long-lasting debate regarding corporate investment sensitivity to cash flow (ISCF). Theoretically, I show that uncertainty, by generating the real-options value of waiting, helps to unify the two opposing views of Fazzari et al. (1988) and Kaplan and Zingales (1997) regarding how financial frictions affect ISCF. My model predicts that a higher degree of financial frictions directly decreases ISCF. However, as introducing real options causes an indirect, enhancing effect of financial frictions on ISCF, the overall impact could reverse. Moreover, besides changing the part played by financial frictions, uncertainty by itself can reduce ISCF. This is because firms with high uncertainty would want to wait for business prospects to become clear before investing. Empirical work studying U.S. listed firms over the past 30 years confirms these results. Further investigating the Bonus Depreciation policy (that boosted after-tax cash flow) suggests that diminishing uncertainty improves policy effectiveness.

Keywords: investment sensitivity to cash flow, uncertainty, real options, financial frictions, bonus depreciation

JEL Classification: D81, E32, E44, E62, G31, G32, G33, G35, H25

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1 Introduction

Understanding corporate investment is important for macroeconomic study of business cycles. Shocks to uncertainty over future economic prospects and frictions in financial markets have been demonstrated to depress the level of investment. In corporate finance, previous work further investigates the sensitivity of investment to firms’ cash flow, as it assesses the propensity to invest of the private sector. Much of the literature pertains to the relationship between the magnitude of investment sensitivity to cash flow and the degree of financial frictions. Among others, Fazzari et al. (1988) find that the sensitivity is larger for more financially-constrained firms. Kaplan and Zingales (1997) disagree based on the contention that, if anything, the relationship is the opposite. In this deliberation and the literature that follows, the discussion mostly concentrates on frictions in financial markets, this paper instead introduces the role of uncertainty.

I first provide a heterogeneous-agent model in which investment sensitivity to cash flow (ISCF) is positive if and only if there exist financial frictions. Furthermore, the model predicts that whether the magnitude of ISCF increases (or decreases) with the degree of frictions relates to uncertainty and the real-options value of waiting it generates. Specifically, the two opposing views in Fazzari et al. (1988) (henceforth, FHP) and Kaplan and Zingales (1997) (henceforth, KZ) can be understood in an amalgamated way: a higher degree of financial frictions alone decreases ISCF directly; however, through diminishing the value of real options, it can indirectly raise ISCF. In addition, I introduce uncertainty, by itself, as an alternative state variable that explains the magnitude of ISCF. I demonstrate that, controlling for the degree of financial frictions, firms facing higher uncertainty feature smaller ISCF. This is based on firms’ spontaneous waiting motives for business prospects to become more clear before investing. I test the model’s predictions using panel data of U.S. individual firms over the past 30 years and find consistent results. Moreover, I evaluate the disproportionally stimulative effects of the Bonus Depreciation policy (that boosted after-tax cash flow) on investment. Via a differences-in-differences method, I observe that firms

1Two seminal strands of literature that study financial frictions start with Kiyotaki and Moore (1997) on collateral constraints and Bernanke et al. (1999) (BGG) on bankruptcy costs. On the other hand, the canonical uncertainty literature focuses on non-convex adjustment costs of capital and the generated real options. For example, see McDonald and Siegel (1986), Dixit and Pindyck (1994), Abel and Eberly (1995), Abel and Eberly (1996), Bloom (2000), Bloom et al. (2007), Magud (2008), Bloom (2009), Bloom et al. (2014) and others. Various recent work takes into account financial frictions and uncertainty at the same time. Arellano et al. (2012), Christiano et al. (2014), and Gilchrist et al. (2014) adopt the BGG setup of bankruptcy costs, arguing that uncertainty reduces investment by pushing firms closer to default boundaries. Others, like Caggese (2007), Khan and Thomas (2011) and Cui (2014), adopt the setup of collateral constraints investigating how they affect investment (disinvestment) decisions along with real options.
with greater uncertainty respond less via investment to a given policy shock. This finding suggests that although liquidity-provision policy is able to ease financial constraints during downturns by generating positive cash-flow shocks, it is not guaranteed that investment will follow. Heightened uncertainty could blur the stimulative effectiveness.

Modigliani and Miller (1958) suggest that ISCF should be zero in a frictionless financial market, as long as the cash-flow shocks are orthogonal to changes in future investment opportunities. However, previous studies have identified a significantly positive ISCF from data. Fazzari et al. (1988) is one of the earliest of the literature that documents this phenomenon, postulating that it signals the presence of financial frictions. Specifically, for firms facing costly or constrained external financing, investment is below the first-best level. Any positive shock to cash flow boosts investment expenditure by adding to the availability of internal funds. Besides this, they further establish evidence for ISCF being larger for more financially-constrained firms and thereby propose that the degree of financial frictions should raise the magnitude of ISCF. Many other papers afterwards confirm the results in FHP, while many others disagree, including Kaplan and Zingales (1997). The “counterpoints” could be broadly summarized by two critiques. For one, financial frictions are sufficient but not necessary for ISCF being positive; instead, this could be a consequence of failing to control for swings in future investment opportunities. Further, even without this omitted-variable bias, it is premature to posit that a higher degree of frictions leads to a greater value of ISCF. This opposing literature has put forward both theoretical and empirical evidence demonstrating that the relationship should instead be negative. Henceforth, the two critiques will be referred to as the “omitted-variable critique” and the “negative-relation critique” towards the FHP view, respectively. My paper is mostly regarding the latter.

To ease the concern over “omitted-variable critique”, I build a model featuring exogenous cash flow whose innovations do not represent swings in future investment opportunities. It provides a unified means of understanding the “negative-relation critique” and the original FHP view. On the one hand, I show analytically that a higher degree of financial frictions decreases ISCF directly by itself, in line with the “negative-relation” view. This is because more frictions reduce the amount of external funds that an additional dollar of internal funds can lever up; as a result, the total amount of extra investment a firm is able to undertake becomes less. On the other hand, I introduce uncertainty over future business prospects as

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2See, for example, Gilchrist and Himmelberg (1995), Carpenter et al. (1998), Allayannis and Mozumdar (2004), Mizen and Vermeulen (2005), Lorenzoni et al. (2007), Carpenter and Guariglia (2008), McLean et al. (2012), Bond and Söderbom (2013) and Muller et al. (2016).

3See, for example, Cleary (1999), Gomes (2001), Alfi (2003), Cooper and Ejarque (2003), Moyen (2004) and Abel (2015). As well, see also Hubbard (1997), Tirole (2010), and Roberts and Whited (2012) for detailed surveys.
an alternative state variable. Interacting with non-convex capital adjustment costs, such as irreversibility, uncertainty generates the real-options value of waiting that changes the part played by financial frictions. An additional, indirect channel via which financial frictions manage to raise ISCF appears. This is because a worsened financial market diminishes the real-options value of waiting, encouraging firms to exercise investment sooner and thereby increasing ISCF. It is referred as the “real-options-induced-financial channel” in this paper, hence offering a new explanation for the FHP view.

Furthermore, my model predicts that uncertainty by itself diminishes ISCF while controlling for the degree of financial frictions, based on firms’ incentives to wait. This effect is named after the “real-options-uncertainty channel.” To the best of my knowledge, only a few previous investigations have concentrated on uncertainty impacting ISCF, but none of them studied real options’ implications in a structural fashion. [Baum et al. (2009)] introduce financial constraints into an otherwise-standard neoclassical model and motivate a panel-regression framework by linearizing the first-order condition. With this, their theoretical work does not explicitly suggest how and why uncertainty affects ISCF. Moreover, based on the convex adjustment costs in the neoclassical setup, their model is unable to address the role of real options. [Li et al. (2015)] document an empirical pattern whereby the ISCF of corporations in emerging markets is smaller when uncertainty in the U.S. (measured by the CBOE VIX index) is higher. We link this phenomenon to the real-options effect of uncertainty without offering a structural explanation. As a model extension, I further discuss possible precautionary motives caused by uncertainty interacting with bankruptcy costs as in [Bernanke et al. (1999)] and discover qualitatively the same lessening impacts on ISCF. In other words, firms exhibiting smaller ISCF when accompanied by higher uncertainty is robust to both real-options and precautionary effects. In Section 4, I show empirical evidence finding the real-options effect to be more important for listed firms.

Using annual data of more than 10,000 listed firms in the U.S. over the past 30 years, I confirm my model’s predictions regarding how uncertainty and financial frictions change ISCF. The empirical model falls within a panel regression framework with interaction terms. Specifically, I regress investment on cash flow and a full set of control variables, including Tobin’s Q for future investment opportunities. This is to estimate the empirical counterpart of ISCF as the coefficient of cash flow. I further let it depend on the measures of uncertainty and financial frictions. Following [Leahy and Whited (1995)], I calculate each individual firm’s volatility of enterprise values as a proxy for uncertainty. To mitigate the potential reverse

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4 Business prospects refer to either productivity or demand conditions, or both. As in [Bloom (2009)], I do not separately distinguish these factors; instead, I assume they simultaneously determine a firm’s future profitability.
causality between volatility and investment, I take the cross-sectional weighted average of
the firm-level volatility year-by-year; the resulting market-wise volatility is treated as exoge-
nous towards individual firms. On the other hand, I use the excess bond premium (EBP) in
Gilchrist and Zakrajšek (2012) as an exogenous measure of financial frictions in general, with
a widened EBP indicating more frictions. Corresponding to the “omitted-variable critique”
in theory, when Tobin’s Q is measured with errors empirically, cash flow could become endo-
dogenous as it is informative to investment opportunities. To deal with this issue, I employ
the method in Erickson et al. (2014) and Erickson et al. (2016) to correct for measurement
errors. When combining it with the use of exogenous volatility and EBP, identification is
accomplished via a measurement-errors-correcting differences-in-differences estimator. This
way of estimating, however, ignores the cross-sectional variation in firms’ individual volatil-
ity and exposure to financial frictions. Inspired by Hennessy and Whited (2007), I use the
size (market value of assets) as an additional proxy for financial frictions; smaller firms are
assumed to face more frictions. I employ the generalized method of moments (GMM) in
Arellano and Bond (1991) as an alternative means of estimating the empirical model to in-
vestigate the impacts of individual volatility and size. All firm-level variables are treated as
endogenous variables and lagged values are utilized as instruments.

Estimation results find that firms have smaller regression coefficients of cash flow when
facing higher volatility, consistent with the “real-options-uncertainty channel” that uncer-
tainty diminishes ISCF. On the other hand, in years with generally tighter financial markets
(indicated by widened EBP), and for firms with smaller sizes (that potentially face greater
financial frictions), the regression coefficients of cash flow are greater. This is in line with the
“real-options-induced-financial channel” that a higher degree of financial frictions enhances
ISCF. The results are significant in economic terms: when individual volatility and EBP
move from their 10-percentile values in a sample to the corresponding medians, the esti-
mated ISCF changes by 23% and 54%, respectively, though the role of size is less prominent.

Studying real options and investment in financial markets with frictions has real-world
implications. For instance, it is useful for evaluating the effectiveness of stimulative policy
during downturns. Considering that recessions are typically periods of both heightened un-
certainty and tighter financial markets, policy that seeks to provide liquidity to financially
constrained firms could be ineffective in finally boosting investment if real options hold firms

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5In any particular year, a firm’s volatility of enterprise values equals the equity volatility obtained from
daily stock returns within this year adjusted by the existing capital structure. The weighted-average volatility
is calculated cross-sectionally using each firm’s market value of assets as the weight.

6This is documented by, among others, Erickson and Whited (2000), Erickson and Whited (2002), Erick-

7I also include real sales growth as an additional proxy to further ease the concern.
back from investing. To illustrate this point, I study the Bonus Depreciation policy in the periods from 2001 to 2004 and from 2008 to 2011, which allowed firms to accelerate the depreciation of equipment investment. It therefore reduced tax payments and boosted after-tax cash flow. I follow Zwick and Mahon (2017) in terms of utilizing industry variation to identify the effects of this policy. Beyond their work, I further show that a given policy shock has disproportionally smaller impacts on investment for firms facing greater uncertainty.\footnote{This result stresses the importance of uncertainty in determining real-world policy effectiveness in encouraging investment.} This result stresses the importance of uncertainty in determining real-world policy effectiveness in encouraging investment.\footnote{See also Dobridge (2016) for a similar finding studying the policy regarding five-year carryback of net operating losses.}

In the rest of the paper, I first illustrate simple empirical evidence via aggregated time series suggesting that ISCF is positive and varies with market-wise uncertainty and financial conditions in Section 2. I then present a heterogeneous-agent model with occasionally-binding constraints in Section 3 to address the “real-options-uncertainty channel” and the “real-options-induced-financial channel” analytically. I empirically test my model’s predictions using data of U.S. listed firms in Section 4. Subsequently, I include policy analyses in Section 5. Finally, I conclude the paper in Section 6.

## 2 Simple Empirical Evidence

In this section, I show simple empirical evidence regarding ISCF being positive and time-varying, reliant upon market-wise uncertainty and financial frictions, while leaving more rigorous empirical work to Section 4. I collect data of more than 10,000 U.S. listed firms from the COMPUSTAT North America database during the time period of 1984-2015. For each given year, I estimate the market-wise ISCF by cross-sectionally regressing investment (measured by capital expenditure as a ratio of the existing capital stock) on cash flow (measured by income plus depreciation as a ratio of the existing capital stock), while controlling for investment opportunities by lagged Tobin’s Q and real sales growth, as well as the potential debt overhang through the existing net leverage.\footnote{I must concede that the Bonus Depreciation policy has much richer effects than generating more cash flow to help firms ease financial constraints. See, among others, House and Shapiro (2008) and Zwick and Mahon (2017). I leave more sophisticated identification to disentangle each of them to future work.} In doing so, I treat firms with identical Q, sales growth and capital structure identically to garner year-by-year estimates of the market-wise ISCF.\footnote{See Section 4 for details with respect to data description and variable definitions.}

\footnote{See McLean and Zhao (2014) for a similar methodology.}
Figure 1 portrays the estimated ISCF with a solid line; the bars over it indicate the 90-percent confidence intervals. In general, ISCF is significantly positive and displays variation year over year. As a comparison, I include a dashed line that presents the dynamics of uncertainty measured by the weighted-average volatility of firms, and a dotted line for the excess bond premium (EBP) from Gilchrist and Zakrašek (2012) that measures investors’ risk aversion. A higher value of EBP indicates that, in general, investors in financial markets are more risk averse and thus impose stricter financial constraints on firms.

A simple regression analysis of ISCF on both average volatility and EBP simultaneously studies their correlations – volatility has a coefficient equal to (-0.100) with a p-value 0.017, indicating a negative correlation with ISCF after controlling for financial frictions. In contrast, EBP has a coefficient equal to 0.010 with a p-value 0.087, referring to a positive correlation. To deal with the “omitted-variable critique” that Tobin’s Q may contain measurement errors, I repeat the whole process with the measurement-error-correcting estimator from Erickson et al. (2014) and Erickson et al. (2016) to obtain ISCF. The correlations become even stronger – when regressing ISCF on both volatility and EBP simultaneously, the coefficient of volatility is (-0.100) with the p-value being 0.007; the coefficient of EBP is 0.016 with the p-value equal to 0.004. In conclusion, I posit that heightened uncertainty is associated with smaller ISCF and tighter financial markets with greater ISCF values.

Besides the general correlations, certain particular years are worth noting. First, from the aftermath of the Asian Financial Crisis to the era of the Internet Bubble (1998-2000), ISCF was insignificant from zero though EBP was high. The volatility spike then could possibly explain this. Secondly, during the Great Recession (2008-2009), both volatility and EBP were near record highs while ISCF was not. In 2010, however, ISCF became larger when both volatility and EBP dropped drastically. Such phenomena are consistent with uncertainty diminishing ISCF. Another two periods in which ISCF was insignificant from zero are 1992 and 2013-2015. The former was right after the early 1990s recession. Throughout 2013-2015, however, EBP was mostly in a negative region, suggesting very

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12 The magnitude of estimated ISCF here is smaller than those values in the previous literature. One reason is that I include more control variables, like sales growth, that may reduce the explaining power of cash flow. It also protects my estimates better from the “omitted-variable critique” as sales growth can capture future investment opportunities. Another factor is that my sample includes data from the most recent years when firms start writing off large amounts of research & development expenses from income. This leads to a significant portion of observations in my sample with negative cash flow, thereby reducing the estimates of ISCF. After adding back R&D expenses, I obtain estimates of ISCF much closer to those in previous studies (see Section 4 for details). I thank Bruce Petersen for offering this constructive suggestion.

13 See Section 4 for the construction of weighted-average volatility and the justification for using EBP to measure financial frictions.

14 Robust standard errors are adopted in this regression. The R-squared value is 18%.

15 Robust standard errors are adopted. The R-squared value increases to 24%.
accommodative financial markets leading to small ISCF. In previous studies, Allayannis and Mozumdar (2004), Ascioglu et al. (2008), Ağca and Mozumdar (2008), Brown and Petersen (2009), Chen and Chen (2012) and McLean and Zhao (2014) also document ISCF being time-varying, but none of them put forward uncertainty and financial frictions leading to such dynamics, which is the focus of this paper. Furthermore, I intend to explain not only the time-series dynamics, but also the cross-sectional distributions of ISCF; namely, I allow heterogeneous firms to have different magnitudes of ISCF even in a same year.

3 A Model

In the previous section, I estimate ISCF via a linear regression of investment on cash flow and show its negative correlation with uncertainty and positive correlation with financial frictions. In this section, I present a theoretical model in which these results will present and be causal. Studying ISCF in a structural way has, among others, two significant advantages. For one, being able to set cash flow orthogonal to future investment opportunities eliminates the information content. Consequently, the ISCF identified in my model is immune to the “omitted-variable critique.” As well, with a structural model, I can separately determine the two opposing impacts of financial frictions on ISCF, corresponding to the two different views in the FHP-KZ debate. In contrast, a reduced-form study can only detect the overall effect, depending on which single impact dominates.

3.1 A Snapshot

I present a three-period, stochastic, heterogeneous-agent model with a number of crucial aspects deviating from the conventional neoclassical framework. First, capital adjustment features an irreversibility constraint – once capital goods are installed, they cannot be uninstalled unless the whole capital stock is sold (at a discounted price). This non-convex adjustment cost, along with uncertainty over future marginal profits, generates a significant real-options value of waiting that restricts back investment. In contrast, the commonly-

\[^{16}\text{Despite I emphasizing the role of uncertainty and financial frictions, they are, by no means, the only factors that matter for ISCF. McLean and Zhao (2014) instead use recession dummies and consumer confidence indices to explain the time-series ISCF and obtain a decent empirical fit. Their focus on recession dummies might account for the low ISCF in 1992. Moreover, Ai et al. (2016) propose moral hazard and managerial compensations to be crucial factors. Limited by the scope of this paper, I neglect these important influences.}\]

\[^{17}\text{See Caggese (2007) for a similar setup. Other types of non-convex adjustment costs will achieve the same goal, like partial irreversibility. I stick to total irreversibility because disinvestment (where firms partially}
adopted convex adjustment costs in the neoclassical literature are neglected in this paper.\footnote{In fact, the structural estimation in Bloom (2009) shows that these costs are indeed small.}

Secondly, financial markets have frictions in the sense that borrowing requires collateral and issuing equity is disallowed.\footnote{As a consequence of the pecking-order theory in Myers and Majluf (1984), equity finance should be much more costly than debt finance, which is why it is omitted here assuming the cost is infinite. However, recent evidence suggests that equity finance could be important for firms facing borrowing constraints; for example, see Brown and Petersen (2009) and Brown et al. (2009). Ignoring equity finance thus tends to make my model over-estimate the effects of borrowing constraints quantitatively. Thanks to the closed-form solution, the essence of my model does not rely on its quantitative performance.} Collateral borrowing imposes a hair-cut ratio \((1 - \theta)\), with a lower \(\theta\) (between 0 and 1) indicating more frictions in financial markets.\footnote{In a model extension, I change the type of financial frictions from collateral constraints to bankruptcy costs and allow for an endogenous default premium. The model’s results remain robust.} A reduction in \(\theta\) is hence modelled as a financial shock that tightens borrowing constraints, as in Quadrini (2011) and Jermann and Quadrini (2012). Hart and Moore (1994) provide a micro-foundation for the hair-cut — it represents the disagreement between borrowers and investors in the value of collateral. Geanakoplos (2010) further links it to the investors’ risk aversion, with more risk-averse investors requiring a higher hair-cut ratio (lower \(\theta\)).\footnote{An alternative cause for hair-cut pertains to agency problems. For instance, if capital goods worth \$K in market value are viewed as \$\theta K from the perspective of investors (creditors) when pledged as collateral, the hair-cut ratio is then \((1 - \theta)\).} To endogenize the hair-cut ratio is beyond the scope of this paper; I instead choose to take \(\theta\) as given and its changes as exogenous financial shocks.\footnote{Specifically in this paper, hair-cut is defined as the difference between an asset’s market value and its collateral value. For instance, if capital goods worth \$K in market value are viewed as \$\theta K from the perspective of investors (creditors) when pledged as collateral, the hair-cut ratio is then \((1 - \theta)\).}

Another feature of my model is with regards to extensive-margin aggregation. In particular, the model is constructed with respect to individual production units rather than firms. A firm, on the other hand, incorporates a group of heterogeneous production units. This design is to overcome a disconnection between theory and data. In the theory of real options, under certain circumstances, it is optimal for firms to choose zero investment rather than any other positive number. However, in the data of consolidated firms, zero investment is almost non-existent. Bloom (2009) documents that even though survey data demonstrates that zeros are common at the unit level, extensive-margin aggregation over different units within reverse the investment) is not the focal point of this paper. Another alternative is fixed costs; including them in the model will further strengthen the effects of real options, but with no qualitative difference.

\footnote{An alternative cause for hair-cut pertains to agency problems. For instance, if capital goods worth \$K in market value are viewed as \$\theta K from the perspective of investors (creditors) when pledged as collateral, the hair-cut ratio is then \((1 - \theta)\).}
a single firm makes them impossible to detect in a consolidated firm’s financial statements. Considering that my sample consists of listed firms that only report their capital expenditures for a consolidated account, I assume investment decisions are made at the unit-level and aggregate the unit-level optimal policy cross-sectionally within a firm to yield firm-level policy rules. A similar procedure is also carried out in [Bloom (2009)](#).

This manner of modelling has both advantages and disadvantages. On the one hand, besides its capacity for justifying the missing zero investment in the data, having a unit-level model ensures the exogeneity of cash flow – the cash flow a production unit receives is not from its own production, but is assigned by the CFO of the parent firm. In other words, the CFO collects income from all production units and redistributes the money. Different units take the money they receive as exogenous when making investment decisions. This process mitigates the omitted-variable bias because a high cash-flow shock has less to do with the production unit’s own profitability, and more to do with the CFO’s optimization. For tractability, I take the redistribution process as exogenous and discuss how this simplification potentially influences my model’s implications. On the other hand, generating a unit-level model but lacking unit-level data leaves me unable to identify the model’s deep parameters. This encourages me to work with a three-period model for which I can attain a closed-form solution without relying heavily on numerical calibration. I leave the extension to an infinite-horizon model (or a life-cycle model) for future research.  

Before delving into further detail, I shall summarize my model’s main results and intuitions. Each consolidated firm incorporates a group of individual production units that possess different variances of future profitability. They can either choose to engage in immediate investment, or to postpone investment until they observe the profitability-realizing next period. The trade-off is with immediate investment, the unit expands capital stock and therefore can benefit from extra production next period; however, as profitability is uncertain, this amount of extra production could be too low to be efficient ex post. However, if a production unit chooses to delay investment, next period it will invest only if it observes the realized profitability being high. As such, it foregoes the extra amount of production but gains an option to avoid inefficient investment. I show analytically that this decision differs across production units depending on their own individual variances - those with low variances choose to exercise investment immediately; those with high variances opt to postpone this decision; hereafter, they are known as investing units and waiting units, respectively.

Financial frictions have different influences on these two types of units. For investing

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24 In principle, keeping with a three-period model should under-estimate the real-options effect because the firm can only delay investment for one period before it exits the market; the benefits of delaying are limited.
units, collateral constraints bind leaving investment sensitive to cash flow, i.e., investing units have positive ISCF. Waiting units, meanwhile, have zero investment regardless of their cash flow; the ISCF is thereby zero. Extensive-margin aggregation makes the firm-level ISCF equivalent to the weighted-average of the investing units’ positive ISCF and waiting units’ zero ISCF. The resulting firm-level ISCF is positive as long as there is at least one unit that invests. Without financial frictions, however, even for investing units, the amount of investment is independent of cash flow. As a result, the ISCF for both types of units and the consolidated firm is zero.

An uncertainty shock to the firm increases the variances of all production units. Therefore, more of them will become waiting units and the consolidated firm’s ISCF will be smaller. This is the “real-options-uncertainty channel.” A financial shock, rather, has two competing forces. On the one hand, it directly decreases the consolidated firm’s ISCF by decreasing the investing unit’s ISCF. This is because by raising the hair-cut ratio, a financial shock reduces the external funds that an additional internal dollar can lever up. The amount of additions to investment hence becomes less. On the other hand, it diminishes the real-options value of waiting because financing conditions in the future deteriorate at the same time. This accelerates exercising investment, encouraging a greater proportion of production units to invest immediately. The consolidated firm’s ISCF thus rises. This is the “real-options-induced-financial channel.”

3.2 Setup

Each firm \( (i) \) consists of \( J \) risk-neutral production units \( \{i_j\}_{j=1}^J \) that exist for three periods. They are heterogeneous both \textit{ex ante} and \textit{ex post}. At the beginning of period 1, unit \( (i_j) \) is given a stochastic cash flow, \( w_{ij} \), from the parent firm, and, at the same time, it learns about the distribution from which it will draw the profitability of operation that lasts in the following two periods; the realized profitability is referred to as \( z_{ij} \). Specifically, assume all units’ future profitability is distributed log-normally and subject to a common mean, \( \bar{z}_i \), but different variances: \( \log z_{ij} \sim N(\log \bar{z}_i - \sigma_{ij}^2/2, \sigma_{ij}^2) \). Furthermore, assume there is a firm-level common factor, \( \sigma_i \), driving all the \( \{\sigma_{ij}\}_{j=1}^J \) through a set of individual-specific factor-loading \( \{\gamma_{ij}\}_{j=1}^J \), i.e., \( \sigma_{ij} = \gamma_{ij} \cdot \sigma_i \) for all \( j \). Call \( \sigma_i \) the firm-level uncertainty. Any increase in \( \sigma_i \) shifts the whole distribution of \( \sigma_{ij} \) to the right. Whether two production units in the same firm have correlated or independent \( z_{ij} \) does not affect the model’s results. This assumption is motivated by Bloom et al. (2014) who note that unit-level uncertainty (measured by TFP shocks) is significantly correlated with the parent firm’s stock volatility; the latter is a widely-adopted
knowledge after being realized. Denote the cross-sectional distributions of \( \{\gamma_{ij}\}_{j=1}^{J} \), \( \{\sigma_{ij}\}_{j=1}^{J} \), and \( \{w_{ij}\}_{j=1}^{J} \) by \( F_{i,\gamma} \), \( F_{i,\sigma} \), and \( F_{i,w} \), respectively.\(^{27}\) For brevity, in the following context, I suppress the subscript \( (i) \), i.e., use \( (j) \) to denote \( (i,j) \). I further employ the subscripts \((1j),(2j),(3j)\) to represent the choice variables in each period 1, 2, and 3 or the stock variables at the beginning of each period for the unit \((j)\).

Capital adjustment features an irreversibility constraint whereby disinvestment is not allowed unless the whole capital stock is sold at a discounted price. Suppose the purchasing price of capital is normalized to 1; the selling price (also called the “liquidation value”) is \( \bar{q} \) between 0 and 1. Besides the irreversibility, there is no other cost to adjust capital. Capital stock will thus spike to the constrained-optimal level during any adjustment process.

Both risk-free savings and borrowings are allowed at a gross interest rate, \( R_f \), per period. Particularly, there exists a two-period bond in period 1 and a one-period bond in period 2 that each production unit can buy or sell. Borrowings are risk-free because of collateral requirements as in Kiyotaki and Moore \((1997)\). Capital goods and financial assets (i.e., savings) could serve as collateral while the former incurs a hair-cut ratio of \((1 - \bar{\theta})\) between 0 and 1.\(^{28}\) On the contrary, there is no hair-cut if financial assets are used as collateral.\(^{29}\)

In period 1, the production unit \((j)\) has an initial capital stock, \( k_{1j} \). It distributes its exogenous cash flow, \( w_{j} \), between dividends, \( d_{1j} \), and investment, \( \iota_{1j} \). It is also able to save in the risk-free bond \((b_{j}^{L} < 0)\) or borrow external debt \((b_{j}^{L} > 0)\) limited by a fraction, \( \bar{\theta} \), of the capital’s liquidation value. The superscript “L” stands for “long”, indicating that the savings or borrowings are in two periods. There is no production or capital depreciation in the first period to simplify the algebra. Formally, see equations \((1)\) to \((5)\). Equations \((1)\) and \((5)\) are the budget constraint and the irreversibility constraint, respectively. Equation \((3)\) is the collateral constraint for two-period debt. Equation \((2)\) describes the capital accumulation process and equation \((4)\) rules out external equity finance.

---

\(^{27}\)Here, I do not impose any restrictions on the relationship between \( F_{i,\sigma} \) and \( F_{i,w} \). In other words, this modelling setup is neutral to any internal competition for resources across different units. In particular, one could imagine that it is after the realization of \( F_{i,\sigma} \) that the CFO of the parent firm redistributes resources so that \( F_{i,w} \) is determined by uncertainty. While modelling the CFO’s optimization is beyond the scope of this paper, I discuss how it could qualitatively affect my empirical predictions at the end of this section.

\(^{28}\)The notation suggests that \( \bar{\theta} \) is common to all production units within a single firm (subscript \( i \) is suppressed). I implicitly assume that the degree of financial frictions depends on firm-level information rather than that from the unit-level.

\(^{29}\)For example, a production unit could save at \( R_f^{2} \) in the first period and borrow against this two-period bond in the second period at \( R_f \); effectively, the unit saves in a one-period bond in period 1 at a rate equal to \( \frac{R_f^{2}}{R_f} = R_f \). Therefore, the setup where there is only a two-period bond in period 1 and a one-period bond in period 2 does not twist the optimal policy.
\[ w_j + b^L_j = d_{1j} + \tau_{1j} \]  
\[ k_{2j} = k_{1j} + \tau_{1j} \]  
\[ R^2_j b^L_j \leq \bar{\theta}q(1 - \delta)^2 k_{2j} \]  
\[ d_{1j} \geq 0 \]  
\[ \tau_{1j} \geq 0 \]

At the beginning of period 2, the profitability of operation, \( z_j \), realizes. All uncertainty resolves in the sense that \( z_j \) will be carried over to the last period. The unit operates and generates profits, \( z_j k_{2j} \), but in the form of account receivables – it cannot obtain the proceeds until the last period. This is a technical assumption to mitigate the Oi-Hartman-Abel effect. Unit \( j \) can choose to invest for the second time. Borrowings and savings are allowed in the one-period bond, \( b^S_j \), as long as the collateral constraint is satisfied, where the superscripted “S” signifies short-term debt. Formally,

\[ b^S_j = d_{2j} + \tau_{2j} \]  
\[ k_{3j} = (1 - \delta)k_{2j} + \tau_{2j} \]  
\[ R^2_j b^L_j + R_f b^S_j \leq \bar{\theta}q(1 - \delta)k_{3j} \]  
\[ d_{2j} \geq 0 \]  
\[ \tau_{2j} \geq 0 \]

\[ ^{30}\text{Specifically, without such a design, if the unit invested in period 1 and received high } z_j \text{ in period 2, it will reinvest the proceeds and generate a convex term of } z_j^2 \text{ in the marginal value of capital. Such convexity will lead to a Jensen’s inequality effect when } z_j \text{ is uncertain – the greater the variance of } z_j, \text{ the higher the expected marginal value of capital in period 1 before the investment decision. Therefore, uncertainty encourages investment, opposing the commonly-accepted empirical evidence, e.g., } [\text{Bloom}(2009)]. \text{ The reason is that in a three-period model, benefits from investing (and reinvesting) is of order-two of } z_j, \text{ but benefits of waiting is only of, at most, order-one of } z_j \text{ (because when the firm waits during the first period and invests in the second period, the benefit is in terms of } R_f z_j). \text{ This effect of convexity is in a similar spirit as } [\text{Oi}(1961), \text{Hartman}(1972), \text{and Abel}(1983)]. \text{ It becomes smaller when the number of periods become larger and will no longer dominate finally. In this paper, in order to obtain a closed-form solution, I instead assume that the proceeds are in the form of account receivables and thus cannot be reinvested. In fact, given the extensive use of trade credit of U.S. firms, this is not a strong assumption. For example, in the COMPUSTAT U.S. sample, the flow of account receivables takes up approximately 25\% of annual cash flow for the median firm, and the number of days sales’ is nearly one-year and a half.}^{31}\]
In period 3, $z_j$ is carried over from period 2. As this is the last period, the production unit operates to generate profits but no longer invests; it instead liquidates the total capital stock. It does not save or issue new debt; account receivables and savings in previous periods, if any, pay back. All proceeds less existing debt will be paid out as dividends. Formally,

$$d_{3j} = z_j(k_{2j} + k_{3j}) + \bar{q}(1 - \delta)k_{3j} - R_f^2b_j^L - R_f b_j^S$$  

$$d_{3j} \geq 0$$

Each production unit maximizes $E[d_{1j} + \beta d_{2j} + \beta^2 d_{3j}]$ subject to (1)–(12), where $\beta$ denotes the time preference common to all units.

### 3.3 Analytical Solution

This section solves the model analytically through backward induction. The last period is trivial – the production unit pays all remaining resources out as dividends. Assumption 1 pins down the solution of period 2 as described by Proposition 1.

**Assumption 1.**

$$\beta R_f < 1$$

**Proposition 1.** Under Assumption 1, in period 2, if the realized $z_j$ is sufficiently high, the production unit will invest all financial resources, including both internal funds and external credit; otherwise, it will maintain $k_{3j} = (1 - \delta)k_{2j}$ and pays out other resources. Formally,

<table>
<thead>
<tr>
<th>If $z_j \leq \psi_1$,</th>
<th>If $z_j &gt; \psi_1$,</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{3j} = (1 - \delta)k_{2j}$</td>
<td>$k_{3j} = \frac{(1 - \delta)k_{2j} - R_f b_j^L}{1 - \frac{\bar{q}(1 - \delta)}{R_f}}$</td>
</tr>
<tr>
<td>$b_j^S = \frac{1}{R_f}[\bar{q}(1 - \delta)^2k_{2j} - R_f^2b_j^L]$</td>
<td>$b_j^S = \frac{1}{R_f}[\bar{q}(1 - \delta)k_{3j} - R_f^2b_j^L]$</td>
</tr>
<tr>
<td>$d_{2j} = b_j^S$</td>
<td>$d_{2j} = 0$</td>
</tr>
</tbody>
</table>

where

$$\psi_1 = \frac{1}{\beta}[1 - \beta \bar{q}(1 - \delta) - (\frac{1}{R_f} - \beta)\bar{q}(1 - \delta)]$$

The value function is given by:

$$V_2(j, b_j^L, z_j) = \begin{cases} 
  [\beta(2 - \delta)z_j + (1 - \delta)(1 - \beta\psi_1)]k_{2j} - R_f b_j^L & \text{if } z_j \leq \psi_1 \\
  [1 - \delta + \beta z_j + \frac{(1 - \delta)\bar{q}(1 - \delta)}{1 - \frac{\bar{q}(1 - \delta)}{R_f}}]k_{2j} - [1 + \frac{\beta(z_j - \psi_1)}{1 - \frac{\bar{q}(1 - \delta)}{R_f}}]R_f b_j^L & \text{if } z_j > \psi_1 
\end{cases}$$  

$$\text{(13)}$$
The production unit distributes limited resources among dividends, savings, and investment. The impatience condition in Assumption 1 states that the production unit prefers dividends to savings, so the borrowings constraint (8) maintains binding. Comparing $z_j$ to $\psi_1$ is, in fact, comparing $[\beta z_j + \beta \bar{q}(1 - \delta) + (\frac{1}{R_f} - \beta)\bar{\theta}\bar{q}(1 - \delta)]$ to 1; the latter represents the price of capital goods in terms of dividends consumption. The former summarizes the benefits of investing – the first term is the extra production, the second term is the liquidation value of the capital goods after depreciation, and the last term instead indicates the utility gains from impatience, wherein the production unit can borrow and consume dividends using the additional capital goods as collateral. Whether to invest therefore relies on whether this whole term exceeds 1. Conditional upon investing, the amount of investment is proportional to the production unit’s net worth because of constant returns to scale, $((1 - \delta)k_{2j} - R_f b^L_j)$. The denominator, $(1 - \frac{\bar{\theta}(1 - \delta)}{R_f})$, indicates the down-payment requirement. If not investing, the production unit pays out all the available resources, letting capital goods depreciate.

**Assumption 2.**

\[
\bar{z} > \max\{\psi_1, \psi_2\}
\]

\[
\psi_2 = \psi_1 + \frac{1}{\beta}[\frac{1}{\beta R_f} - 1][1 - \frac{\bar{\theta}(1 - \delta)}{R_f}]
\]

**Proposition 2.** Under Assumption 2, in period 1 if the expected marginal value of capital is greater than the expected marginal value of savings, the production unit will invest all resources including both internal funds and external credit; otherwise, it will save internal funds in the two-period bond, i.e. $b^L_j < 0$. Formally,

<table>
<thead>
<tr>
<th>If $x_{kj} \leq (-x_{bj})$</th>
<th>If $x_{kj} &gt; (-x_{bj})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{2j} = k_{1j}$</td>
<td>$k_{2j} = \frac{w_j + k_{1j}}{1 - \frac{\bar{\theta}(1 - \delta)}{R_f^2}}$</td>
</tr>
<tr>
<td>$b^L_j = (-w_j)$</td>
<td>$b^L_j = \frac{\bar{\theta}(1 - \delta)^2}{R_f^2}k_{2j}$</td>
</tr>
<tr>
<td>$d_{1j} = 0$</td>
<td>$d_{1j} = 0$</td>
</tr>
</tbody>
</table>

where

\[
x_{kj} = \mathbb{E}[\frac{\partial V_2}{\partial k_{2j}}(k_{2j}, b^L_j, z_j)]
\]

\[
x_{bj} = \mathbb{E}[\frac{\partial V_2}{\partial b^L_j}(k_{2j}, b^L_j, z_j)]
\]

This expression is common in the literature of collateral constraints. Specifically, for each unit of capital goods that is worth 1, only an amount equal to $\frac{\bar{\theta}(1 - \delta)}{R_f}$ is allowed to be paid by external funds; the rest has to be paid by internal funds as down payments.
Under Assumption 2, Proposition 2 summarizes the optimal policy in period 1. I use $x_{kj}$ to denote the expected marginal value of capital. It is indeed comparable to the marginal $q$ in the neoclassical literature. \(^{33}\) I use $x_{bj}$ to denote the expected marginal value of debt. Conditional on debt being treated as negative savings, $(-x_{bj})$ is the expected marginal value of savings, which can be referred to as the real-options value of waiting. Assumption 2 is derived from $\beta(-x_{bj}) > 1$ for all $j$, which is to compare the expected marginal value of savings with the value of dividends. Specifically, as long as the expected profitability $\bar{z}$ is high enough, the production unit is willing to save rather than to consume dividends conditional on not investing. It is because by saving the production unit keeps an option to invest in period 2 and a chance to benefit from high returns in production (if $z_j$ turns out to be high). As a consequence, the optimal policy is to choose between investing and saving, which is determined by comparing $x_{kj}$ and $(-x_{bj})$ based on the first-order homogeneity of the value function. If $x_{kj}$ is greater, the production unit will invest all financial resources rendering the borrowing constraint binding. The amount of new capital equals to the ratio between the production unit’s net worth $(w_j + k_{1j})$ and the down-payment requirement. Otherwise, the production unit will wait by saving all resources in the two-period bond.

**Proposition 3.** $(x_{kj} + x_{bj})$ is a decreasing function of $\sigma_j$.

Proposition 3 shows that the investment determinant, $(x_{kj} + x_{bj})$, decreases with the individual variance, $\sigma_j$. Specifically, the production unit tends to wait by saving when the variance is large. This is because by saving, the unit only invests after observing high $z_j$ ex post. It thereby generates a marginal value function of saving convex in $z_j$. Greater variance therefore raises the expected marginal value of savings $(-x_{bj})$ from a Jensen’s inequality effect, which decreases $(x_{kj} + x_{bj})$. Figure 2 and Figure 3 depict the case graphically under standard calibration in Table 1. Thanks to the closed-form solution, Proposition 3 does not rely on these numerically set parameters as long as Assumption 1 and 2 are satisfied.

**Assumption 3.**

$$\max\{\psi_1, \psi_3\} < \bar{z} < \psi_4$$

$$\psi_3 = (R_f - (1 - \delta))(1 - \frac{\beta \psi_1}{1 - \frac{R_f(1 - \delta)}{R_f}})$$

$$\psi_4 = \frac{R_f - (1 - \delta) + \beta \psi_1 (1 - \delta)}{\beta [1 + \frac{1 - \delta}{R_f} (1 - \delta - R_f)]}$$

\(^{33}\)For example, see Hayashi (1982) and Abel (1983).
\[ \beta > (1 - \delta) \]
\[ R_f < (1 - \delta) + (1 - \bar{q}(1 - \delta)) \]

**Proposition 4.** Under Assumption 3, there exists a unique root, \( \sigma^* \), for \( (x_{kj} + x_{bj})(\sigma_j) = 0 \). For any production unit, \( j \), if \( \sigma_j < \sigma^* \), then \( x_{kj} > -x_{bj} \); if \( \sigma_j > \sigma^* \), then \( x_{kj} < -x_{bj} \).\(^{34}\)

Given that \( (x_{kj} + x_{bj}) \) continuously declines with \( \sigma_j \), the existence and uniqueness of \( \sigma^* \) comes directly from the Intermediate Value Theorem. It requires \( (x_{kj} + x_{bj}) \) to be positive at \( \sigma_j = 0 \) and negative at \( \sigma_j = +\infty \). Assumption 3 ensures this – \( \bar{z} \) must be in a moderate region. If \( \bar{z} \) is too small, \( (x_{kj} + x_{bj}) \) will always be negative – the production unit will always opt to wait because of the expectation that the benefits of investing are too minimal to overcome real options. In contrast, if \( \bar{z} \) is too large, \( (x_{kj} + x_{bj}) \) will always be positive (the production unit will always choose to invest immediately). The other two conditions in Assumption 3 are sufficient (not necessary) to make a choice of \( \bar{z} \) exist, i.e., \( \max\{\psi_1, \psi_2, \psi_3\} < \psi_4 \).\(^{35}\) If the production unit is too impatient, it will never choose to wait because forgoing near-future profits is just too costly. Moreover, if saving is too profitable, the production unit will always choose to save (wait) but never invest.

### 3.4 Investment Sensitivity to Cash Flow

The optimal investment policy under uncertainty (i.e., in period 1) has direct implications in ISCF. Specifically, the unit-level ISCF is defined as the partial derivative of investment to cash flow, with both terms normalized by the existing capital stock, namely \( \frac{\partial \iota_j}{\partial w_j/k_1} \). Taking Propositions 2 and 4 together:

\[
ISCF_j = \begin{cases} 
0 & \text{if } \sigma_j > \sigma^* \\
-\frac{1}{\bar{q}(1 - \delta)^2} & \text{if } \sigma_j < \sigma^* 
\end{cases}
\]

(14)

Depending on their individual variances, production units can be classified into two groups; they have different investment behavior and display distinct ISCF. For those with variances greater than \( \sigma^* \), investment is zero regardless of cash flow; ISCF is therefore zero. For those possessing variances smaller than \( \sigma^* \), in contrast, investment is sensitive to cash flow (ISCF being positive) because of binding collateral constraints. It is straightforward

\(^{34}\)\( \sigma^* \) is specific to a particular consolidated firm. It is common to all production units within this firm. Again, my notation suppresses the firm subscript \( i \) here.

\(^{35}\)Assumption 2 has to be satisfied simultaneously; hence, there is a \( \psi_2 \).
from (14) that when $\bar{\theta}$ is set to $+\infty$, namely when borrowing constraints are removed, ISCF is zero for both types of units. This evidence proves that in my model, ISCF being positive is a pure result of financial frictions. As such, it is immune to the “omitted-variable critique.”

For the firm-level ISCF, consider all the production units that belong to this firm, $\{j\}_{j=1}^J$. Their ISCF is defined by (14), with $\sigma^*$ denoting the investment threshold specific to this firm, common to all $j$. Employ $\sigma$ to denote the firm-level uncertainty. Suppose an individual unit, $j$, has a variance, $\sigma_j = \gamma_j \sigma$. Based on Propositions 2 and 4, define a set of investing units by $\mathcal{I} = \{j|\gamma_j < \frac{\sigma^*}{\sigma}\}$. Aggregated firm-level investment is thus $\iota_1 = \Sigma_{\mathcal{I}} \iota_1 j$; aggregated cash flow is $w = \Sigma_{j=1}^\prime w_j$; aggregated initial capital stock is $k_1 = \Sigma_{j=1}^J k_{1j}$. The firm-level ISCF, $\frac{\partial \iota_1 / k_1}{\partial w / k_1}$, is thus given by the following equation (16) – it is equal to a ratio between the share of cash flow in the hands of investing production units and the down-payment requirement imposed by the collateral constraint in period 1. Effectively, it is the weighted average of the two distinct unit-level ISCF values in (14), with unit-level cash flow serving as the weight.

$$\frac{\iota_1 / k_1}{1 - \frac{\bar{\theta} (1-\delta)^2}{R_f^2}} \left(\Sigma_{\mathcal{I}} \frac{w_j}{w} + \frac{\bar{\theta} (1-\delta)^2}{R_f^2} \left(\Sigma_{\mathcal{I}} \frac{k_{1j}}{k_1}\right)\right)$$

(15)

$$ISCF = \frac{\Sigma_{\mathcal{I}} \frac{w_j}{w}}{1 - \frac{\bar{\theta} (1-\delta)^2}{R_f^2}}$$

(16)

Next, I show how an increase in the firm-level uncertainty ($\sigma$) could alter (16), in particular the “real-options-uncertainty channel.” Suppose $\sigma$ rises and suppose temporarily that the distribution of cash flow across each production unit ($F_w$, or explicitly, $\{w_j\}_{j=1}^J$) remain unchanged after this increase in uncertainty. From equation (16), it is clear that higher $\sigma$ reduces the numerator by shrinking the set, $\mathcal{I}$. Intuitively, when the parent firm’s uncertainty increases, it drives up the individual uncertainty (variances of $z_j$) of all production units by shifting the whole distribution to the right ($\sigma_j = \gamma_j \sigma, \forall j$). This in turn raises the value of real options relative to the benefits of investment. An investing production unit could thus become a waiting unit and has zero ISCF as a consequence. Given that the firm-level ISCF is the weighted average of unit-level ISCF, more units having zero ISCF values diminishes the average value.

A higher degree of financial frictions (lower $\bar{\theta}$) instead has two competing impacts on (16), corresponding to the two opposing views in the FHP-KZ debate. On the one hand, it is clear that a direct effect of lower $\bar{\theta}$ is to reduce the firm-level ISCF from the denominator,
consistent with the “negative-relation critique” by KZ. This effect acts through the intensive
margin – investing production units have lower, positive ISCF as a result of tighter collateral
constraints. Intuitively, as the hair-cut ratio and down-payment requirements become
stricter, a given amount of cash flow can purchase less additional capital goods because the
amount of external funds it can lever up become less. On the other hand, a lower $\bar{\theta}$ has an
additional extensive-margin effect via increasing the $\sigma^*$. This is because tighter collateral
constraints negatively impact borrowing in the future (i.e., in period 2); the real-options
value of delaying investment becomes lower. More production units will hence invest im-
nediately, having positive ISCF; the firm-level ISCF increases as a result, consistent with
the FHP view. This is the “real-options-induced-financial channel.” Previous studies did
not identify this channel because real options and extensive-margin aggregation over het-
erogeneous production units were not included in their discussion. Before illustrating this
channel, an additional remark is worth discussing. In the conventional literature of real op-
tions, the changing option value of waiting comes from mean-preserving uncertainty shocks.
In the “real-options-induced-financial channel”, however, the uncertainty holds constant – it
is the financial shock that influences the option value. A similar impact is also documented
by Boyle and Guthrie (2003), Bolton et al. (2014) and Gilchrist et al. (2014), though the
mechanism is different.

Even though $\sigma^*$ does not have an explicit expression, how it changes with $\bar{\theta}$ can be shown
via the Implicit Function Theorem. Given the downward slope of $(x_{kj} + x_{bj})$ in $\sigma_j$, lower $\bar{\theta}$
leads to higher $\sigma^*$ if and only if it generates smaller $(x_{kj} + x_{bj})$. In the Appendix I describe
the analytical conditions under which this “real-options-induced-financial channel” exists.
A simplest sufficient (not necessary) condition is to have $\beta$ close to $1/R_f$. Intuitively, it
depends on how financial frictions influence the expected benefits from immediate investment
$(x_{kj})$ and real-options value of waiting $(-x_{bj})$ differently. $(x_{kj} + x_{bj})$ being smaller means
the production unit prefers more to waiting. According to Proposition 1, there exist two
stochastic states in period 2: an investing state with $z_j$ being greater than $\psi_1$ and a consuming
state when the opposite is true. A financial shock affects $x_{kj}$ and $(-x_{bj})$ by influencing the
marginal value added by capital and saving, respectively, in the two separate states, and the
probability of states.

First, as lower $\bar{\theta}$ induces higher $\psi_1$, the probability of the investing state becomes smaller
when facing a negative financial shock. Secondly, conditional on the investing state, lower $\bar{\theta}$
reduces the production unit’s capacity of borrowing to invest. Given the expected marginal
value added by capital and saving coming majorly from investing in the good state, these two
effects dampen both $x_{kj}$ and $(-x_{bj})$. However, the magnitude is different. This is because
the ability to invest depends also on net worth acting as down-payment: an additional unit of capital good (that worth $1) installed in period 1 is worth $(1 - \delta)$ in period 2 after depreciation; in contrast, saving one extra dollar in bond leads to an increment of $R_f$ in net worth. Based on $R_f$ being greater than $(1 - \delta)$, these two effects cause a reduction in $(-x_{b_j})$ bigger than that in $x_{k_j}$. Besides, the third effect of a financial shock is to influence the marginal value added conditional on the consuming state. If the unit chose to save an extra dollar in period 1, it consumes the resulting $R_f$ as dividend when encountering the consuming state in period 2. Tighter financial constraints have no effect in this case because it does not incur collateralized borrowing. In contrast, when the unit chose to install additional capital in period 1, it could borrow against the extra capital in period 2 without investing, but to increase dividend consumption. Lower $\bar{\theta}$ limits the value gained in this borrow-to-consume effect of extra capital. Therefore, the third effect reduces only $x_{k_j}$ but not $(-x_{b_j})$. Taking all three impacts together, how $(x_{k_j} + x_{b_j})$ changes with $\bar{\theta}$ relies on the relative strength of effect-1 and -2 together, compared to effect-3. When the production unit has limited impatience (e.g. when $\beta$ is close to $1/R_f$), effect-3 is dwarfed by the first two. $(x_{k_j} + x_{b_j})$ will become higher and the unit prefers more to immediate investment. Figure 4 illustrates this channel graphically under the calibration in Table 1: it plots the corresponding $\sigma^*$ against various values of $\bar{\theta}$, indicating that as $\bar{\theta}$ is reduced, the value of $\sigma^*$ rises.

Based on the two competing forces, the net effect of financial frictions on ISCF depends on the distribution of production units, $\{\gamma_j\}_{j=1}^J$. For example, if many production units were clustered at the bottom of the waiting region (i.e., their values of $\gamma_j$ are only slightly higher than $\sigma^*/\sigma$), a financial shock (lower $\bar{\theta}$) that increases $\sigma^*$ could change these units from waiting units to investing units. The firm-level ISCF will become larger. On the contrary, if the distribution of production units was relatively far from the threshold, changing the threshold will have a limited impact via the extensive margin and then the firm-level ISCF will decline upon lower $\bar{\theta}$ because of the direct, intensive-margin effect.

Lastly, I discuss the potential influences of ignoring the redistribution of cash flow by the parent firm’s CFO. This is actually about how the distribution of $\{w_j\}_{j=1}^J (F_w)$ changes with $\sigma$ and $\bar{\theta}$. Consider the following three possibilities:

1. There is no strategic redistribution, i.e., $F_w$ does not change.

2. The CFO solves an optimization to adjust $F_w$. The optimal policy always puts all liquid resources in the active units, i.e., $\Sigma_I w_j = w$.

3. The CFO solves an optimization to adjust $F_w$. The optimal policy places partial resources in the active units ($I$), which changes after uncertainty or financial shocks.
Previous arguments assume the case-1 scenario. Instead, under case 2, the extensive margin will no longer matter because the CFO’s redistribution keeps the numerator of (16) constant (though the set $I$ still changes). In other words, uncertainty will not have any real-options effect on the firm-level ISCF, while financial shocks will unambiguously diminish it. Meanwhile, in case 3, what matters is whether the CFO optimally selects injecting money into the group of active units ($I$) or to take money out of them upon uncertainty or financial shocks. If the former, it counter-balances the extensive-margin effect of the uncertainty shocks and reinforces that of the financial shocks, and vice versa. While solving this optimal redistribution is beyond the scope of this paper, I contend that in reality the CFO’s optimal choice could be to take money out of the group of active units upon heightened uncertainty for precautionary motives and inject money into the group of active units when facing tighter constraints to ease financing conditions. Under this conjecture in case 3, compared to case 1, uncertainty shocks should have stronger effects in reducing ISCF and financial shocks should have more profound impacts with regards to increasing ISCF. I will return to this discussion in the empirical portion later. My estimation results are actually informative in distinguishing the three scenarios.

3.5 An Extension: Endogenous Default and Credit Spread

The baseline model described assumes that all borrowings are risk-free because of collateral constraints. Default therefore does not take place in equilibrium. Now, I drop this assumption by letting production units borrow beyond collateral constraints. The extra debt will hence become risky and default will be unavoidable if the realized $z_j$ is not high enough to service the debt. In particular, suppose the units can borrow risky debt after the collateral constraints bind if they are willing to pay credit spreads to compensate the possible loss of lenders in default. The risky debt is assumed to be less senior than the collateralized debt. This extension seeks to incorporate the potential precautionary effects of uncertainty shocks that were neglected before. I show that it further decreases the firm-level ISCF beyond the “real-options-uncertainty channel.”

Following Bernanke et al. (1999), I model the bankruptcy process by employing the costly state-verification setup in Townsend (1979). Specifically, production units in default are taken over by lenders and liquidated. During this process, a fraction of value, $\xi < 1$, is

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36 Bates et al. (2009) documents that precautionary motives lead firms to hoard cash when facing heightened uncertainty. Hoarding cash means taking money away from actively investing production units.

37 See also Hennessy and Whited (2007), Arellano et al. (2012), Christiano et al. (2014), and Gilchrist et al. (2014) for a similar setup.
destroyed as dead-weight loss. Lenders need to break even so they require higher compensation from production units, rendering the risky borrowing costly. Such costs stimulate a precautionary motive in production units – as uncertainty increases the default probability, production units tend to reduce borrowing (risky leverage) and investment ex ante to avoid bankruptcy. This motive bestows uncertainty shocks with a new intensive-margin effect – ISCF becomes smaller for the investing units while remaining unchanged for the waiting units because they do not incur risky debt. Consequently, the firm-level ISCF declines as it is the average of the unit-level ISCF (see details in the Appendix). The solution to this extension is no longer in closed-form. Numerical work with calibration is instead provided. In the empirical part, I further show that the sub-sample estimation can disentangle this precautionary channel from the real-options-uncertainty channel.

4 Firm-Level Empirical Work

This section examines the model’s predictions using data of more than 10,000 U.S. listed firms over the past 30 years. Suggested by the linear investment rule in equation (15), I estimate an empirical counterpart of the ISCF in theory to be the coefficient of regressing investment on cash flow. How it changes with uncertainty and financial frictions is instead captured by including interaction terms into the regression. After carefully taking into account the potential endogeneity problems caused by measurement errors and reverse-causality, I find that the estimated ISCF is smaller when firms face higher uncertainty, consistent with the “real-options-uncertainty channel.” On the contrary, it rises with the degree of financial frictions, which is in agreement with the “real-options-induced-financial channel.”

4.1 Data and Empirical Specifications

In equation (15), firm-level investment is linear in cash flow, with both terms normalized by the existing capital stock. I thus start with a linear investment regression in (17), where $\iota_{i,t}$ refers to the investment of firm $i$ in year $t$. $w_{i,t}$ is the corresponding cash flow; $k_{i,t-1}$ is the existing capital stock; $X_{i,t-1}$ is a group of control variables; $\rho_i$ and $\eta_t$ refer to firm- and year-fixed effects, respectively, while $\epsilon_{i,t}$ is the disturbance term. Each firm-level variable is recorded in consolidated accounts corresponding to those in the theoretical model after the extensive-margin aggregation. The time frequency is annual. For $X_{i,t-1}$, I utilize Tobin’s Q and real sales growth to control for expected future profitability (investment opportunities), i.e., $\bar{z}_i$ in the theoretical model. Moreover, previous studies indicate that existing debt could
crowd out current investment, known as the debt-overhang problem.\(^\text{38}\) This is outside of my theoretical model; I therefore include the existing net leverage to control for it.

\[
\frac{\iota_{i,t}}{k_{i,t-1}} = (\rho_i + \eta_t) + \zeta_{i,t}(\frac{w_{i,t}}{k_{i,t-1}}) + X'_{i,t-1}\phi + \epsilon_{i,t} \quad (17)
\]

The coefficient of cash flow, \(\zeta_{i,t}\), is the estimated ISCF.\(^\text{39}\) The “real-options-uncertainty channel” and “real-options-induced-financial channel” suggest that \(\zeta_{i,t}\) depends on firm-level uncertainty \((\sigma_{i,t})\) and the degree of financial frictions \((f f_{i,t})\).\(^\text{40}\) Approximating the relationship with a linear equation, \(\zeta_{i,t} = \varsigma + \tau_1\sigma_{i,t} + \tau_2 f f_{i,t}\), and substituting it into (17) generates a regression with interaction terms in equation (18). I include the level of \(\sigma_{i,t}\) and \(f f_{i,t}\) into the regression to isolate the effects of interaction terms. The “real-options-uncertainty channel” suggests that \(\tau_1\) should be negative, while the “real-options-induced-financial channel” infers that \(\tau_2\) could be positive.

\[
\frac{\iota_{i,t}}{k_{i,t-1}} = (\rho_i + \eta_t) + \varsigma(\frac{w_{i,t}}{k_{i,t-1}}) + X'_{i,t-1}\phi + \mu \sigma_{i,t} + \chi f f_{i,t} \\
+ \tau_1(\frac{w_{i,t}}{k_{i,t-1}}) \times \sigma_{i,t} + \tau_2(\frac{w_{i,t}}{k_{i,t-1}}) \times f f_{i,t} + \epsilon_{i,t} \quad (18)
\]

I employ annual data of U.S. listed firms in the COMPUSTAT North America database from 1984-2015. To focus on spontaneous capital investment decisions, I drop firms in the financial, utility, and public sectors as they either do not invest majorly in capital goods or are highly regulated. After a standard selection procedure following the literature, the sample consists of more than 10,000 individual firms in an unbalanced panel.\(^\text{41}\) On average, the sample contains more than 3,000 firms cross-sectionally in each year; each firm stays in the panel for approximately 15 years. I use capital expenditure to measure investment \((\iota)\) and employ property, plant, and equipment as a measure of capital stock \((k)\). Cash flow \((w)\) is income before extraordinary items plus depreciation and amortization.\(^\text{42}\) Tobin’s Q is the macroeconomic Q defined by Erickson and Whited (2006). It equals the sum of debt and

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\(^{38}\)See, among others, Myers (1977) and Hennessy et al. (2007).

\(^{39}\)Henceforth to be succinct, I use the abbreviation “ISCF” for both the theoretical definition in equation (16) and the estimated empirical counterpart. Such a linear investment specification is justified by the modelling assumptions in Section 3; however, non-linearity because of monopoly power or decreasing returns to scale could break it down in reality.

\(^{40}\)Corresponding to the theoretical model, \(f f_{i,t}\) measures \((1 - \tilde{\theta}_{i,t})\) representing the hair-cut ratio, with a higher value indicating tighter collateral constraints \((lower \tilde{\theta}_{i,t})\).

\(^{41}\)Observations are required to have positive values of the following variables: total assets, property, plant and equipment, book value of common equity, sales, capital expenditure, and cash and cash equivalents. Moreover, daily stock prices have to be higher than $1 and lower than $2,000. Firms also need to have daily data on stock returns for at least 150 trading days in each year.

\(^{42}\)See Kaplan and Zingales (1997) for a similar choice of variables.
market capitalization less total inventories, divided by capital stock. Real sales growth is the growth rate of sales (net) adjusted to the 1982 dollar using the Producer Price Index. Net leverage is equivalent to the sum of debt and market capitalization less the stock of cash, divided by market capitalization.

I construct the measure of firm-level uncertainty ($\sigma$) in two steps. Initially, I obtain the realized equity volatility from daily stock returns for each individual firm. As an alternative, I estimate the underlying conditional volatility using the Exponential GARCH model in \textit{Nelson} (1991). Next, I divide the obtained equity volatility by capital structure (one plus net leverage). This is in an effort to follow \textit{Leahy and Whited} (1995): the resulting variable captures the volatility over the enterprise values, namely the present values of all the firm’s future projects. Without this adjustment, equity volatility is not a reliable measure of uncertainty (over business prospects) for this study – two firms with the same volatility of enterprise values, thereby equally volatile business prospects, could have different equity volatilities because of distinct leverage. In addition, \textit{Bloom et al.} (2014) observe that this measure of uncertainty constructed by listed firms’ daily stock returns (adjusted or unadjusted by capital structure) is highly correlated with the uncertainty of production units (measured by TFP shocks) belonging to the corresponding firm. It is therefore in agreement with my theoretical model, wherein the firm-level uncertainty drives the dynamics of all unit-level uncertainty (called “variances” in the theoretical model).

To measure the degree of financial frictions, $ff$, I first employ the excess bond premium (EBP) in \textit{Gilchrist and Zakrajšek} (2012) by exploiting the data of publicly traded corporate bonds. It is built as the residual in credit spreads after partialling out expected default loss and averaged cross-sectionally across the market.\footnote{Three points are worth noting regarding the measure of Q. First, it corresponds to the average Q in theory rather than the marginal Q. This is because \textit{Erickson and Whited} (2006) state that average Q is more empirically relevant when there exist financial frictions. Secondly, except for the market capitalization, all terms in constructing the macroeconomic Q here are measured in book values. \textit{Erickson and Whited} (2006) provide a detailed survey regarding alternative ways in the literature (e.g., to estimate the replacement costs of capital stock as in \textit{Salinger and Summers} (1983)) and compare these different methods. They determine that utilizing book values does not underperform other alternatives but serves as the most convenient way. In addition, besides the macroeconomic Q, another widely-adopted measure is the market-to-book ratio of assets (called financial Q). \textit{Erickson and Whited} (2006) show that using this alternative definition introduces unnecessary errors into the regression as the denominator of Q (total assets) is different from that of the investment rate (capital stock). My baseline findings remain robust if I replace the macroeconomic Q with the financial Q. Using macroeconomic Q does lead to a better fit, indicated by a higher R squared value.}

\footnote{The EGARCH model is selected by the Akaike Information Criterion independently for each individual firm. The resulting conditional volatility is on a daily basis. I take the average across all trading days within the corresponding year. Based on the consistency of EGARCH and the large amount of daily observations, the estimation process is quite precise. I therefore ignore the standard errors associated with this step.}

\footnote{Data is available from the website of Board of Governors of the Federal Reserve System.}
garding “credit spread puzzle”, this portion of the variation in credit spreads that cannot be explained by expected default appears to reflect time-varying liquidity premium, the tax treatment of corporate bonds, and most importantly, investors’ risk aversion. A widened EBP indicates a reduction in the risk-bearing capacity of financial markets and, as a result, a contraction in credit supply. This is supported by the theoretical work of He and Krishnamurthy (2013) and related empirical evidence of Adrian et al. (2010a) and Adrian et al. (2010b). In addition, Geanakoplos (2010) highlight that increasing investors’ risk aversion could worsen their disagreement with borrowers over the value of collateral, rendering tighter collateral constraints. This further lays a theoretical foundation for using EBP as an empirical counterpart proportional to the hair-cut ratio \(1 - \bar{\theta}\) in Section 3. Despite many strong features, EBP is still not a perfect measure of financial frictions. For one, it is constructed using a small group of firms with publicly traded debt available. Even though Gilchrist and Zakrajšek (2012) argue that it has implications far beyond influencing those that are used in construction, like performing well in forecasting macroeconomic fluctuations, it introduces a sample-selection bias. For instance, I have more than 10,000 listed firms in my sample, but only around 1,000 of them were used to construct EBP. What worsens the situation is these firms with publicly traded debt are in general bigger than others.

Further, EBP is a market-wise measure common to all firms in a single year. It ignores that firms could have individual exposure to financial frictions. To mitigate the concern, I include firms’ sizes (the market value of assets in 1982 dollar adjusted by the Producer Price Index) as an additional, individual proxy, assuming that smaller firms potentially face tighter financial constraints. This could be rooted in a lack of information, reputation, economies of scale, or other reasons. Besides size, existing literature has put forward many other choices for individual measures of frictions, for example, whether the firm pays dividends (Fazzari et al. (1988)), the Kaplan-Zingales index (Kaplan and Zingales (1997)), the Cleary index (Cleary (1999)), and the Whited-Wu index (Whited and Wu (2006)). Hennessy and Whited (2007) employ structural estimation to show that the size is superior to these alternatives when capturing the “deep” characteristic of facing more financial frictions. Those latter instead represent the needs for external financing. In addition, the existence of bond rating has also been used widely as an indicator of finance constraints. As a consequence of

\[46\] For example, see Collin-Dufresn et al. (2001), Houweling et al. (2005) and Driessen (2004).

\[47\] Needing external financing does not necessarily mean facing significant frictions. For instance, a creditworthy international corporation based in the U.S. but with a huge amount of foreign income might have the necessity to issue corporate bonds in the U.S. against its foreign cash holdings to pay dividends, and, at the same time, avoid repatriation costs. On the other hand, a firm with more frictions when seeking external financing may have the incentive to hoard cash and ends up with few needs for external funds. However, hoarding cash does not resolve any fundamental reason that led to such frictions ex ante, like agency costs or information asymmetry.
the lack of data access, I leave it to future research.

Table 2 presents the summary statistics of the corporate-level variables. Two points are worth discussing. For one, a major fraction of observations have values of macroeconomic Q greater than one; the latter acts as the investment threshold in neoclassical theory. This phenomenon, however, is consistent with real options raising the conventional investment hurdle denominated in Q as in Dixit and Pindyck (1994). It therefore justifies my modelling of non-convex adjustment costs (that generate real options). Erickson and Whited (2006) further stipulate that besides this structural reason, there could be measurement errors in the macroeconomic Q leading to its large values. Specifically, the numerator is the firm’s market value, including both tangible and intangible assets; in contrast, the denominator is the book value of tangible assets (capital goods) solely. If the firm holds valuable intangible assets, the macroeconomic Q for measuring investment opportunities in tangible assets could be overvalued. Seeing that roughly 25% of my sample consists of firms in the service sector, which potentially hold significant amounts of intangible assets, this measurement issue could be significant. To mitigate such a concern, I conduct robustness checks in three different ways. First, I drop observations with the values of macroeconomic Q greater than 40. Secondly, I replace the macroeconomic Q by the financial Q, putting the book value of total assets (both tangible and intangible) in the denominator. Alternatively, I admit the existence of the measurement errors and employ the measurement-error-correcting estimator in Erickson et al. (2014) and Erickson et al. (2016) to correct for the impacts.

Another point worth considering is regarding some observations presenting large, negative numbers for the cash-flow ratio. Besides the operating losses, a potential reason could be certain firms writing off serious amounts of expenses, like R&D, leading to negative income. However, such expenses could be regarded as part of the internal funds and R&D investment itself has become more and more crucial in recent years. To account for this, I add back the R&D expenditure to both cash flow and capital expenditure to obtain the “total cash-flow ratio” and “total investment ratio.” I check the robustness of my baseline results in terms of these two alternative definitions of variables.

48 To remove the impacts of possible outliers, I winsorize each variable to its top and bottom 2%.
49 This is even stricter, and thus more conservative, than the criterion in Erickson and Whited (2006) which is 60.
50 See, among others, Brown and Petersen (2009) and Brown et al. (2009).
4.2 Measurement-Errors-Correcting Estimation

Leaving aside the measure of size temporarily, estimating equation (18) with the enterprise-value volatility as a proxy for uncertainty ($\sigma$) and the EBP to capture financial frictions ($ff$) could incur two potential endogeneity problems. First, there could be a reverse causality that more investment leads to firm values being more volatile (higher $\sigma$). For instance, investing aggressively may incur certain riskier projects rendering equity-holders nervous and consequently generating greater stock-return volatility. To mitigate such a concern, I replace the firm-level volatility with the cross-sectional weighted-average values year-by-year using market value of assets as the weight. The resulting weighted-average volatility is assumed to be exogenous to any individual firm’s investment decisions. Another potential endogeneity problem is with respect to the measurement errors in Tobin’s Q. If Q cannot fully capture future investment opportunities, cash flow would be correlated with the regression’s error term. Erickson and Whited (2000) point out that such measurement errors can bias the estimated coefficients of not only Q, but also other variables, such as $\tau_1$ and $\tau_2$, even when the weighted-average volatility and EBP are exogenous by construction. Erickson et al. (2014) and Erickson et al. (2016) further put forward an estimator that utilizes higher-order cumulants to correct for measurement errors. The identification requires the data to have at least non-zero skewness and a sufficient sample size to ensure valid estimates of higher-order cumulants. My sample meets these requirements.

I therefore combine the measurement-errors-correcting method with the use of weighted-average volatility and EBP as exogenous variables. This results in a measurement-errors-correcting differences-in-differences estimator that should mitigate the two aforementioned endogeneity concerns simultaneously. In particular, the periods with soaring volatility and widened EBP act as the treatment group, while those with low volatility and EBP are the control group. Cash flow after measurement errors in Q being corrected for perform as the continuous-treatment variable. Investment responses upon cash-flow shocks in the two classified periods are compared.

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51 Alfaro et al. (2016) instead utilize industry-level variation, specifically the different exposure of firms towards political uncertainty and oil price shocks, as instruments to obtain identification.

52 See also, among others, Erickson and Whited (2002), Erickson and Whited (2012) and Abel (2015).

53 Compared to the earlier work utilizing higher-order moments for estimation, such as Erickson and Whited (2000), the cumulants estimator has a closed-form solution, thereby insensitive to numerical optimization.

54 Erickson and Whited (2012) demonstrate that when the skewness is around one, the cross-sectional $N$ is equal to 1,500 and the time-series length, $T$, equals 10, hence the estimator performs well in simulations. My sample has on average $N > 3,000$ and $T > 15$, and all my variables have skewness (in absolute values) greater than one.

55 See McLean and Zhao (2014) for a similar method where their way to distinguish the treatment and control groups (of years) is through recession dummies.
Before showing the results, I quickly summarize the sources of measurement errors and the identification strategy (details found in Erickson and Whited (2000), Erickson et al. (2014), and Erickson et al. (2016)). The measurement errors come from the differences between measured Q and the true future investment opportunities that are unobservable. Such differences could be rooted in either theoretical reasons, like Q being an insufficient statistic for investment, or measurement issues, such as the challenges in estimating replacement costs and overestimation by neglecting intangible assets, or both. As a consequence, the measured Q is assumed to follow a classical errors-in-variables (EIV) model. In particular, it equals the sum of an unobserved variable (that stands for the true investment opportunities) and a mean-zero error term; the unit slope and zero intercept in this measurement equation is crucial for identification. The estimation process takes place via two steps. The first step separately regresses the investment rate (dependent variable) and the measured Q on all other perfectly measured explanatory variables. The two regression residuals depend on the unobserved variable, the perfectly measured variables, the measurement error, and the error term in the original regression equation. The second step takes powers of these two residual equations, multiplying the results together, and takes expectations on both sides. Under a set of assumptions on the error terms, this leads to moment conditions expressed by the regression coefficients, the higher-order moments of both the data and the unobserved variable where these latter moments are treated as parameters. A moment estimator could be constructed by using subsets of these moment conditions (i.e., based on moments up to a certain order); however, the numerical optimization required to minimize the approximated moment conditions could render the estimation process sensitive to starting points. Based on Geary (1941), Erickson et al. (2014) put forward cumulant estimators that are asymptotically equivalent to the moment estimators, but with a convenient closed form. The key identification assumption falls within a requirement that the mismeasured Q is non-normally distributed, as in, for example, non-zero skewness, which plausibly holds in corporate finance.

Table 3 lists the estimation results. Columns (1), (2) and (3) utilize cumulants up to order-five, six and seven for estimation, respectively. Regarding these choices, Erickson et al. (2016) suggest that the order-five estimator is a worthwhile start and the more observations one has, the higher the order of cumulants that could be used. Given the relatively large sample size I have, I select the order-seven estimator as that which is preferred and show how the results could be altered if I replace the means of calculating volatility from the realized

56Reasons could be imperfect competition, non-linear production functions, non-convex adjustment costs, financial frictions, and other deviations from the neoclassical framework.

57Cumulants are polynomial functions of moments.
values to the EGARCH-inferred values in column (4).

In all the specifications, firm-level variables are demeaned and year dummies are included to account for both firm- and year-fixed effects; standard errors are clustered at the firm level. The levels of weighted-average volatility and EBP are therefore omitted to avoid multi-collinearity. Estimation results indicate that cash flow, lagged Q, and sales growth contribute positively to investment as expected, while the existing net leverage has a negative impact, suggesting a potential debt-overhang problem; all these estimates are highly significant from a statistical perspective. More importantly, cash flow interacting with volatility has a negative coefficient ($\tau_1$), which is significant at the 1% statistical level. In contrast, the coefficient of its interaction with EBP is positive ($\tau_2$), with a 1% statistical significance. These estimates are equivalent to firms exhibiting smaller ISCF values when uncertainty is heightened (represented by spikes of the weighted-average volatility) and greater ISCF when facing more financial frictions (indicated by the widened EBP). Further, they are in line with the “real-options-uncertainty channel” and the “real-options-induced-financial channel” put forward by the model in the previous section, respectively. Last but not least, the R squared value is equivalent to 0.3, which is indicative of a decent fit.

4.3 Generalized Method of Moments Estimation

Despite its various strong aspects, the measurement-errors-correcting estimator has some unappealing features. First and foremost, the identification based on the exogeneity of the market weighted-average volatility and EBP, ignores the cross-sectional variation in the firm-specific volatility and exposure to financial frictions. In other words, results in the previous section should be interpreted as ISCF being less during periods of heightened uncertainty and larger when financial markets are generally tighter. Rather, they have little to offer regarding whether a more volatile (or financially constrained) firm should have larger or smaller ISCF compared to others. Introducing firm-level volatility or an individual proxy for financial frictions (size), however, could cause reverse causality that the measurement-errors-correcting estimator cannot fix. Specifically, a firm investing heavily in capital goods might render equity holders nervous and thereby stock returns more volatile; it also simultaneously enlarges the firm’s size.

Secondly, the moment conditions in the measurement-errors-correcting estimator are

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58 Indeed, for both the realized and the EGARCH-inferred volatility, results are similar for all the specifications from order-four to order-eight. Results are available upon request. According to Erickson et al. (2016), the order must be at least four to obtain overidentification; orders higher than eight are not advised to be used because of the computational burden.
based on a group of orthogonality assumptions imposed by Erickson et al. (2014) and Erickson et al. (2016). They can be examined by a Hansen-J over-identifying test. The test, however, strongly rejects the null hypothesis for all the specifications in Table 3 (p-value smaller than 1%). Such rejections are also present in Erickson et al. (2014) and Erickson et al. (2016) for different datasets and empirical specifications. It could suggest a violation of one of the conditions imposed by those assumptions, such as the strong exogeneity of the perfectly measured corporate-level variables (e.g., sales growth, net leverage). Even though those variables are used in a one-year lag, if contemporaneously they correlate to the error term, cancelling firm-level fixed effects by the demeaning process can still introduce endogeneity according to Nickell (1981). This is because both the demeaned control variables and the demeaned error term include impacts from the contemporaneous period; they are thereby correlated. Lastly, only firm-level clustered standard errors are allowed in the current algorithm of the measurement-errors-correcting estimator. However, firms within the same sector might receive common investment shocks so that their error terms could be correlated. Adopting sector-clustered standard errors would thus be more robust.

To deal with these issues, I estimate equation (18) by the Generalized Method of Moments (GMM) from Arellano and Bond (1991) instead, treating all firm-level variables as endogenous variables. Based on the inclusion of firm-specific volatility, size, and their interaction terms with cash flow, this estimation investigates not only via a time-series but also cross-sectionally how uncertainty and financial frictions influence ISCF. Specifically, I transform equation (18) by taking the first difference to remove firm-fixed effects. Unlike demeaning, first differencing guarantees that lagged variables are valid instruments for the transformed equation conditional on the error term having limited serial correlations. The latter is examined by the Arellano-Bond autocorrelation test (AR test) and is cross-checked by the Hansen-J over-identifying test. Particularly with respect to my sample and specification, the AR test provides evidence for order-two but not order-three serial correlations in the error term. I therefore use the lagged three-period and earlier firm-level variables as instruments. I estimate the GMM using the two-step method with standard errors clustered at the sector level.

Table 4 shows the estimation results. Individual volatility serves as a proxy for firm-level uncertainty and the size represents the potential financial frictions an individual firm might face. The element of EBP is considered simultaneously as it captures the general tightness of financial markets. Year dummies are employed to control for other macroeconomic factors.

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59 Specifically, see Assumption 1 in Erickson et al. (2016).
60 The relevance of instruments is guaranteed by the autocorrelation nature of firm-level variables.
61 For clustered standard errors, sectors are classified by the 4-digit SIC codes.
common to all firms. Column (1) includes the estimate adopting lagged three-period and four-period firm-level variables as instruments for the transformed regression equation. In theory, earlier lags are valid instruments, as well; however, when the number of lags rises, the relevance of instruments becomes weaker. In column (2), I check the robustness for earlier lags by utilizing lagged three-period to six-period firm-level variables. In column (3), I instead portray the estimate when the EGARCH-inferred volatility measures individual uncertainty. Column (4) replaces the year dummies with a full set of sector-year dummies to control for potential industry-specific-time-varying shocks.

The estimation results confirm previous conclusions for all specifications. Specifically, cash flow interacting with individual-specific volatility has a negative coefficient; also, that of its interaction with EBP remains positive; both terms are significant statistically. Moreover, the coefficient of size interacting with cash flow is significantly negative. Based on the smaller size representing more exposure to financial frictions, such findings are in agreement with those of EBP whereby financial frictions enlarge ISCF. In summary, introducing cross-sectional variation and GMM estimators supplies further evidence for supporting the “real-options-uncertainty channel” (such that uncertainty diminishes ISCF) and the “real-options-induced-financial channel” (such that financial frictions elevate ISCF). Besides this, the negative coefficient of individual volatility itself illustrates the depressing role of uncertainty at the levels of investment, which is consistent with previous studies (e.g., Bloom (2009)). The AR tests ensure the validity of the instruments used, which are also cross-checked by the Hansen-J over-identifying test.

To gauge the economic significance of the estimated channels, I fit the values of ISCF numerically conditional on different levels of uncertainty and financial frictions. It utilizes the linear combinations of regression coefficients in column (4) of Table 4 namely, that with sector-year dummies included (thereby the most robust to omitted variables). In particular, I differentiate investment in equation (18) with respect to cash flow, leading to the following linear representation of ISCF:

$$ ISCF_{i,t} = \frac{\partial(\tau_{i,t}/k_{i,t-1})}{\partial(w_{i,t}/k_{i,t-1})} = \varsigma + \tau_1 \sigma_{i,t} + \tau_{21}EBP_t + \tau_{22}Size_{i,t} $$

Next, I substitute different sample percentile values for $\sigma_{i,t}$, $EBP_t$, and $Size_{i,t}$, respectively; when changing one of these, I fix the other two at their sample median values to isolate the impacts. The fitted ISCF is listed in Table 5. The first column is based on

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62 See Brown et al. (2009) for the same choice of lags as instruments.
63 For dummy variables, sectors are classified by the 10 major SIC divisions.
64 Regarding the impacts of size, see also Gilchrist and Himmelberg (1995) and Abel and Eberly (2011).
when $\sigma_{i,t}$ is valued at its 10-, 20-, 30-, ... and 90-percentile values, respectively, while at the same time fixing $EBP_t$ and $Size_{i,t}$ at the corresponding sample median values. The second column is to change $EBP_t$ while fixing $\sigma_{i,t}$ and $Size_{i,t}$. The last column is to change $Size_{i,t}$ but to fix $\sigma_{i,t}$ and $EBP_t$. The results show that the estimated impacts are, in fact, large in economic terms. First, when uncertainty moves from its 10-percentile value in the sample to the sample median, the fitted ISCF decreases by 23%. As uncertainty passes its median value, the fitted ISCF is even no longer significantly different from zero. Moreover, as EBP increases from its median to the 90-percentile value, the fitted ISCF rises by more than a half (54%). In contrast, the influence of size on the fitted ISCF is not as prominent – when changing from the median value to its 10-percentile value, the fitted ISCF only increases by 30 basis points. One possible explanation for this is that a firm’s size only acts as an indirect proxy for the financial frictions it faces. Based on the structural estimation in Hennessy and Whited (2007), I put forth that smaller firms face more costly external financing; however, this relationship is not causal but rather a consequence of simultaneity. In other words, a firm facing more frictions is not because of being small *ceteris paribus*, but rooted in certain “deep” characteristics that co-exist with the small size, like information asymmetry. Given that the estimates in Table 5 are conditional on a full set of control variables, the part of size as a proxy for the underlying “deep” characteristics could be undermined if those control variables correlate to the unobservable characteristics at the same time.

Another point regarding the fitted ISCF is worth noting. In Table 3, when all three determinants of ISCF are valued at their sample medians, the fitted ISCF is 0.0157. This number is much smaller than what the existing literature has documented (usually between 0.1 to 0.6). Besides the inclusion of more control variables, such as sales growth mitigating the role of cash flow, a potential reason is that my sample includes a large proportion of firms in the service sectors. For them, capital investment might not play as major of a role as the conventional manufacturing firms. Instead, the Research & Development (R&D) investment could be vital, especially in recent years. As R&D is expensed out from cash flow and is not included in the measurement of investment, the ISCF value of 0.0157 could be underestimated. To account for this, I add back the R&D expenses to both investment and cash flow. After repeating the estimation process, the fitted ISCF equals 0.178, when volatility, EBP, and size are valued at their sample medians and year dummies are included (with sector-year dummies, the value instead is 0.169). This value falls into the range suggested by the existing literature. Moreover, as suggested by column (1) of Table 6, all the regression coefficients remain the same sign and achieving statistical significance. In other words,

\[65\] I thank Bruce Petersen for highlighting this constructive suggestion.
the “real-options-uncertainty channel” and the “real-options-induced-financial” channel are robust to including R&D investment into the discussion.

My empirical results also survive other robustness checks, depicted by Table 6. First, as an alternative way of dealing with the overestimation of macroeconomic Q, I drop observations with a measured Q greater than 40. Secondly, I replace the macroeconomic Q with the financial Q, another widely-adopted control variable for investment opportunities. Moreover, I remove the period of the Great Recession from my sample to make certain the results are not driven by specific features of crises besides uncertainty and financial frictions. In addition, I include lagged investment into the regression as an additional explanatory variable as per (Eberly et al., 2012) estimating a dynamic panel model. In each of the scenarios, my main findings remain unchanged – uncertainty reduces ISCF and EBP enlarges ISCF. However, the effects of size are less robust; they are preserved only by adding back R&D, dropping the Great Recession and adopting the dynamic panel specification.

4.4 Further Discussion

Before moving on, more discussions are provided surrounding the empirical results. First, ISCF declining with volatility further shields my estimation process from the “omitted-variable critique.” In particular, suppose that ISCF is positive because cash flows were informative about future investment opportunities. Such information revealed should be more relevant to investment decisions upon heightened uncertainty when information from other sources is scarce. Uncertainty should thus enlarge ISCF, opposing what is observed in the data. See Abel and Eberly (2011) for a similar argument that the information-based ISCF increases with the value of growth options.

Secondly, Section 3 proves that the “real-options-uncertainty channel” leads uncertainty to diminish ISCF. However, as the model’s extension demonstrates, the precautionary motives to avoiding bankruptcy might result in the same empirical patterns. Sub-sample estimates are useful in disentangling these two explanations. Specifically, the precautionary motives do not appear in my theoretical model until the leverage reaches a threshold (that binds collateral constraints). This is in line with a commonly accepted argument in the literature of precautionary savings that agents with weaker balance sheets have stronger

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66 Based on the AR test indicating an order-3 serial correlation (p-value equal to 0.0499) but not order-4, I use corporate-level variables in the lagged-4 and -5 periods as instruments. Maintaining the previous choice of instruments, however, do not change the estimate qualitatively.

67 Results are similar (available upon request) if: (1) I only include manufacturing firms in the sample; and (2) I follow Leahy and Whited (1995) in dropping observations with M&A greater than 15% of total assets.
precautionary motives. I therefore divide my sample into two sub-samples according to the existing net leverage (based on the median value), with higher values positing weaker balance sheets. If the role of uncertainty in reducing ISCF comes primarily from the precautionary motives, it should be more prominent for the sub-sample with high leverage.

Table 7 features the sub-sample estimation results, with the first column containing the sub-samples with low existing net leverage and the second column those for the high existing net leverage. After comparing the two estimates, I find that cash flow interacting with volatility is significantly negative only for the sub-sample with low leverage. In other words, uncertainty diminishes ISCF only when firms have stronger, rather than weaker, balance sheets. This rejects the precautionary motives driving the empirical patterns and therefore indirectly justifies the “real-options-uncertainty channel.” With this, the term of cash flow interacting with EBP is significantly positive only for the sub-sample with low leverage, as well; the interaction term between cash flow and size, however, reverses its sign for the sub-sample with high leverage (smaller firms with potentially more financial frictions having smaller ISCF). This summarizes the “real-options-induced-financial channel” holding only for the financially strong firms. Both pieces of evidence supports that the real-options effects are more relevant to firms with stronger balance sheets. Li et al. (2015) find similar results using data of corporations in the emerging markets. Despite balance-sheet strength affecting real options being beyond the scope of this paper, such a phenomenon could be explained by my modelling framework. Real options are valuable because by delaying investment, firms forego the near-term profits in exchange for the benefits of avoiding inefficient investment in the future. Firms with weak current balance sheets should thus have less motives to do so because they value near-term profits more than those with strong current balance sheets. For instance, investing immediately helps expand their business, generating more profits that could service debt or operating expenses. Stated another way, the real-options effects are less prominent for financially weak firms and so are the two real-options-related channels.

A last point of discussion is centered around the redistribution of resources by the consolidated firms’ CFOs upon uncertainty and financial shocks. This redistribution process is neglected by the theoretical model; it, however, could affect interpretations of the empirical results. The “case-2” scenario at the end of Section 3 is clearly rejected by the data. This is

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68See Carroll (1997) for the discussions of consumer behavior.
69The strength of balance sheets should not be regarded as the degree of financial frictions. It instead measures the needs for external funds rather than the costs to obtain external funds. It is advised to consult Hennessy and Whited (2007) for more discussion.
70Another piece of evidence disfavoring the precautionary explanation is that the listed firms in my sample are typically far from their default boundaries. I calculate their default probability following Bharath and Shumway (2008). Only less than 10% of firms have a probability of defaulting in one year greater than 1%.
because if a firm’s CFO always redistributes all resources into the active production units, uncertainty should have no impact on ISCF. With regards to the “case 3” scenario, I postulate that taking money away from active production units upon heightened uncertainty (for precautionary motives) and injecting money into active production units when facing more frictions (to ease financing constraints) could lead to uncertainty diminishing ISCF and financial frictions enlarging ISCF, the same as the roles relevant to real options. If the CFOs’ redistribution is indeed the underlying reason of the observed empirical patterns, such phenomena should be more prominent for firms with weaker current balance sheets because they are those who possess stronger precautionary motives and higher needs for funds. Sub-sample estimates in Table 7 again rule out this possibility. The real-options effects, hence, serve as the most possible candidate for clarifying the observed influences of uncertainty and financial frictions on ISCF.

5 Policy Implications

Based on the pertinence of uncertainty and real options to firms investing in financial markets with frictions, this section reviews the policy implications in the real world. Seeing that recessions are typically periods of both heightened uncertainty and tighter financial markets, how stimulative policy could overcome these obstacles to investment is of great policy importance. \cite{Hubbard1997} has documented that a crucial channel through which monetary and fiscal policy could boost investment in financial markets with frictions is by strengthening firms’ internal funds (cash flow).\footnote{In contrast, the traditional channel in frictionless financial markets is via affecting the users’ costs of capital, or, equivalently, the value of Q.} For instance, expansionary monetary policy that lowers interest rates reduces firms’ debt-service burden, thereby adding to cash flow. Fiscal policy, like tax incentives, enhances after-tax cash flow. However, as suggested in the previous sections, the final effectiveness of policy in boosting investment could be dwarfed by heightened uncertainty because of the associated small ISCF. I utilize the \textit{Bonus Depreciation} policy enacted during and in the aftermath of the Internet Bubble (2001-2004) and Great Recession (2008-2011) as instances to illustrate this point. I show that the investment of firms with higher volatility responses less in the case of a common policy shock. Identification is achieved via a differences-in-differences method following \cite{Zwick2017}; I further correct for the potential endogeneity problems caused by firm-level controls.

Bonus depreciation is a temporary tax incentive on equipment investment. Its stimulative role is rooted in allowing firms to depreciate equipment expenditure faster, thereby
reducing the present value of tax payments. Conceptually, suppose a firm purchases $1 million worth of computers, a five-year item. Under normal tax policy, right after the purchase, the firm can deduct 20 percent of the value for depreciation ($200,000). From the first to the fifth year, the corresponding deduction values are: $320,000, $192,000, $115,000, $115,000, and $58,000, respectively. The values of tax shields resulting from such deductions are therefore (under a 35% corporate tax rate): $70,000, $112,000, $67,200, $40,300, $40,300, and $20,200. The total tax benefit is calculated as the sum of present values of these flows of tax shields. Bonus depreciation permits a firm to deduct more at the time of investment and then depreciate the remainder according to the normal schedule. In the context of the computer example, a 50-percent bonus depreciation endows the firm with the privilege to deduct 50% × $1 million right after the purchase beyond the normal schedule, leading to a total immediate deduction of $600,000. As a result, each subsequent deduction falls by half. Even though the total value of deductions does not change, each of them has been accelerated. The total tax benefit, as the sum of present values of tax shields, therefore increases. In spite of bonus depreciation only applying to equipment investment, it acts as a cash windfall that eases financing constraints and therefore should promote investment across all categories. Moreover, given that it does not change future investment opportunities, this policy serves as a suitable example that is immune to the “omitted-variable critique.”

To identify the impacts of bonus depreciation on investment, Zwick and Mahon (2017) present a differences-in-differences specification. In particular, because bonus depreciation contributes via modifying the sum of present values of tax shields, its de facto impact grows with the duration of equipment goods that firms invest in. Therefore, those in industries with most of their investment in long-duration equipment are influenced more, acting as the treatment group, while those in the short-duration industries serve as the control group. They construct a continuous-treatment variable (called “z_{N,t}”) by combining such industry-level variation with the changes of deduction rules imposed by bonus depreciation; N represents the four-digit NAICS industries and t signifies years. In technical terms, z_{N,t} is the present discounted value of one dollar of deductions for investment eligible for bonus depreciation. During normal periods without policy incentives, they estimate the industry-level z_N utilizing disaggregated-level data from the Internal Revenue Service. In bonus years, however, z_N is adjusted by the size of the bonus \(B_t\): \(z_{N,t} = B_t + (1 - B_t) \times z_N\). \(B_t\) is the additional expense allowed per dollar of investment; in the instance of a 50-percent bonus depreciation,
$B_t$ is 0.5. At different points in time, $B_t$ was set to different values: 0, 0.3, 0.5, or 1.\textsuperscript{76} By regressing firm-level investment on the treatment variable, $z_{N,t}$, they establish that $z_{N,t}$ increasing temporarily in bonus years significantly encourages investment.\textsuperscript{77}

$$\frac{\iota_{i,t}}{k_{i,t-1}} = (\rho_i + \eta_t) + \zeta(w_{i,t}/k_{i,t-1}) + X'_{i,t-1}\phi + \mu z_{N,t} + \lambda z_{N,t} \times \sigma_{i,t} + \epsilon_{i,t}$$

(20)

However, the heterogeneous responses conditional on volatility has not been investigated. To do this, I incorporate $z_{N,t}$ interacting with volatility $\sigma_{i,t}$ into the regression in equation (20). Like before, $\iota$, $w$ and $k$ represent capital investment, cash flow and capital stock, respectively. $X$ is a vector of controls including lagged Tobin’s Q, sales growth and net leverage. $\rho$ and $\eta$ are firm- and year-level fixed effects. The coefficient of treatment variable $z_{N,t}$ interacting with volatility captures the disproportional impacts of a common policy shock. Seeing the estimated coefficient $\varrho$ being negative indicates weaker policy effectiveness under high uncertainty, and vice versa. Given the endogeneity issues regarding individual volatility and Tobin’s Q, I estimate the regression adopting the same GMM estimator as in Section 4.3, employing the lagged three and four-period corporate variables as instruments. Table 8 includes the estimation results where standard errors are clustered at the four-digit NAICS level. The estimation results show that the policy stimulus indeed promotes investment with the coefficient of $z_{N,t}$ being positive, but less so if volatility is higher. This is suggested by a significantly negative coefficient of the interaction term. To gauge the economic significance, Table 9 lists the fitted investment responses to a unit policy shock in $z_{N,t}$, conditional on volatility valued at the corresponding sample percentiles:

$$\frac{\partial(\iota_{i,t}/k_{i,t-1})}{\partial z_{N,t}} = \lambda + \varrho \sigma_{i,t}$$

(21)

As volatility $\sigma_{i,t}$ moves from its 10-percentile value to the sample median value, the fitted investment response in equation (21) drops by more than 8%; when volatility continues growing to its 90-percentile value, the reduction is by almost 80% and the fitted response is no longer significant in statistical terms. This indicates that higher uncertainty might attenuate the stimulating effects of the bonus depreciation policy.

Even though, as Hubbard (1997) documents, strengthening internal funds to ease financing constraints is a crucial channel in which tax credits, like bonus depreciation, can boost investment, it is by no means the only channel. The heterogeneous responses I have

\textsuperscript{76}Specifically, $B_t$ was 0.3 in 2001 and 2002; $B_t$ was 0.5 in 2003, 2004, 2008, and 2009; $B_t$ was 1 in 2010 and 2011. In other years, $B_t$ was 0 (non-bonus years).

\textsuperscript{77}The results hold for both eligible equipment investment and total investment across all categories.
identified before should only be interpreted as the different total responses by firms with individual-specific volatility. Lack of access to disaggregated-level data restricts my ability to fully disentangle the effect of easing financing constraints with others; however, additional discussions conceptually might still be of assistance. In particular, Zwick and Mahon (2017) point out that “bonus depreciation provides a temporary reduction in the after-tax price and a temporary increase in the first-year deduction for eligible investment goods.” I therefore roughly group the total effects into these two categories. Besides the neoclassical channel regarding the users’ costs of capital, the first in fact enhances the after-tax cash flow in terms of capital goods, thereby easing financing constraints. Its diminishing role in boosting investment is in line with the “real-options-uncertainty channel” proposed in the previous sections. The second category, however, should not be the cause of the estimated differences in Table 9. This is because the benefits of increasing the first-year deduction grow with the effective discount rate that firms employ to discount tax benefits. Those with high discount rates should value the acceleration more than those with low discount rates. As volatility enlarges discount rates, the effects of this category should be stronger for firms with higher volatility, which is not observed in the data.

In summary, the differential responses I have identified should mostly come from the reduced after-tax prices of capital goods, as well as the associated rising after-tax cash flow in terms of capital goods. The “real-options-uncertainty channel” provides a possible explanation. Although after-tax cash flow rising eases financing constraints, the investment of firms facing high uncertainty could be delayed by real options; those firms therefore respond less to the policy stimulus. Further extricating it from the neoclassical channel requires more detailed disaggregated-level data. For example, if I had access to firms’ investments in eligible capital goods versus ineligible capital goods, estimating the empirical model in equation (20) using only ineligible investment would presumably isolate the financing effect. This is because the only reason that ineligible investment can rise is because of eased financing constraints. I leave this to future research.

6 Conclusions

During the past 10 years since the Great Recession, investigating uncertainty and financial frictions driving business cycles has seen greater popularity in the macroeconomic literature.

78In the previous model, the price of capital goods is used as the numeraire. Therefore cash flow in theory is valued in terms of capital goods. As a result, a reduction in the after-tax price of capital goods in reality indicates a positive cash-flow shock that eases financing constraints.
For instance, as Stock and Watson (2012) have written: “shocks that produced the recession primarily were associated with financial disruptions and heightened uncertainty.” Numerous policies have been put forward to surmount these obstacles to stimulating the economy. Among others, boosting capital investment by the private (corporate) sector is a crucial aspect of achieving this goal. Previous studies in macroeconomics have found investment to be low when uncertainty soars and financial markets tighten. However, an even more important question is whether firms cannot invest because of financial constraints or because they voluntarily choose not to. Describing firms’ propensity to invest, ISCF could answer this question, but is largely neglected by the macroeconomic literature.

This paper studies the relationships among uncertainty, financial frictions, and ISCF. The latter two items also factor into a long-lasting debate in corporate finance between Fazzari et al. (1988) and Kaplan and Zingales (1997), which can be summarized by two arguments: (1) the cause of positive ISCF; whether it originates from financial frictions (the FHP view) or purely an omitted-variable bias (the KZ view); and (2) the direction in which financial frictions change ISCF – positive (the FHP view) or negative (the KZ view). This paper focuses predominantly on the second argument: conditional on the omitted variables being well controlled for, why and how financial frictions influence the magnitude of ISCF. Through a heterogeneous-agent model introducing uncertainty and the real-options value of waiting, I provide a unified way to understand the two opposing views in the FHP-KZ debate. Specifically, a higher degree of financial frictions directly shrinks ISCF by imposing stricter down-payment requirements. However, via reducing the real-options value of waiting, it also accelerates investment leading to a rise in ISCF indirectly. This channel is the “real-options-induced-financial channel.” Their relative strength depends on the extensive-margin distribution of production units within a consolidated firm. The model further suggests that, besides altering the part played by financial frictions, uncertainty itself diminishes ISCF. This is because the real-options value of waiting renders firms resistant to external shocks, including additions to cash flow (known as the “real-options-uncertainty channel”). Studying a group of U.S. listed firms over the past 30 years finds evidence supporting the “real-options-induced-financial channel” and the “real-options-uncertainty channel”, with the results significant in both statistical and economic terms. The former demonstrates that my model offers a theoretical explanation for the empirical patterns in Fazzari et al. (1988), while the latter is useful in predicting the effectiveness of liquidity-provision policy. I also examine the de facto effects of the Bonus Depreciation policy (that enhances after-tax cash flow) on individual firms’ investment. The estimation results arrived at through a differences-in-differences method find that firms respond less to policy shocks
when accompanied by higher uncertainty.

In conclusion, I determine that ISCF is an additional characteristic of firms that is relevant to macroeconomic study of business cycles. Furthermore, uncertainty is an extra state variable that determines ISCF. Both have almost been neglected by their related literature. Investment levels and ISCF being low together with heightened uncertainty suggests the low investment to be rooted in firms’ own willingness. Therefore, the role of liquidity-provision policy seeking to ease financial constraints could be lessened by uncertainty. Reducing uncertainty should therefore be accounted for to a great extent in policy considerations.
Figure 1: Time-varying ISCF, Uncertainty and Financial Frictions

This figure compares the year-by-year estimates of ISCF (the left y-axis) to the market weighted-average volatility (the right y-axis) and excess bond premium (the right y-axis). In each year, ISCF is estimated as the cross-sectional regression coefficient of investment ratio on cash flow ratio, controlling for lagged Tobin’s Q, real sales growth and net leverage. The bars over the estimated ISCF indicate the 90-percent confidence intervals. The market weighted-average volatility is the cross-sectional average value of firm-level volatility of enterprise values, employing the market value of assets as the weight. Excess bond premium (EBP) is estimated by [Gilchrist and Zakrajšek (2012)]. Investment rate is capital expenditure divided by the year-start value of property, plant and equipment. Cash flow rate is income before extraordinary items plus depreciation and amortization, divided by the year-start value of property, plant and equipment. Tobin’s Q is the sum of debt and market capitalization less total inventories, divided by property, plant and equipment. Real sales growth is the growth rate of sales (net) adjusted to the 1982 U.S. dollar using the Producer Price Index. Net leverage equals to the sum of debt and market capitalization less the stock of cash, divided by market capitalization. Firm-level volatility of enterprise values is obtained through dividing the realized volatility of daily stock returns by capital structure (one plus net leverage).
This figure portrays the realized marginal value of capital and savings as functions in $z_j$, at the beginning of period 2 for production unit $j$, under the calibration in Table 1. The line with circular markers plots the marginal value of capital, $MVK_{2j} = \frac{\partial V_{2j}(k_{2j},b_j^L,z_j)}{\partial k_{2j}}$. The line without markers represents the marginal value of savings, $(-MVB_{2j})$, where $MVB_{2j} = \frac{\partial V_{2j}(k_{2j},b_j^L,z_j)}{\partial b_j^L}$ refers to the marginal value of debt; savings are regarded as negative debt.
This figure portrays the expected marginal value of capital and savings as functions in $\sigma_j$, in period 1 for production unit $j$, under the calibration in Table 1. The line with circular markers plots the expected marginal value of capital, $x_{kj} = \mathbb{E}\left[\frac{\partial V_j(k_{j},b_{Lj},z_{j})}{\partial k_{j}}\right]$. The solid line without markers represents the expected marginal value of savings, $(-x_{bj})$, where $x_{bj} = \mathbb{E}\left[\frac{\partial V_j(k_{j},b_{Lj},z_{j})}{\partial L_{j}}\right]$ refers to the expected marginal value of debt; savings are regarded as negative debt. The dash line instead shows the investment determinant in period 1: $(x_{kj} + x_{bj})$. 
This figure portrays the negative relationship between $\bar{\theta}$ and $\sigma^*$ under the calibration in Table 1, with $\bar{\theta}$ capturing the tightness of collateral constraint and $\sigma^*$ being the root of $x_{kj} + x_{bj} = 0$. As financial shocks reduce $\bar{\theta}$, the corresponding $\sigma^*$ rises, representing the real-options-induced-financial channel.
Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbols</th>
<th>Values</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected profitability (per capital, annual)</td>
<td>( \bar{z} )</td>
<td>0.3</td>
<td>COMPUSTAT</td>
</tr>
<tr>
<td>Risk-free rate (gross, annual)</td>
<td>( R_f )</td>
<td>1.04</td>
<td>FRED</td>
</tr>
<tr>
<td>Time preference (annual)</td>
<td>( \beta )</td>
<td>0.96</td>
<td>Bloom et al. (2014)</td>
</tr>
<tr>
<td>Depreciation (annual)</td>
<td>( \delta )</td>
<td>0.1</td>
<td>Bloom et al. (2014)</td>
</tr>
<tr>
<td>Liquidity price of capital</td>
<td>( \bar{q} )</td>
<td>0.85</td>
<td>Cui (2014)</td>
</tr>
<tr>
<td>Collateral constraint</td>
<td>( \theta )</td>
<td>0.65</td>
<td>-</td>
</tr>
</tbody>
</table>

This table summarizes the model calibration that generates Figure 2, Figure 3, and Figure 4. Expected profitability (per unit of capital goods) is set to 0.3 matching the median value of cash flow (as a ratio of capital stock) of COMPUSTAT U.S. listed firms from 1984 to 2015. Cash flow rate is income before extraordinary items plus depreciation and amortization, divided by the year-start value of property, plant and equipment. Gross interest rate \( R_f \) is set to 1.04 per year, equal to the average 2-year Treasury bond yield from 1984 to 2015. The choice of maturity is to match the modelling setup that firms borrow in a two-period bond. Time preference parameter \( \beta \) and annual depreciation \( \delta \) are taken from Bloom et al. (2014). Liquidation price of capital is set to 85% of the purchasing price, as in Cui (2014). In Figure 2 and Figure 3, the collateral constraint parameter \( \bar{\theta} \) is set to 0.65; together with \( \bar{q} \), it generates a fire-sale value equal to 55% of the purchasing price of capital goods (0.85 \times 0.65), belonging to the range 40%-60% suggested by the existing literature. In Figure 4, the value of \( \bar{\theta} \) instead moves all the way from 0 to 1.
Table 2: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Obs.</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment Ratio ($\iota_{i,t}/k_{j,t-1}$)</td>
<td>94,592</td>
<td>0.351</td>
<td>0.403</td>
<td>0.226</td>
</tr>
<tr>
<td>Cash-flow Ratio ($w_{j,t}/k_{j,t-1}$)</td>
<td>95,340</td>
<td>0.102</td>
<td>2.690</td>
<td>0.305</td>
</tr>
<tr>
<td>Macroeconomic Q</td>
<td>95,209</td>
<td>18.797</td>
<td>39.297</td>
<td>5.885</td>
</tr>
<tr>
<td>Financial Q</td>
<td>91,722</td>
<td>1.890</td>
<td>1.456</td>
<td>1.403</td>
</tr>
<tr>
<td>Real Sales Growth</td>
<td>95,340</td>
<td>0.151</td>
<td>0.451</td>
<td>0.064</td>
</tr>
<tr>
<td>Net Leverage</td>
<td>95,340</td>
<td>0.594</td>
<td>1.071</td>
<td>0.257</td>
</tr>
<tr>
<td>$\sigma_{i,t}$ by RVOL</td>
<td>95,340</td>
<td>0.320</td>
<td>0.385</td>
<td>0.176</td>
</tr>
<tr>
<td>$\sigma_{i,t}$ by EGARCH</td>
<td>95,340</td>
<td>0.329</td>
<td>0.374</td>
<td>0.192</td>
</tr>
<tr>
<td>Size</td>
<td>95,316</td>
<td>1,040.265</td>
<td>2,475.345</td>
<td>150.264</td>
</tr>
</tbody>
</table>

Investment ratio is capital expenditure normalized by year-start property, plant and equipment. Cash-flow ratio is income before extraordinary items plus depreciation and amortization, normalized by year-start property, plant and equipment. Macroeconomic Q is the sum of debt and market capitalization less total inventories, normalized by property, plant and equipment. Financial Q is the market-to-book ratio of total assets. Real sales growth is the growth rate of sales (net) adjusted to the 1982 U.S. dollar using the Producer Price Index. Net leverage is the sum of debt and market capitalization less cash and cash equivalents over market capitalization. $\sigma_{i,t}$ by RVOL is the realized equity volatility normalized by (1+Net Leverage). $\sigma_{i,t}$ by EGARCH is the EGARCH-inferred equity volatility normalized by (1+Net Leverage). Size is market value of asset, calculated by book value of assets minus total common equity plus market capitalization (in millions U.S. dollar).
<table>
<thead>
<tr>
<th></th>
<th>(1) Inv Ratio</th>
<th>(2) Inv Ratio</th>
<th>(3) Inv Ratio</th>
<th>(4) Inv Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash-flow Ratio (CF)</td>
<td>0.0177***</td>
<td>0.0176***</td>
<td>0.0178***</td>
<td>0.0201***</td>
</tr>
<tr>
<td></td>
<td>(0.00283)</td>
<td>(0.00275)</td>
<td>(0.00284)</td>
<td>(0.00327)</td>
</tr>
<tr>
<td>Sigma × CF</td>
<td>-0.0641***</td>
<td>-0.0659***</td>
<td>-0.0640***</td>
<td>-0.0803***</td>
</tr>
<tr>
<td></td>
<td>(0.0187)</td>
<td>(0.0182)</td>
<td>(0.0187)</td>
<td>(0.0219)</td>
</tr>
<tr>
<td>EBP × CF</td>
<td>0.00864***</td>
<td>0.00876***</td>
<td>0.00863***</td>
<td>0.00957***</td>
</tr>
<tr>
<td></td>
<td>(0.00247)</td>
<td>(0.00242)</td>
<td>(0.00248)</td>
<td>(0.00255)</td>
</tr>
<tr>
<td>lag Q</td>
<td>0.00905***</td>
<td>0.00863***</td>
<td>0.00909***</td>
<td>0.00908***</td>
</tr>
<tr>
<td></td>
<td>(0.000111)</td>
<td>(0.000100)</td>
<td>(8.43e-05)</td>
<td>(8.41e-05)</td>
</tr>
<tr>
<td>lag Real Sales Gr</td>
<td>0.0165***</td>
<td>0.0191***</td>
<td>0.0163***</td>
<td>0.0163***</td>
</tr>
<tr>
<td></td>
<td>(0.00389)</td>
<td>(0.00379)</td>
<td>(0.00384)</td>
<td>(0.00384)</td>
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<tr>
<td>lag Net Leverage</td>
<td>-0.0399***</td>
<td>-0.0407***</td>
<td>-0.0398***</td>
<td>-0.0398***</td>
</tr>
<tr>
<td></td>
<td>(0.00161)</td>
<td>(0.00159)</td>
<td>(0.00159)</td>
<td>(0.00159)</td>
</tr>
</tbody>
</table>

Number of Firms: 12,278 12,278 12,278 12,278
R-squared: 0.305 0.293 0.306 0.306

Uncertainty Type: Realized Realized Realized EGARCH
Firm Fixed Effects: YES YES YES YES
Year Dummies: YES YES YES YES
Method: Cumulants 5 Cumulants 6 Cumulants 7 Cumulants 7

Robust standard error in parentheses; *** p<0.01, ** p<0.05, * p<0.1
Table 4: Generalized Method of Moments Estimation

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inv Ratio</td>
<td>Inv Ratio</td>
<td>Inv Ratio</td>
<td>Inv Ratio</td>
</tr>
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<td>-0.0748***</td>
<td>-0.0858***</td>
<td>-0.0887*</td>
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<td>(0.0268)</td>
<td>(0.0272)</td>
<td>(0.0324)</td>
<td>(0.0508)</td>
</tr>
<tr>
<td>Cash-flow Ratio (CF)</td>
<td>0.0193**</td>
<td>0.0175**</td>
<td>0.0250**</td>
<td>0.0210**</td>
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<tr>
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<td>(0.00929)</td>
<td>(0.00881)</td>
<td>(0.0104)</td>
<td>(0.0105)</td>
</tr>
<tr>
<td>Sigma × CF</td>
<td>-0.0182*</td>
<td>-0.0123*</td>
<td>-0.0241**</td>
<td>-0.0251**</td>
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<td>(0.00936)</td>
<td>(0.00656)</td>
<td>(0.00946)</td>
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<tr>
<td>EBP × CF</td>
<td>0.0105**</td>
<td>0.00834**</td>
<td>0.0102**</td>
<td>0.0104**</td>
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<td>(0.00389)</td>
<td>(0.00437)</td>
<td>(0.00420)</td>
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<td>0.00546***</td>
<td>0.00540***</td>
<td>0.00542***</td>
<td>0.00567***</td>
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<tr>
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<td>(0.000242)</td>
<td>(0.000246)</td>
<td>(0.000300)</td>
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<td>0.0903***</td>
<td>0.104***</td>
<td>0.0954***</td>
<td>0.0584**</td>
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<td>(0.0178)</td>
<td>(0.0218)</td>
<td>(0.0269)</td>
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<td>-0.106***</td>
<td>-0.107***</td>
<td>-0.113***</td>
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<td>(0.00834)</td>
<td>(0.00857)</td>
<td>(0.0151)</td>
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<td>(7.54e-07)</td>
<td>(7.42e-07)</td>
<td>(7.27e-07)</td>
<td>(1.01e-06)</td>
</tr>
<tr>
<td>Size × CF</td>
<td>-6.46e-07**</td>
<td>-6.06e-07**</td>
<td>-6.76e-07**</td>
<td>-5.36e-07*</td>
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<tr>
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<td>(2.57e-07)</td>
<td>(2.41e-07)</td>
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<td>(2.99e-07)</td>
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<td>9,263</td>
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<tr>
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<td>0.999</td>
<td>1.000</td>
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<tr>
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<td>AR(3) Test</td>
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<td>0.167</td>
<td>0.158</td>
<td>0.562</td>
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<td>Realized</td>
<td>EGARCH</td>
<td>Realized</td>
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<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Year Dummies</td>
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<td>YES</td>
<td>YES</td>
<td>NO</td>
</tr>
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<td>Sector-year Dummies</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
</tr>
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<td>Method</td>
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<td>A-Bond 3-6</td>
<td>A-Bond 3-4</td>
<td>A-Bond 3-4</td>
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</table>

Robust standard error in parentheses; *** p<0.01, ** p<0.05, * p<0.1
This table shows the estimated ISCF, i.e. equation (19), employing the regression results in column (4) of Table 4. Pct10, Pct20, ... and Pct90 refer to the sample percentile values of the corresponding Variable, which is the realized volatility of enterprise values, excess bond premium and size, respectively, in column (1)-(3). Each ISCF is estimated when the corresponding Variable is valued at the corresponding sample percentile value, while the other two determinants are valued at their sample median values. The p-values are obtained via testing the estimated ISCF (as a linear combination of regression coefficients) being different from zero.
Table 6: Robustness

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<thead>
<tr>
<th></th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inv Ratio</td>
<td>-0.238***</td>
<td>0.0239</td>
<td>-0.104***</td>
<td>-0.0953***</td>
<td>-0.0847***</td>
</tr>
<tr>
<td></td>
<td>(0.115)</td>
<td>(0.0364)</td>
<td>(0.0335)</td>
<td>(0.0322)</td>
<td>(0.0292)</td>
</tr>
<tr>
<td>Uncertainty (Sigma)</td>
<td>0.214***</td>
<td>0.0683***</td>
<td>0.0328**</td>
<td>0.0321**</td>
<td>0.0158*</td>
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<tr>
<td></td>
<td>(0.0267)</td>
<td>(0.0162)</td>
<td>(0.0152)</td>
<td>(0.0134)</td>
<td>(0.00872)</td>
</tr>
<tr>
<td>Cash-flow Ratio (CF)</td>
<td>-0.0631*</td>
<td>-0.0535**</td>
<td>-0.0259***</td>
<td>-0.0225**</td>
<td>-0.0173*</td>
</tr>
<tr>
<td></td>
<td>(0.0361)</td>
<td>(0.0222)</td>
<td>(0.00841)</td>
<td>(0.0111)</td>
<td>(0.00891)</td>
</tr>
<tr>
<td>Sigma × CF</td>
<td>0.0414***</td>
<td>0.0367***</td>
<td>0.0112**</td>
<td>0.0176***</td>
<td>0.0105***</td>
</tr>
<tr>
<td></td>
<td>(0.0130)</td>
<td>(0.0127)</td>
<td>(0.00533)</td>
<td>(0.00504)</td>
<td>(0.00388)</td>
</tr>
<tr>
<td>EBP × CF</td>
<td>0.0123***</td>
<td>0.0164***</td>
<td>0.0998***</td>
<td>0.00595***</td>
<td>0.00488***</td>
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<tr>
<td></td>
<td>(0.000791)</td>
<td>(0.00154)</td>
<td>(0.00619)</td>
<td>(0.000355)</td>
<td>(0.000245)</td>
</tr>
<tr>
<td>lag Q</td>
<td>0.209***</td>
<td>0.118***</td>
<td>0.103***</td>
<td>0.0954***</td>
<td>0.0364</td>
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<tr>
<td></td>
<td>(0.0526)</td>
<td>(0.0279)</td>
<td>(0.0235)</td>
<td>(0.0269)</td>
<td>(0.0251)</td>
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<td>lag Real Sales Gr</td>
<td>-0.0850***</td>
<td>-0.0722***</td>
<td>-0.0959***</td>
<td>-0.112***</td>
<td>-0.0927***</td>
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<tr>
<td></td>
<td>(0.0134)</td>
<td>(0.00830)</td>
<td>(0.00925)</td>
<td>(0.00937)</td>
<td>(0.00817)</td>
</tr>
<tr>
<td>lag Net Leverage</td>
<td>4.16e-06***</td>
<td>-1.79e-07</td>
<td>-1.35e-06</td>
<td>2.39e-06***</td>
<td>3.71e-07</td>
</tr>
<tr>
<td></td>
<td>(1.25e-06)</td>
<td>(6.32e-07)</td>
<td>(9.42e-07)</td>
<td>(7.49e-07)</td>
<td>(7.00e-07)</td>
</tr>
<tr>
<td>Size</td>
<td>-1.45e-06**</td>
<td>3.14e-09</td>
<td>2.10e-07</td>
<td>-1.30e-06***</td>
<td>-4.87e-07*</td>
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<tr>
<td></td>
<td>(6.02e-07)</td>
<td>(9.01e-07)</td>
<td>(3.32e-07)</td>
<td>(2.31e-07)</td>
<td>(2.64e-07)</td>
</tr>
<tr>
<td>lag Inv Ratio</td>
<td>0.127***</td>
<td>0.0240</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

| Observations         | 81,519    | 68,633    | 78,371    | 69,143    | 81,212    |
| Number of Firms      | 9,261     | 8,068     | 9,204     | 9,028     | 9,232     |
| Hansen J Test        | 0.998     | 0.998     | 0.999     | 0.235     | 1.000     |
| AR(2) Test           | 0.000     | 0.000     | 0.000     | 0.000     | 0.358     |
| AR(3) Test           | 0.236     | 0.0823    | 0.0964    | 0.177     | 0.716     |
| AR(4) Test           | 0.529     |           |           |           |           |

<table>
<thead>
<tr>
<th>Robustness</th>
<th>Add R&amp;D Drop Big Q Fin Q Drop GR Dynamic Panel</th>
</tr>
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<tbody>
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<td>Uncertainty Type</td>
<td>Realized</td>
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<tr>
<td>Firm Fixed Effects</td>
<td>YES</td>
</tr>
<tr>
<td>Year Dummies</td>
<td>YES</td>
</tr>
<tr>
<td>Method</td>
<td>A-Bond 3-4</td>
</tr>
</tbody>
</table>

Robust standard error in parentheses; *** p<0.01, ** p<0.05, * p<0.1
### Table 7: Sub-sample Estimation

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<th>Inv Ratio</th>
<th>Inv Ratio</th>
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<tbody>
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<td><strong>Uncertainty (Sigma)</strong></td>
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<tr>
<td></td>
<td>(0.0424)</td>
<td>(0.0671)</td>
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<tr>
<td><strong>Cash-flow Ratio (CF)</strong></td>
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<td>-0.00489</td>
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<td></td>
<td>(0.00948)</td>
<td>(0.0111)</td>
</tr>
<tr>
<td><strong>Sigma × CF</strong></td>
<td>-0.0260**</td>
<td>0.00183</td>
</tr>
<tr>
<td></td>
<td>(0.0102)</td>
<td>(0.0191)</td>
</tr>
<tr>
<td><strong>EBP × CF</strong></td>
<td>0.00773**</td>
<td>0.0147</td>
</tr>
<tr>
<td></td>
<td>(0.00388)</td>
<td>(0.0117)</td>
</tr>
<tr>
<td><strong>lag Q</strong></td>
<td>0.00506***</td>
<td>0.00825***</td>
</tr>
<tr>
<td></td>
<td>(0.000263)</td>
<td>(0.000647)</td>
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<tr>
<td><strong>lag Real Sales Gr</strong></td>
<td>0.0314</td>
<td>0.0275</td>
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<td>(0.0278)</td>
<td>(0.0215)</td>
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<td><strong>lag Net Leverage</strong></td>
<td>-0.177***</td>
<td>-0.1000***</td>
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<td>(0.0231)</td>
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<td><strong>Size</strong></td>
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<tr>
<td></td>
<td>(8.82e-07)</td>
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<tr>
<td><strong>Size × CF</strong></td>
<td>-4.40e-07</td>
<td>2.03e-06***</td>
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<td>(2.74e-07)</td>
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<td>6,274</td>
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<td><strong>Hansen J Test</strong></td>
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<td>1.000</td>
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<tr>
<td><strong>AR(2) Test</strong></td>
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<td>0.000</td>
</tr>
<tr>
<td><strong>AR(3) Test</strong></td>
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<td>Firm Fixed Effects</td>
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<td>YES</td>
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<tr>
<td>Year Dummies</td>
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<td>YES</td>
</tr>
<tr>
<td>Method</td>
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<td>A-Bond 3-4</td>
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Robust standard error in parentheses; *** p<0.01, ** p<0.05, * p<0.1
Table 8: Bonus Depreciation Estimation

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<tr>
<td>(Sigma)</td>
<td>(0.503)</td>
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<tr>
<td>$z_{N,t}$</td>
<td>1.734**</td>
</tr>
<tr>
<td>(Sigma $\times z_{N,t}$)</td>
<td>-0.918*</td>
</tr>
<tr>
<td>(0.554)</td>
<td></td>
</tr>
<tr>
<td>Cash-flow Ratio</td>
<td>0.00297</td>
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<tr>
<td>(0.00673)</td>
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</tr>
<tr>
<td>lag Q</td>
<td>0.00530***</td>
</tr>
<tr>
<td>(0.000267)</td>
<td></td>
</tr>
<tr>
<td>lag Real Sales Gr</td>
<td>0.0996***</td>
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<tr>
<td>(0.0215)</td>
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<tr>
<td>lag Net Leverage</td>
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<td>(0.00975)</td>
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<td>Size</td>
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<td>(5.93e-06)</td>
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<td>AR(3) Test</td>
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<tr>
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Robust standard error in parentheses; *** p<0.01, ** p<0.05, * p<0.1
Table 9: Bonus Depreciation: Estimated Investment Responses

<table>
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<tbody>
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<td>Pct10</td>
<td>1.694*</td>
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<td>Pct20</td>
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<td>Pct30</td>
<td>1.641*</td>
<td>0.896</td>
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<tr>
<td>Pct40</td>
<td>1.605*</td>
<td>0.907</td>
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<tr>
<td>Pct50</td>
<td>1.556*</td>
<td>0.923</td>
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<td>Pct60</td>
<td>1.490</td>
<td>0.944</td>
</tr>
<tr>
<td>Pct70</td>
<td>1.395</td>
<td>0.978</td>
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<tr>
<td>Pct80</td>
<td>1.246</td>
<td>1.034</td>
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<td>Pct90</td>
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<td>1.161</td>
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<td>Pct10/Pct50</td>
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<td>Pct10/Pct90</td>
<td>1.791</td>
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</table>

*** p<0.01, ** p<0.05, * p<0.1

This table shows the estimated investment responses to a unit shock in $z_{N,t}$, i.e. equation (21), employing the regression results in Table 8. Pct10, Pct20, ..., and Pct90 refer to the sample percentile values of the realized volatility of enterprise values. Each response is estimated when the volatility is valued at the corresponding sample percentile value. The column Std Err lists instead the standard errors of the estimated responses. The p-values are obtained via testing the estimated responses (as a linear combination of regression coefficients) being different from zero.


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