Estimating Downside Risk in Stock Returns under Structural Breaks

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\textbf{Abstract:}
We show with simulations that inducing structural breaks in the volatility of returns causes non-normality by significantly increasing kurtosis. We endogenously detect significant structural breaks in the volatility of US stock returns and incorporate this information to estimate Value-at-Risk (VaR) to measure the downside risk. Out-of-sample performance results indicate that our proposed model, which incorporates both time varying volatility and structural breaks in volatility, produces more accurate VaR forecasts than several benchmark methods. We highlight the economic importance of our results by calculating the daily capital charges using the Basel Accords.

\textbf{JEL Classification:} G1

\textbf{Key Words:} Volatility, structural breaks, GARCH.

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1. Introduction

Accurately measuring and forecasting risk is very important, especially during crisis periods such as those observed in the past twenty years in many major financial markets. There is abundant evidence that stock returns are characterized by structural breaks in volatility (Starica and Granger, 2005). Structural breaks in volatility produce an upward bias in estimated volatility persistence (Hillebrand, 2005) and contribute to higher estimates of kurtosis which produces non-normality in the data (Karoglou, 2010). Since models of risk are primarily concerned with extreme events, it is essential that they are able to account for structural breaks.

The primary measure of downside risk is Value-at-Risk (VaR). VaR is the expected loss in value a portfolio would incur with a specific probability value over a particular holding period. For example, a one quarter, 1% VaR of $2 million means there is a 1% probability that the portfolio would lose 2 million dollars or more in a single quarter. VaR forecasts depend on the estimated distribution of underlying returns. Excessively conservative estimates of VaR reduce profitability because capital will be withheld from profitable, although risky, investments. Excessively liberal estimates increase the frequency of regulation violations and the possibility of bankruptcy. Therefore, any VaR model which incorrectly forecasts the downside risk because it ignores structural breaks will have serious consequences.

In this paper we show with Monte Carlo simulations that inducing a structural break in normally distributed data causes excess kurtosis. This excess kurtosis drives the non-normality in stock returns. We endogenously detect structural breaks in volatility using the modified iterated cumulative sums of squares (ICSS) algorithm on daily US stock market index returns using data from July 1, 1996 to June 30, 2016. We argue that a forecasting method which accounts for structural breaks in volatility will improve the forecasts of VaR. We test six models for
specifying the volatility to forecast VaR, two of which utilize structural breaks. We compare their out-of-sample VaR estimates with unconditional and conditional tests. We find that our proposed semi-parametric Filtered Historical Simulation approach (FHS) filtered with a GARCH model that incorporates structural breaks provides the best estimates as it provides the least number of violations. This model captures current market conditions through time varying volatility without making any assumption on the underlying distribution of returns. This is particularly advantageous as there is no consensus in the literature about the ‘true’ underlying distribution of stock returns. We highlight the economic importance of our results by calculating the daily capital charges from the estimated VaRs using the Basel II framework. The results show that incorporating structural breaks to a model results in less violations with lower daily capital charges.

2. Literature Review

Traditional volatility approaches for estimating VaR assume asset returns are normally distributed. Although this assumption substantially simplifies the computation of VaR, it is not consistent with the empirical evidence on stock returns, which finds that the distribution of returns is negatively skewed (French, Schwert, and Stambaugh, 1987), fat-tailed (Bollerslev, 1987), and peaked around the mean (Engle and Gonzalez-Rivera, 1991). This implies that extreme negative returns are much more likely to occur in practice than would be predicted by the symmetric thinner tailed normal distribution. Thus, several studies have experimented with fat-tailed and asymmetric return distributions for VaR estimation. Bali and Theodossiou (2007) compare different GARCH specifications and provides strong evidence that skewed fat-tailed distribution yield a more precise and robust approach in VaR calculations than the normal

It is widely documented that volatility is time varying and GARCH models are popularly used to model the time varying volatility for VaR estimation (Giot and Laurent, 2004). GARCH models are estimated under the assumption that the unconditional variance of returns is constant and thus volatility is generated by a stable GARCH process. However, markets often experience structural breaks in the unconditional variance due to political, social, and economic events (Aggarwal, Inclan, and Leal, 1999). Lamoureux and Lastrapes (1990) document that volatility persistence is overestimated when standard GARCH models are applied to a series which has structural breaks in variance. Hillebrand (2005) provides further robust evidence that volatility persistence is biased upward if structural breaks in the GARCH parameters are ignored. Starica and Granger (2005) find shifts in the unconditional variance in daily US stock returns and report that forecasts based on their non-stationary model are superior to those provided by a stationary GARCH model. Further, Rapach and Strauss (2008) document that forecasts generated from models that incorporate structural breaks, detected with the modified ICSS algorithm, improve the forecasts of exchange rate volatility. Thus we conclude that a properly specified GARCH model should account for structural breaks, especially if the model is to be used for forecasting purposes.
Interestingly, there are only a few papers which entertain the possibility of structural breaks in volatility in forecasting VaR. Billio and Pelizzon (2000) examine the application of a switching volatility model to forecast the distribution of returns and estimation of VaR. They find that the switching regime model is preferable to other methods. Berens, Weiss, and Wied (2015) use various conditional correlation models to test the accuracy for forecasting VaR of financial portfolios and they find that correlation models can be improved by tests for structural breaks in co-movements. However, the models in both of these papers are restricted, by construction, to only two regimes. Recently, Chen and Spokoiny (2015) propose a new methodology with a focus on volatility estimation that is able to account for non-stationarity and heavy tails simultaneously. Our approach could be viewed as a logical extension of their work. The majority of the existing literature experiment with different distributions (i.e. heavy tails), but we directly incorporate the structural breaks without making assumptions on the underlying distribution as we argue that structural breaks are the underlying cause of the observed non-normality. To the best of our knowledge, this is the first paper which estimates VaR using FHS with time varying volatility incorporated with structural breaks.

3. Monte Carlo Simulations

In this section we use simulations to explore the effect of a structural break in the unconditional variance on the non-normality of returns, estimated volatility persistence, and the estimated VaR forecasts.
To simulate daily market returns, we generate samples of normally distributed returns (N=4,000) with zero mean and an annualized volatility of 20%. Then, we induce a structural break in the middle of the sample by doubling the value of unconditional variance and the entire series with structural breaks is investigated. Drawing on 5,000 simulations, we find that although the series is normally distributed before and after the structural break, the entire series is leptokurtic as the mean kurtosis is 4.07 with standard error of 0.15. Figure 1 shows the impact of a structural break in variance on kurtosis and Jarque-Bera test. The Jarque-Bera test compares a sample’s skewness and kurtosis to expectations for samples from a normal distribution. The confidence ellipse for a sample of observations is centered on a skewness of 0 and a kurtosis of 3. The black ellipse in Figure 1 encompasses the 99 percent confidence range for a random sample of 4,000 observations from a normal distribution. It is clear that ignoring a structural break that doubles the variance causes the appearance of excess kurtosis.

We double the volatility in the simulations since our empirical results, reported later in the paper, show this upward variance shift to be typical of those detected. Experiments with different levels of shifts in variance show the tail of the distribution becomes fatter as the magnitude of the structural break increases. We also found that halving the variance has the same effect on kurtosis as doubling the variance. These results are shown in Figure 2, which is pertinent because volatility shifts found in empirical data vary in magnitude and direction.

As GARCH is the most popular method of estimating VaR forecasts, we proceed by generating returns \( R_t \) by the following GARCH model for our next set of simulations:

\[
R_t = \varepsilon_t
\]

\[1\]

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2 We used this value of variance because our empirical results, later reported in the paper, show the annualized volatility \( \sqrt{\frac{250\omega}{(1-\alpha-\beta)}} \) to be 20%.
\[ h_t = \omega + 0.1 \epsilon_{t-1}^2 + 0.85 h_{t-1} \]  \quad (2)

where \( \epsilon_t = v_t \sqrt{h_t} \) (\( v_t \) is drawn from a N(0,1) distribution) and \( h_t \) denotes conditional variance.

The model is estimated with a sample size of 4,000. We induce a structural break in the unconditional variance in the middle of the sample; which yields 2,000 observations in each sub-sample to avoid the small sample bias documented by earlier studies [See Lundblad (2007)]. The annualized volatility in the first sub-sample of the simulated data is kept at 20%. We induce a structural break in variance in the second sub-sample so its annualized volatility becomes 40%.

We then estimate a GARCH model 5,000 times using the full sample to capture the effect of a structural break on parameter estimates. We repeat the above procedure over simulated data where we keep the annualized volatility unchanged at 20% in the second sub-sample. We find that the mean volatility persistence is 0.99 when structural breaks are induced although the data is generated by volatility persistence of 0.95. This shows that ignoring structural breaks creates a bias in the estimated volatility persistence. We extend our simulations by adding a dummy variable for the induced structural break in variance in accordance with our empirical methodology. We re-estimate the model 5,000 times and find the mean volatility persistence to be 0.95. This shows that adding dummy variable to account for a structural break gives correct volatility persistence.

VaR estimates based on the GARCH model are directly affected by the parameters determining the volatility persistence, as later outlined in the methodology section. Therefore, miscalculating the volatility persistence will directly affect the VaR forecast. In order to see the magnitude of this forecast error, we find the mean 1-day VaR forecast at the 1% level to be 5.75% (with a standard error of 1.2) based on the 5,000 simulations when structural breaks are induced. The ‘true’ 1% VaR (mean of the first percentile of each set of simulated returns in the
5,000 simulations) in the simulated data is 5.25% (with a standard error of 0.33). The VaR estimate is clearly biased when structural breaks are ignored. This bias is expected to increase as the magnitude of the break increases or the VaR forecast horizon increases (like a 5-day or 10-day VaR).

Our simulations suggest that overlooking structural breaks in volatility produces misleading results, but accurately accounting for these breaks remedies the problem.³

4. Empirical Methodology

4.1. Detecting structural breaks

Hillebrand (2005) provides robust evidence that a structural break in the GARCH process leads to a structural break in the unconditional variance. Inclan and Tiao (1994) propose a cumulative sum of squares (IT) statistic to test the null hypothesis of a constant unconditional variance against the alternative hypothesis of a break in the unconditional variance for iid processes. However, Sanso, Arrago, and Carrionet (2004) show this statistic is significantly oversized when applied to a dependent process like GARCH, but it can be modified through a nonparametric adjustment.

Inclan and Tiao (1994) propose an iterated cumulative sum of squares (ICSS) algorithm based on their IT statistic for testing multiple breaks in the unconditional variance. This algorithm can also be applied to the modified IT statistic with the nonparametric adjustment to avoid the problems when it is applied to a dependent process. In this paper, we apply the ICSS algorithm to the modified IT statistic for detecting structural breaks in the unconditional variance

³ We only report the baseline case here and the detailed results of all of our Monte Carlo Simulations are not reported for the sake of brevity but are available on request.
of stock returns. The standard 5% significance level is used to test for multiple breaks in the unconditional variance of stock returns.\(^4\)

### 4.2. Definition of Value at Risk

Let the asset return process be denoted by

\[
R_t = \mu_t + \varepsilon_t
\]

where \(\varepsilon_t | I_{t-1} \sim (0, h_t)\), \(I_{t-1}\) is the information set at time \(t-1\) and \(h_t\) is the variance at time \(t\). The VaR measure with coverage probability \((p)\) is defined as the conditional quantile, \(VaR_{p|t-1}(p)\), where

\[
P(R_t \leq VaR_{p|t-1}(p) | I_{t-1}) = p
\]

Equation 4 states the proportion of exceptions when the actual loss exceeds the 99% VaR is at most 1%. Throughout the paper we assume \(\mu_t = 0\) in Equation 3, so that \(R_t = \varepsilon_t\), which is a reasonable assumption for daily data and is also consistent with the literature (Christoffersen, 2009). We focus on the portfolio VaR with the 1% coverage probability, which is regularly used in the literature for computing risk exposure (Basel Committee on Banking Supervision, 1995).

Value-at-Risk is estimated using univariate models (Giot and Laurent, 2004; Kuester, Mittnik and Paolella, 2006) and multivariate models (McAleer and da Veiga, 2008). Berkowitz and O’Brien (2002) conclude that a simple univariate model is able to improve the accuracy of portfolio VaR in the case of US commercial banks, and Brooks and Persand (2003) conclude there is no gain from using multivariate models. Christoffersen (2009) argues that univariate models are more appropriate for the purpose of risk measurements, such as computing VaR

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\(^4\) Interested readers are referred to Rapach and Strauss (2008) for a detailed description of the methodology as they use the same methodology to detect structural breaks in the unconditional variance of exchange rates.
forecasts, while multivariate models are more suitable for risk management, such as portfolio selection. Based on this evidence we only use univariate models in our empirical analysis. We use the following six specifications of volatility.\(^5\)

### 4.3. RiskMetrics

The RiskMetrics approach calibrates the variance using an Exponentially Weighted Moving Average, which corresponds to the following Integrated GARCH model:

\[
h_t = (1 - \lambda) \varepsilon_{t-1}^2 + \lambda h_{t-1}
\]

where the contribution to the long-term persistence of unity, namely \(\lambda\), is set to 0.94 for daily data, and hence is not estimated (see JP Morgan, 1996). RiskMetrics assumes the standardized residuals are normally distributed, so the VaR measure is:

\[
VaR_{RM,t-1}^{\text{RM}}(p) = Z_p \sqrt{h_t}
\]

where \(Z_p\) denotes the \(p\)-th percentile of a standard normal variable.

### 4.4. GARCH

The conditional variance in the Gaussian GARCH(1,1) model of Bollerslev (1986) evolves as:

\[
h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}
\]

The one-step ahead conditional quantile with coverage probability \(p\) is given as

\(^5\) All VaR calculations reported in the paper were calculated with the help of files which were graciously provided by Peter Christoffersen. We used quasi-maximum likelihood estimation for the GARCH models and robust standard errors were calculated by the method given by Bollerslev and Wooldridge (1992). We also computed VaR using the historical simulation approach, which is a very simple method and quite popular among banks and financial institutions (Perignon and Smith, 2010). The empirical results are not reported but are available on request.
\[ \text{VaR}^{GARCH}_{t-1}(p) = Z_p \sqrt{h_t} \quad (8) \]

where the forecast of \( h_t \) is obtained from Eq. (7).

### 4.5. GARCH-T

The assumption of normality on non-normal data produces incorrect measures of the true risk faced by the financial institutions or portfolio managers. Thus, we also estimate VaR thresholds assuming a t-distribution given as:

\[ \text{VaR}^{GARCH-T}_{t-1}(p) = T_p(\hat{v}_t), \frac{\sqrt{\hat{v}_t - 2}}{\hat{v}_t} \sqrt{h_t} \quad (9) \]

where \( T_p(\hat{v}_t) \) denotes the p-th percentile of a student t random variable with \( \hat{v}_t \) degrees of freedom, and \( h_t \) is the forecast obtained from the GARCH model given in equation 7.

### 4.6. FHS

The assumption of normally distributed daily stock returns is problematic, yet there is no consensus in the literature for an alternative distribution. Rather than imposing such a choice, the Filtered Historical Simulation method in financial risk management relies on a simple resampling scheme. The historical simulations are “filtered” in that raw returns are not simulated, instead a set of shocks \((z_t)\) are simulated which are then filtered by a GARCH model. The procedure begins with filtering a parametric GARCH model to generate a sequence of standardized returns \((\hat{z}_t = R_t / \sqrt{\hat{h}_t})\), where \( \hat{h}_t \) denotes the in-sample fitted conditional volatility estimate from a GARCH model. VaR is then estimated as:

\[ \text{VaR}^{FHS}_{t-1}(p) = \hat{Z}_p \sqrt{\hat{h}_t} \quad (10) \]
where $\hat{Z}_p$ is the empirical p-th percentile of the fitted standardized returns ($\hat{Z}_t$) over the last 250 trading days [see Christoffersen (2009) for further details].

4.7. GARCH – Breaks

Here the VaR is computed by Equation 8 where the forecast of $h_t$ is obtained from the following equation:

$$h_t = \omega + d_1 D_1 + \cdots + d_n D_n + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \quad (11)$$

where, following Aggarwal, Inclan, and Leal (1999) and Ewing and Malik (2005), $D_1, \ldots, D_n$, are a set of dummy variables taking a value of one from each structural break point in variance onwards and zero elsewhere. This model accounts for structural breaks, but assumes normality of stock returns.

4.8. FHS – Breaks

The VaR for the Filtered Historical Simulation with breaks is estimated by Equation 10 where the forecast of $h_t$ is obtained from Equation 11. Thus, this method accounts for time varying volatility through GARCH modelling as the returns are filtered through a GARCH model while structural breaks in volatility are accounted for by the endogenously determined dummy variables. The advantage of this method is that it captures current market conditions through volatility dynamics without making any assumptions about the underlying distribution of return shocks.

5. Data
We use the daily returns of the S&P 500 index from July 1, 1996 to June 30, 2016 (5,035 observations). The first 15 years (July 1, 1996 to June 30, 2011) are used for estimation and the last 5 years (July 1, 2011 to June 30, 2016) are used for out-of-sample forecasting. Table 1 provides basic descriptive statistics on the data for these two subsamples. Both sets of data are negatively skewed with excessive kurtosis, which implies that the lower tails will differ dramatically from the normal distribution. The skewness and kurtosis measures produce very large and significant Jarque-Bera statistics, strongly rejecting the null hypothesis of normality. These findings of negative skewness, excessive kurtosis and non-normality are well documented in the literature.

6. Empirical Results

6.1. GARCH and structural breaks

The modified ICSS algorithm identifies six break points in the estimation sample (July 1, 1996 to June 30, 2011) as shown in Table 2. We focus on the effect of these detected break points on volatility dynamics and VaR forecasts, not the political or economic events that may be the root causes of the volatility shifts.

The next step is to incorporate these break points in variance into the benchmark GARCH model by including a set of dummy variables in the variance equation corresponding to these break points. The volatility persistence drops considerably, from 0.991 to 0.976, when structural breaks are accounted for in the GARCH model for the fifteen-year estimation window (see Table 3). This is consistent with earlier empirical findings and simulation results reported in Section 3. The half-life of shocks falls from 76 days to 28 days and the log likelihood increases from

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6 The frequency and start date of the sample was selected to be consistent with earlier relevant studies as most of them have used 15 years of recent daily data for estimation.
11,751 to 11,767, which indicates that models that accounts for structural breaks give a better fit. The skewness, kurtosis, and Jarque-Bera statistics for the two models indicate that the GARCH model with structural breaks reduces some of the non-normality in the returns and better fits the data. Specifically, standardized residuals from the model with structural breaks show less kurtosis than the model which ignores breaks. This is consistent with our results from the simulations in Section 3 which show that a structural break increases the kurtosis.

6.2. Out-of-sample VaR forecasting

In order to assess the out-of-sample performance of the VaR measures, we follow the following procedure. A 15-year rolling sample, starting from July 1, 1996, is used to estimate the VaR measures and a 1-year holdout sample (year subsequent to the estimation) is used to evaluate the performance. Specifically, the first rolling (estimation) sample includes the returns from July 1, 1996 to June 30, 2011 and the first holdout sample includes the returns from July 1, 2011 to June 30, 2012. Next, the estimated sample and hold out sample are both rolled forward by one year. The procedure continues through to the end of the sample. Our two models which incorporate structural breaks are rolled similarly but account for the detected breaks that were found in the estimation sample from July 1, 1996 to June 30, 2011. This estimation procedure yields a total forecasting sample of 5 individual years which yields 1,258 daily observations.

7 The significance of considering structural breaks is further supported by the likelihood ratio statistic which is calculated as LR = 2[L(Θ1)-L(Θ0)] where L(Θ1) and L(Θ0) are the maximum log likelihood values obtained from the GARCH models with and without structural breaks, respectively. In our case LR = 2(11767.4- 11751.6) = 32, so we reject the null of no change even at the 1% significance level suggesting that the model with structural break is significantly better.

8 In order to see robustness of our results, we also generated out-of-sample forecasts without rolling the estimation sample and generated forecasts of all five years at one time with the given original estimation sample. Our overall results reported in the paper remain unchanged with this alternative procedure. Results are available on request.
The results for the out-of-sample VaR for the one-day ahead forecast at the 1% level for the six different estimation methods are shown in Figure 3 with the actual stock index returns. It is interesting that different VaR measures provide similar estimates when stock returns are less volatile (like the middle part of our forecasting period) while the VaR estimates across methods diverge considerably when stock returns are more volatile (like the earlier and later part of our forecasting period). We conclude that when markets are stable (less volatile) then the choice of method used to estimate VaR is less relevant but in periods of high volatility, which is typically associated with a crisis, the model choice matters considerably. It is also interesting that the VaR based on the RiskMetrics model gives the highest number of violations (30) while the least number of violations (16) are given by our proposed FHS-Breaks model (See Table 4). It is important to note that the number of violations decrease when structural breaks are incorporated into the GARCH and the FHS model.

6.3. Unconditional Coverage Test

The unconditional coverage test compares the frequency of violations (i.e. actual losses exceed predicted losses) against the frequency that is expected under the null hypothesis that the proportion of exceptions when the actual loss is greater than the 99% VaR is equal to 1%. A rejection of the null hypothesis suggests that the model is not adequate. We employ the Likelihood Ratio test of Kupiec (1995) which is known as the unconditional coverage test as:

\[
LR_{uc} = -2 \ln \left( (1 - \hat{p})^{T - X} \hat{p}^X \right) + 2 \ln \left( (1 - \hat{p})^{T - X} (\hat{p})^X \right)
\]

where \( p = 0.01 \) is the target exception rate, \( \hat{p} \) the sample proportion of exceptions, \( X \) is the total number of exceptions, \( T \) is the total number of observations, and \( LR \) is asymptotically distributed as chi-square with one degree of freedom.
The results presented in Table 4 shows that only the RiskMetrics model fails the unconditional coverage test at the 1% significance level. Therefore, this standard backtesting test indicates that most models produce VaR measures that do not systematically understate or overstate the portfolio’s underlying level of risk. This is important because, as will be shown in section 7, the daily capital charges incurred by financial institutions are directly linked to these results.

The $LR_{UC}$ given in Equation 12 is an unconditional test statistic because it simply counts violations over the entire period. However, in the presence of volatility clustering, the VaR models that ignore mean-volatility dynamics may provide the correct unconditional coverage, but at any given time, may have inadequate conditional coverage. Thus we proceed with following serial independence test.

6.4. Independent Coverage Test

The conditional coverage test developed by Christoffersen (1998) inspects serial independence of VaR estimates. For a given VaR estimate, the indicator variable ($I_t$) is constructed such that $I_t$ is 1 if a violation occurs and 0 if no violation occurs. Christoffersen (1998) proposes the following likelihood ratio test statistic for the null hypothesis of serial independence against the alternative of first-order Markov dependence:

$$LR_{IND} = 2\left[ n_{00}\ln(\Pi_{00}/(1-\Pi)) + n_{01}\ln((1-\Pi_{00})/\Pi) + n_{10}\ln(\Pi_{10}/(1-\Pi)) + n_{11}\ln((1-\Pi_{10})/\Pi) \right]$$ (13)

where $n_{ij}$ is the number of observations with value $i$ followed by $j$, $\Pi_{00} = n_{00}/(n_{00}+n_{01})$, $\Pi_{10} = n_{10}/(n_{10}+n_{11})$, and $\Pi = (n_{01}+n_{11})/N$, respectively. The $LR_{IND}$ statistic has an asymptotic chi-square distribution with one degree of freedom. The results for the likelihood ratio test of
independence are presented in Table 4. The $LR_{IND}$ test shows that all models provide an adequate VaR as their sequence of violations are independent.

### 6.5. Conditional Coverage Test

Christoffersen (1998) argues that violations should be independent and identically distributed over time. However, what is most pressing is to simultaneously test if the VaR violations are independent and the number of violations are adequate as well. The conditional coverage test jointly tests for independence and correct coverage. The joint test statistic ($LR_{CC}$) of conditional coverage is the sum of the two individual test statistics for unconditional coverage and independence [see Christoffersen (2003) for details].

The results for $LR_{CC}$ are also presented in Table 4. Both the RiskMetrics model and the GARCH model fail the conditional coverage test at the 1% significance level. Models with lower values for the test statistics are preferable because they have smaller probabilities of rejecting the null hypothesis that the VaR model is inadequate. The lowest unconditional and conditional coverage test statistics are given by the FHS-Breaks model, even though it has the highest independence test statistic.

### 7. Calculating Daily Capital Charges Based on VaR Forecasts

Under the framework of Basel II, banks must report the VaR estimates to domestic regulatory authorities. These estimates are used to compute the amount of regulatory capital requirements in order to control a financial institution’s market risk exposure and provide a cushion against adverse market conditions. The Basel Accord stipulates that the daily capital charge must be set at the higher of the previous day’s VaR or the average VaR over the last 60
business days, multiplied by a factor $k$ as shown in Table 5. Thus the Basel Accord imposes penalties in the form of a higher multiplicative factor $k$ on banks that use models that lead to a greater number of violations. The empirical evidence presented by Berkowitz and O’Brien (2002) and Perignon, Deng, and Wang (2008) show that banks systematically overestimate their VaR which leads to excessive amount of regulatory capital which negatively affects their profitability. Therefore, using a model which accurately estimates capital requirements can lead to an increase in efficiency and will yield better results for financial risk management.\(^9\)

We calculate the required daily capital charges based on our VaR forecasts and report these results in Table 6. The FHS model gives the largest mean daily capital charge and the GARCH-Breaks model provides the least mean daily capital charge. Capital charges that are larger than necessary are undesirable because they reduce profitability while capital charges that are smaller than necessary are undesirable because they may lead to a bank failure as the capital requirements may be insufficient to cover future losses. Our results show that portfolio managers who want to follow a conservative strategy should calculate VaR using FHS as this will yield fewer violations although at the cost of lower profitability. It is very important to note that incorporating structural breaks into the GARCH and FHS models not only decreases the number of violations, but also lowers the daily mean capital charge as well. This is an interesting finding as typically models that are able to reduce the number of violations tend to have a higher mean capital charge.

8. Conclusion

\(^9\) McAleer, Jimenez-Martin, and Perez-Amaral (2010) propose a decision rule for calculating daily capital charges in light of these competing forces.
This paper shows that the best way to forecast downside risk in the stock market is through a model which includes both time varying volatility and structural breaks. This paper is based on the abundant robust evidence in the literature that stock returns contain structural breaks in volatility, which causes an upward bias in estimated volatility persistence. We show with Monte Carlo simulations that inducing structural breaks in normally distributed data creates excessive kurtosis, which is a primary driver of the non-normality in stock index returns. We identify structural breaks in the volatility of a recent sample of US stock market index returns using an iterative algorithm. We incorporate these breaks into different models, which are used to compute out-of-sample forecasts of Value-at-Risk. We propose a semi-parametric Filtered Historical Simulation model, where returns are filtered using a GARCH model incorporated with structural breaks. This approach has the advantage that it captures the current market conditions through the volatility dynamics without making any assumptions on the underlying distribution of return shocks. This approach is particularly useful as there is no consensus in the literature about the ‘true’ underlying distribution of stock returns.

Our newly proposed method provides the least number of violations of VaR and consistently shows superior results when compared with benchmark methods using different backtesting test procedures. The economic importance of our results is highlighted by calculating the daily capital charges from the estimated VaRs using the Basel Accords. The results show that incorporating structural breaks in both the GARCH and FHS models yields fewer violations with smaller average daily capital charges. We conclude that incorporating structural breaks results in improved downside risk forecast which has major implications regarding the financial risk management.
References


Table 1  
Descriptive Statistics

<table>
<thead>
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<th></th>
<th>July 1, 1996 to June 30, 2011 (Estimation sample)</th>
<th>July 1, 2011 to June 30, 2016 (Forecasting sample)</th>
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</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0001</td>
<td>0.0003</td>
</tr>
<tr>
<td>Maximum</td>
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<td>0.0463</td>
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<td>Minimum</td>
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<td>-0.0689</td>
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<td>Std. Dev.</td>
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<td>Kurtosis</td>
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<tr>
<td>Jarque-Bera</td>
<td>8648.68</td>
<td>1156.59</td>
</tr>
</tbody>
</table>

Notes: The total sample of daily returns of the S&P 500 is from July 1, 1996 to June 30, 2016 covering 20 years (5035 observations). First 15 years (July 1, 1996 to June 30, 2011) is used for estimation and last 5 years (July 1, 2011 to June 30, 2016) is used for out-of-sample forecasting. The Jarque-Bera statistic is used to test whether or not the series resembles normal distribution, both samples reject normality at the one percent level of significance.
### Table 2
Structural Breaks in Volatility

<table>
<thead>
<tr>
<th>Break Points</th>
<th>Time Period</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 1, 1996 - July 27, 1998</td>
<td></td>
<td>0.0098</td>
</tr>
<tr>
<td>July 28, 1998 - June 29, 2002</td>
<td></td>
<td>0.0133</td>
</tr>
<tr>
<td>June 30, 2002 - October 15, 2002</td>
<td></td>
<td>0.0223</td>
</tr>
<tr>
<td>October 16, 2002 - March 31, 2003</td>
<td></td>
<td>0.0143</td>
</tr>
<tr>
<td>April 1, 2003 - September 8, 2003</td>
<td></td>
<td>0.0097</td>
</tr>
<tr>
<td>September 9, 2003 - July 25, 2006</td>
<td></td>
<td>0.0068</td>
</tr>
<tr>
<td>July 26, 2006 - June 30, 2011</td>
<td></td>
<td>0.0158</td>
</tr>
</tbody>
</table>

**Note:** Time periods detected by modified ICSS algorithm.
### Table 3
Estimation Results for Univariate GARCH Models

<table>
<thead>
<tr>
<th>Model</th>
<th>( \omega )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \alpha + \beta )</th>
<th>Half-life (days)</th>
<th>Log likelihood</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Jarque-Bera</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breaks Ignored</td>
<td>1.4E-06</td>
<td>0.082</td>
<td>0.909</td>
<td>0.991</td>
<td>76.66</td>
<td>11751.60</td>
<td>-0.41</td>
<td>4.54</td>
<td>484</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Breaks accounted for</td>
<td>2.7E-06</td>
<td>0.080</td>
<td>0.896</td>
<td>0.976</td>
<td>28.53</td>
<td>11767.41</td>
<td>-0.37</td>
<td>4.31</td>
<td>363</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** The estimation sample period is from July 1, 1996 to June 30, 2011. The P-values in parenthesis are based on robust standard errors calculated from the method given by Bollerslev and Wooldridge (1992). \( \alpha + \beta \) measures the volatility persistence. Half-life gives the point estimate of half-life \((j)\) in days given as \((\alpha + \beta)^j = \frac{1}{2}\). Estimated variance equation without structural breaks for GARCH model is \( h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} \). The Skewness, Kurtosis and Jarque-Bera test statistics are based on the standard residuals from the estimated models. The Jarque-Bera statistics are used to test the null hypothesis that the series are normally distributed and both reject this hypothesis at the one percent level of significance.
**Table 4**

Backtesting VaR

<table>
<thead>
<tr>
<th></th>
<th>Violations</th>
<th>$LR_{uc}$</th>
<th>$LR_{ind}$</th>
<th>$LR_{cc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskMetrics</td>
<td>30</td>
<td>17.578*</td>
<td>1.656</td>
<td>19.234*</td>
</tr>
<tr>
<td>GARCH</td>
<td>22</td>
<td>5.839</td>
<td>3.609</td>
<td>9.449*</td>
</tr>
<tr>
<td>GARCH-T</td>
<td>17</td>
<td>1.420</td>
<td>5.502</td>
<td>6.923</td>
</tr>
<tr>
<td>FHS</td>
<td>18</td>
<td>2.089</td>
<td>5.067</td>
<td>7.157</td>
</tr>
<tr>
<td>GARCH-Breaks</td>
<td>21</td>
<td>4.752</td>
<td>3.936</td>
<td>8.689</td>
</tr>
<tr>
<td>FHS-Breaks</td>
<td>16</td>
<td>0.870</td>
<td>5.972</td>
<td>6.843</td>
</tr>
</tbody>
</table>

**Notes:** The test statistics for the likelihood ratio tests for unconditional coverage, independence coverage, and conditional coverage are reported with the total number of violations during the forecasting period. * denotes that we reject the null hypothesis at the 1% level implying that the model is inadequate. $LR_{uc}$ is the unconditional coverage test given by Kuipic (1995). $LR_{ind}$ and $LR_{cc}$ are conditional coverage tests given in Christoffersen (1998, 2003).
Table 5
Basel Accord Penalty Zones

<table>
<thead>
<tr>
<th>Zone</th>
<th>Number of Violations</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>0 to 4</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.50</td>
</tr>
<tr>
<td>Yellow</td>
<td>7</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.85</td>
</tr>
<tr>
<td>Red</td>
<td>10+</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Note:** The number of violations in the past 250 business days determines the penalty zone.
Table 6
Daily Capital Charges

<table>
<thead>
<tr>
<th>RiskMetrics</th>
<th>Violations</th>
<th>Mean</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH</td>
<td>30</td>
<td>0.0660</td>
<td>0.0432</td>
<td>0.1154</td>
</tr>
<tr>
<td>GARCH-T</td>
<td>22</td>
<td>0.0637</td>
<td>0.0436</td>
<td>0.1025</td>
</tr>
<tr>
<td>FHS</td>
<td>17</td>
<td>0.0675</td>
<td>0.0469</td>
<td>0.1159</td>
</tr>
<tr>
<td>GARCH-Breaks</td>
<td>21</td>
<td>0.0620</td>
<td>0.0456</td>
<td>0.0995</td>
</tr>
<tr>
<td>FHS-Breaks</td>
<td>18</td>
<td>0.0703</td>
<td>0.0471</td>
<td>0.1322</td>
</tr>
</tbody>
</table>

Notes: Descriptive statistics for the daily capital charges for the six models, with the number of violations in the forecasting period. The daily capital charge is the higher of the negative of the previous day’s VaR or the average VaR over the last 60 business days multiplied by \((3+k)\), where \(k\) is the penalty given in Table 5.
Figure 1
Impact of Volatility Breaks on Kurtosis

Notes: This figure shows the impact of a structural break in variance on kurtosis and Jarque-Bera test. The Jarque-Bera test compares a sample’s skewness and kurtosis to expectations for samples from a normal distribution. The confidence ellipse for a sample of observations is centered on a skewness of 0 and a kurtosis of 3. The black ellipse shown above encompasses the 99 percent confidence range for a random sample of 4,000 observations from a normal distribution.
Figure 2
Impact of the Magnitude and Direction of Volatility Break on Kurtosis

Notes: The above graph shows the effect on kurtosis of different magnitude and direction of volatility breaks. It shows that the tail of the distribution becomes fatter as the magnitude of the structural break increases and also shows that doubling the variance has the same effect on kurtosis as halving the variance.
Figure 3
Out-of-Sample VaR Forecasts