Error-in-Variables and Tests of Asset Pricing Models with Liquidity Risk*

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Abstract

This paper diagnoses the impact of error-in-variables (EIV) on inferences in asset pricing models. I test the CAPM and the LCAPM in a manner that explicitly accounts for EIV, without pooling stocks into portfolios. I find that the single-factor CAPM beta is not priced. I prove that the aggregate liquidity risk in the liquidity-adjusted CAPM of Acharya and Pedersen (2005) is priced, and the portfolio-based approach is unable to capture this relationship. The cumulant-based approach to handle EIV enables me to test the effects of the individual components of aggregate liquidity risk, and I document that the risk associated with the commonality in illiquidity has a positive premium and the risk associated with the sensitivity of a stock’s illiquidity to the value-weighted market return has a negative premium. I also show that for microcap stocks, the risk attributable to the covariance between stock return and market-wide liquidity has a negative relationship with average returns. I prove that the LCAPM cannot be rejected when the betas are estimated at the stock-level.

Keywords: Error-in-Variables; Cumulants; Liquidity Risk

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1 Introduction

True security betas are unobservable. However, accurate measurement of betas is of paramount importance for evaluating the performance of asset pricing models in explaining the cross-section of average returns. In the traditional two-pass estimation methodology, which is usually used to test beta-pricing models, the betas estimated from the first stage are used as the explanatory variables in the second stage. Estimation error in the first stage leads to error-in-variables (EIV) in the second pass.

The EIV problem is widespread in economic statistics and distorts inferences if it is not taken into account explicitly. In a linear regression model, with one mismeasured regressor and an intercept, measurement error biases the slope coefficient towards zero. This is known as the attenuation effect due to EIV. This error may also have an effect in the opposite direction on the intercept, and bias the intercept away from zero. This is a large issue in asset pricing, where finding a significant intercept means the model is a failure.

In a model with one mismeasured regressor, EIV does not change the sign on the coefficient estimate. This does not always hold true when more than one regressor is affected by EIV. In a model with many mismeasured regressors, the slope coefficient on a mismeasured regressor is affected not only by its own measurement error, but also by the measurement error in the other mismeasured regressors. The second effect, which may bias the coefficients away from zero, is known as the contamination effect and is severe if the measurement errors are correlated. Because of the two opposing effects, all the coefficient estimates in a multiple regression are inconsistent, and no definite conclusion can be drawn about the direction of the bias or whether the coefficients are significantly different from zero.

EIV correction is of utmost importance in multivariate models plagued with measurement error. In asset pricing tests assessing the role of beta in explaining expected returns, researchers recognize this problem and are willing to trade-off power for precision. Therefore, in most of these tests, portfolios are used instead of individual stocks to address measurement error. It is argued that grouping stocks into portfolios diversifies away the estimation error in the betas. However, this approach has its own drawbacks. Ang et al.(2010) show that aggregating stocks into portfolios
decreases the cross-sectional dispersion of betas. Thus using portfolios as test assets lowers the efficiency of the tests due to the inherent loss of information. Liang (2000) contends that if the sorting variable used to form portfolios is also measured with error, then this approach biases the results. Lewellen et. al (2010) argue that the portfolios commonly used by researchers in the tests of asset pricing models bias these tests towards accepting the model. These papers highlight the need for a viable alternate technique to address EIV in the betas in asset pricing tests.

The primary implication of the single-factor capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), and Black (1972) suggests that market beta, which captures the return covariance of an individual security with the return on the market portfolio, should be priced. Nevertheless, prior research has suggested that market betas have limited ability to explain asset returns. Despite its repeated failure to explain the cross-section of expected stock returns, the popularity of the CAPM persists because of its intuitive appeal and simplicity. Since the beta estimates in CAPM are measured with error, it is crucial to determine whether the failure of the market beta to explain average stock returns is a result of model misspecification or the attenuation effect of EIV.

Liquidity is another attribute that could affect an investment’s required return. It describes the ease of buying or selling an asset without affecting its price. It is a multi-dimensional concept that is difficult to measure. Empirical studies have used various proxies for liquidity. Some of the different measures of liquidity used in literature, such as bid-ask spreads (Amihud and Mendelson, 1986), price-impact (Brennan and Subrahanyam, 1996; Amihud, 2002), and share turnover (Brennan, Chordia and Subrahmanyam, 1998) have been shown to affect asset returns. While each of these proxies likely captures some aspect of the theoretical concept of “liquidity”, none of them embeds their proxy explicitly into an asset pricing model.

In contrast, Acharya and Pedersen (2005), henceforth referred to as AP2005, propose a liquidity-adjusted capital asset pricing model (LCAPM), which is a generalization of the CAPM in an economy with trading frictions. In the LCAPM, the “net market risk” is given by the sensitivity of stock returns net of its illiquidity costs to the market return net of the market illiquidity cost. This “net market risk” can be decomposed into four components, three of which are related to stock and market-wide liquidity. AP2005 start with a simple theoretical model based on the
CAPM and show that the overall risk in a market with trading costs can be attributed to the beta due to the sensitivity of stock returns to fluctuations in market return, and three liquidity betas stemming from the commonality in liquidity, the sensitivity of stock returns to fluctuations in market-wide liquidity, and the sensitivity of stock illiquidity to fluctuations in market return. Similar to the market beta, these liquidity betas are unobservable. They test the LCAPM using a two-pass procedure, where liquidity betas are estimated in the first stage from the moments of past returns and illiquidity. These beta estimates are used in the second stage regressions. Hence, their test of LCAPM suffers from EIV.

This paper tests the CAPM and the LCAPM in a manner that explicitly accounts for EIV, without pooling stocks into portfolios. I employ a unique method based on cumulants developed in Geary (1942) to deal with EIV in asset pricing tests. One of the main advantages of this methodology is that it enables me to use individual stocks as test assets, and eliminates the need to form portfolios. I use the information contained in the third and higher order cumulants of the joint distribution of the observable variables. This technique has previously been used by Erickson, Jiang and Whited (2014), henceforth referred to as EJW2014, to address EIV in areas of corporate finance. Their paper developed a convenient two-step minimum distance estimator with a simple closed-form solution. The applications covered in their paper dealt with a maximum of two mismeasured regressors, whereas this paper will deal with up to five mismeasured regressors. I use simulated data to highlight the importance of accounting for measurement error in a model with multiple mismeasured regressors. A comparison of small sample performance of cumulant-based estimators with OLS is also presented, which highlights the advantages of using the cumulant approach.

I estimate the CAPM with portfolio as well as stock-level betas. I find that the relation between expected returns and portfolio beta is flat. Then I estimate the market beta for individual stocks and use the cumulant-based approach to correct for EIV. Even after correcting for measurement error in the market beta, I find that the CAPM is still misspecified.

Next, I present the empirical results of tests of the LCAPM. Previous papers have estimated the betas in the LCAPM against an equal-weighted market portfolio. But, the CAPM states that the value-weighted portfolio of all stocks in the economy should be the tangency portfolio. This
motivates the estimation of betas in my paper based on a value-weighted instead of an equal-weighted market portfolio. Using the more theoretically appropriate regressor leads to results that are stronger, more intuitive and more consistent than results found using the equal-weighted market portfolio.

First, I group stocks and estimate betas at the portfolio level, as done in previous papers, and find that liquidity risk is not priced. But grouping stocks not only diversifies away the estimation error, but also diversifies away information contained in individual stock-level betas. Next, I estimate the betas for individual stocks and address the inherent EIV in these stock-level betas using the cumulant-based approach. Using these stock-level betas, I find that liquidity risk is priced and the LCAPM cannot be rejected. This is the first paper to document the positive relation between aggregate liquidity risk and expected returns after accounting for the estimation error in the stock-level betas.

This aggregate liquidity beta can be decomposed into three separate betas arising from different components of a stock’s exposure to systematic risk. AP2005 find that the three liquidity betas are highly correlated with each other and a model estimated with the individual betas suffers from severe multicollinearity. Thus the existing papers have not been able to disentangle the effects of these betas on stock returns. However, when I estimate the betas at the stock level, based on a value-weighted market portfolio, I find that the correlations between the betas are quite low. Therefore, another advantage of the technique used in my paper is that it allows me to disaggregate the liquidity beta into its individual components. Thus the statistical identification of the separate effects of different liquidity risks is possible using the cumulant-based approach. I find that the return premium due to the risk associated with the commonality in illiquidity is positive and significant. I also find that the risk due to the covariance between an asset’s illiquidity and the market return is negatively priced. To the best of my knowledge, this is the first paper to document the relationship between individual liquidity risks and expected returns after controlling for measurement error explicitly.

Next, I investigate whether the pricing ability of liquidity risk is driven by microcap stocks. Fama and French (2008) define the microcaps as stocks whose market capitalization is below the 20th NYSE percentile. They show that though these stocks account for only 3% of the market
capitalization of the NYSE-Amex-NASDAQ universe, they account for about 60% of the stocks. Hou et al. (2015) note that the microcap stocks are highly illiquid and have higher transaction costs. I find that the coefficient on liquidity beta is much smaller when we exclude the microcaps from the sample. I also find evidence of a negative premium due to the risk associated with the return sensitivity to market illiquidity.

The remainder of the paper is structured as follows. Section 2 reviews the existing literature. Section 3 explains the classical EIV model and how it affects returns. It also describes the EJW2014 method to handle the bias due to EIV. Section 4 discusses the data and methodology employed to estimate the LCAPM. Section 5 uses simulations to compare the small-sample performance of OLS and higher order cumulants. Section 6 describes the empirical applications and results of the tests of the CAPM and the LCAPM. Section 7 conducts robustness checks. Section 8 concludes.

2 Relevant Literature

This paper forms a nexus between two different lines of research, namely, the literature on asset pricing models that seek to explain the relationship between risk and expected returns of securities and the literature on measurement error and techniques adopted to get consistent estimates in error-laden models.

The first strand of literature explores whether systematic risk is priced. It takes us back to the seminal question in asset pricing that asks whether differences in exposure to market-wide risk factors can explain the differences in the expected returns of assets. A variety of asset pricing models have been proposed in the literature to understand why different assets earn different rates of return and if it can be attributable to the difference in their sensitivities to systematic risk. The factors used in these models to explain returns are often different.

According to the single-factor CAPM, the return on the market portfolio is the only source of non-diversifiable risk and an asset’s exposure to this factor determines its expected return. However, Reinganum (1981), Lakonishok and Shapiro (1986) and Fama and French (1992) find that a relation between expected returns and market beta does not exist in the 1963-1990 period.
Richer models have been proposed that use other economic variables as systematic risk factors. Unanticipated changes in the term premium, default premium, the growth rate of industrial production and inflation constitute the factors in Chen, Roll and Ross (1986). Jagannathan and Wang (1996) extend the proxy for market return to include the return on human capital as a factor. Firm characteristics have also been used to create factors that affect expected returns. Fama and French (1993) show that a three-factor model, consisting of the market factor, a factor based on the market value of equity (small-minus-big, SMB), and a factor based on the book-to-market equity (high-minus-low, HML), jointly do a reasonable job at explaining the cross section of stock returns. However, post 1990s, the model’s performance deteriorates. The three-factor model fails to account for the significant alphas generated by momentum strategies. Carhart (1997) introduces a momentum factor, which is based on the prior returns (high prior returns-minus-low prior returns, MOM), as an extension to the Fama-French three-factor model and he finds that short-term persistence in equity mutual fund returns can be explained by the MOM factor.

Most of these models assume frictionless markets. But, in reality there are no truly frictionless markets since trading is always associated with certain costs or restraints. Hence researchers have incorporated the effect of trading frictions into factor models. Pastor and Stambaugh (2003) measure illiquidity as the return reversal in response to volume shocks and examine whether market-wide liquidity is a priced factor. They find that the innovations in market illiquidity is a priced factor that is related to the average returns of an asset. The LCAPM proposed by AP2005 provides a unified framework that encompasses the different channels via which liquidity affects stock returns. In the LCAPM, the expected return of a security depends on its expected illiquidity level, market risk and three liquidity risks. But, as mentioned earlier, it is difficult to investigate the pricing ability of the liquidity risks because they are highly correlated with each other. The results in AP2005 are not robust to various portfolio formation techniques and not consistent across different model specifications. I find that the incongruity is a consequence of the measurement error embedded in these liquidity-based models. The measurement error in these models stem from the error in the liquidity estimate as well as the estimation error in beta estimates. Thus it is crucial to address EIV in tests of the LCAPM.

The second strand of literature studies the effect of measurement error in economic models
and explores methods to overcome the EIV problem. Papers in corporate finance often use proxy
variables when the independent variables in a predicted relationship are not observable. Traditionally,
instrumental variables have been used to address the bias due to measurement error. However,
it is often difficult to find a good instrument that is correlated with the mismeasured variable, but
uncorrelated with any other determinant of the dependent variable. Erickson and Whited (2000,
2002), henceforth referred to as EW2000 and EW2002 respectively, circumvent the need to find
suitable instruments in such models. They develop consistent estimators using information in third
and higher order moments of the observable variables. Nevertheless, the estimating equations used
in these papers are complicated non-linear functions of the parameters to be estimated, and estimation
is sensitive to the starting values used in the numerical optimization process, as local optima
may exist.

Geary (1942) proposes a method to derive the true relationship between variables, when the
independent variables are measured with error. He develops a system of estimating equations using
high order cumulants to determine the true underlying relation between the variables. Building
on Geary, EJW2014 propose simple linear estimating equations with a closed-form solution us-
ing high-order cumulants, instead of moments. Furthermore, their paper extends Geary’s results
by employing minimum distance estimation to solve models which have overidentifying cumulant
equations.

In asset pricing papers that seek to estimate the price of risk in linear factor models, the
most common approach used to address EIV is to form diversified portfolios based on common
characteristics. Fama and MacBeth (1973) show that this method reduces the estimation error in
the individual beta estimates. However, the portfolio grouping method may conceal information
that exists in individual stocks. Roll (1977) proposes that mispricing in individual assets can be
diversified away in portfolios. Shanken (1992) suggests a correction factor under the assumption of
conditional homoscedasticity to account for the estimation error in betas. Kim and Skoulakis (2015)
use a regression-calibration method to correct the betas estimated in the first pass in the two-pass
cross-sectional regression (CSR) method. A correction factor is used to calibrate the betas. These
calibrated betas, which are used in the second-pass, satisfy the orthogonality conditions necessary
for N-consistency. The authors estimate risk premia using individual stock-level data and over
short time horizons. They develop an entirely new set of asymptotic results specialized for their regression-calibration approach.

The approach in this paper is much simpler. I use the higher order cumulants technique introduced by Geary (1942) to tackle the bias and possible inconsistency in asset pricing tests due to measurement error. This method eliminates the need to form portfolios, and allows me to use individual stock level data. It also relies on standard asymptotic distribution theory. EJW2014 use a maximum of two mismeasured regressors in their applications. I apply the EJW2014 methodology to asset pricing models which have greater numbers of mismeasured regressors.

3 Errors in Variables and Estimated Betas

A common procedure for investigating the relation between betas and expected asset returns is based on Fama and MacBeth (1973), henceforth referred to as FM1973. It involves two steps. For the single-factor CAPM, risk factor betas for individual stocks are estimated in the first step from time-series regressions given by

\[ R_{i,t} = \alpha_i + \beta_i F_{m,t} + \nu_{i,t} \]

where \( F_{m,t} \) is the market realization for month \( t \), and \( \beta_i \) is the beta for stock (or portfolio) \( i \). The second step estimates risk premia from monthly cross-sectional regressions of returns on the beta estimates obtained from the first step

\[ R_{i,t} = \rho_0 + \lambda_1 \hat{\beta}_i + u_{i,t} \]

where \( \lambda_1 \) is the market factor risk premium and \( \hat{\beta}_i \) is the estimated beta of each stock (or portfolio) from the first pass. A time series of \( \hat{\lambda}_1 \) is obtained from monthly estimates of Equation 2 and the sample mean of this distribution is used as the final estimate \( \lambda_1 \). The standard error of the estimate is based on the assumption that \( \hat{\lambda}_1 \) estimates are independent and identically distributed. Inferences are drawn based on these values.

This method is easy to implement and has become a standard methodology in the finance literature. The true beta, \( \beta \), is unobservable and the estimated beta, \( \hat{\beta} \), from the first pass is used
as the independent variable in the second pass cross-sectional regression. The difference between $\beta$ and $\hat{\beta}$ constitutes the measurement error in the model. Hence this method suffers from EIV. The next subsection describes a model with EIV and how to get consistent estimates in such a model.

### 3.1 Classical Error-in-variables problem

Let $(y_{i,t}, x_{i,t}, z_{i,t}), i = 1, ..., n, t = 1, ...T$, be sequences of observable variables. $(u_{i,t}, \epsilon_{i,t}, \chi_{i,t})$ are sequences of unobservable variables. The classical EIV model is described by

\begin{align*}
y_{i,t} &= z_{i,t} \rho + \chi_{i,t} \lambda + u_{i,t} \\
x_{i,t} &= \chi_{i,t} + \epsilon_{i,t}
\end{align*}

where $\lambda$ and $\rho$ are the unknown vectors. Regressing $y_{i,t}$ on $x_{i,t}$ gives inconsistent estimates of $\lambda$. This issue is categorically addressed in EW2000, EW2002 and EJW2014, which develop consistent estimators for models plagued with EIV. They use estimating equations involving higher order moments (EW2000, EW2002) and higher order cumulants (EJW2014), which are functions of moments, in order to estimate consistently the slope parameters in Equation 3. The next subsection gives a brief summary of cumulants and the advantages of using cumulant based estimators.

### 3.2 Cumulant estimators

Cumulants were first studied in 1889 by T.N. Thiele, who had termed them as semi-invariants. They are polynomial functions of moments. Cumulants $\kappa_r$ of a random variable $X$ are defined by a cumulant-generating function $K(\xi)$, which is the natural logarithm of the moment-generating function $M(\xi)$. $\kappa_r$ are the coefficients in the Taylor expansion of the cumulant generating function about the origin, and are given by

\begin{equation}
K_x(\xi) \equiv \ln M(\xi) = \ln E[e^{\xi X}] = \sum_{r=0}^{\infty} \kappa_r \xi^r / r!
\end{equation}

where $\kappa_0 = 0$, $\kappa_1 = E[X]$, $\kappa_2 = E[X^2] - E[X]^2$, $\kappa_3 = E[X^3] - 3E[X^2]E[X] + 2E[X]^3$.

A property of cumulants that makes them useful is additivity - the $r$th cumulant of the sum of
two independent random variables equals the sum of the \( rth \) cumulant of the individual variables. This property also makes cumulants an attractive choice in estimating models with measurement error.

EJW2014 exploits properties of cumulants to derive two-step minimum distance estimators. To apply this method, the variables in Equations 3 and 4 should satisfy the following assumptions: (i) the elements of \( (u_{i,t}, \chi_{i,t}, z_{i,t}) \) should have finite moments of every order, (ii) \( (u_{i,t}, \epsilon_{i,t}) \) should be independent of \( (z_{i,t}, \chi_{i,t}) \), (iii) the elements of \( (u_{i,t}, \epsilon_{i,t}) \) should be independent of each other, (iv) \( E(u_{i,t}) = 0 \) and \( E(\epsilon_{i,t}) = 0 \), (v) \( E[(\chi_{it}, z_{it})'(\chi_{it}, z_{it})] \) should be positive definite.

EJW2014 use the relations between cumulants to form an over-identified system of estimating equations. Using a minimum distance estimator to efficiently combine information from the high order cumulants, they solve the equations for \( \lambda \). In a model with mismeasured and perfectly measured regressors, as described in Equations 3 and 4, the perfectly measured regressors are first partialled out and the system is expressed in terms of regression residuals. Population linear regression of \( x_{i,t} \) on \( z_{i,t} \) yields the residual \( x_{i,t} - z_{i,t} \mu_x \), where \( \mu_x \equiv [E(z_{it}'z_{it})]^{-1}[E(z_{it}'x_{it})] \). Linear regression of \( \chi_{i,t} \) on \( z_{i,t} \) yields the residual \( \chi_{i,t} - z_{i,t} \mu_\chi \). But since \( z_{i,t} \) and \( \epsilon_{i,t} \) are independent of each other,

\[
\mu_x = [E(z_{it}'z_{it})]^{-1}[E(z_{it}'(\chi_{it} + \epsilon_{i,t}))] = [E(z_{it}'z_{it})]^{-1}[E(z_{it}'\chi_{it})] \equiv \mu_\chi
\]

The residual \( \chi_{i,t} - z_{i,t} \mu_\chi \) is denoted by \( \eta_{i,t} \).

Subtracting \( z_{i,t} \mu_x \) from both sides of Equation 4 yields \( \dot{x} \) which is defined as

\[
\dot{x} \equiv x_{i,t} - z_{i,t} \mu_x = \chi_{i,t} - z_{i,t} \mu_x + \epsilon_{i,t} = \eta_{i,t} + \epsilon_{i,t}
\]

The residual of population linear regression of \( y_i \) on \( z_i \) is \( y_i - z_i \mu_y \), where \( \mu_y \equiv [E(z_{it}'z_{it})]^{-1}[E(z_{it}'y_{it})] \).

Subtracting \( z_{i,t} \mu_y \) from both sides of Equation 3 yields \( \dot{y} \) which is defined as

\[
\dot{y} \equiv y_{i,t} - z_{i,t} \mu_y = \eta_{i,t} \lambda + u_{i,t}
\]

In the two-step minimum distance estimation, the first step comprises of substituting the least
square estimates of $\hat{\mu}_x$ and $\hat{\mu}_y$ in Equations 7 and 8 respectively, as they are consistent estimates of $\mu_x$ and $\mu_y$. The second step estimates $\lambda$ using the sample cumulants of $y_i - z_i \hat{\mu}_y$ and $x_i - z_i \hat{\mu}_x$. The $\lambda$ estimation approach is based on the results derived in Geary(1942). EJW2014 show that the system of cumulant based estimating equations is identified if $\eta_{i,t}$ is skewed. Thus the third moment, and thereby the third order cumulant of $\eta_{i,t}$ should be non-zero.

Consider a model with $J$ mismeasured regressors. If $\kappa(s_0, s_1, ..., s_J)$ is the multivariate cumulant of order $s_0$ in $\hat{y}$, order $s_1$ in $\hat{x}_1$ and order $s_J$ in $\hat{x}_J$, then Geary proves that

$$\kappa(s_0 + 1, s_1, s_2, \ldots, s_J) = \lambda_1 \kappa(s_0, s_1 + 1, s_2, \ldots, s_J) + \ldots + \lambda_J \kappa(s_0, s_1, s_2, \ldots, s_J + 1)$$  \hspace{1cm} (9)

as long as at least two elements in $(s_0, s_1, s_2, \ldots, s_J)$ are different from zero. The system of equations based on Equation 9 can be represented by

$$K_y = K_x \lambda$$  \hspace{1cm} (10)

$J$ independent equations can identify $\lambda$. If the number of equations in the system, $M$, is less than $J$, then $\lambda$ is indeterminate. If $M > J$, then the system is overidentified and EJW2014 discusses a method of estimating $\lambda$ from minimum distance estimation of

$$\hat{\lambda} \equiv \text{argmin}_l (\hat{K}_y - \hat{K}_x l)^\prime \hat{W} (\hat{K}_y - \hat{K}_x l)$$  \hspace{1cm} (11)

where $\hat{W}$ is a symmetric positive definite weighting matrix.

The cumulants in Equation 10 can be obtained from the moments of the distribution of the observable variables. Cumulants can be expressed as the sums of products of moments. Chapter 2 of McCullagh (1987) gives an expression for any cumulant of a distribution as a function of the moments of the distribution. An example of the relationship between third order cumulants and moments involving four random variables is given by:

$$\kappa(1, 1, 1, 0) \equiv E(\hat{y}\hat{x}_1\hat{x}_2) - E(\hat{y}\hat{x}_1)E(\hat{x}_2) - E(\hat{y}\hat{x}_2)E(\hat{x}_1) - E(\hat{x}_2\hat{x}_3)E(\hat{y}) + 2E(\hat{y})E(\hat{x}_1)E(\hat{x}_2) \equiv E(\hat{y}\hat{x}_1\hat{x}_2)$$  \hspace{1cm} (12)
where \( \kappa(1, 1, 1, 0) \) is a third order cumulant of degree 1 in \( \hat{y} \), degree 1 in \( \hat{x}_1 \), degree 1 in \( \hat{x}_2 \) and degree 0 in \( \hat{x}_3 \). Thus the matrices \((\hat{K}_y, \hat{K}_x)\) can be estimated from the sample moments of \( \hat{y} \equiv y_{i,t} - z_{i,t}\hat{\mu}_y \) and \( \hat{x} \equiv x_{i,t} - z_{i}\hat{\mu}_x \).

In this paper, I consider models with one, three and five mismeasured regressors. In a model with three mismeasured regressors, \( \chi \) contains three elements. Examples of third order cumulant estimating equations represented by Equation 10 are:

\[
K(2, 1, 0, 0) = \lambda_1 K(1, 2, 0, 0) + \lambda_2 K(1, 1, 1, 0) + \lambda_3 K(1, 1, 0, 1) \tag{13}
\]

\[
K(2, 0, 0, 1) = \lambda_1 K(1, 1, 0, 1) + \lambda_2 K(1, 0, 1, 1) + \lambda_3 K(1, 0, 0, 2) \tag{14}
\]

With three mismeasured regressors, we have 18, 66 and 159 estimating equations using cumulants up to degree three, four and five respectively. Hence, the model is always overidentified, and the minimum distance estimator of \( \lambda \) in Equation 11 can be obtained from

\[
\hat{\lambda} = (\hat{K}_x' \hat{W} \hat{K}_x)' \hat{K}_x' \hat{W} \hat{K}_y \tag{15}
\]

I use this method to obtain consistent estimates in tests of asset pricing models with mismeasured regressors. The cumulant based estimating equations have a closed-form solution, which eliminates the need to find suitable starting values and iterating to a numerical minimization of the objective function given in Equation 11. As demonstrated in EW2012, performance of moment estimators is highly sensitive to the selection of starting values when numerical optimization is used. This is one of the main advantages of using cumulant over moment based estimators.

An increase in the order of cumulants substantially increases the number of estimating equations. However, it does not necessarily improve the accuracy of the estimation. The performance of different orders of cumulant estimators can be compared by comparing the percentage of cumulants that are statistically different from zero. The next section describes the data and beta estimation methodology used in this paper.
4 Data and Methodology

4.1 Data

I use daily return and volume data from January 1st, 1963 until December 31st, 2014 for all common stocks listed on NYSE, AMEX and NASDAQ, available from CRSP. Accounting information is obtained from Compustat Annual and Quarterly Fundamental Files. A firm’s book-to-market ratio is computed by dividing its last fiscal year-end book value by its fiscal year-end market equity. Stocks with share prices in the top 1% or bottom 1% at the end of the previous month are excluded. Volume is measured in millions of dollars. The rate on 30-day US Treasury bill is used as the risk-free rate. I define an all-but-microcap sample as the universe of stocks excluding the microcaps. The next subsection describes the LCAPM proposed in AP2005, which is estimated using these data.

4.2 Liquidity-adjusted Capital Asset Pricing Model

The LCAPM is a CAPM in returns net of illiquidity costs. In this model, the expected net return of a stock (stock return net of its illiquidity cost) depends on its net market beta, which is defined as the sensitivity of the net return of the stock to the market net return (return on market portfolio net of market illiquidity cost). In other words, it is the relation in gross returns that must hold if the CAPM holds in net returns. A conditional version of the LCAPM is given by

\[
E_t(R_{i,t+1} - C_{i,t+1}) = R_{f,t} + \lambda_t \frac{\text{Cov}_t(R_{i,t+1} - C_{i,t+1}, R_{m,t+1} - C_{m,t+1})}{\text{Var}_t(R_{m,t+1} - C_{m,t+1})}
\]

where \( R_{i,t+1} \) is an asset’s gross return, \( C_{i,t+1} \) is the illiquidity cost, \( R_{f} \) is the risk free rate, \( \lambda_t \) is the risk premium associated with net beta, \( R_{m,t+1} \) is the market return and \( C_{m,t+1} \) is the market illiquidity cost.

Following Lesmond, Ogden and Trzcinka (1999), henceforth referred to as LOT1999, I use the percentage of zero daily returns as a proxy of illiquidity cost. This measure of illiquidity has also been used in Bekaert, Harvey, and Lundblad (2007) and Lee (2011). Fong, Holden, and Trzcinka (2011) find that zero returns efficiently capture the time-series patterns of stock market liquidity.
compared to effective spread-based benchmarks. The percentage of zero returns is defined by

\[ ZR_{i,t} = \frac{N_{i,t}}{T_t} \]  

(17)

where \( N_{i,t} \) is the number of trading days of stock \( i \) in month \( t \) that experience no price movement from the prior end-of-day price. \( T_t \) is the number of trading days in month \( t \), which is defined by the number of days with non-missing returns. \( ZR_{i,t} \) is estimated using CRSP daily stock returns. If a stock has less than ten trading days in a month, then it is dropped from the data. I also exclude observations that have a zero-return proportion greater than 80%. The average of the monthly percentage of zero returns over the past 12 months is denoted by \( ZR_{-12} \) and is used as a proxy for the illiquidity cost, \( C \), in Equation 16.

The net beta in Equation 16 can be decomposed into four separate betas and the gross return on a stock can be expressed as

\[ E_t(R_{i,t+1}) - R_{f,t} = E_t(C_{i,t+1}) + \lambda_t \frac{Cov_t(R_{i,t+1}, R_{m,t+1})}{\text{var}_t(R_{m,t+1} - C_{m,t+1})} + \lambda_t \frac{Cov_t(C_{i,t+1}, C_{m,t+1})}{\text{var}_t(R_{m,t+1} - C_{m,t+1})} - \lambda_t \frac{Cov_t(R_{i,t+1}, C_{m,t+1})}{\text{var}_t(R_{m,t+1} - C_{m,t+1})} - \lambda_t \frac{Cov_t(C_{i,t+1}, R_{m,t+1})}{\text{var}_t(R_{m,t+1} - C_{m,t+1})} \]  

(18)

Equation 18 gives a straightforward relation between the expected gross return, \( E_t(R_{i,t+1}) \), the expected illiquidity cost, \( E_t(C_{i,t+1}) \), and four covariances which represent the components of a stock’s sensitivity to systematic risk. An unconditional LCAPM described by

\[ E(R_{i,t} - R_{f,t}) = E(C_{i,t}) + \lambda \beta_1^1 + \lambda \beta_2^2 - \lambda \beta_3^3 - \lambda \beta_4^4 \]  

(19)

can be obtained by assuming constant \( \lambda \) and constant conditional covariances of innovations in illiquidity and returns, where

\[ \beta_1^1 = \frac{\text{cov}(R_{i,t+1}, R_{m,t+1})}{\text{var}(R_{m,t+1} - [C_{m,t+1} - E_t(C_{m,t+1})])} \]  

(20)

\[ \beta_2^2 = \frac{\text{cov}(C_{i,t+1} - E_t(C_{i,t+1}), C_{m,t+1} - E_t(C_{m,t+1}))}{\text{var}(R_{m,t+1} - [C_{m,t+1} - E_t(C_{m,t+1})])} \]  

(21)
\[ \beta_i^3 = \frac{\text{cov}(R_{i,t+1}, C_{m,t+1} - E_t(C_{m,t+1}))}{\text{var}(R_{m,t+1} - [C_{m,t+1} - E_t(C_{m,t+1})])} \]  

(22)

\[ \beta_i^4 = \frac{\text{cov}(C_{i,t+1} - E_t(C_{i,t+1}), R_{m,t+1})}{\text{var}(R_{m,t+1} - [C_{m,t+1} - E_t(C_{m,t+1})])} \]  

(23)

\[ \beta^1 \] is proportional to the covariance of a stock’s return with the return on the market portfolio. Hence it is similar in flavor to the market beta. \( \beta^2 \) represents the risk due to commonality in illiquidity. Chordia et al. (2000) and Hasbrouck and Seppi (2001) find that variation in stock liquidity is positively correlated with variation in market liquidity. Thus, Equation 21 suggests that if shocks to liquidity cannot be diversified away, then the sensitivity of a stock to such shocks could be regarded as a component of a stock’s exposure to systematic risk. Hence stocks with higher sensitivities to broad illiquidity shocks may demand a higher return. \( \beta^3 \) depends on the sensitivity of the stock return to fluctuations in market illiquidity. A stock with high \( \beta^3 \) has a high return when the market is illiquid, which works as a hedge against market illiquidity. Pastor and Stambaugh (2003) document that after controlling for exposure to other priced factors, the average return for stocks with high covariation with market liquidity exceeds that for stocks with low covariation with market liquidity by 7.5% annually. However, their paper does not control for the other components of liquidity risk. \( \beta^4 \) captures the sensitivity of a security’s illiquidity to the market return. A stock with high \( \beta^4 \) is illiquid when market returns are high and is liquid in a down market. Thus an asset with high \( \beta^4 \) works as a hedge against market downturns since it has low illiquidity costs during states of low market return.

In AP2005, the total effect of systematic risk is captured by the combination of the three liquidity betas and the market beta. The authors argue that the liquidity betas are highly correlated with each other, and including them separately in the cross-sectional regression leads to a collinearity problem. Thus they aggregate the betas, and their framework does not allow them to identify the effects of the individual liquidity risks on asset returns. They do this by defining the net liquidity beta as

\[ \beta_{i}^{net} = \beta_i^2 - \beta_i^3 - \beta_i^4 \]  

(24)

and the condensed LCAPM is given by

\[ E(R_{i,t} - R_{f,t}) = E(C_{i,t}) + \lambda^1 \beta_i^1 + \lambda^{net} \beta_i^{net} \]  

(25)
The next subsection describes how betas in Equations 24 and 25 are estimated.

4.3 Beta Estimation

This paper studies how liquidity affects stock returns using test portfolios (as done in previous papers) as well as individual stocks. Grouping stocks into portfolios is the standard method adopted to address EIV. Black et al. (1972) show that grouping can substantially reduce the bias due to measurement error and for large sample size, sampling error in the estimated betas can be eliminated. Their paper suggests estimating the group risk parameter (portfolio beta) on sample data that is not used in the ranking procedure in order to prevent an association of the measurement error in the $\beta$ estimates with the errors in the coefficients used in ranking the portfolios. To implement this technique, $\hat{\beta}_{pre}$ is computed from five years of previous monthly data for each stock. Individual securities are then assigned to groups based on their $\hat{\beta}_{pre}$ ranking. Portfolio data from a subsequent time period is then used to estimate portfolio $\hat{\beta}_k$.

However, Liang (2000) contends that sorting based on variables computed using a preceding sample does not completely eliminate the possibility of biased inferences due to measurement error. In the portfolio formation process, estimation errors embedded in the sorting variable can cause systematic bias in the results. He shows that grouping stocks may aggregate these measurement errors, which results in positive or negative errors for extreme portfolios that further biases the results. Moreover, grouping stocks into portfolios also causes loss of information present in individual stock data.

Though the existing literature has identified problems associated with portfolio formation procedures in asset pricing tests, this method is still used to deal with measurement error. In contrast, I use higher order cumulant based estimators to tackle the bias and inconsistency caused by EIV. This method does away with the need to use portfolios as test assets and I use individual stock-level data. To compare the performance of the two approaches mentioned above, I also use portfolio betas in the tests in this paper.

To contrast the ability of portfolio betas with that of individual betas to explain returns, betas are estimated at both the portfolio and the stock level as described below. I compute the
market return for each month $t$ based on a value-weighted average of returns on all stocks in the market portfolio in that month. Similarly, market illiquidity for month $t$ is defined as the value-weighted average of $ZR_{i,t}$ in that month. The first order autocorrelation in market illiquidity is 0.99. Thus innovations in market illiquidity, $C_{m,t} - E_{t-1}(C_{m,t})$ are obtained from the first-differences in illiquidity levels. Similarly, for stock $i$, the change in illiquidity is used as its innovation.

Tests using individual stock level data use individual stock-level betas, which are obtained as follows. For each stock $i$ in month $t$, $\beta_{i,t}^{k}$ where $k = (1, 2, 3, 4)$, is computed from monthly returns and innovations in illiquidity for stock $i$ and for the value-weighted market portfolio, over months $t - 60$ to $t - 1$, using Equations 20-23. The 60-month window rolls forward every month. Individual stock windows with less than 36 prior monthly returns or innovations in illiquidity are dropped.

Next, I describe the portfolio formation approach used in this paper to obtain the portfolio betas. For each stock $i$, the pre-ranking beta, $\beta_{i,t}^{k,pre}$, $(k = 1, 2, 3, 4)$ of month $t$ is estimated using the time-series of monthly returns and innovations in illiquidity for the previous 60 months. If a stock has less than 36 valid observations in the $t - 60$ to $t - 1$ monthly window, then $\beta_{i,t}^{k,pre}$ for that stock is set to missing. Thus the $\beta_{i,t}^{k,pre}$ of a stock is the same as $\beta_{i,t}^{k}$ described earlier. Stocks are then sorted monthly into ten equal-weighted portfolios based on $\beta_{i,t}^{k,pre}$ for month $t$. The post-ranking portfolio beta, $\beta_{p}^{k}$, is then estimated for each of the ten portfolios over the entire sample period using Equations 20-23.

The post-ranking portfolio beta estimation procedure to obtain $\beta_{p}^{1}$ is illustrated as follows. First, I calculate the pre-ranking $\beta_{i,t}^{1,pre}$ for stock $i$ in month $t$ using Equation 20, based on the time-series of previous 60 months of returns and illiquidity innovations. Then, at the beginning of month $t$, stocks are sorted into ten equally weighted portfolios based on $\beta_{i,t}^{1,pre}$. Subsequently, the post-ranking beta for portfolio $p$, denoted by $\beta_{p}^{1}$, is estimated over the entire sample period, using Equation 20. $\beta_{p}^{1}$ is then assigned to all the individual stocks $i$, which belong to portfolio $p$ in a given month $t$. The same technique is repeated to sort stocks into deciles based on $\beta_{i,t}^{k,pre}$ and calculate $\beta_{p}^{k}$ for $k = 2, 3, 4$. This $\beta_{p}^{k}$ is assigned to all stocks belonging to portfolio $p$ ranked on the basis of $\beta_{i,t}^{k,pre}$. This approach is similar to that used in Lee (2011). But he uses an equal-weighted market portfolio instead of a value-weighted market portfolio to estimate the betas. Additionally, the 5-year window for $\beta_{i,t}^{k,pre}$ estimation in Lee (2011) rolls forward on a yearly basis, and the decile
portfolios are formed at the beginning of every year, instead of every month as I do here.

5 Simulations

In this section I use simulations in order to highlight the importance of an EIV correction. Small sample performance of the cumulant estimators is compared with that of OLS estimators. I consider two models, similar to the models estimated later in the paper using real data. The first model has three mismeasured regressors and two perfectly measured regressors. This corresponds to the condensed LCAPM. The second model has five mismeasured regressors and two correctly measured regressors, thus corresponding to the LCAPM with individual liquidity risks.

5.1 Simulation Setup

The data generating process (DGP) should match the characteristics of the real data set as closely as possible. Hence, I generate panel data of length 564, which is equal to the number of months in the second stage CSR, and width 3000, which is equal to the average number of stocks per month. For the first model, I select values for \( \lambda_1, \lambda_{net}, \lambda_{zr,12} \) and \( \rho \) which are the unknown parameters in the model. The entire panel is then generated from a system of equations as follows:

\[
\begin{align*}
\chi_j^{i,t} &= \delta \chi_j + \nu \chi_{j,t} \\
\zeta_p^{i,t} &= \delta \zeta_p + \nu \zeta_{p,t} \\
u_{i,t} &= \nu_{i,t} \\
\epsilon_j^{i,t} &= \nu \epsilon_{j,t}
\end{align*}
\]

where \((\nu \chi_{j,t}, \nu \zeta_{p,t}, \nu u_{i,t}, \nu \epsilon_{j,t})\) are drawn from zero-mean, unit-variance gamma distributions, \(j\) is the number of mismeasured regressors and \(p\) is the number of perfectly measured regressors in the model. Gamma distributions are used to ensure that all the assumptions stated in Section 1.3 are satisfied. \((\delta \chi_j, \delta \zeta_p)\) are chosen such that the means of the simulated \((\chi_j^{i,t}, \zeta_p^{i,t})\) equal the means of \((x_j^{i,t}, z_p^{i,t})\) in the real data. The simulated \((x_j^{i,t}, y_{i,t})\) are generated from the simulated \((\chi_j^{i,t}, \zeta_p^{i,t})\)
using Equations 3 and 4.

The measurement errors ($\epsilon_{i,t}^j$) and the regression error ($u_{i,t}$) in the simulation can be controlled by adjusting the shape parameters of the gamma distributions for ($\epsilon_{i,t}^j, u_{i,t}$). The measurement quality of the proxy variable is given by the $R^2$ of Equation 4. In order to test the power of the methodology used in this paper, I simulate data with high as well as low levels of measurement error in the mismeasured regressors. The results help us determine if higher order estimators are effective in eliminating bias in parameter estimates due to EIV in models with varying levels of measurement error. The following two sections give the simulation results using three and five mismeasured regressors.

5.1.1 Three mismeasured regressors

Table 1 presents the results from a model that is similar to the condensed LCAPM. In this set-up, $j = 3$ and $p = 2$. The table reports the slope estimates ($\lambda_1, \lambda_2, \lambda_3$), which are the coefficients on the mismeasured regressors, the slope estimates ($\rho_1, \rho_2$), which are the coefficients on the two correctly measured regressors and the intercept. The measurement quality of the mismeasured regressor is given by its coefficient of determination, which indicates the proportion of the variance in the mismeasured regressor that is explained by the true regressor. This is set to range from 45% to 97%, with $\chi^1$ having the highest degree of measurement error and $\chi^3$ the lowest. The values of the parameters used in the DGP are $\lambda_1 = 0.70$, $\lambda_2 = 0.80$, $\lambda_3 = 0.90$, $\rho_1 = 0.40$ and $\rho_2 = 0.50$. The intercept in the DGP is designed to be zero.

The first row labeled Fama-MacBeth exhibits the attenuation bias due to measurement error. The parameter estimates of the mismeasured regressors are biased downward. The intercept $\rho_0$ is biased upward and appears to be statistically significant when estimated using FM. The true DGP has an intercept of zero. Thus, the FM technique rejects a correctly specified asset pricing model due to measurement error. The results obtained from third, fourth and fifth order cumulant estimators are denoted by CUMD3, CUMD4 and CUMD5 respectively, and are reported in the second, third and fourth rows in Table 1. In contrast to FM, the intercept is not significantly different from zero for CUMD3, CUMD4 and CUMD5.
Next, I compare the estimates of $\lambda_1$, $\lambda_2$ and $\lambda_3$ obtained from Fama-MacBeth technique with those obtained from cumulant-based estimators. I find that that $\lambda_1$ and $\lambda_2$ are statistically different from zero, but biased downward (attenuation bias), when estimated using FM. However, the $\lambda_1$, $\lambda_2$ and $\lambda_3$ estimates obtained from CUMD3, CUMD4 and CUMD5 are close to the true values used in the DGP and are statistically different from zero. The results in Table 1 indicate that cumulant estimators perform better than FM in addressing bias caused by EIV.

5.1.2 Five mismeasured regressors

This section compares the performance of the Fama-MacBeth technique with that of estimators using higher order information in a model with five mismeasured regressors and two perfectly measured regressors. The slope estimates used in the DGP are $\lambda_1=0.1$, $\lambda_2=0.2$, $\lambda_3=0.3$, $\lambda_4=0.4$, $\lambda_5=0.5$, $\rho_1=1.0$, $\rho_2=0.5$ and the intercept is zero. I generate data with low quality proxy for $\chi_1$, $\chi_3$, $\chi_5$ and high quality proxy for $\chi_2$ and $\chi_4$.

The first row in Table 2 highlights the contamination bias and the attenuation bias induced by EIV. The FM estimates are biased. The estimates of $\lambda_1$ and $\lambda_2$ have the wrong sign and $\lambda_2$ is statistically insignificant. $\lambda_3$, $\lambda_4$ and $\lambda_5$ estimates are biased downward. Similar to the model with three mismeasured regressors, the intercept is biased upward and is statistically different from zero, though in the DGP, the intercept is zero. In contrast, the estimates from CUMD3, CUMD4 and CUMD5 are unbiased and similar to each other. The values of $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$ and $\lambda_5$ are close to the true values used in the DGP, and the intercept is statistically insignificant.

The results from Table 2 show that in a model with many mismeasured regressors, the slope parameters obtained from FM may even have the wrong sign. These results highlight the need for EIV correction in multivariate models. The simulations give evidence of the superior performance of higher order cumulant estimators compared to FM estimators. The next section applies these estimators to asset pricing models.
6 Results

6.1 Summary Statistics

Descriptive statistics of the sample are reported in Table 3. The number of firm-month observations is 1,303,337 for the full sample and 655,590 for the all-but-microcap sample. Panel A1 and Panel A2 give the summary statistics of $\beta_1^i$, $\beta_2^i$, $\beta_3^i$ and $\beta_4^i$ estimated at the stock-level based on a value-weighted market portfolio, using Equations 20-23. Panel A1 reports the characteristics for all stocks and Panel A2 excludes the microcap stocks. The striking difference between the two panels is that the average $\beta_4^i$ is negative for the whole sample, but positive when we exclude the microcap stocks. This shows that the $\beta_4^i$ for the microcap stocks is strongly negative. The table also indicates that the average $\beta_3^i$ is smaller for the microcap stocks than for the larger stocks, which results in the larger value of average $\beta_3^i$ for the all-but-microcap sample. As expected, the illiquidity cost, ZR_12 is higher in case of the full sample than for the all-but-microcap sample. The average market capitalization increases and the book-to-market ratio decreases when we exclude the microcap stocks. Univariate and multivariate tests of normality are performed to ensure that the stock-level betas are non-normal. Doornik-Hansen (2008) test of $\beta_k^i$ and ZR_12 rejects the null hypothesis of normality for all the variables.

Panels B1 and B2 give the descriptive statistics of $\beta_1^p$, $\beta_2^p$, $\beta_3^p$ and $\beta_4^p$ for the ten equal-weighted portfolios for the full sample and the all-but-microcap sample respectively. Doornik-Hansen (2008) test is used to check for univariate and multivariate normality of the portfolio betas, and the p-values (not reported) from the test are greater than 0.10. Thus the portfolio betas fail to reject univariate and multivariate normality. The differences between these two panels are similar to the differences when using stock-level betas. I find that the average $\beta_4^p$ is more negative for the whole sample than for the all-but-microcap sample. This shows that the $\beta_4^p$ for microcap stocks is very negative.

Table 4 reports the correlations between different measures of liquidity risk. Panel A presents the correlations between the portfolio betas, $\beta_1^p$, $\beta_2^p$, $\beta_3^p$ and $\beta_4^p$ for the entire sample. Panel B reports the correlations between the stock-level betas. Similar to AP2005, I calculate the cross-sectional
pair-wise correlations between the betas for individual stocks for each month. The averages of the time-series of these correlations give the correlations between the beta measures for the entire sample period.

The correlations between the portfolio betas are very small, ranging from 2.5% to 7.4%. The correlations between the stock-level betas are also very small, ranging from 2.4% to 7.9%. I find that the correlations between the portfolio betas reported in this paper are significantly lower than those reported in AP2005, which ranged from 44.1% to 97.1%. In AP2005, stocks are sorted into 25 illiquidity portfolios and the betas for each portfolio are computed as per Equations 20-23 based on an equal-weighted market portfolio, using the entire monthly time-series from 1964 to 1999. Thus, the higher correlations in AP2005 may be driven by the greater emphasis on small stocks.

6.2 Pricing of market risk

This section tests the validity of the single-factor CAPM and investigates whether market risk is priced. Market beta, $\beta_{mkt}^{i,t}$ for each firm $i$ in month $t$ is estimated by using the previous 60 monthly returns. To obtain the portfolio betas, stocks are then sorted monthly into decile portfolios based on $\beta_{mkt}^{i,t}$, and these ten equally weighted portfolios are used as test assets. The following model is then estimated using portfolio or stock-level data to estimate the market risk premium

$$E(R_{i,t} - R_{f,t}) = \alpha_0 + \lambda_{mkt}\beta_{mkt}^{i,t}$$ (30)

Panel A of Table 5 reports the estimated risk premium using the portfolio approach. The Fama-MacBeth results indicate that market risk is not priced. The coefficient estimate of market beta is insignificant and $\alpha_0$ is positive and highly significant.

Panel B of Table 5 reports the results when the CAPM is estimated using stock level data. FM cross-sectional regression results are presented in the first row of Panel B. The second, third and fourth rows present the results obtained by using cumulants of order three, four and five respectively. Standard errors clustered by time are reported in parenthesis.

The Fama-MacBeth results show that the relation between expected return and market beta is
flat. The intercept is positive and highly significant. Thus the CAPM is rejected. For the cumulant-based methodology, an important check is whether the assumptions underlying high order cumulant estimators are satisfied. Geary (1942) states that in order to use the cumulant based estimators, at least two of the cumulants should be different from zero. The results using CUMD3 are similar to the FM results mainly because CUMD3 has only one cumulant estimator equation, and this cumulant is not statistically different from zero. CUMD4 has three cumulants and only one of them is different from zero. Three out of the six cumulants in CUMD5 are different from zero, and hence it is the only specification that satisfies the necessary condition in Geary(1942). The results of CUMD5 show that the coefficient estimate of market beta is negative. However, alpha is positive and highly significant. According to this test, the CAPM is misspecified.

6.3 Is liquidity risk priced?

This section explores the relationship between expected asset returns and liquidity risk in the condensed LCAPM. Subsection 1.6.3.1 examines portfolio betas, and Subsection 1.6.3.2 examines stock level liquidity betas. The results are reported in Table 6.

6.3.1 Condensed LCAPM estimated with portfolio betas

This section reports empirical results of the test of the condensed LCAPM, which seeks to answer whether liquidity risk is priced. First, I follow the portfolio-based approach to compute the four liquidity betas based on a value-weighted market portfolio. These portfolio betas, $\beta_p^k$, are then assigned to each stock in portfolio $p$. This method is traditionally used in asset pricing models to address EIV in the estimated betas.

The 12-month average illiquidity cost $ZR_{12}$, the log of market capitalization and the log of the book-to-market ratio of each stock at the end of the previous month is also included in the model. The 12-month average zero-return proportion for the lagged month is used as the proxy for expected illiquidity at time $t$, $E(ZR_{12,t})$. Fama and French (1992) find that size and the book-to-market ratio can explain the cross-section of stock returns. To control for these effects, I include the log of market capitalization and the log of the book-to-market ratio in the regression.
These two variables may be considered as refinements to the intercept. If we find that the intercept is non-zero, then this specification allows us to check whether the significant intercept is due to stylized deviations in the model caused by the well-researched effects of size and book-to-market.

The risk premia are estimated from the following model

\[
E(R_{i,t} - R_{f,t}) = \rho^0 + \lambda^1 \beta^1_{i,p,t} + \lambda^{\text{net}} \beta^{\text{net}}_{i,p,t} + \lambda^{\text{nr}} E(ZR_{12,i,t}) + \rho^1 \ln(MV)_{i,t} + \rho^2 \ln(B/M)_{i,t} \quad (31)
\]

where \( \beta^{\text{net}}_{i,p,t} = \beta^2_{i,p,t} - \beta^3_{i,p,t} - \beta^4_{i,p,t} \) and \( \beta^2_{i,p,t} \) is defined as the \( \beta^2 \) of portfolio \( p \) to which stock \( i \) belongs in month \( t \). \( \beta^3_{i,p,t} \) and \( \beta^4_{i,p,t} \) are defined similarly. Panel A of Table 6 reports the means of the estimated premia. This specification separates the premium due to liquidity risk from that due to market beta and level of illiquidity cost of an asset. The FM results indicate that \( \beta^\text{net}_p \) is insignificant and has a coefficient of 0.0004 with a t-statistic of 0.667. \( \beta^1_p \) and \( ZR_{12} \) are insignificant, and so is the intercept. Thus neither the liquidity risk nor the market risk is priced and the illiquidity cost is insignificant. The log of the book-to-market ratio is used in Equation 31 as a stylized deviation to the intercept. Hence, the significance on \( \ln(B/M) \) is equivalent to the model’s intercept being significant. Thus according to this test using portfolio betas, the LCAPM is misspecified.

Panel B1 of Table 3 shows that the skewness and kurtosis of the portfolio liquidity betas are low. Hence, we cannot apply the portfolio-beta method using higher order cumulants to estimate risk premia in the condensed specification. However, Panel A1 of Table 3 shows that the stock-level liquidity betas are non-normal. Thus we can apply the cumulant approach to the model comprising of individual betas. The next section implements this approach.

### 6.3.2 Condensed LCAPM estimated with stock-level betas

Panel B of Table 6 reports the empirical results illustrating the EIV correction method developed in Section 1.3. Betas are estimated from monthly data over the prior five years. Individual stock-level data are used to estimate the risk premia

\[
E(R_{i,t} - R_{f,t}) = \rho^0 + \lambda^1 \beta^1_{i,t} + \lambda^{\text{net}} \beta^{\text{net}}_{i,t} + \lambda^{\text{nr}} E(ZR_{12,i,t}) + \rho^1 \ln(MV)_{i,t} + \rho^2 \ln(B/M)_{i,t} \quad (32)
\]
where \( \beta_{net}^{i,t} = \beta_{2}^{i,t} - \beta_{3}^{i,t} - \beta_{4}^{i,t} \). Results from the FM cross-sectional regressions are reported in the first row of Panel B. These results are similar to those in Panel A. The results using third through fifth order cumulant estimators are reported in second, third and fourth rows of Panel B. Standard errors clustered by time are reported in parenthesis.

FM produces a small coefficient on \( \beta_{net} \) and it is insignificant. In contrast, the results from the cumulant estimators are sharply different from the FM results but nearly identical to each other. The coefficient on \( \beta_{net} \) in CUMD3 through CUMD5 is larger in magnitude than the coefficient in FM. This difference stems from the attenuation bias in the FM estimate and it highlights the need to control for the measurement error in the betas computed at the stock-level.

I find that \( \beta_{net} \) is positive and significant for all the specifications using the cumulant-based approach. This draws our attention to the advantages of using stock-level betas. In CUMD3, \( \beta_{net} \) is positive and significant, with a t-statistic of 2.21. Thus liquidity risk is priced. The coefficient on \( \beta_{1} \) is negative and significant after controlling for EIV. This is similar to the results in Kan, Robotti and Shanken (2013) and Shanken and Zhou (2007), which employ alternative methods to address model misspecification in beta-pricing models. The coefficients on \( ZR_{12}, \ln(MV) \) and \( \ln(BM) \) are insignificant. Thus the model is not rejected in explaining the cross-section of expected returns. Employing fourth order cumulant estimators, I get similar results. \( \beta_{net} \) is positive with a t-statistic of 3.65. The results get stronger as we move from CUMD3 to CUMD4. In CUMD5, \( \beta_{1}, \beta_{net}, ZR_{12} \) and \( \ln(MV) \) are significant.

Using higher order cumulants, identification comes from the non-normality of the true regressors. When all of the high order cumulants are different from zero, then all of the cumulant equations fully contribute to identifying the parameters. In both CUMD3 and CUMD4, 54% of the cumulants are statistically different from zero. However, in CUMD5, only 37% of cumulants are different from zero. This may indicate that CUMD3 and CUMD4 are better specifications than CUMD5.

A striking result in Table 6 is that the LCAPM fails to explain expected stock returns when we use the portfolio-based approach, as shown in Panel A. However, as evident from Panel B, when we use stock-level betas and use the cumulant-based approach to handle EIV, the LCAPM is not
mis-specified. These results show that the portfolio-based approach was unable to capture the true positive relation between liquidity risk and expected stock returns. Overall, I find that the LCAPM is not rejected as an explanation of the cross-section of average returns.

### 6.4 Pricing of the individual liquidity risks

As discussed earlier, Acharya and Pedersen (2005) reported a severe multi-collinearity problem when they included the three liquidity betas separately in the cross-sectional regressions. This is mainly because of the high correlation coefficients between the portfolio betas when they are estimated using the entire monthly series and based on an equal-weighted market portfolio. Their paper states that due to this reason, statistical identification of the separate effects of the three liquidity betas is difficult and they cannot conclude which of these risks are empirically relevant. To answer this question, Lee (2011) estimates the model with one liquidity beta at a time along with the market beta and cost of illiquidity. However, his method omits two of the three theoretically relevant liquidity risks in each model specification. Hence, the results may suffer from omitted variable bias.

Panel A of Table 7 reports the coefficients from FM regression of the full LCAPM with all betas estimated at the portfolio level. It shows that $\beta^4$ is negatively priced and significant ($t$-statistic=3.75). However, the direction on $\beta^3$ is counter-intuitive. Despite using portfolio betas, these results seem to suffer from contamination bias due to the measurement error in this multivariate model.

The correlations between the individual stock-level liquidity betas, as shown in Panel B of Table 4, are much lower than the correlations reported in previous literature. Previous papers did not deal with individual stock betas due to the noise introduced in the betas when they are estimated at stock level. But I address this error in estimated betas using the cumulant approach described in Section 1.3. Thus it is possible to empirically test the LCAPM with the individual liquidity risks, without the need of aggregating the betas. This section investigates which of the liquidity risks is significant. I test the LCAPM given in Equation 33 using individual stock level
data

\[ E(R_{i,t} - R_{f,t}) = \rho^0 + \lambda^1 \beta^1_{i,t} + \lambda^2 \beta^2_{i,t} + \lambda^3 \beta^3_{i,t} + \lambda^4 \beta^4_{i,t} + \lambda^5 E(ZR_{12,t}) + \rho^1 \ln(MV)_{i,t} + \rho^2 \ln(B/M)_{i,t} \]

(33)

Results are reported in Panel B of Table 7. The Fama-MacBeth results show that \( \beta^4 \) is negative and significant. However, the other liquidity betas are not significant. Moreover, the signs on \( \beta^2 \) and \( \beta^3 \) are opposite to theoretical implications, which may be attributable to the contamination bias in the model.

The second and third rows report the results obtained from CUMD3 and CUMD4 respectively. The signs on all of the liquidity betas align with theory. I find that the return premium due to \( \beta^2 \), which represents \( \text{cov}(c_i, c_m) \), is positive and significant at the 10% level in CUMD3 and at the 1% level in CUMD4. This documents that investors demand a premium for holding a stock that is illiquid when the overall market is illiquid. Thus commonality in illiquidity is priced.

I find that \( \beta^4 \), which represents \( \text{cov}(c_i, r_m) \), is negatively priced. It is significant at the 5% level in CUMD3 and 1% level in CUMD4. This demonstrates that investors are willing to pay a premium to hold a security that becomes more liquid when the market return is low. Stocks with high values of \( \beta^4 \) have lower illiquidity costs in states of poor market return and hence work as a hedge against market downturns. Thus investors have a preference for these stocks. Panel B of Table 7 also shows that \( \beta^3 \) is insignificant. Thus, in general, investors are not willing to accept a lower expected return on stocks that have a higher return when the market as a whole is more illiquid. The coefficient on the illiquidity cost is insignificant. A notable result in this table is that the intercept, the coefficient on \( \ln(MV) \) and the coefficient on \( \ln(B/M) \) are not significant. This implies that the model is not rejected.

Overall, I find that the risks due to \( \text{cov}(c_i, c_m) \) and \( \text{cov}(c_i, r_m) \) are the two most important sources of systematic liquidity risks that are related to expected stock returns and the LCAPM cannot be rejected. Moreover, the results get stronger as we move from third order to fourth order estimators. I also find that 44% of the cumulants in CUMD3 and 47% of the cumulants in CUMD4 are statistically different from zero. This may explain the reason behind the stronger results in CUMD4.
7 Robustness Tests

To check the robustness of the results in Tables 6 and 7, I consider different specifications and portfolios. First, I consider whether the results are robust after I exclude the microcap stocks from the sample. The results are presented in Section 1.7.1. As a further robustness check, I re-estimate the model with an equal-weighted market portfolio in Section 1.7.2.

7.1 Controlling for microcap stocks

7.1.1 Pricing of liquidity risk after excluding the microcap stocks

This section aims to explore if the results in Tables 6 and 7 are driven by the microcap stocks. In my sample, I find that the microcap stocks account for around 50% of the observations but less than 5% of the total market capitalization. These stocks have high illiquidity and transaction costs. It is interesting to investigate whether the pricing ability of liquidity risk holds for all stocks or is limited to these highly illiquid stocks that represent only a small portion of market wealth.

This section presents empirical results of the test of the condensed LCAPM after excluding the microcap stocks from the sample. The results from the portfolio-based approach are reported in Panel A of Table 8. I find that $\beta_{\text{net}}$ is still insignificant. However, the intercept on the model is positive and significant. This is in contrast to the insignificant intercept obtained in Panel A of Table 6 using the full sample. The insignificant intercept in Table 6 may have been caused by the contamination bias due to the mismeasured betas in the model. This indicates that the grouping method does not completely eliminate the measurement error in the betas for the microcap stocks.

Panel B reports the results from the cumulant-based approach with betas estimated at the stock level. The coefficient on $\beta_{\text{net}}$ for the all-but-microcap sample is less than half its magnitude for the full sample. Furthermore, this coefficient is not significant in CUMD3 and CUMD4. Thus the positive relation between $\beta_{\text{net}}$ and expected returns is mostly driven by the microcap stocks and the price of liquidity risk decreases substantially after excluding these stocks. The proportion of cumulants that are different from zero is 54% in CUMD3 and 37% in CUMD4. The intercept and the coefficient on $\beta_{\text{net}}$ are significant in CUMD5. However, only 18% of the fifth order cumulants...
are different from zero, which may be the reason behind the difference in results between CUMD5 and the lower order cumulants. Overall, the results in Panel B are in line with the effect we would expect to see when we exclude the tiny stocks, which are also usually the most illiquid, from the sample.

### 7.1.2 Pricing of the individual liquidity betas after excluding the microcap stocks

The effect of excluding the microcap stocks in the estimation of the LCAPM with disaggregated betas is shown in Table 9. The results from the portfolio-based approach are reported in Panel A. I find that \( \beta_4 \) is still negatively priced and significant at the 5% level. However, similar to the result obtained in the previous section, I find that the intercept on the model is positive and significant. This suggests that for the microcap stocks, the grouping method may not be very efficient in eliminating the measurement errors in the individual betas.

Panel B presents the results when the model is estimated with stock-level betas. The coefficient on \( \beta_2 \) for the all-but-microcap sample is insignificant and much smaller in magnitude than the coefficient for the full sample. Thus the microcap stocks drive the risk premium due to the commonality in liquidity. The coefficient on \( \beta_3 \) is positive for the all-but-micro sample but insignificant for the full sample. This implies that the coefficient on \( \beta_3 \) for the microcap stocks must be very large and negative in order to cause the insignificant coefficient on \( \beta_3 \) for the full sample. \( \beta_4 \) is still negative, but the coefficient has a smaller magnitude than that in the full sample.

This table indicates that the effects of microcap stocks are very important when analyzing the relation between liquidity risk and expected returns. The main inference that stands out from these results is that the risk due to the covariation between a stock’s return and the market liquidity is negatively priced for the microcap stocks. Thus investors are willing to accept a lower return to hold these microcap stocks that have a high return when market is illiquid.

### 7.2 LCAPM estimation with betas based on an equal-weighted market portfolio

AP2005 and Lee (2011) use an equal-weighted market portfolio to estimate the liquidity betas. They estimate the betas at the beginning of every year, based on the monthly data for the last five
years. However, prior literature has highlighted the shortcomings of using an equal-weighted market portfolio. Equal-weighted market return and market illiquidity are dominated by the microcap stocks. When I use an equal-weighted instead of a value-weighted market portfolio to estimate the portfolio liquidity betas, the correlations between these betas are quite substantial, ranging from 10.35% to 42.49%. Thus, it is problematic to correctly identify the separate effects of these three liquidity betas. To circumvent this multicollinearity problem, AP2005 estimates the LCAPM with $\beta^{net}$, which is a linear combination of the three liquidity betas. Using monthly data from January 1988 to December, Lee (2011) finds that $\beta^{net}$ is positive and significant at the 5% level.

Table 10 reports the results when the beta estimation is based on an equal-weighted market portfolio. The five-year rolling window for beta estimation rolls forward every month. I find that for the entire sample from January 1963 to December 2014, $\beta^{net}$ is positive but insignificant. Panel B reports the results of the test of the LCAPM with stock-level betas. I find that the coefficient on $\beta^{net}$ in CUMD3 and CUMD4 is positive and has a higher magnitude than in the value-weighted results reported in Panel B of Table 6. This indicates that the results using an equal-weighted market portfolio are driven to a greater degree by the microcap stocks. The coefficients on net liquidity risk and illiquidity cost are positive and significant with t-statistics of 3.83 and 3.16 respectively in CUMD4. These coefficients are insignificant in CUMD3 and CUMD5. Amongst the three specifications, the proportion of cumulants different from zero is the highest in CUMD4, which may be the reason behind the differences in coefficients estimated from using cumulants of different orders.

Table 11 reports the results obtained when the net liquidity beta is decomposed into its individual components. Panel A presents the coefficients from FM regression of the LCAPM using portfolio betas. I find that $\beta^4$ is negatively priced and significant. The intercept is positive and significant. Panel B reports the results when betas are calculated at the stock-level. I find that the risk due to the covariation between a stock’s return and the market illiquidity, $\beta^3$, is significant and negatively priced for both CUMD3 and CUMD4. This is because these results are dominated by the microcap stocks, which have been shown to have a negative price of risk for $\beta^3$. The risk due to commonality in liquidity, $\beta^2$, is positive and insignificant. Furthermore, unlike the results obtained using CUMD3 and CUMD4 in Panel B of Table 7, I find that the intercept is positive
and significant when I use an equal-weighted market portfolio. Thus the LCAPM is misspecified when an equal-weighted portfolio is cast as the market.

8 Conclusion

Measurement error is endemic in asset pricing models employing the standard two-pass cross-sectional regression methodology. The sensitivities to factors are estimated in the first stage, and used as independent variables in the second stage. Error in the estimates may lead to inconsistent estimates of the slope coefficients and the intercept in the second stage.

This paper implements a method that is new to the estimation of risk premia in asset pricing models to address EIV. I propose an alternative to the cross-sectional regression step in a Fama-MacBeth framework, and use higher order information in the data to estimate the price of risk. This is achieved by using a system of equations that express higher order cumulants of observable variables as a linear function of the coefficients to be estimated and other higher order cumulants. It does not necessitate sorting stocks into groups to test hypotheses and hence circumvents the information loss caused due to portfolio formation.

I apply this methodology to the CAPM and LCAPM. I find that the single factor CAPM still fails to explain the cross-section of average returns. In testing the LCAPM, I use stock level betas and find that liquidity risk is priced. Then I decouple the net liquidity risk into three components and empirically test the significance of these factors. Betas estimated at the stock level are not highly correlated with each other, and hence we can bypass the collinearity problem that was a challenge in the previous studies.

I find that the risk premium due to the commonality in liquidity is positive. I also find that the premium due to the sensitivity of a stock’s illiquidity to the market return is negatively related to the cross-section of expected return. This paper sheds light on the true relationship between liquidity, liquidity risk and asset returns after controlling for contamination and attenuation biases. I also show that $\beta^3$, which measures the exposure of a stock’s return to the market illiquidity, is negatively related to expected returns for microcap stocks.
The scope of applications of higher-order cumulants in handling EIV is expansive. Future research could investigate whether measurement error distorts the inferences in the q-factor model or the Fama-French five-factor model.


Table 1 Simulation Results: Three mismeasured regressors

The data are simulated by the following model:

\[ y_{i,t} = 0.7 \chi_{1,i,t}^1 + 0.8 \chi_{1,i,t}^2 + 0.9 \chi_{1,i,t}^3 + 0.4 z_{1,i,t}^1 + 0.5 z_{1,i,t}^2 \]

The estimated model is

\[ y_{i,t} = \lambda_1 \chi_{1,i,t}^1 + \lambda_2 \chi_{1,i,t}^2 + \lambda_3 \chi_{1,i,t}^3 + \rho_0 + \rho_1 z_{1,i,t}^1 + \rho_2 z_{1,i,t}^2 \]

The total number of time periods is 564, and each time period has 3000 observations. This table reports the estimates obtained by applying the Fama-MacBeth methodology and higher order cumulant estimators. The standard errors are reported in parenthesis. ** and *** denote significance at the 5% and 1% level respectively.

<table>
<thead>
<tr>
<th>Three mismeasured regressors</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
<th>( \rho_0 )</th>
<th>( \rho_1 )</th>
<th>( \rho_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fama-MacBeth</td>
<td>0.3198***</td>
<td>0.6159***</td>
<td>0.8737***</td>
<td>0.3653***</td>
<td>0.4002***</td>
<td>0.4999***</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
<td>(0.0007)</td>
<td>(0.0003)</td>
<td>(0.0017)</td>
<td>(0.0006)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>CUMD3</td>
<td>0.6989***</td>
<td>0.8006***</td>
<td>0.8993***</td>
<td>0.0020</td>
<td>0.3999***</td>
<td>0.4997***</td>
</tr>
<tr>
<td></td>
<td>(0.0032)</td>
<td>(0.0017)</td>
<td>(0.0009)</td>
<td>(0.0035)</td>
<td>(0.0007)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>CUMD4</td>
<td>0.6950***</td>
<td>0.8008***</td>
<td>0.8992***</td>
<td>0.0056</td>
<td>0.3999***</td>
<td>0.4998***</td>
</tr>
<tr>
<td></td>
<td>(0.0027)</td>
<td>(0.0014)</td>
<td>(0.0007)</td>
<td>(0.0031)</td>
<td>(0.0007)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>CUMD5</td>
<td>0.6969***</td>
<td>0.8010***</td>
<td>0.8993***</td>
<td>0.0038</td>
<td>0.3999***</td>
<td>0.4998***</td>
</tr>
<tr>
<td></td>
<td>(0.0023)</td>
<td>(0.0012)</td>
<td>(0.0007)</td>
<td>(0.0028)</td>
<td>(0.0007)</td>
<td>(0.0003)</td>
</tr>
</tbody>
</table>
Table 2 Simulation Results: Five Mismeasured regressors

The data are simulated by the following model:

\[ y_{i,t} = 0.1x_{1,i,t} + 0.2x_{2,i,t} + 0.3x_{3,i,t} + 0.4x_{4,i,t} + 0.5x_{5,i,t} + 0.4z_{1,i,t} + 0.5z_{2,i,t} \]

The estimated model is

\[ y_{i,t} = \lambda_1 x_{1,i,t} + \lambda_2 x_{2,i,t} + \lambda_3 x_{3,i,t} + \lambda_4 x_{4,i,t} + \lambda_5 x_{5,i,t} + \rho_0 + \rho_1 z_{1,i,t} + \rho_2 z_{2,i,t} \]

The total number of time periods is 564, and each time period has 3000 observations. I report the estimates obtained by applying the Fama-MacBeth methodology and higher order cumulant estimators. The standard errors are reported in parenthesis. ** and *** denote significance at the 5% and 1% level respectively.

<table>
<thead>
<tr>
<th></th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
<th>( \lambda_4 )</th>
<th>( \lambda_5 )</th>
<th>( \rho_0 )</th>
<th>( \rho_1 )</th>
<th>( \rho_2 )</th>
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<tr>
<td>Fama-MacBeth</td>
<td>-0.0476***</td>
<td>-0.0219</td>
<td>0.1483***</td>
<td>0.3619***</td>
<td>0.3979***</td>
<td>1.1090***</td>
<td>1.0001***</td>
<td>0.5002***</td>
</tr>
<tr>
<td></td>
<td>(0.0148)</td>
<td>(0.0178)</td>
<td>(0.0119)</td>
<td>(0.0111)</td>
<td>(0.0118)</td>
<td>(0.1244)</td>
<td>(0.0164)</td>
<td>(0.0253)</td>
</tr>
<tr>
<td>CUMD3</td>
<td>0.1005***</td>
<td>0.1989***</td>
<td>0.2945***</td>
<td>0.4019***</td>
<td>0.4971***</td>
<td>-0.0035</td>
<td>1.0000***</td>
<td>0.5005***</td>
</tr>
<tr>
<td></td>
<td>(0.0032)</td>
<td>(0.0033)</td>
<td>(0.0041)</td>
<td>(0.0034)</td>
<td>(0.0031)</td>
<td>(0.0193)</td>
<td>(0.0007)</td>
<td>(0.0011)</td>
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<tr>
<td>CUMD4</td>
<td>0.1011***</td>
<td>0.2011***</td>
<td>0.2938***</td>
<td>0.4035***</td>
<td>0.4968***</td>
<td>-0.0109</td>
<td>1.0000***</td>
<td>0.5005***</td>
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<tr>
<td></td>
<td>(0.0026)</td>
<td>(0.0026)</td>
<td>(0.0035)</td>
<td>(0.0028)</td>
<td>(0.0023)</td>
<td>(0.0153)</td>
<td>(0.0007)</td>
<td>(0.0011)</td>
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Table 3 Summary Statistics: Stock-level betas

This table gives the descriptive statistics of the data. Panels A1 and A2 report the mean, standard deviation, skewness and kurtosis of the stock-level betas in the LCAPM estimated from January 1963 to December 2014 for all and the all-but-microcap stocks respectively. For each stock $i$ in month $t$, $\beta_{k,t}^i$, where $k = (1,2,3,4)$, is estimated from monthly returns and innovations in illiquidity for stock $i$ and for the value-weighted market portfolio, over months $t - 60$ to $t - 1$, using Equations 20-23. The innovations in illiquidity are obtained from the first differences in illiquidity over the 60-month window. This window rolls forward every month. Individual stock windows with less than 36 prior monthly returns or innovations in illiquidity are dropped. $ZR_{12}$ is the average zero-return proportion. $\ln(MV)$ is the log of the market capitalization and $\ln(B/M)$ is the log of the book-to-market ratio.

<table>
<thead>
<tr>
<th>Panel A1: All stocks</th>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Skewness</th>
<th>Kurtosis</th>
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</thead>
<tbody>
<tr>
<td>$\beta_1^i$</td>
<td>1.1181</td>
<td>0.7006</td>
<td>1.1115</td>
<td>4.6298</td>
<td></td>
</tr>
<tr>
<td>$\beta_2^i$</td>
<td>0.0581</td>
<td>0.1509</td>
<td>0.6012</td>
<td>4.7382</td>
<td></td>
</tr>
<tr>
<td>$\beta_3^i$</td>
<td>0.0169</td>
<td>0.1422</td>
<td>0.3248</td>
<td>18.5410</td>
<td></td>
</tr>
<tr>
<td>$\beta_4^i$</td>
<td>-0.0099</td>
<td>0.3667</td>
<td>-0.1060</td>
<td>2.4426</td>
<td></td>
</tr>
<tr>
<td>$ZR_{12}$</td>
<td>0.1729</td>
<td>0.1479</td>
<td>0.9632</td>
<td>0.6384</td>
<td></td>
</tr>
<tr>
<td>$\ln(MV)$</td>
<td>4.9541</td>
<td>2.1104</td>
<td>0.3374</td>
<td>-0.2008</td>
<td></td>
</tr>
<tr>
<td>$\ln(B/M)$</td>
<td>-0.4251</td>
<td>0.8929</td>
<td>-0.5936</td>
<td>2.1015</td>
<td></td>
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<table>
<thead>
<tr>
<th>Panel A2: Excluding microcap stocks</th>
<th>Variable</th>
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<th>Kurtosis</th>
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<tr>
<td>$\beta_1^i$</td>
<td>1.1087</td>
<td>0.6274</td>
<td>1.3814</td>
<td>5.0206</td>
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<tr>
<td>$\beta_2^i$</td>
<td>0.0610</td>
<td>0.1302</td>
<td>1.0014</td>
<td>4.5511</td>
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<tr>
<td>$\beta_3^i$</td>
<td>0.0197</td>
<td>0.1023</td>
<td>0.3787</td>
<td>9.9942</td>
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<tr>
<td>$\beta_4^i$</td>
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<td>0.3065</td>
<td>-0.0178</td>
<td>2.8852</td>
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<tr>
<td>$ZR_{12}$</td>
<td>0.1118</td>
<td>0.1079</td>
<td>1.4589</td>
<td>3.5614</td>
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<tr>
<td>$\ln(MV)$</td>
<td>6.6272</td>
<td>1.6393</td>
<td>0.4499</td>
<td>0.2057</td>
<td></td>
</tr>
<tr>
<td>$\ln(B/M)$</td>
<td>-0.6232</td>
<td>0.8048</td>
<td>-0.6386</td>
<td>1.8132</td>
<td></td>
</tr>
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Table 3 B Summary Statistics: Portfolio betas

This table gives the descriptive statistics of the data. Panels B1 and B2 report the mean, standard deviation, skewness and kurtosis of portfolio betas in the LCAPM estimated from January 1963 to December 2014 for all and the all-but-microcap stocks respectively. For each stock $i$, the pre-ranking beta, $\beta_{t,t}^{k,pre}$, ($k = 1, 2, 3, 4$) of month $t$ is estimated using the time-series of monthly returns and innovations in illiquidity for the previous 60 months with respect to either the value-weighted market return or the innovations in value-weighted market illiquidity. The innovations in illiquidity are obtained from the first differences in illiquidity over the 60-month window. This window rolls forward every month. If a stock has less than 36 valid observations in the $t - 60$ to $t - 1$ monthly window, then $\beta_{t,t}^{k,pre}$ for that stock is set to missing. Stocks are then sorted into ten equal-weighted portfolios based on $\beta_{t,t}^{k,pre}$ for month $t$. The post-ranking portfolio beta, $\beta_{p}^{k}$, is then estimated for each of the ten portfolios over the entire sample period using Equations 20-23.

<table>
<thead>
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<th>Mean</th>
<th>Std Dev</th>
<th>Skewness</th>
<th>Kurtosis</th>
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<tr>
<td>$\beta_{p}^{1}$</td>
<td>1.0969</td>
<td>0.5958</td>
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<td>$\beta_{p}^{2}$</td>
<td>0.0549</td>
<td>0.0824</td>
<td>0.0199</td>
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<td>$\beta_{p}^{3}$</td>
<td>0.0061</td>
<td>0.0859</td>
<td>-0.2119</td>
<td>0.3176</td>
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<td>$\beta_{p}^{4}$</td>
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<td>0.3200</td>
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<table>
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<th>Skewness</th>
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<td>0.5056</td>
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<tr>
<td>$\beta_{p}^{4}$</td>
<td>-0.0014</td>
<td>0.2666</td>
<td>-0.0708</td>
<td>0.3918</td>
</tr>
</tbody>
</table>
This table reports the correlations between the different beta measures. The correlations are computed monthly for all eligible stocks and then averaged over the sample period. Panel A presents the correlations of $\beta_{p}^{1}, \beta_{p}^{2}, \beta_{p}^{3}$ and $\beta_{p}^{4}$ for the portfolio betas formed each month using data from January 1963 to December 2014. For each stock $i$, the pre-ranking beta, $\beta_{i,t}^{k,pre}, (k = 1, 2, 3, 4)$ of month $t$ is estimated using the time-series of monthly returns and innovations in illiquidity for the previous 60 months with respect to either the value-weighted market return or the innovations in value-weighted market illiquidity. Stocks are then sorted into ten portfolios based on $\beta_{i,t}^{k,pre}$ for month $t$. The post-ranking portfolio beta, $\beta_{p}^{k}$, is then estimated for each of the ten equal-weighted portfolios over the entire sample period using Equations 20-23. Panel B reports the correlations of $\beta_{i}^{1}, \beta_{i}^{2}, \beta_{i}^{3}$ and $\beta_{i}^{4}$ estimated at the stock-level. For each stock $i$ in month $t$, $\beta_{i}^{k,t}$ where $k = (1,2,3,4)$, is computed from monthly returns and innovations in illiquidity for stock $i$ and for the value-weighted market portfolio, over months $t−60$ to $t−1$, using Equations 20-23.

### Panel A:
Beta correlations for portfolios

<table>
<thead>
<tr>
<th></th>
<th>$\beta_{p}^{1}$</th>
<th>$\beta_{p}^{2}$</th>
<th>$\beta_{p}^{3}$</th>
<th>$\beta_{p}^{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{p}^{1}$</td>
<td>1.000</td>
<td>0.025</td>
<td>0.058</td>
<td>-0.067</td>
</tr>
<tr>
<td>$\beta_{p}^{2}$</td>
<td>1.000</td>
<td>-0.074</td>
<td>0.059</td>
<td></td>
</tr>
<tr>
<td>$\beta_{p}^{3}$</td>
<td>1.000</td>
<td></td>
<td>0.026</td>
<td></td>
</tr>
<tr>
<td>$\beta_{p}^{4}$</td>
<td></td>
<td></td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

### Panel B:
Beta correlations for individual stocks

<table>
<thead>
<tr>
<th></th>
<th>$\beta_{i}^{1}$</th>
<th>$\beta_{i}^{2}$</th>
<th>$\beta_{i}^{3}$</th>
<th>$\beta_{i}^{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{i}^{1}$</td>
<td>1.000</td>
<td>0.026</td>
<td>0.058</td>
<td>-0.073</td>
</tr>
<tr>
<td>$\beta_{i}^{2}$</td>
<td>1.000</td>
<td>-0.079</td>
<td>0.061</td>
<td></td>
</tr>
<tr>
<td>$\beta_{i}^{3}$</td>
<td>1.000</td>
<td></td>
<td>0.024</td>
<td></td>
</tr>
<tr>
<td>$\beta_{i}^{4}$</td>
<td></td>
<td></td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>
This table presents the estimated coefficients of the single-factor CAPM. I consider the following model

\[ E(R_p - R_{f,t}) = \alpha_0 + \lambda_{mkt} \beta_{mkt} \]

I report the estimates obtained using Fama-MacBeth methodology and correcting for EIV using third through fifth order cumulants. The standard errors are reported in parenthesis. Panel A reports the estimated coefficients from Fama-MacBeth cross-sectional regressions of the CAPM based on portfolio beta. Market beta, \( \beta_{mkt} \) for each firm \( i \) in month \( t \) is estimated by using the previous 60 monthly returns. To obtain the portfolio betas, stocks are then sorted into deciles portfolios based on \( \beta_{i,t}^{mkt} \), and these ten equally weighted portfolios are used as test assets. Panel B reports the estimates from estimating market beta at the stock level and then controlling for EIV explicitly using third through fifth order cumulants. ** and *** denote significance at the 5% and 1% level respectively.

<table>
<thead>
<tr>
<th>Panel A: CAPM estimated with portfolio beta</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta_{mkt}^{p} )</td>
<td>( \alpha_0 )</td>
</tr>
<tr>
<td>Fama-MacBeth</td>
<td>0.0023</td>
<td>0.0095***</td>
</tr>
<tr>
<td></td>
<td>(0.0019)</td>
<td>(0.0009)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: CAPM estimated with stock level beta</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta_{i,t}^{mkt} )</td>
<td>( \alpha_0 )</td>
</tr>
<tr>
<td>Fama-MacBeth</td>
<td>-0.0001</td>
<td>0.0091***</td>
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<tr>
<td></td>
<td>(0.0011)</td>
<td>(0.0013)</td>
</tr>
<tr>
<td>CUMD3</td>
<td>-4.609</td>
<td>4.915***</td>
</tr>
<tr>
<td></td>
<td>(4.474)</td>
<td>(0.0020)</td>
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<tr>
<td>CUMD4</td>
<td>-0.5225***</td>
<td>0.5689***</td>
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<tr>
<td></td>
<td>(0.1727)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>CUMD5</td>
<td>-0.4444***</td>
<td>0.4858***</td>
</tr>
<tr>
<td></td>
<td>(0.0658)</td>
<td>(0.0002)</td>
</tr>
</tbody>
</table>
Table 6 Estimation of the Condensed LCAPM

This table presents the estimated coefficients of the LCAPM using data on all stocks from January 1963 to December 2014. I consider the following model

\[ E(R_{i,t} - R_{f,t}) = \rho^0 + \lambda^1 \beta^1_{i,t} + \lambda^{net} \beta^{net}_{i,t} + \lambda^{zar} E(ZR_{12,i,t}) + \rho^1 \ln(MV_{i,t}) + \rho^2 \ln(B/M_{i,t}) \]

I report the estimates obtained using Fama-MacBeth methodology and correcting for EIV using third and fourth order cumulant estimators. The standard errors are reported in parenthesis. Panel A reports the coefficients obtained from estimating the LCAPM using portfolio betas, \( \beta^k_p \). For each stock \( i \), the pre-ranking beta, \( \beta^{k,pre}_{i,t} \), \((k = 1, 2, 3, 4)\) of month \( t \) is estimated using the time-series of monthly returns and innovations in illiquidity for the previous 60 months with respect to either the value-weighted market return or the innovations in value-weighted market illiquidity. Stocks are then sorted into ten portfolios based on \( \beta^{k,pre}_{i,t} \) for month \( t \). The post-ranking portfolio beta, \( \beta^k_p \), is then estimated for each of the ten equally weighted portfolios over the entire sample period using Equations 20-23. This \( \beta^k_p \) is assigned to all stocks belonging to portfolio \( p \) ranked on the basis of \( \beta^{k,pre}_{i,t} \). \( \beta^{net}_p = \beta^2_p - \beta^3_p - \beta^4_p \). Panel B reports the coefficients obtained from estimating the LCAPM using stock-level betas and then controlling for EIV explicitly using third and fourth order cumulant estimators. For each stock \( i \) in month \( t \), \( \beta^k_{i,t} \) where \( k = (1,2,3,4) \), is computed from monthly returns and innovations in illiquidity for stock \( i \) and for the value-weighted market portfolio, over months \( t - 60 \) to \( t - 1 \), using Equations 20-23. \( \beta^{net}_{i,t} = \beta^2_{i,t} - \beta^3_{i,t} - \beta^4_{i,t} \). \( ZR_{12} \) is the previous month’s average zero-return proportion. \( \ln(MV) \) is the log of the market capitalization and \( \ln(B/M) \) is the log of the book-to-market ratio at the end of the previous year. ** and *** denote significance at the 5% and 1% level respectively.

<table>
<thead>
<tr>
<th>Panel A : Condensed LCAPM estimated with portfolio betas</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta^1_p )</td>
</tr>
<tr>
<td>Fama-MacBeth</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Condensed LCAPM estimated with stock-level betas</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta^1 )</td>
</tr>
<tr>
<td>Fama-MacBeth</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>CUMD3</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>CUMD4</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>CUMD5</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

** and *** denote significance at the 5% and 1% level respectively.
Table 7 Pricing of the individual liquidity betas

This table presents the estimated coefficients of the LCAPM using data on all stocks from January 1963 to December 2014. I consider the following model

\[ E(R_{i,t} - R_{f,t}) = \rho^0 + \lambda^1 \beta^1_{1,t} + \lambda^2 \beta^2_{1,t} + \lambda^3 \beta^3_{1,t} + \lambda^4 \beta^4_{1,t} + \lambda^{12} E(ZR_{12,t}) + \rho^1 \ln(MV_{i,t}) + \rho^2 \ln(B/M_{i,t}) \]

I report the estimates obtained using Fama-MacBeth methodology and correcting for EIV using third and fourth order cumulant estimators. The standard errors are reported in parenthesis. Panel A reports the coefficients obtained from estimating the LCAPM using portfolio betas, \( \beta^k_p \). For each stock \( i \), the pre-ranking beta, \( \beta^k_{i,t,pre} \), of month \( t \) is estimated using the time-series of monthly returns and innovations in illiquidity for the previous 60 months with respect to either the value-weighted market return or the innovations in value-weighted market illiquidity. Stocks are then sorted into ten portfolios based on \( \beta^k_{i,t,pre} \) for month \( t \). The post-ranking portfolio beta, \( \beta^k_p \), is then estimated for each of the ten equally weighted portfolios over the entire sample period using Equations 20-23. This \( \beta^k_p \) is assigned to all stocks belonging to portfolio \( p \) ranked on the basis of \( \beta^k_{i,t,pre} \). Panel B reports the coefficients obtained from estimating the LCAPM using stock-level betas and then controlling for EIV explicitly using third and fourth order cumulant estimators. For each stock \( i \) in month \( t \), \( \beta^k_{i,t} \) where \( k = (1,2,3,4) \), is computed from monthly returns and innovations in illiquidity for stock \( i \) and for the value-weighted market portfolio, over months \( t - 60 \) to \( t - 1 \), using Equations 20-23. \( ZR_{12} \) is the previous month’s average zero-return proportion. \( \ln(MV) \) is the log of the market capitalization and \( \ln(B/M) \) is the log of the book-to-market ratio at the end of the previous year. ** and *** denote significance at the 5% and 1% level respectively.

<table>
<thead>
<tr>
<th>Panel A: LCAPM estimated with portfolio betas</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta^1_p )</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Fama-MacBeth</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: LCAPM estimated with stock-level betas</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta^1 )</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Fama-MacBeth</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>CUMD3</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>CUMD4</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
Table 8: Robustness Check: Estimation of the LCAPM excluding microcap stocks

This table presents the estimated coefficients of the LCAPM using data from January 1963 to December 2014, excluding microcap stocks. I consider the following model

\[ E(R_{i,t} - R_{f,t}) = \rho^0 + \lambda^1 \beta_{i,t}^1 + \lambda^{net} \beta_{i,t}^{net} + \lambda^{ZR} E(ZR_{12,i,t}) + \rho^1 \ln(MV_{i,t}) + \rho^2 \ln(B/M_{i,t}) \]

I report the estimates obtained using Fama-MacBeth methodology and correcting for EIV using third through fifth order cumulant estimators. The standard errors are reported in parenthesis. Panel A reports the coefficients obtained from estimating the LCAPM using portfolio betas, \( \beta_{i,t}^p \). For each stock \( i \), the pre-ranking beta, \( \beta_{i,t}^{pre,k} \), \((k = 1, 2, 3, 4)\) of month \( t \) is estimated using the time-series of monthly returns and innovations in illiquidity for the previous 60 months with respect to either the value-weighted market return or the innovations in value-weighted market illiquidity. Stocks are then sorted into ten portfolios based on \( \beta_{i,t}^{pre,k} \) for month \( t \). The post-ranking portfolio beta, \( \beta_{i,t}^p \), is then estimated for each of the ten equal-weighted portfolios over the entire sample period using Equations 20-23. \( \beta_{i,t}^p \) is assigned to all stocks belonging to portfolio \( p \) ranked on the basis of \( \beta_{i,t}^{pre,k} \).

\[ \beta_{net,p}^i = \beta_{i,t}^2 - \beta_{i,t}^3 - \beta_{i,t}^4. \]

Panel B reports the coefficients obtained from estimating the LCAPM using stock-level betas and then controlling for EIV explicitly using third through fifth order cumulant estimators. For each stock \( i \) in month \( t \), \( \beta_{i,t}^k \) where \( k = (1,2,3,4) \), is computed from monthly returns and innovations in illiquidity for stock \( i \) and for the value-weighted market portfolio, over months \( t - 60 \) to \( t - 1 \), using Equations 20-23. \( \beta_{i,t}^{net} = \beta_{i,t}^2 - \beta_{i,t}^3 - \beta_{i,t}^1 \). \( ZR_{12} \) is the previous month’s average zero-return proportion. \( \ln(MV) \) is the log of the market capitalization and \( \ln(B/M) \) is the log of the book-to-market ratio at the end of the previous year. ** and *** denote significance at the 5% and 1% level respectively.
Table 9 Robustness Check: Pricing of the individual liquidity betas excluding the microcap stocks

This table presents the estimated coefficients of the LCAPM using data from January 1963 to December 2014, excluding microcap stocks. I consider the following model

\[
E(R_{i,t} - R_{f,t}) = \rho^0 + \lambda^1 \beta^1_{i,t} + \lambda^2 \beta^2_{i,t} + \lambda^3 \beta^3_{i,t} + \lambda^4 \beta^4_{i,t} + \lambda^{ZR} E(ZR_{12,i,t}) + \rho^1 \ln(MV_{i,t}) + \rho^2 \ln(B/M_{i,t})
\]

I report the estimates obtained using Fama-MacBeth methodology and correcting for EIV using third and fourth order cumulant estimators. The standard errors are reported in parenthesis. Panel A reports the coefficients obtained from estimating the LCAPM using portfolio betas, \(\beta^k_p\). For each stock \(i\), the pre-ranking beta, \(\beta^k_{i,t,pre}\) of month \(t\) is estimated using the time-series of monthly returns and innovations in illiquidity for the previous 60 months with respect to either the value-weighted market return or the innovations in value-weighted market illiquidity. Stocks are then sorted into ten portfolios based on \(\beta^k_{i,t,pre}\) for month \(t\). The post-ranking portfolio beta, \(\beta^k_p\), is then estimated for each of the ten equal-weighted portfolios over the entire sample period using Equations 20-23. This \(\beta^k_p\) is assigned to all stocks belonging to portfolio \(p\) ranked on the basis of \(\beta^k_{i,t,pre}\). Panel B reports the coefficients obtained from estimating the LCAPM using stock-level betas and then controlling for EIV explicitly using third and fourth order cumulant estimators. For each stock \(i\) in month \(t\), \(\beta^k_{i,t}\) where \(k = (1,2,3,4)\), is computed from monthly returns and innovations in illiquidity for stock \(i\) and for the value-weighted market portfolio, over months \(t - 60\) to \(t - 1\), using Equations 20-23. \(ZR_{12}\) is the previous month’s average zero-return proportion. \(\ln(MV)\) is the log of the market capitalization and \(\ln(B/M)\) is the log of the book-to-market ratio at the end of the previous year. ** and *** denote significance at the 5% and 1% level respectively.

### Panel A: LCAPM estimated with portfolio betas

<table>
<thead>
<tr>
<th></th>
<th>(\beta^1_p)</th>
<th>(\beta^2_p)</th>
<th>(\beta^3_p)</th>
<th>(\beta^4_p)</th>
<th>(ZR_{12})</th>
<th>Intercept</th>
<th>(\ln(MV))</th>
<th>(\ln(B/M))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fama-MacBeth</td>
<td>0.0008</td>
<td>0.0024</td>
<td>0.0081</td>
<td>-0.0013**</td>
<td>0.0438</td>
<td>0.0233***</td>
<td>-0.0025***</td>
<td>-0.0005</td>
</tr>
<tr>
<td></td>
<td>(0.0020)</td>
<td>(0.0020)</td>
<td>(0.0077)</td>
<td>(0.0006)</td>
<td>(0.0222)</td>
<td>(0.0031)</td>
<td>(0.0004)</td>
<td>(0.0006)</td>
</tr>
</tbody>
</table>

### Panel B: LCAPM estimated with stock-level betas

<table>
<thead>
<tr>
<th></th>
<th>(\beta^1)</th>
<th>(\beta^2)</th>
<th>(\beta^3)</th>
<th>(\beta^4)</th>
<th>(ZR_{12})</th>
<th>Intercept</th>
<th>(\ln(MV))</th>
<th>(\ln(B/M))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fama-MacBeth</td>
<td>0.0008</td>
<td>0.0008</td>
<td>0.004</td>
<td>-0.0015</td>
<td>0.0398</td>
<td>0.0229***</td>
<td>-0.0025***</td>
<td>-0.0006</td>
</tr>
<tr>
<td></td>
<td>(0.0021)</td>
<td>(0.0074)</td>
<td>(0.0153)</td>
<td>(0.0008)</td>
<td>(0.0206)</td>
<td>(0.0032)</td>
<td>(0.0004)</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>CUMD3</td>
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<td>-0.0118</td>
<td>0.0072</td>
<td>-0.0025</td>
<td>0.0256</td>
<td>0.0189</td>
<td>-0.0007</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.0039)</td>
<td>(0.006)</td>
<td>(0.019)</td>
<td>(0.0017)</td>
<td>(0.0238)</td>
<td>(0.0131)</td>
<td>(0.0014)</td>
<td>(0.0017)</td>
</tr>
<tr>
<td>CUMD4</td>
<td>-0.0116***</td>
<td>0.0015</td>
<td>0.0553***</td>
<td>-0.0027***</td>
<td>0.0220**</td>
<td>0.0258***</td>
<td>-0.0009</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>(0.0019)</td>
<td>(0.0022)</td>
<td>(0.0074)</td>
<td>(0.0010)</td>
<td>(0.0098)</td>
<td>(0.0079)</td>
<td>(0.001)</td>
<td>(0.0018)</td>
</tr>
</tbody>
</table>
Table 10 Condensed LCAPM estimated with an equal-weighted market portfolio

This table presents the estimated coefficients of the LCAPM using data from January 1963 to December 2014 with the beta estimation based on an equal-weighted market portfolio. I consider the following model

\[ E(R_{i,t} - R_{f,t}) = \beta^0 + \lambda_1 \beta_{i,t}^1 + \lambda_{net} \beta_{i,t}^{net} + \lambda_2 E(ZR_{12,i,t}) + \lambda_3 \ln(MV_{i,t}) + \lambda_4 \ln(B/M_{i,t}) \]

I report the estimates obtained using Fama-MacBeth methodology and correcting for EIV using third through fifth order cumulant estimators. The standard errors are reported in parenthesis. Panel A reports the coefficients obtained from estimating the LCAPM using portfolio betas, \( \beta_{i,t}^p \). For each stock \( i \), the pre-ranking beta, \( \beta_{i,t}^{k,pre} \), \((k = 1, 2, 3, 4)\) of month \( t \) is estimated using the time-series of monthly returns and innovations in illiquidity for the previous 60 months with respect to either the equal-weighted market return or the innovations in equal-weighted market illiquidity. Stocks are then sorted into ten portfolios based on \( \beta_{i,t}^{k,pre} \) for month \( t \). The post-ranking portfolio beta, \( \beta_{i,t}^p \), is then estimated for each of the ten equal-weighted portfolios over the entire sample period using Equations 20-23. This \( \beta_{i,t}^p \) is assigned to all stocks belonging to portfolio \( p \) ranked on the basis of \( \beta_{i,t}^{k,pre} \).

\( \beta_{i,t}^{net} = \beta_{i,t}^p - \beta_{i,t}^2 - \beta_{i,t}^3 \). Panel B reports the coefficients obtained from estimating the LCAPM using stock-level betas and then controlling for EIV explicitly using third through fifth order cumulant estimators. For each stock \( i \) in month \( t \), \( \beta_{i,t}^k \) where \( k = (1, 2, 3, 4) \), is computed from monthly returns and innovations in illiquidity for stock \( i \) and for the equal-weighted market portfolio, over months \( t - 60 \) to \( t - 1 \), using Equations 20-23. \( \beta_{i,t}^{net} = \beta_{i,t}^1 - \beta_{i,t}^2 - \beta_{i,t}^3 \). \( ZR_{12} \) is the previous month’s average zero-return proportion. \( \ln(MV) \) is the log of the market capitalization and \( \ln(B/M) \) is the log of the book-to-market ratio at the end of the previous year. ** and *** denote significance at the 5% and 1% level respectively.

<table>
<thead>
<tr>
<th>Panel A: Condensed LCAPM estimated with portfolio betas</th>
<th>( \beta^1 )</th>
<th>( \beta^{net} )</th>
<th>ZR_12</th>
<th>intercept</th>
<th>ln(MV)</th>
<th>ln(B/M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fama-MacBeth</td>
<td>0.0009</td>
<td>0.0011</td>
<td>0.0041</td>
<td>0.0048**</td>
<td>0.00004</td>
<td>0.0006</td>
</tr>
<tr>
<td></td>
<td>(0.0028)</td>
<td>(0.0009)</td>
<td>(0.0079)</td>
<td>(0.0023)</td>
<td>(0.0003)</td>
<td>(0.0004)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Condensed LCAPM estimated with stock-level betas</th>
<th>( \beta^1 )</th>
<th>( \beta^{net} )</th>
<th>ZR_12</th>
<th>Intercoet</th>
<th>ln(MV)</th>
<th>ln(B/M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fama-MacBeth</td>
<td>-0.0005</td>
<td>0.0007</td>
<td>0.0045</td>
<td>0.0042</td>
<td>0.0003</td>
<td>0.0008**</td>
</tr>
<tr>
<td></td>
<td>(0.0024)</td>
<td>(0.0012)</td>
<td>(0.0087)</td>
<td>(0.0023)</td>
<td>(0.0003)</td>
<td>(0.0004)</td>
</tr>
<tr>
<td>CUMD3</td>
<td>-0.0595***</td>
<td>0.0204</td>
<td>0.7897</td>
<td>-0.2419***</td>
<td>0.0321***</td>
<td>-0.0159***</td>
</tr>
<tr>
<td></td>
<td>(0.0219)</td>
<td>(0.0294)</td>
<td>(0.4954)</td>
<td>(0.0081)</td>
<td>(0.0072)</td>
<td>(0.0073)</td>
</tr>
<tr>
<td>CUMD4</td>
<td>-0.0441***</td>
<td>0.0181***</td>
<td>0.1171***</td>
<td>0.0072</td>
<td>0.0036</td>
<td>-0.0034</td>
</tr>
<tr>
<td></td>
<td>(0.0034)</td>
<td>(0.0047)</td>
<td>(0.0371)</td>
<td>(0.0049)</td>
<td>(0.0031)</td>
<td>(0.0033)</td>
</tr>
<tr>
<td>CUMD5</td>
<td>-0.0079***</td>
<td>-0.0051</td>
<td>0.0371</td>
<td>0.0020</td>
<td>0.0016**</td>
<td>0.0024</td>
</tr>
<tr>
<td></td>
<td>(0.0022)</td>
<td>(0.0036)</td>
<td>(0.0267)</td>
<td>(0.0039)</td>
<td>(0.0008)</td>
<td>(0.0015)</td>
</tr>
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</table>
Table 11 Pricing of the individual liquidity betas with an equal-weighted market portfolio

This table presents the estimated coefficients of the LCAPM using data from January 1963 to December 2014 with the beta estimation based on an equal-weighted market portfolio. I consider the following model

\[ E(R_{i,t} - R_{f,t}) = \rho^0 + \lambda^1 \beta^1_{i,t} + \lambda^2 \beta^2_{i,t} + \lambda^3 \beta^3_{i,t} + \lambda^4 \beta^4_{i,t} + \lambda^{sr} E(ZR_{12,i,t}) + \rho^1 \ln(MV_{i,t}) + \rho^2 \ln(B/M_{i,t}) \]

I report the estimates obtained using Fama-MacBeth methodology and correcting for EIV using third and fourth order cumulant estimators. The standard errors are reported in parenthesis. Panel A reports the coefficients obtained from estimating the LCAPM using portfolio betas, \( \beta^k_p \). For each stock \( i \), the pre-ranking beta, \( \beta^k_{i,t}^{pre} \) \((k = 1, 2, 3, 4)\) of month \( t \) is estimated using the time-series of monthly returns and innovations in illiquidity for the previous 60 months with respect to either the equal-weighted market return or the innovations in equal-weighted market illiquidity. Stocks are then sorted into ten portfolios based on \( \beta^k_{i,t}^{pre} \) for month \( t \). The post-ranking portfolio beta, \( \beta^k_p \), is then estimated for each of the ten equal-weighted portfolios over the entire sample period using Equations 20-23. This \( \beta^k_p \) is assigned to all stocks belonging to portfolio \( p \) ranked on the basis of \( \beta^k_{i,t}^{pre} \). Panel B reports the coefficients obtained from estimating the LCAPM using stock-level betas and then controlling for EIV explicitly using third and fourth order cumulant estimators. For each stock \( i \) in month \( t \), \( \beta^k_{i,t} \) where \( k = (1,2,3,4) \), is computed from monthly returns and innovations in illiquidity for stock \( i \) and for the equal-weighted market portfolio, over months \( t - 60 \) to \( t - 1 \), using Equations 20-23. \( ZR_{12} \) is the previous month’s average zero-return proportion. \( \ln(MV) \) is the log of the market capitalization and \( \ln(B/M) \) is the log of the book-to-market ratio at the end of the previous year. ** and *** denote significance at the 5% and 1% level respectively.

<table>
<thead>
<tr>
<th>Panel A: LCAPM estimated with portfolio betas</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta^1_p )</td>
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<tr>
<td>Fama-MacBeth</td>
</tr>
<tr>
<td>(0.0029)</td>
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<table>
<thead>
<tr>
<th>Panel B: LCAPM estimated with stock-level betas</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta^1 )</td>
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<tr>
<td>(0.0051)</td>
</tr>
<tr>
<td>CUMD4</td>
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<tr>
<td>(0.0021)</td>
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</table>