Hedging, Speculation and Forward Market Equilibrium under Ambiguity

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Abstract

This paper considers a two-agent, single-asset forward market consisting of a producer (with an underlying long position) and a speculator (with no underlying position). The agents face both risk and ambiguity relating to the future spot price of a commodity. Agents have multiple priors relating to the precision of a signal they receive relating to the price and make their respective decisions regarding optimal forward position using “maxmin” expected utility. In this setting, we derive (i) the relationship between the forward price and each agent’s optimal position, and (ii) the equilibrium forward price and quantity. In the process, the paper sheds light on how differences between the agents in beliefs relating to the ambiguity of the signal shape the equilibrium outcome. It lays out the conditions under which different kinds of equilibria will obtain, such as no-trade equilibrium as well as equilibria in which the producer is fully hedged (“full-trade equilibrium”), partially hedged and over-hedged. In the special case in which both agents share the same multiple priors, it is seen that market equilibrium precludes both no-trade and full-trade, but allows for partial trade and thus for partial risk-transfer. The main contribution of the paper is that it brings out some nuances in the relationship between ambiguity and hedging/trading behavior. In the process, it clarifies the differences between the effects of risk aversion and ambiguity. If ambiguity is measured in terms of the range of possibilities, it is seen that an increase in ambiguity, unlike an increase in risk aversion, does not necessarily lead to more conservative behavior. Instead, agents react to changes in ambiguity in what a trader may refer to as a “directional” manner. Depending on whether the nature of the ambiguous information is favorable to the asset or not, and on whether the change in agents’ beliefs reflects greater optimism or pessimism about the accuracy of the signal, an increase in ambiguity could result in less hedging and more speculation.

Key words: Ambiguity, Hedging, Maxmin expected utility
1 INTRODUCTION AND LITERATURE REVIEW

Most decision-making models in Finance are based on the Expected Utility framework of von Neumann and Morgenstern (1947) and the Subjective Expected Utility (SEU) framework of Savage (1954). In these models, probabilities of future states of the world are taken as given by assuming either that they are objectively known or that they have been subjectively assessed by agents. It has been known for a long time that these frameworks do not always provide a good description of agents’ behavior especially in situations in which probabilities are not precisely known. A famous example is the urn experiment described by Ellsberg (1961).

The basic distinction between situations in which probabilities are known and unknown is often referred to as risk vs ambiguity (with the latter also referred to as Knightian uncertainty). A number of approaches have been devised to model such situations, such as the two-stage approach pioneered by Segal (1987), the Maxmin Expected Utility or Multiple Priors approach of Gilboa and Schmeidler (1989), the Rank-Dependent or Choquet Utility of Schmeidler (1989), the Smooth Ambiguity approach of Klibanoff et al (2006), as well as Robust Control, Multiplier Utility and other generalizations (for example, Hansen and Sargent, 2001 and Maccheroni et al., 2006). Epstein and Schneider (2010) provide an excellent overview of different approaches to modeling ambiguity especially as it relates to Finance.

The current study adopts the Maxmin Expected Utility (or Multiple Priors) approach of Gilboa and Schmeidler (1989), and builds upon the studies by Epstein and Schneider (2008) (henceforth ES) and Illeditsch (2009). The main contributions of the paper are that it sheds light on: (i) how differences in ambiguity faced by agents shape the equilibrium outcome and (ii) how changes in ambiguity faced by agents affect their actions and the equilibrium outcome.

The current paper is based on the framework developed by ES to study how news of uncertain quality may affect asset prices. A risk-neutral but ambiguity-averse agent receives an ambiguous signal. Unable to choose among a set of multiple conditional probability distributions relating to the accuracy of the signal, the agent maximizes expected utility assuming a worst-case scenario. That is to say, for a given value of the decision variable, the agent chooses the probability distribution that results in the lowest expected utility (the worst-case probability distribution), and makes her choice of the decision variable by maximizing expected utility over these worst-case distributions. ES use their model to illustrate (among other things) that investors react more to “bad news” (i.e., news unfavorable to the asset) than “good news” and require an ambiguity premium to compensate for poor information quality.

Illeditsch (2009) builds upon this approach by allowing for heterogeneous investors who are both risk-averse and ambiguity-averse, and investigates how an ambiguous signal about the future value of an asset affects market participation and equilibrium. The model illustrates that even a small change in information can lead to considerable price swings. Also, it is seen that under certain circumstances, there is a price range over which investors do not change their position in the asset – the phenomenon of portfolio inertia. Perhaps most surprisingly, a higher value of the signal does not always result in a higher price.

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This study makes use of the same approach as ES and Illleditsch, but applies it to a forward market context. In a single-period setting, a producer and a speculator face both risk and ambiguity relating to the future spot price of a good (also referred to as an asset) and need to decide what position to take in a forward contract on this good. The producer has a fixed quantity of the good (normalized to unity for convenience) to sell at the end of the period. Both agents have a common initial prior relating to the future price (i.e., the spot price of the good at the end of the period). At the start of the period both agents perceive a noisy, ambiguous signal relating to the future spot price. Each has a set of beliefs, possibly although not necessarily different from one another, about the posterior probability distribution of the future spot price conditional on the signal. Based on the signal and the forward price, each has to decide on their respective forward position. It is assumed that the agents will use maxmin utility for this purpose. The equilibrium forward price is endogenously determined as the market clearing price, if one exists. The paper derives the conditions under which full, partial and no trade equilibria will hold.

The current study has connections to the literature on hedging using forward and/or futures contracts. In particular, it is closely related to Lien (2000) and Lien and Wang (2003) (henceforth referred to LW), which were among the first studies of how ambiguity affects hedging and the equilibrium futures price. The model in LW is set in the Newberry and Stiglitz (1981) framework. There are two agents in this single period model – a producer and a speculator. The former seeks to hedge a given quantity of output by selling futures contracts to the speculator, and the focus is on how the equilibrium futures position and price are determined. Both agents face ambiguity or Knightian uncertainty relating to the future spot price. Ambiguity is modeled using the \( \varepsilon \)-contamination method. The distribution of the future spot price is modeled as a mixture of a known distribution (truncated normal) and an unknown distribution with compact support. “\( \varepsilon \)” is a parameter that reflects the decision-maker’s subjective probability of correctness of the known distribution. The uncertainty (as reflected by \( 1 - \varepsilon \)) relating to the unknown distribution in the mixture is dealt with by the decision-maker using the maxmin method, which basically means that the agent assumes the worst case parameter value for the future spot price distribution. Using this model, LW first analyze the producer’s hedging and the speculator’s trading decision, and derive the demand and supply schedules of the speculator and producer respectively (i.e., the quantity of the futures position each agent would demand or supply at different futures prices). In the process, they show that the well-known portfolio inertia results hold for each of the agents. The producer’s optimal position remains a full hedge over a certain range of the futures price and the speculator’s optimal position remains zero over a certain (and in general a different) range. LW also show that larger the Knightian uncertainty faced by the agents (i.e., smaller the value of \( \varepsilon \)), larger the producer’s hedge position and smaller the speculator’s trading position at any given futures price. Next, LW go on to analyze equilibrium in the futures market and derive the conditions under which a full trade-equilibrium and a no-trade equilibrium are likely. Finally, they also explore the impact of the degree of Knightian uncertainty and agents’ risk aversion on the equilibrium futures quantity and price.

In many ways, the current study is similar to LW; however, there are important differences. The main difference between the model in LW and the one in the current paper is in how ambiguity or Knightian uncertainty is modeled. In their model, the ambiguity faced by the agent is very

\[2\] They analyze a single period model and there is no marking to the market. Thus, their study is effectively about forward contracts.
broad. The agents are assumed to be uncertain about the distribution of the future price, and consequently, in accordance with the extreme pessimism in the maxmin approach, make the worst case assumption regarding the future price – that it will just take on the worst case value.

In the current study, the agents have a common initial belief regarding the distribution of the future spot price. They then receive a noisy signal relating to the future spot price. The ambiguity they face is regarding the precision of this signal, although they do know that the distribution belongs to the normal family. The ambiguity faced by the agents is thus more specific than in LW, but it is seen to allow for a rich set of possible behaviors and outcomes.

The current study confirms many prior results. As demonstrated in many prior studies including LW and ES, in a setting of ambiguity, there are typically one or more ranges of prices in which there is non-participation as well as other ranges in which agents display portfolio inertia – small changes in price do not result in any change in agents’ positions. These results are visible in the current study as well.

Another result pertains to the counter-intuitive relationship between the signal and the equilibrium outcome (price and quantity). A higher (lower) value for the signal does not necessarily imply a higher (lower) equilibrium price. As in Illeditsch (2009), there is a discontinuity in the mapping between the signal and the equilibrium price.

LW show that only if the agents “incur sufficiently different degrees of Knightian uncertainty” is it possible (and indeed, it then becomes more likely) that either no-trade or full-trade equilibrium will prevail. This too is confirmed by the current study. If both agents share the same multiple priors, market equilibrium precludes both no-trade equilibrium and full-trade equilibrium, but allows for a partial trade equilibrium and thus for partial risk-transfer. This result is interesting for the following reason. In a well-known paper by Billot, Chateauneuf, Gilboa and Tallon (2000), it is shown that any overlap in the priors held by agents leads to a no-trade equilibrium. To quote them, “commonality of beliefs is the necessary and sufficient condition to explain the absence of betting.” As noted by Neilson (2007) and confirmed by the results of the current study, the existence of a hedging motive makes this result inapplicable in a context such as the one in this study. As mentioned above, if agents have exactly the same beliefs, a no-trade equilibrium is seen to be impossible.

To reiterate the point, the current study confirms and illustrates in a hedging context several well-known results from prior studies. However, it also makes some noteworthy contributions of its own of which the main ones are the following: (i) It investigates in some detail how differences in ambiguity faced by agents shape the equilibrium outcome. (ii) It yields some novel insights into how changes in ambiguity faced by agents affect their actions and the equilibrium outcome. In the process, it sheds light on the differences between the effects of risk aversion and ambiguity on agent behavior and the equilibrium outcome.

Unlike prior studies of forward/futures market equilibrium under ambiguity (such as LW), the model in this paper allows one to study the impact of new information on the equilibrium outcome. Consider this statement by LW: “Knightian uncertainty and risk aversion have similar qualitative effects on a partial trade market equilibrium.” In their model, agents react to increased ambiguity in much the same way as they would to increased risk – by increasing their caution.
The main difference between ambiguity and risk in the LW setting is that an increase in ambiguity makes no-trade and full-trade equilibria more likely.

By introducing ambiguity through a signal pertaining to the future spot price rather than in the prior distribution of the price, the current study brings out interesting differences between ambiguity and risk aversion and shows that ambiguity and risk aversion can have very different effects. First, agents need not always react to greater ambiguity with increased caution. They may well (although not always) respond by adding to their risk. For example, an increase in ambiguity in the context of “bad news” may make an asset more attractive at the current price and lead to a larger, riskier position. Second, when there is a change in the ambiguity beliefs of agents, it may not be the increase or decrease in the extent of ambiguity – that is to say, in the range of possibilities considered by agents – that is important. What may truly matter are the following: (i) the type of information – whether and to what extent the signal is favorable to the asset; and (ii) the type of change in belief - whether the change reflects the belief that the signal may now possibly be more accurate or less accurate than previously thought. It is interesting to note that an increase in ambiguity may not always be viewed unfavorably by agents.

The rest of the paper proceeds as follows. The next section develops the model with one sub-section devoted to the producer and the next sub-section to the speculator. The following section describes different types of equilibria that may result as well as the effects of changes in risk aversion and ambiguity parameters on the equilibrium outcome. The last section concludes with a summary.

2 THE MODEL

Consider a single-period setting in which there are two agents, a producer and a speculator. The first sub-section concentrates on the producer, who, depending on the context may also referred to as the hedger. The second sub-section focuses on the speculator. The third explores the conditions under which a full trade, partial trade and no trade equilibrium will prevail in this market, and discusses how changes in the signal and the parameters affect the equilibrium outcome.

The Producer:

The producer has a single unit of output to sell at the end of the period and faces a stochastic future spot price, \( p \). The risk can be hedged using an infinitely divisible forward contract with forward price, \( f \). The decision the producer needs to take is the size of the forward position, \( x \). The cost relating to this single unit of output is taken to be a known constant, and the agent’s profits, \( \pi \), are therefore given by:

\[
\pi = p(1 - x) + fx - c
\]

Note that a positive value for \( x \) denotes a short position in the forward contract. For convenience, \((1-x)\) will be referred to as the unhedged position (or the net position in the underlying or the net exposure to the future spot price, \( p \)). However, no restrictions are being imposed on \( x \), and the
producer is freely allowed to under-hedge and over-hedge, and may even take up a long forward position. \( x \) can therefore take any value in \((\infty, \infty)\).

Her initial (prior) distribution relating to the future spot price is normal (Gaussian) with a known mean and variance.

\[
p \sim N(\mu_p, \sigma^2_p)
\]

(2)

It is assumed that \( \sigma^2_p \) is small enough in relation to \( \mu \) so that the probability of a negative draw of \( p \) is rendered negligible.

At the start of the period, she receives a noisy signal, \( s \), relating to the future price.

\[
s = p + \epsilon, \quad \epsilon \sim N(0, \sigma^2)
\]

(3)

The error term, \( \epsilon \) is independent of the price. Also, \( \sigma^2 \) is assumed to be small enough in relation to \( p \) to render the probability of a negative draw for the signal negligible.

The signal is referred to as “good news” if its value is greater than the prior mean \((s < \mu_p)\) and as “bad news” if its value is less \((s < \mu_p)\).

In addition to being noisy, the signal is also of doubtful precision. The producer recognizes this and has a set of multiple priors about the variance of the signal.

\[
\sigma^2 \in [\sigma^2_a, \sigma^2_b] \subset (0, \infty)
\]

(4)

The producer uses maxmin utility and will therefore, for any given position, \( x \), assume the value of \( \sigma^2 \) to be that which results in minimum expected utility. She will then choose her optimal position, \( x^* \), to maximize this minimum expected utility.

For a given \( \sigma^2 \), the agent’s updated distribution for the future price (conditional on the observed signal) is as follows:

\[
p | s \sim N_p(\mu_p + \beta(s - \mu_p), \sigma^2_p(1 - \beta)) \quad \beta = \frac{\sigma^2_p}{\sigma^2_p + \sigma^2}
\]

(5)

As in Illeditsch (2009), \( \beta \) is a measure of the accuracy of the signal and, depending on the value of \( \sigma^2 \) chosen by the agent, it can take a value strictly between 0 and 1. The set of conditional distributions can be conveniently summarized by the set \([\beta_a, \beta_b]\), and for the producer, choosing a \( \sigma^2 \) is equivalent to choosing the corresponding \( \beta \). Here, the parameters \( \beta_a \) and \( \beta_b \) are defined as follows:

\[
\beta_a = \frac{\sigma^2_a}{\sigma^2_a + \sigma^2_b} \text{ and } \beta_b = \frac{\sigma^2_b}{\sigma^2_a + \sigma^2_b}
\]

(6)

Note that \( \beta_a \) corresponds to the low precision signal and \( \beta_b \), to the high precision signal. For future reference, note that a decrease in \( \beta_a \) and/or an increase in \( \beta_b \) would reflect an increase in the range of possibilities and thus an increase in the extent of ambiguity faced by the agent. Also, an increase in either \( \beta_a \) or \( \beta_b \) would (in some sense) imply an increase in the agent’s confidence in the accuracy of the signal, and a decrease in either \( \beta_a \) or \( \beta_b \) would (in some sense) imply an decrease in the agent’s confidence in the accuracy of the signal. For convenience, let us refer to
an increase in $\beta_a$ or $\beta_b$ as an “optimistic” change in belief about signal accuracy, and a decrease as a “pessimistic” change.

Owing to the normality of the future price, for any given value of $\beta$, the distribution of future profits, $\pi$, is also normal.

The updated expected value of profits is:

$$E(\pi|s) = (\mu_p + \beta(s - \mu_p))(1-x) + fx - c \quad (7)$$

The updated variance of profits is:

$$Var(\pi|s) = (1-x)^2 \sigma_p^2(1-\beta) \quad (8)$$

Taking the forward price as given, the producer’s objective can be stated as follows:

$$\max_x \min_\beta E[u(\pi)] \quad (9)$$

Suppose that $u$ is a Constant Absolute Risk Aversion (CARA) utility function with risk-aversion parameter, $\gamma > 0$:

$$u(z) = -\exp(-\gamma z) \quad (10)$$

As the utility function is strictly increasing, the optimization problem above is equivalent to optimizing the Certainty Equivalent, $CE$.

$$\max_x \min_\beta CE(1-x, \beta; f, s) \quad (11)$$

Above, the $CE$ is being viewed as a function of $(1-x)$ rather than $x$ as that turns out to be more convenient.

As previously noted, the producer’s profits, $\pi$, are also normally distributed. This combined with a CARA utility function implies that for a given value of $\beta$, the $CE$ is given by:

$$CE(1-x; \beta, f, s) = E(\pi|s) - \frac{1}{2} \gamma Var(\pi|s) \quad (12)$$

Now, consider the optimization problem of the producer after observing the signal. This part of the paper proceeds along the same lines as the build-up to and the proof of Proposition 1 in Illeditsch (2009). As noted previously, at this point, the forward price is being taken as given. The problem of how the equilibrium forward price is determined endogenously by the joint behavior of the producer and the speculator is discussed later.

After observing the signal, the first step for the producer is to decide on the rule by which to pick the appropriate $\sigma^2$, or equivalently, the appropriate $\beta$, for each possible value of $x$. Only after that can she decide on the optimal value of $x$. Regarding the first step, the appropriate $\beta$ needs to be chosen to minimize the certainty equivalent, $CE$.

Now, the $CE$ contains two components – the expected value and the variance of profits. Let us first examine the impact of $\beta$ on each of the components. On the one hand, the value of $\beta$ that will minimize the expected value of profits depends on whether the unhedged position is long or short (that is to say, on whether the value, $1-x$, is positive or negative) and also on whether the
signal is “good news” \((s > \mu_p)\) or “bad news” \((s < \mu_p)\). If the unhedged position is long, the worst case \(\beta\) is the low precision value of \(\beta_a\) if the signal is good news and the high precision value of \(\beta_b\) if the signal is bad news. For a short position, it is the other way around. On the other hand, the value of \(\beta\) that will maximize the variance of profits (so as to minimize the CE) will be \(\beta_a\) regardless of whether the position is long or short. Since a single worst case \(\beta\) has to be chosen for a given pair of \(x\) and \(s\) (as opposed to choosing two separate \(\beta\)'s, one to minimize the expected value and another to maximize the variance), the appropriate choice of \(\beta\) may involve trading off the effect on the expected value with that on the variance. The same value of \(\beta\) will not work for all values of \(x\) and \(s\). The following proposition, which is very similar to Proposition 1 in Illeditsch (2009), lays down the rules for how \(\beta\) should be chosen for a given value of \(x\).

**Proposition 1**: Fix the signal \(s\) and the forward price \(f\). Define \(\hat{\theta}\) as follows:

\[
\hat{\theta} \equiv \frac{-2(s - \mu_p)}{\gamma \sigma_p^2}
\]  

(13)

Let the producer’s net position \((1 - x)\) be given. The appropriate beta, \(\beta^*\), in order to minimize the certainty equivalent, \(CE\), is determined as follows:

\[
\text{If } (1 - x) \leq \min(0, \hat{\theta}), \text{ then } \beta^* = \beta_a
\]

\[
\text{If } \min(0, \hat{\theta}) \leq (1 - x) \leq \max(0, \hat{\theta}), \text{ then } \beta^* = \beta_b
\]

\[
\text{If } (1 - x) \geq \max(0, \hat{\theta}), \text{ then } \beta^* = \beta_a
\]

(14)

**Proof**: Please refer to Appendix A.

As mentioned in the proposition above, the \(CE\)-minimizing \(\beta\) is always either \(\beta_a\) or \(\beta_b\), and the change from one to the other happens at \((1-x) = 0\) and \((1-x) = \hat{\theta}\). The minimum certainty equivalent, \(CE\), is a continuous and concave function of \((1-x)\), the unhedged position. Moreover, assuming that \(s \neq \mu_p\), it is continuously differentiable except at \((1-x) = 0\) and \((1-x) = \hat{\theta}\). Below is a graph that illustrates the relationship between the certainty equivalent, \(CE\), and the unhedged position, \((1-x)\).
The figures above show, for different values of $\beta$, the producer’s Certainty Equivalent, $CE$, as a function of her unhedged position, $1-x$. In each case, the dashed curve shows the $CE$ for $\beta_a$, the dotted curve, the $CE$ for $\beta_b$, and the solid curve the minimum of the two curves. Thus, the solid curve shows the “worst case” $CE$ for any given forward position, $x$. Both graphs are based on the following numerical values of the model parameters: $\mu_p = 100$, $\sigma_p^2 = 25$, $\beta_a = 0.2$, $\beta_b = 0.6$, and $\gamma = 2$. In Figure 1a, the left graph, the observed signal and the forward price are assumed to be, respectively, $s = 150$ and $f = 125$. The optimal hedge is $x = 1$, a full hedge, as the minimum $CE$ is maximized at $1-x = 0$ as seen in the graph. In Figure 1b, the right graph, the observed signal and the forward price are assumed to be, respectively, $s = 55$ and $f = 65$. In this case, the minimum $CE$ is maximized at $1-x = 0.4$, and the optimal hedge is therefore $x = 0.6$, a partial hedge. In the first case, the optimal hedge occurs at one of the two kinks in the curve, while in the second case, the optimum occurs at an interior point. Of course, other possibilities exist as well, as is apparent from Propositions 2a and 2b.

The next proposition lays out the relationship between the forward price and the agent’s optimal hedge position. For ease of exposition, the proposition is divided into two parts. Part (a) provides the optimal hedge in case the signal is “good news” ($s > \mu_p$) and part (b), in case the signal is “bad news” ($s < \mu_p$) and part (c) the “knife-edge” case of $s = \mu_p$.

Define $\theta_a$ and $\theta_b$ as follows:

$$
\theta_a \equiv \text{argmax } CE(1-x; \beta_a) = \frac{\mu_p + \beta_a(s - \mu_p) - f}{\gamma \sigma_p^2(1 - \beta_a)}
$$

$$
\theta_b \equiv \text{argmax } CE(1-x; \beta_b) = \frac{\mu_p + \beta_b(s - \mu_p) - f}{\gamma \sigma_p^2(1 - \beta_b)}
$$

Further, let $CE_a$ denote the $CE$ curve corresponding to $\beta_a$ and $CE_b$, that corresponding to $\beta_b$.

Viewing the minimum $CE$ as a function of the unhedged position, $(1-x)$, $\hat{\theta}$ denotes the value of $(1-x)$ at one of the two kinks in this curve, the other kink being at $(1-x) = 0$. $\theta_a$ and $\theta_b$ are the values at which $CE_a$ and $CE_b$ attain their respective global maximum.

**Proposition 2 (Parts a, b and c):** Let the signal $s$ and the forward price $f$ be given.

**Part (a):** Suppose that $s > \mu_p$. Depending on the range in which the forward price lies, the producer’s optimal forward position, $x^*$, is given by one of the five following cases:
\[ x_a \equiv 1 - \theta_a \quad \text{if} \quad f \leq \mu_p + \beta_a(s - \mu_p) \]
\[ 1 \quad \text{if} \quad \mu_p + \beta_a(s - \mu_p) \leq f \leq \mu_p + \beta_b(s - \mu_p) \]
\[ x_b \equiv 1 - \theta_b \quad \text{if} \quad \mu_p + \beta_b(s - \mu_p) \leq f \leq \mu_p + (2 - \beta_b)(s - \mu_p) \]
\[ \hat{x} \equiv 1 - \hat{\theta} \quad \text{if} \quad \mu_p + (2 - \beta_b)(s - \mu_p) \leq f \leq \mu_p + (2 - \beta_a)(s - \mu_p) \]
\[ x_a \equiv 1 - \theta_a \quad \text{if} \quad f \geq \mu_p + (2 - \beta_a)(s - \mu_p) \]

**Part (b):** Suppose that \( s < \mu_p \). Depending on the range in which the forward price lies, the producer’s optimal forward position, \( x^* \), is given by one of the five following cases:

\[ x_a \equiv 1 - \theta_a \quad \text{if} \quad f \leq \mu_p + (2 - \beta_a)(s - \mu_p) \]
\[ \hat{x} \equiv 1 - \hat{\theta} \quad \text{if} \quad \mu_p + (2 - \beta_a)(s - \mu_p) \leq f \leq \mu_p + (2 - \beta_b)(s - \mu_p) \]
\[ x_b \equiv 1 - \theta_b \quad \text{if} \quad \mu_p + (2 - \beta_b)(s - \mu_p) \leq f \leq \mu_p + \beta_b(s - \mu_p) \]
\[ 1 \quad \text{if} \quad \mu_p + \beta_b(s - \mu_p) \leq f \leq \mu_p + \beta_a(s - \mu_p) \]
\[ x_a \equiv 1 - \theta_a \quad \text{if} \quad f \geq \mu_p + (2 - \beta_a)(s - \mu_p) \]

**Part (c):** Suppose that \( s = \mu_p \).

The producer’s optimal position, \( x^* \), is given by

\[ 1 - \theta_a = 1 - \frac{(\mu_p - f)}{\gamma \sigma_p^2 (1 - \beta_a)} \]

If \( f > \mu_p \), the producer will over-hedge and the net position \((1 - x^*)\) will be short. If \( f < \mu_p \), then the net position will be long. If \( f = \mu_p \), then the net position is zero indicating a full hedge.

**Proof:** Please refer to Appendix A.

Taking the signal \( s \) as given, Proposition 2 provides the producer’s optimal forward position for any given forward price. Given that we are primarily thinking of the producer as hedging an underlying long position with a short forward position, the relationship between the forward price and the producer’s optimal position may be viewed as the supply schedule of the forward contract. (Of course, as discussed below, the model does allow the producer to undertake a long forward position. In such a case, the producer would be not hedging but speculating. It is worth noting that referring to the producer as a hedger is convenient for exposition, but not strictly accurate.)

What follows below is a discussion of the implications of Proposition 2 – in other words, a discussion of various possible hedging behaviors by the producer.

**No hedging, full hedging, under-hedging and over-hedging:**

Observe that the five cases in each of parts 2a and 2b are arranged in ascending order of the forward price. It is easily verified that the producer’s optimal (short) position viewed as a function of the forward price is continuous and weakly increasing. (The qualification “weakly” is needed owing to “portfolio inertia” for certain ranges of the forward price as discussed below.)
In part (a), the optimal position is unity or less in the first case (underhedging) and unity or more in the last three cases (overhedging). In part (b), the optimal position is unity or less in the first three cases and unity or more in the last case. Note for future reference that for a given forward price \( f \), the optimal position \( x^* \) is unique.

As may be expected, if the forward price is low enough, the agent will underhedge (\( x^* < 1 \)). As noted above, this occurs in the first case of part (a) and the first three cases of part (b) – that is to say, if

\[
f < \mu_p + \beta(s - \mu_p), \forall \beta \in [\beta_a, \beta_b]
\]

In fact, if the forward price is really low, far from hedging, the agent will find it optimal to augment her original long position in the underlying by entering into a long forward position. For example, if \( s > \mu_p \), this will occur (as may be seen in the first case of Part (a) of Proposition 2) if \( f \) takes a value such that \( \theta_a > 1 \):

\[
f < \mu_p + \beta_a(s - \mu_p) - \gamma \sigma_p^2(1 - \beta_a)
\]

On the other hand, if the forward price is high enough, the agent will overhedge (\( x^* > 1 \)) as may be seen in the last three cases of part (a) and the last case of part (b). For this to happen,

\[
f > \mu_p + \beta(s - \mu_p), \forall \beta \in [\beta_a, \beta_b]
\]

For a given signal, \( s \), there are generally only a few cases in which the optimal hedge position is exactly zero. If \( s > \mu_p \), this will occur only if \( f \) is a specific value in the range specified in the first case of part (a) – namely, if \( f \) takes a value such that \( \theta_a = 1 \):

\[
f = \mu_p + \beta_a(s - \mu_p) - \gamma \sigma_p^2(1 - \beta_a)
\]

If \( s < \mu_p \), the optimal hedge would be zero if \( f \) is in the range specified in the first of the five cases mentioned in part (b) and takes the value such that \( \theta_a = 1 \), or if \( f \) is in the range specified in the second case and the signal, \( s \), takes the value such that \( \tilde{\theta} = 1 \) or if \( f \) is in the range specified in the third case and takes the value such that \( \theta_b = 1 \).

It is interesting to compare this with standard results in the literature dealing with portfolio selection under ambiguity. As apparently first pointed out by Dow and Werlang (1992), subsequently confirmed in many different studies (including classic studies such as Epstein and Wang, 1994 and Mukerji and Tallon, 2001), and illustrated by Epstein and Schneider (2010) in a typical portfolio selection problem in a setting of ambiguity, there is a range of prices at which agents will stay out of the market – the so-called nonparticipation result. In the current setting, the agent has a hedging motivation, and this clearly makes a difference. It is only for a few specific values of the forward price for which the result is possible – effectively making this a zero probability event.

On the other hand, there is a range of forward prices at which the optimal hedge is a full hedge. As seen from both part (a) (second case) and part (b) (fourth case), provided that the forward price is in between the minimum and maximum possible values for the updated mean of the future spot price, the optimal hedge position is unity.

\[
If s > \mu_p:\quad \mu_p \leq f \leq \mu_p + \beta_b(s - \mu_p)
\]
If \( s < \mu_p \):
\[
\mu_p + \beta_b(s - \mu_p) \leq f \leq \mu_p + \beta_a(s - \mu_p)
\] (24)

Also, note that in this range of prices, the agent exhibits *portfolio inertia*. By being fully hedged, the agent totally shields herself from all ambiguity, and at that point, small changes in the forward price that occur within this range are not sufficient to make her change her position.

Further, another range of forward prices within which portfolio inertia is seen is the one in which the hedge ratio is \( 1 - \hat{\theta} \):

If \( s > \mu_p \):
\[
\mu_p + (2 - \beta_b)(s - \mu_p) \leq f \leq \mu_p + (2 - \beta_a)(s - \mu_p)
\] (25)

If \( s < \mu_p \):
\[
\mu_p + (2 - \beta_a)(s - \mu_p) \leq f \leq \mu_p + (2 - \beta_b)(s - \mu_p)
\] (26)

Thus, we see portfolio inertia if the optimal hedge position occurs at either of the two kinks in the minimum CE curve.

**Effect of changes in risk aversion and ambiguity parameters on the hedging decision:**

We will now look at the effect of a change in the producer’s risk aversion coefficient on her optimal position. First, consider the cases in which the optimal position is \( (1 - \theta_a) \).

\[
\frac{\partial(1 - \theta_a)}{\partial \gamma} = \frac{\theta_a}{\gamma}
\]
\[
\begin{cases} 
< 0 \text{ if } f > \mu_p + \beta_a(s - \mu_p) \\
> 0 \text{ if } f < \mu_p + \beta_a(s - \mu_p)
\end{cases}
\] (27)

Now, if \( f > \mu_p + \beta_a(s - \mu_p) \), then \((1 - \theta_a)\) can be the optimal position only in the fifth case of both parts a and b of Proposition 2. In these cases, \( x^* \) is greater than unity indicating that the producer has overhedged. Therefore, the negative sign of the derivative above indicates that an increase in risk aversion will cause the producer to decrease her short position from above unity.

If \( f < \mu_p + \beta_a(s - \mu_p) \), then \((1 - \theta_a)\) can be the optimal position only in the first case of both parts a and b. In these cases, \( x^* \) is less than unity indicating that the producer has underhedged. Therefore, the positive sign of the derivative indicates that an increase in risk aversion will cause the producer to increase her short position (or decrease her long position) from below unity.

The main point is that in every one of these instances, an increase in risk aversion moves the producer’s optimal hedge closer to unity. Not surprisingly, higher risk aversion is seen to induce more cautious behavior.

Below are the derivatives relating to other possible optimal hedges, namely \( x_b \) and \( \hat{x} \). (The case in which the optimal hedge is unity is ignored. Owing to portfolio inertia, small changes in risk aversion will in general have no impact on the producer’s optimal position.)

\[
\frac{\partial(1 - \theta_b)}{\partial \gamma} = \frac{\theta_b}{\gamma}
\] (28)

\[
\frac{\partial(1 - \hat{\theta})}{\partial \gamma} = \frac{\hat{\theta}}{\gamma}
\] (29)
The analysis and results in both these cases are identical to the one above.

Now, consider the effect of a change in the ambiguity parameters on the producer’s optimal position. For future reference, note that other things constant, an increase in $\beta_b$ and/or a decrease in $\beta_a$ can be interpreted as an increase in ambiguity faced by the agent as either would reflect an increase in the range of possible values of the signal variance. Likewise, a decrease in $\beta_b$ and/or an increase in $\beta_a$ can be interpreted as a decrease in ambiguity faced by the agent.

If the optimal hedge is at either of the two kinks in the minimum CE curve – that is to say, if the optimal hedge is unity or $1 - \hat{\theta}$, small changes in the parameters, as noted earlier, will not result in any changes in the agent’s position. However, the situation is different at either of the interior optima, $1 - \theta_a$ or $1 - \theta_b$.

$$\frac{\partial (1 - \theta_a)}{\partial \beta_a} = \frac{-(s - f)}{\gamma \sigma_p^2 (1 - \beta_a)^2}$$

$$\frac{\partial (1 - \theta_b)}{\partial \beta_b} = \frac{-(s - f)}{\gamma \sigma_p^2 (1 - \beta_b)^2}$$

In both cases, if the signal value is higher (lower) than the forward price (which, for now, is exogenously determined), a marginal increase in the relevant ambiguity parameter results in a decrease (increase) in the producer’s optimal position.

To understand the implications of the above, suppose that the signal value is lower than the forward price, and that the forward price is in the range specified in the fifth case of either part (a) or part (b) of Proposition 2. Observe that in these cases, $\partial (1 - \theta_a) / \partial \beta_a$ would be positive, implying that a marginal decrease in $\beta_a$ would result in a decrease in the optimal (short) forward position. Note that in the context of the fifth case of either part, the optimal position is greater than unity indicating overhedging. A decrease in the position would therefore move the producer’s forward position closer to unity. This result is perfectly in accord with the intuition that an increase in ambiguity should result in more cautious behavior. Similarly, if the signal value is higher than the forward price and the forward price is in the range specified in the first case of either part (a) or part (b) (in which case the optimal position is less than unity indicating underhedging), a decrease in $\beta_a$ would result in an increase in the optimal position, which would again move the producer closer to a full hedge.

For yet another example, suppose that the signal value is higher than the forward price, and that the forward price is in the range specified in the third case of part (a) (assuming that $s > \mu_p$) in which case the optimal position is greater than unity indicating overhedging. In this case, a marginal increase in $\beta_b$ would result in a decrease in the optimal position, which would move the producer’s position closer to unity or a full hedge.

Does all this mean that changes in ambiguity always have the same kind of effect on hedging behavior as changes in risk aversion? Perhaps surprisingly, the answer turns out to be in the negative. For a counter-example, suppose that the signal value is still higher than the forward
price, but that the forward price is in the range specified in the third case of part (b) (assuming that \( s < \mu_p \)) in which case the optimal position is less than unity indicating underhedging. In this case too, an increase in \( \beta_b \) would result in a decrease in the optimal position, but now this would move the producer farther away from a full hedge.

Interestingly, the results in this last example would be exactly reversed if the signal value were to be lower than the forward price. In such a scenario, an increase in \( \beta_b \) would result in an increase in the optimal position. If this occurs in the context of the third case of part (a), the producer would move farther from a full hedge (as she would now be overhedged to a greater extent). But if this occurs in the context of the third case of part (b), the producer would move closer to a full hedge (as she would now be underhedged to a lesser extent).

The main point here is that (unlike in LW) an increase in ambiguity does not necessarily lead to more cautious behavior. Changes in ambiguity impact hedging behavior differently than changes in risk aversion.

For convenience (as stated previously), let us refer to a decrease in either of the ambiguity parameters, \( \beta_a \) and \( \beta_b \), as a “pessimistic” change as it would, in some sense, reflect a decrease in the agent’s confidence in the accuracy of the signal. Similarly, let us refer to an increase in either parameter as an “optimistic” change.

What the examples above demonstrate is that what matters is not whether ambiguity – measured by the range of possibilities – has increased or decreased. Rather, the effect of a change in ambiguity on the hedge position depends on two factors: (i) whether the nature of the information is favorable to the asset or unfavorable – that is to say, whether the signal value is higher or lower than the current forward price, and (ii) whether the change in the ambiguity parameter is optimistic or pessimistic. Note that the terms “optimistic change” and “pessimistic change” refer purely to changes in beliefs about the possible range of signal accuracy, not to changes in beliefs about the eventual outcome.

If the signal value is higher (lower) than the forward price, an increase in pessimism about signal accuracy (due to a decrease in the relevant parameter, \( \beta_a \) or \( \beta_b \) ) leads to a larger (smaller) short position or smaller (larger) long position, and an increase in optimism about signal accuracy (due to an increase in the relevant parameter, \( \beta_a \) or \( \beta_b \) ) leads to a smaller (larger) short position or a larger (smaller) long position. An intuitive way of interpreting these results is as follows.

Assuming that the signal value is higher (lower) than the forward price, suppose that the agent for some reason becomes more pessimistic about the accuracy of the signal. Roughly speaking, this means that she now regards high values of the spot price to be less (more) likely. Consequently, she reacts by increasing (decreasing) her short position or decreasing (increasing) her long position. An optimistic revision in her beliefs has the opposite effects.

Note that whether such behavior will appear to be cautious or speculative cannot be predicted without information about the agent’s current position (that is to say, her position before the change in her beliefs about signal accuracy). If her current position is that she is underhedged, an increase in her short position will appear conservative although the agent is not really reacting out of motives relating to caution. If her current position is that she is overhedged, the same increase in her short position will appear speculative.
There is thus an important difference between how an agent reacts to changes in risk aversion vs ambiguity. Our producer always reacts to higher risk (or rather increased risk aversion in the context of this model) by moving closer to a full or one-for-one hedge. On the other hand, her reactions to changes in ambiguity are directional; whether she decides to increase or decrease her position (whatever it may happen to be, long or short) depends (roughly speaking) on the change in her beliefs about the future spot price viewed in relation to the current forward price.

The Speculator:

This subsection introduces the speculator, lays out the information available to him and derives his optimal trading position. (We refer to this second agent as a speculator as unlike the producer, he has no underlying position.)

Based on the prevailing forward price, the speculator needs to decide what position to take in the forward contract. His sole cash flows consist of the profits or losses on his forward position and his objective function is to maximize the expected utility of these future cash flows. Let \( \pi_s \) denote his future cash flows and \( y \) denote his forward position.

\[
\pi_s = (p - f)y
\]

Note that a positive value of \( y \) denotes a long forward position and a negative value, a short position.

Suppose that the speculator has exactly the same prior information as the producer. Thus, his initial (prior) distribution relating to the future price is the same as that of the producer.

\[
p \sim N(\mu_p, \sigma_p^2)
\]

Further, the speculator is assumed to observe the same noisy, ambiguous signal relating to the future price.

\[
s = p + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2)
\]

However, the speculator may have a different set of beliefs regarding its precision and therefore of the future conditional distribution of the price of the commodity. His set of multiple priors relating to the variance of the signal are given by:

\[
\sigma^2 \in [\sigma_m^2, \sigma_n^2] \subseteq (0, \infty)
\]

Also, the speculator has different risk preferences. Although the form of the speculator’s utility function is the same, his risk-aversion parameter is \( \gamma_s \):

\[
u_s(z) = -\exp(-\gamma_s z)
\]

The speculator too uses maxmin utility and will therefore, for any given position, \( y \), assume the value of \( \sigma^2 \) to be that which results in minimum expected utility. He will then choose his optimal position, \( y^* \), to maximize this minimum expected utility.

For a given \( \sigma^2 \), the speculator’s updated distribution for the future price (conditional on the observed signal) has the same form as the producer’s:
\[ p(s) \sim N_p(\mu_p + \beta(s - \mu_p), \sigma_p^2(1 - \beta)) \quad \beta = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_s^2} \]  

(37)

The difference is that the speculator’s set of conditional distributions is summarized by the set \([\beta_m, \beta_n]\), with \(\beta_m\) and \(\beta_n\) defined as follows:

\[ \beta_m = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_s^2} \quad \text{and} \quad \beta_n = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_m^2} \]  

(38)

Note that \(\beta_m\) corresponds to the low precision signal and \(\beta_n\), to the high precision signal.

Owing to the normality of the future price, for any given value of \(\beta\), the distribution of future profits, \(\pi_s\), is also normal.

The updated expected value of profits is:

\[ E(\pi_s|s) = (\mu_p + \beta(s - \mu_p) - f)y \]  

(39)

The updated variance of profits is:

\[ \text{Var}(\pi_s|s) = y^2\sigma_p^2(1 - \beta) \]  

(40)

Taking the forward price as given, the speculator’s objective is as follows:

\[ \max_y \min_\beta E[u_s(\pi_s)] \]  

(41)

Again, as the utility function is strictly increasing, the optimization problem is equivalent to optimizing the Certainty Equivalent, \(CE_s\).

\[ \max_y \min_\beta CE_s(y, \beta; f, s) \]  

(42)

For a given value of \(\beta\), \(CE_s\) is given by:

\[ CE_s(y; \beta, f, s) = E(\pi_s|s) - \frac{1}{2} \gamma_s \text{Var}(\pi_s|s) \]  

(43)

Similar to the process for the producer, the optimization proceeds in two steps. First, rules for determining the appropriate certainty-equivalent-minimizing \(\beta\) for any given value of \(y\) need to be derived. Second, the optimal trading position needs to be determined. The next proposition covers the first step. Obviously, it closely parallels Proposition 1 relating to the producer.

**Proposition 3:** Fix the signal \(s\) and the forward price \(f\). Define \(\hat{y}\) as follows:

\[ \hat{y} = \frac{-2(s - \mu_p)}{\gamma_s \sigma_p^2} \]  

(44)

Let the speculator’s position \(y\) be given. The appropriate beta in order to minimize the certainty equivalent, \(CE_s\), is then determined as follows:

\[ \text{If} \ (y) \leq \min(0, \hat{y}), \text{then} \ \beta^* = \beta_m \]

\[ \text{If} \ \min(0, \hat{y}) \leq (y) \leq \max(0, \hat{y}), \text{then} \ \beta^* = \beta_n \]

\[ \text{If} \ (y) \geq \max(0, \hat{y}), \text{then} \ \beta^* = \beta_m \]  

(45)

**Proof:** Please refer to Appendix A.
The minimum certainty equivalent, $CE_s$, is a continuous and concave function of $y$. Moreover, assuming that $s \neq \mu_p$, it is continuously differentiable except at $y = 0$ and $y = \hat{y}$.

The next proposition lays out the speculator’s optimal position. For ease of exposition, the proposition is divided into two parts. Part (a) provides the optimal trading position in case the signal is “good news” ($s > \mu_p$), part (b), in case the signal is “bad news” ($s < \mu_p$), and part (c) the “knife-edge” case of $s = \mu_p$.

Define $y_m$ and $y_n$ as follows:

$$
y_m \equiv \arg\max_y CE_s(y; \beta_m) = \frac{\mu_p + \beta_m(s - \mu_p) - f}{\gamma_s \sigma_p^2(1 - \beta_m)}
$$

$$
y_n \equiv \arg\max_y CE_s(y; \beta_n) = \frac{\mu_p + \beta_n(s - \mu_p) - f}{\gamma_s \sigma_p^2(1 - \beta_n)}
$$

Further, let $CE_m$ denote the $CE$ curve corresponding to $\beta_m$ and $CE_n$, that corresponding to $\beta_n$.

Viewing the minimum $CE$ as a function of the trading position, $y$, $\hat{y}$ denotes the value of $y$ at one of the two kinks in this curve, the other kink being at $y = 0$. $y_m$ and $y_n$ denote the value of $y$ at the maximum value of the $CE$ curves corresponding to $\beta_m$ and $\beta_n$ respectively.

**Proposition 4 (Parts a, b and c):** Let the signal $s$ and the forward price $f$ be given.

**Part (a):** Suppose that $s > \mu_p$. Depending on the range in which the forward price lies, the speculator’s optimal trading position, $y^*$, is given by one of the five following cases:

$$
y_m \quad \text{if} \quad f \leq \mu_p + \beta_m(s - \mu_p) \\
0 \quad \text{if} \quad \mu_p + \beta_m(s - \mu_p) \leq f \leq \mu_p + \beta_n(s - \mu_p) \\
y_n \quad \text{if} \quad \mu_p + \beta_n(s - \mu_p) \leq f \leq \mu_p + (2 - \beta_n)(s - \mu_p) \\
\hat{y} \quad \text{if} \quad \mu_p + (2 - \beta_n)(s - \mu_p) \leq f \leq \mu_p + (2 - \beta_m)(s - \mu_p) \\
y_m \quad \text{if} \quad f \geq \mu_p + (2 - \beta_m)(s - \mu_p)
$$

**Part (b):** Suppose that $s < \mu_p$. Depending on the range in which the forward price lies, the speculator’s optimal trading position, $y^*$, is given by one of the five following cases:

$$
y_m \quad \text{if} \quad f \leq \mu_p + (2 - \beta_m)(s - \mu_p) \\
\hat{y} \quad \text{if} \quad \mu_p + (2 - \beta_m)(s - \mu_p) \leq f \leq \mu_p + (2 - \beta_n)(s - \mu_p) \\
y_n \quad \text{if} \quad \mu_p + (2 - \beta_n)(s - \mu_p) \leq f \leq \mu_p + \beta_n(s - \mu_p) \\
0 \quad \text{if} \quad \mu_p + \beta_n(s - \mu_p) \leq f \leq \mu_p + \beta_m(s - \mu_p) \\
y_m \quad \text{if} \quad f \geq \mu_p + \beta_m(s - \mu_p)
$$

**Part (c):** Suppose that $s = \mu_p$. The speculator’s optimal position, $y^*$, will equal $y_m = \frac{\mu_p - f}{\gamma_s \sigma_p^2(1 - \beta_m)}$. 

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If $f > \mu_p$, the speculator will take up a short position. If $f < \mu_p$, he will choose a long position. If $f = \mu_p$, his optimal position is zero.

**Proof:** Please refer to Appendix A.

Taking the signal $s$ as given, Proposition 4 provides the speculator’s optimal forward position for any given forward price. Given that we are thinking of the speculator as taking the other side of the producer’s hedging transaction, and as such typically taking a long position, the relationship between the forward price and the speculator’s optimal position may be viewed as the demand schedule of the forward contract. (Of course, as discussed below, the model does allow the speculator to undertake a short forward position.)

What follows below is a discussion of the implications of Proposition 4 – in other words, a discussion of various possible trading positions that the speculator may take.

**Possible speculative positions:**

Observe that the five cases in each of parts 4a and 4b are arranged in ascending order of the forward price. It is easily verified that the speculator’s optimal (long) position viewed as a function of the forward price is continuous and weakly decreasing. (The qualification “weakly” is needed owing to “portfolio inertia” for certain ranges of the forward price as discussed below.) In part (a), the optimal position is zero or higher in the first case (long position) and zero or lower in the last three cases (short position). In part (b), the optimal position is zero or higher in the first three cases and zero or lower in the last case. Note for future reference that for a given forward price $f$, the optimal position is unique.

If the forward price is below a certain threshold, the agent will take up a long trading position ($y^* > 0$). This will occur if the forward price satisfies the first case in part (a) of Proposition 4 or any of the first three cases of part (b) - that is to say, if

$$f < \mu_p + \beta(s - \mu_p), \forall \beta \in [\beta_m, \beta_n]$$  \hspace{1cm} (49)

In fact, if the forward price is low enough, the agent will take up a long trading position in excess of unity ($y^* > 1$). This is possible (although not guaranteed) in the first case in part (a), and in any of the first three cases of part (b). For example, for this to happen in the first case of part (a), $f$ should take a value such that $y_m > 1$:

$$f < \mu_p + \beta_m(s - \mu_p) - \gamma_s \sigma_p^2(1 - \beta_m)$$  \hspace{1cm} (50)

On the other hand, if the forward price is high enough, the agent will take up a short position ($y^* < 0$) as may be seen in last three cases of Part (a) of Proposition 4 and the last case of Part (b) of Proposition 4 – that is to say, if

$$f > \mu_p + \beta(s - \mu_p), \forall \beta \in [\beta_m, \beta_n]$$  \hspace{1cm} (51)
For a given signal, \( s \), there are generally only a few values for the forward price at which the optimal trading position is exactly unity. For example, if \( s > \mu_p \), this will occur only if \( f \) is in the range specified in the first case of part (a) – namely, if \( f \) takes a value such that \( y_m = 1 \):

\[
f = \mu_p + \beta_m (s - \mu_p) - \gamma_s \sigma_p^2 (1 - \beta_m)
\]

(52)

If \( s < \mu_p \), this can occur either if \( f \) is in the range specified in the first case of part (b) and takes a value such that \( y_m = 1 \) or if \( f \) is in the range specified in the second case and the signal, \( s \), takes a value such that \( \hat{y} = 1 \) or if \( f \) is in the range specified in the third case and takes a value such that \( y_n = 1 \).

However, observe that there is range of forward prices at which the optimal position is zero, the nonparticipation result. As seen from both part (a) (second case) and part (b) (fourth case) of Proposition 4, provided the forward price is between the minimum and maximum values of the updated mean of the future spot price, the optimal position is zero. Further, in this range of prices, the agent exhibits portfolio inertia - changes in the forward price within this range will not induce the agent to enter the market, or more generally to change his position.

If \( s > \mu_p \):

\[
\mu_p + \beta_n (s - \mu_p) \leq f \leq \mu_p + \beta_n (s - \mu_p)
\]

(53)

If \( s < \mu_p \):

\[
\mu_p + \beta_m (s - \mu_p) \leq f \leq \mu_p + \beta_m (s - \mu_p)
\]

(54)

Another range of forward prices within which portfolio inertia is seen is the one in which the optimal position is \( \hat{y} \):

If \( s > \mu_p \):

\[
\mu_p + (2 - \beta_n) (s - \mu_p) \leq f \leq \mu_p + (2 - \beta_m) (s - \mu_p)
\]

(55)

If \( s < \mu_p \):

\[
\mu_p + (2 - \beta_m) (s - \mu_p) \leq f \leq \mu_p + (2 - \beta_n) (s - \mu_p)
\]

(56)

Thus, we see portfolio inertia if the optimal trading position occurs at either of the two kinks in the minimum CE curve.

Effect of changes in risk aversion and ambiguity parameters on the speculator’s trading decision:

We will now look at how a change in the speculator’s risk aversion coefficient affects his optimal position. First, consider the cases in which the optimal position is \( y_m \).

\[
\frac{\partial y_m}{\partial \gamma_s} = -\frac{y_m}{\gamma_s} \begin{cases} < 0 & \text{if } f < \mu_p + \beta_m (s - \mu_p) \\ > 0 & \text{if } f > \mu_p + \beta_m (s - \mu_p) \end{cases}
\]

(57)

Now, if \( f > \mu_p + \beta_m (s - \mu_p) \), then \( y_m \) can be the optimal position only in the fifth case of both parts a and b of Proposition 4. In these cases, \( y^* \) is negative indicating that the speculator has gone short. Therefore, the positive sign of the derivative above indicates that an increase in risk aversion will cause the speculator to decrease his short position.
If \( f < \mu_p + \beta_m (s - \mu_p) \), then \( y_m \) can be the optimal position only in the first case of both parts a and b. In these cases, \( y^* \) is positive indicating that the speculator has gone long. Therefore, the negative sign of the derivative indicates that an increase in risk aversion will cause the producer to decrease his long position.

The main point is that in every one of these instances, an increase in risk aversion moves the speculator’s optimal position closer to zero. As one would expect, higher risk aversion induces more cautious behavior.

Below are the derivatives relating to other possible optimal hedges, namely \( y_n \) and \( \hat{y} \). (The case in which the optimal position is zero is ignored. In this case, owing to portfolio inertia, small changes in risk aversion will have no impact on the speculator’s optimal position.)

\[
\frac{\partial y_n}{\partial \gamma_s} = -\gamma_n \\
\frac{\partial \hat{y}}{\partial \gamma_s} = -\gamma \\
\frac{\partial y_n}{\partial \beta} = \frac{(s - f)}{\gamma_s \sigma_p^2 (1 - \beta_m)^2} \\
\frac{\partial y_n}{\partial \beta} = \frac{(s - f)}{\gamma_s \sigma_p^2 (1 - \beta_n)^2}
\]

The analysis and results in both these cases are identical to the one above.

Now, consider the effect of a change in the ambiguity parameters on the speculator’s optimal position. For future reference, note that other things constant, an increase in \( \beta_n \) and/or a decrease in \( \beta_m \) can be interpreted as an increase in ambiguity faced by the agent as either would reflect an increase in the range of possible values of the signal variance. Likewise, a decrease in \( \beta_n \) and/or an increase in \( \beta_m \) can be interpreted as a decrease in ambiguity faced by the agent.

At each of the two kinks in the minimum CE curve – that is to say, when the optimal position is zero or \( 1 - \hat{y} \), small changes in the parameters, as noted earlier, will not result in any changes in the agent’s position. However, the situation is different at any of the interior optima, \( y_m \) or \( y_n \).

\[
\frac{\partial y_m}{\partial \beta_m} = \frac{(s - f)}{\gamma_s \sigma_p^2 (1 - \beta_m)^2} \\
\frac{\partial y_n}{\partial \beta_n} = \frac{(s - f)}{\gamma_s \sigma_p^2 (1 - \beta_n)^2}
\]

In both cases, if the signal value is higher (lower) than the forward price (which, for now, is exogenously determined), an increase in the relevant ambiguity parameter results in an increase (decrease) in the speculator’s optimal position.

Note that an increase in \( \beta_n \) and/or a decrease in \( \beta_m \) can be interpreted as an increase in the ambiguity faced by the agent. Therefore, we see that (unlike in LW) an increase in ambiguity does not necessarily lead to a smaller trading position.

As mentioned previously, we will refer to a decrease in either of the ambiguity parameters, \( \beta_m \) and \( \beta_n \), as a “pessimistic” change as it would, in some sense, reflect a decrease in the agent’s
confidence in the accuracy of the signal. Similarly, we will refer to an increase in either parameter as an “optimistic” change.

We see from the partial derivatives above that the effect of a change in the ambiguity parameters on the optimal position depends both on whether the news is favorable to the asset or not (that is to say, whether the signal value is higher or lower than the forward price) and on whether the change in the ambiguity parameter is optimistic or pessimistic.

If the signal is higher (lower) than the forward price, an increase in pessimism about the signal accuracy (whether due to a decrease in $\beta_m$ or $\beta_n$) leads to a smaller (larger) long position or a larger (smaller) short position; and an increase in optimism about signal accuracy (whether due to an increase in $\beta_m$ or $\beta_n$) leads to a larger (smaller) long position or a smaller (larger) short position.

As previously discussed in connection with the producer, there is an important difference between the way agents react to changes in risk/risk aversion and ambiguity. Higher risk aversion induces more conservative behavior. However, in response to a change in ambiguity, whether an agent decides to increase or decrease his position (whether long or short) depends (roughly speaking) on the change in his beliefs about the future spot price viewed in relation to the current forward price.

### 3 EQUILIBRIUM

Thus far, the optimal positions of the representative producer and the speculator have been calculated taking the forward price as given. In this section, the conditions for the existence of market equilibrium are discussed.

Take the signal $s$ as given. Market equilibrium is defined as a pair of values of the forward price and quantity, $(f, q)$, such that given this forward price, the optimal position of the producer is equal in magnitude but opposite in sign to that of the speculator — in other words, $f$ is such that $x^* = y^* = q$. (Recall that by definition, a positive value for $x$ denotes a short position and a positive value for $y$ denotes a long position.) If both these variables are positive and equal, it would mean that the producer is taking a short position in the forward that is matched by the long position of the speculator. (However, the model also allows for the producer to go long and the speculator to go short.)

From Proposition 2, we know that that the producer’s optimal position is either $x_a$, 1, $x_b$ or $\hat{x}$. From Proposition 4, the speculator’s optimal position is either $y_m$, 0, $y_n$ or $\hat{y}$. Thus, (at least at first sight), there are 16 possible equilibrium combinations or types.

Appendix B contains a list of all the possible equilibrium types and the conditions for each to obtain. The following section explores some of the different equilibrium possibilities based on those conditions.

But first, a natural question to ask is the following: given a signal $s$, is the equilibrium unique or are there multiple equilibria? In other words, could there more than one $(f, q)$ pair that satisfies the market equilibrium criterion? As noted earlier in connection with Propositions 2 and 4, for a
given forward price, the producer’s optimal position and the speculator’s optimal position are each unique. So the question is whether the equilibrium forward price is unique. The answer is that the equilibrium forward price is indeed unique for the most part. The only exception is a knife-edge case in which the signal takes on this specific value:

\[ s = \mu_p - \frac{\gamma y \sigma_p^2}{2(\gamma + \gamma_s)} \]  

In this case, the producer’s optimal position is \( \hat{x} \) and the speculator’s optimal position is \( \hat{y} \), and any of a whole range of forward prices can constitute an equilibrium. Refer to Appendix B for details. It is worth reiterating that except for this one case, the equilibrium forward price is uniquely determined by the signal value. This can be verified by going through each one of the 16 cases or equilibrium types in Appendix B and calculating the corresponding forward price. The discussion following Proposition 5 provides an illustration.

**No-trade equilibrium:**

**Proposition 5:** If \( s > \mu_p \), then the conditions for no-trade equilibrium to obtain are as follows:

\[ \beta_a > \beta_m \quad \text{and} \quad \frac{\gamma \sigma_p^2 (1 - \beta_a)}{\beta_a - \beta_m} < s - \mu_p \]  

Additionally, if \( \beta_a > \beta_n \), then \( s - \mu_p < \frac{\gamma \sigma_p^2 (1 - \beta_a)}{\beta_a - \beta_n} \)

If \( s < \mu_p \), the conditions are as follows:

\[ \hat{\theta} > 1, \beta_b < \beta_n \quad \text{and} \quad \frac{\gamma \sigma_p^2 (1 - \beta_b)}{\beta_n - \beta_b} < -(s - \mu_p) \]  

Additionally, if \( \beta_b < \beta_m \), then \( -(s - \mu_p) < \frac{\gamma \sigma_p^2 (1 - \beta_b)}{\beta_m - \beta_b} \)

**Proof:** Here is an outline of the proof. First consider the case \( s > \mu_p \) (good news). For \( x^* \) to be zero, the first of the five cases in part (a) of Proposition 2 has to obtain, and \( \theta_a \) must equal 1, which implies that

\[ f = \mu_p + \beta_a(s - \mu_p) - \gamma \sigma_p^2 (1 - \beta_a) \]  

\( y^* = 0 \) requires that the second of the five cases in part (a) of Proposition 4 must obtain.

\[ \mu_p + \beta_m(s - \mu_p) < f < \mu_p + \beta_n(s - \mu_p) \]

These requirements can be jointly expressed as

\[ \beta_m(s - \mu_p) < \beta_a(s - \mu_p) - \gamma \sigma_p^2 (1 - \beta_a) < \beta_n(s - \mu_p) \]  

which is equivalent to the conditions as stated in the proposition above. The case of bad news \( s < \mu_p \) can be dealt with in similar fashion. For this case (refer to Appendix B), the only possible equilibrium type turns out to be the one in which the producer’s optimal position is
given by $1 - \theta_b$, and the equilibrium forward price is found by solving the equation $1 - \theta_b = 0$. This completes the proof.

Observe that for each of the cases, $s > \mu_p$ and $s < \mu_p$, the equilibrium outcome is unique. For example, for the case $s > \mu_p$, the equilibrium forward price is uniquely given by (65), and the equilibrium volume is of course just zero.

The proposition implies that a necessary condition for no-trade equilibrium to prevail subsequent to good news ($s > \mu_p$) is that the producer should be more optimistic about the worst case scenario regarding signal accuracy than the speculator ($\beta_a > \beta_m$). Intuitively, when the news is good and the producer is more optimistic about its accuracy than the speculator, then the producer is likely to ask for a high forward price in order to go short while the speculator may demand a lower forward price for going long, and no trade may result.

Under what conditions is no-trade equilibrium likely? Assuming good news ($s > \mu_p$), observe that a lower value for $\beta_m$ and a higher value for $\beta_n$ each increases the likelihood of a no-trade equilibrium. This can be roughly restated as follows: other things constant, greater the ambiguity faced by the speculator, larger the likelihood of a no-trade equilibrium. On the other hand, the speculator’s degree of risk aversion, $\gamma_s$, has no bearing on this matter. It is interesting to note that these results are somewhat similar to the ones in Proposition 4 of LW. However, unlike in their model, there is no simple relationship between the ambiguity faced by the producer (or the producer’s degree of risk aversion) and the likelihood of a no-trade equilibrium. In their study, LW show that a no-trade equilibrium is more likely if the producer faces less Knightian uncertainty or if the producer’s risk aversion were to decrease. It is apparent that this is not true in the current model. A large enough change in either $\beta_a$ or $\gamma$ (in either direction) can cause a violation of the bounds specified above. For example, if the producer’s optimism as compared to the speculator is too great ($\beta_a \gg \beta_m$), then it is possible that the producer decides to go long and the speculator short so that trade results but with roles reversed. A decrease in the producer’s risk aversion may also have a similar effect, while on the other hand an increase in her risk aversion may push down her asking price enough to result in some trade.

The case of bad news ($s < \mu_p$) can be analyzed in similar fashion. A necessary condition for no-trade equilibrium to prevail subsequent to bad news ($s < \mu_p$) is that the speculator should be more optimistic (or confident) about the best possible signal accuracy than the producer ($\beta_n > \beta_b$). However, if this difference in confidence is too great ($\beta_m \gg \beta_b$), then the equilibrium type may well shift from no-trade to some trade but with roles reversed. Roughly speaking, when the news is bad and the speculator is more optimistic (or confident) about its accuracy than the producer, then the speculator may demand a relatively low forward price, thus making a no-trade outcome more likely.

Now, consider the special case, $\beta_a = \beta_m$ and $\beta_b = \beta_n$. Clearly, the conditions required for a no-trade outcome cannot be satisfied. Thus, a no-trade equilibrium is impossible if both agents hold identical beliefs regarding the ambiguity of the future price. This is an interesting result and we elaborate upon it below.
Trade equilibria:

We will explore the conditions for various types of trade equilibria (i.e., equilibria in which the volume is non-zero), and start with a special case.

Equilibrium with identical beliefs and risk aversion:

For an explicit calculation of equilibrium, consider the special case in which the two agents hold the same set of beliefs (that is to say, $\beta_a = \beta_m$ and $\beta_b = \beta_n$) and also have the same coefficient of risk aversion ($\gamma = \gamma_s$). Appendix C contains the details. The main result is that, in this special case, the equilibrium volume is $\frac{1}{2}$, while the equilibrium forward price depends on the value of the signal. (A constant equilibrium volume independent of the signal value is a feature of this special case and not generally true.) The relationship between the signal value and the equilibrium forward price is discussed later.

This is an interesting result for the following reason. Prior studies, notably, Billot et al (2000), have shown that in the context of purely speculative agents holding multiple priors, any overlap between the set of beliefs held by traders leads to a no-trade equilibrium. The example above illustrates how this result changes when one of the agents has a hedging motive. In fact, the presence of the hedging motive turns the result on its head. In the current setting, it is seen that trade will necessarily result if the beliefs of both traders are identical.

Full-trade equilibrium:

The following proposition relates to full-trade equilibrium – i.e., an equilibrium in which the producer opts for a full hedge ($x^* = y^* = 1$).

**Proposition 6:** Suppose that $s > \mu_p$. The conditions for a full-trade equilibrium are:

$$\beta_m > \beta_a \text{ and } \frac{y_s \sigma_p^2(1 - \beta_m)}{(\beta_m - \beta_a)} < s - \mu_p \quad (68)$$

and if $\beta_m > \beta_b$, then $s - \mu_p < \frac{y_s \sigma_p^2(1 - \beta_m)}{(\beta_m - \beta_b)}$

Supposing $s < \mu_p$, the conditions are as follows:

$$\hat{y} > 1, \beta_n < \beta_b \text{ and } \frac{y_s \sigma_p^2(1 - \beta_n)}{(\beta_b - \beta_n)} < -(s - \mu_p) \quad (69)$$

and if $\beta_n < \beta_a$, then $-(s - \mu_p) < \frac{y_s \sigma_p^2(1 - \beta_n)}{(\beta_a - \beta_n)}$

**Proof:** Here is an outline of the proof. First, consider the case in which the signal is good news ($s > \mu_p$). From part (a) of Propositions 2, the producer’s hedge position will be unity if

$$\mu_p + \beta_a (s - \mu_p) \leq f \leq \mu_p + \beta_b (s - \mu_p) \quad (70)$$

From part (a) of Proposition 4, the speculator will choose a unit long position ($y_m = 1$) if...
\[ f = \mu_p + \beta_m(s - \mu_p) - \gamma s \sigma_p^2 (1 - \beta_m) \]  

These requirements can be jointly expressed as

\[ \beta_a(s - \mu_p) < \beta_m(s - \mu_p) - \gamma s \sigma_p^2 (1 - \beta_m) < \beta_b(s - \mu_p) \]

which is equivalent to the conditions as stated in the proposition above. The conditions for the case \( s < \mu_p \) can be derived in similar fashion, and this completes the proof.

The proposition implies that a necessary condition for full-trade equilibrium to prevail subsequent to good news \((s > \mu_p)\) is that speculator should be more optimistic (or confident) about the worst possible signal accuracy than the producer \((\beta_m > \beta_a)\). Intuitively, when the news is good and the speculator is more optimistic about its accuracy than the producer, then the speculator is likely to be willing to accept a high forward price, and this makes a full-trade equilibrium more likely.

Observe that (assuming good news) a lower value for \( \beta_a \) and a higher value for \( \beta_b \) each increases the likelihood of a full-trade equilibrium. This can be roughly restated as follows: other things constant, greater the ambiguity faced by the producer, larger the likelihood of a full-trade equilibrium. On the other hand, the producer’s degree of risk aversion, \( \gamma \), has no bearing on this matter. These results are similar to those in Proposition 4 of LW. However, unlike in their model, there is no simple relationship between the ambiguity faced by the speculator (or the speculator’s degree of risk aversion) and the likelihood of a full-trade equilibrium. In their model, LW show that a full-trade equilibrium is more likely if the speculator faces less Knightian uncertainty or if his risk aversion were to decrease. It is apparent that this is not true in the current model. A large enough change in either \( \beta_m \) or \( \gamma s \) (in either direction) can cause a violation of the bounds specified in the condition above. For example, if the speculator’s optimism as compared to the producer is too great \((\beta_m \gg \beta_b)\), then the speculator is likely to wish to increase his long position even beyond unity which may shift the equilibrium from a full hedge to an overhedged one (that is to say, a shift from an equilibrium in which the producer has a full hedge to one in which the producer is overhedged). A decrease in the speculator’s risk aversion may also have a similar effect, while on the other hand an increase in his risk aversion may push down his demand schedule enough to shift the equilibrium type from full hedge to partial.

The case of bad news \((s < \mu_p)\) can be analyzed in similar fashion. A necessary condition for full-trade equilibrium to prevail subsequent to bad news \((s < \mu_p)\) is that the speculator should be more pessimistic about the best possible signal accuracy than the producer \((\beta_n < \beta_b)\), but not so much so \((\beta_n \ll \beta_a)\) that the equilibrium type shifts to an overhedged one. Intuitively, when the news is bad, but the speculator is more pessimistic (or less confident) about its accuracy than the producer, then the speculator would be willing to accept a higher forward price than otherwise, thus making a full-trade outcome more likely.

However, consider again the special case in which the two agents share the same set of beliefs \((\beta_a = \beta_m)\). In this case, the condition for the speculator implies that

\[ f = \mu_p + \beta_a(s - \mu_p) - \gamma s \sigma_p^2 (1 - \beta_a) \]

This implies that

\[ f < \mu_p + \beta_a(s - \mu_p) \]
However, this is incompatible with the condition for the producer. Thus, it is seen that complete risk-transfer (i.e., a full hedge) may not always be possible. A necessary condition is that the two agents need to have different beliefs regarding the set of possible conditional distributions.

**Over-hedging equilibrium:**

Next are derived conditions under which the producer would over-hedge in equilibrium: $x^* = y^* > 1$.

**Proposition 7:** Suppose that $s > \mu_p$. The conditions for an over-hedging equilibrium are as follows:

\[
\frac{1 - \beta_b}{1 - \beta_m} - 1 \left(s - \mu_p\right) > \gamma_s \sigma_p^2
\]

\[
\frac{1 - \beta_m}{1 - \beta_b} - 1 \left(s - \mu_p\right) < \gamma \sigma_p^2
\]

(75)

On the other hand, if $s < \mu_p$, then the conditions are:

\[-(s - \mu_p) \left(1 - \frac{1 - \beta_n}{1 - \beta_a}\right) < \gamma \sigma_p^2\]

\[-(s - \mu_p) \left[\frac{1 - \beta_n}{1 - \beta_a} + 1 + \frac{2\gamma}{\gamma_s}\right] > \gamma \sigma_p^2\]

\[-(s - \mu_p) \left(1 - \frac{1 - \beta_a}{1 - \beta_n}\right) > \gamma_s \sigma_p^2\]

(76)

**Proof:** Here is an outline of the proof. Consider the case, $s > \mu_p$. From part (a) of Proposition 2, $x^* > 1$ requires one of the last three cases to hold and therefore $f > \mu_p + \beta_b(s - \mu_p)$. From part (a) Proposition 4, for $y^* > 1$, the first of the five cases must hold – that is, $y^* = y_m$ and $f < \mu_p + \beta_m(s - \mu_p)$. It can be worked out from the conditions in the Appendix B table that the only possible overhedging equilibrium is $1 - \theta_b = y_m$. Solving this equation for $f$, and substituting this value into the requirement $\mu_b < f < \mu_m$ leads to the conditions specified above for the case $s > \mu_p$.

The derivation of the conditions for $s < \mu_p$ is similar. From part (b) of Proposition 2, $x^* > 1$ requires the last of the five cases to hold and therefore $f > \mu_p + \beta_a(s - \mu_p)$. From part (b) of Proposition 4, for $y^* > 1$, one of the first three cases must hold and therefore $f < \mu_p + \beta_n(s - \mu_p)$. It can be worked out from the conditions in the Appendix B table that the only possible overhedging equilibrium is $1 - \theta_a = y_n$. Substituting the equilibrium value of $f$ into the relevant conditions on $f$ specified in the table leads to the conditions above, which completes the proof.
For the case $s > \mu_p$, the conditions require that $\beta_m > \beta_b$. That is to say, the speculator should be considerably more optimistic (or confident) about signal accuracy than the producer, in that the speculator’s worst case estimate of the signal variance should be lower than the producer’s best case estimate ($\sigma^2_m < \sigma^2_a$).

For the case $s < \mu_p$, the conditions require that $\beta_a > \beta_n$. That is to say, the producer should be considerably more optimistic (or confident) about signal accuracy than the speculator, in that the producer’s worst case estimate of the signal variance should be lower than the speculator’s best case estimate ($\sigma^2_b < \sigma^2_m$).

Thus, roughly speaking, in order for the equilibrium to involve overhedging by the producer, either the speculator must be relatively more confident about the accuracy of good news or the producer should be relatively more confident about the accuracy of bad news.

**Speculative equilibrium:**

Next are derived conditions for a purely speculative equilibrium – an equilibrium in which both parties take purely speculative positions – the producer, a long position in forwards and the speculator, a short position: $x^* = y^* < 0$.

**Proposition 8:** Suppose that $s > \mu_p$. The conditions for a purely speculative equilibrium are as follows:

\[
\left(\frac{1 - \beta_n}{1 - \beta_a} - 1\right)(s - \mu_p) > \gamma \sigma^2_p \\
\left(\frac{1 - \beta_a}{1 - \beta_n} - 1\right)(s - \mu_p) < \gamma_s \sigma^2_p
\]  \hspace{1cm} (77)

Supposing that $s < \mu_p$, the conditions are as follows:

\[
-(s - \mu_p)\left(1 - \frac{1 - \beta_m}{1 - \beta_b}\right) > \gamma \sigma^2_p \\
-(s - \mu_p)\left[\frac{1 - \beta_b}{1 - \beta_m} + 1 + \frac{2\gamma s}{\gamma}\right] > \gamma_s \sigma^2_p \\
-(s - \mu_p)\left(1 - \frac{1 - \beta_b}{1 - \beta_m}\right) < \gamma_s \sigma^2_p
\]  \hspace{1cm} (78)

**Proof:** Again, we offer a sketch of the proof. Consider the case $s > \mu_p$. From part (a) of Proposition 2, $x^* < 0$ requires the first case to hold and therefore $f < \mu_p + \beta_a(s - \mu_p)$. From part (a) of Proposition 4, for $y^* < 0$, one of the last three must hold and $f > \mu_p + \beta_n(s - \mu_p)$. It can be worked out from the conditions in the Appendix B table that only possible speculative equilibrium is $1 - \theta_a = y_n$, and the conditions in the proposition above can be derived using the restrictions on $f$ in the table.
Next, suppose that \( s < \mu_p \). From part (b) of Proposition 2, \( x^* < 0 \) requires one of the first three cases to hold and therefore \( f < \mu_p + \beta_b (s - \mu_p) \). From part (b) of Proposition 4, for \( y^* < 0 \), the last of the five cases must hold and therefore \( f > \mu_p + \beta_m (s - \mu_p) \). It can be worked out from the conditions in the Appendix B table that that only possible speculative equilibrium is \( 1 - \theta_b = y_m \), and the conditions in the proposition above can be derived using the restrictions on \( f \) in the table. This completes the proof.

In the case \( s > \mu_p \), the conditions require that \( \beta_a > \beta_n \). That is to say, the producer should be considerably more optimistic (or confident) about signal accuracy than the speculator, in that the producer’s worst case estimate of the signal variance should be lower than the speculator’s best case estimate \( \sigma_b^2 < \sigma_m^2 \).

In the case \( s < \mu_p \), the conditions require that \( \beta_m > \beta_b \). That is to say, the speculator should be considerably more optimistic (or confident) about signal accuracy than the producer, in that the speculator’s worst case estimate of the signal variance should be lower than the producer’s best case estimate \( \sigma_n^2 < \sigma_a^2 \).

Thus, in order for the equilibrium to involve totally speculative positions by both parties, either the producer must be considerably more confident about the accuracy of good news or the speculator should be considerably more confident about the accuracy of bad news.

**Signal to forward price map:**

Within any particular equilibrium type, the relationship between the signal value and the equilibrium forward price is monotonic. A higher value of the signal results in a higher equilibrium forward price. However, if a change in the signal value results in a movement from one equilibrium type to another, then it is possible that a higher signal value may result in a lower forward price. As in Illeditsch (2009), there may be discontinuities in the mapping between the signal and the equilibrium forward price.

Consider the simple, special case in Appendix 3 in which the two agents hold the same set of beliefs (\( \beta_a = \beta_m \) and \( \beta_b = \beta_n \)) and have the same coefficient of risk aversion \( \gamma = \gamma_s \). As shown in Appendix C, the relationship between the signal and the equilibrium forward price is as follows:

\[
\begin{align*}
    f &= \mu_p + \beta_b (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_b)}{2} \quad \text{iff } s - \mu_p < -\frac{\gamma \sigma_p^2}{4} \\
    f &= \mu_p + \beta_a (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_a)}{2} \quad \text{iff } s - \mu_p > -\frac{\gamma \sigma_p^2}{4} \\
    \mu_p + \beta_a (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_a)}{2} < f < \mu_p + \beta_b (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_b)}{2} \quad \text{iff } s - \mu_p = -\frac{\gamma \sigma_p^2}{4}
\end{align*}
\]

\[(79)\]

Above, the forward price can be interpreted as the “worst case” expected value minus a risk premium.

It is seen that the relationship has a discontinuity at
In fact, if the signal goes up from a little under this critical value to a little over, then the equilibrium forward price actually decreases. The underlying reason for this is that the agents treat “good news” and “bad news” differently. When the signal is below the critical value, it is viewed as “bad news” and assumed to be of high accuracy (or of low variance). When the signal goes above the critical value, agents switch to treating it as a low accuracy (or high variance) signal. The higher variance calls for a higher risk premium and therefore results in a lower equilibrium forward price.

Impact of a change in risk and ambiguity parameters on the equilibrium:

The discussion in this section is similar to the previous discussions in section 2 in which we went over the effect of changes in risk aversion and ambiguity individually on the producer and the speculator. However, there are some new points to be made as well.

The effect of a change in the risk aversion parameters on the equilibrium outcome is explored in Appendix D. The main points are as follows. An increase in the producer’s risk aversion tends to move the producer closer to a full hedge, regardless of whether the news is good or bad. If the producer is only partially hedged, she will increase her short forward position, and other things constant, the equilibrium forward price will decrease and the volume will increase. If the producer is overhedged, she will decrease her short forward position, and other things constant, the equilibrium price will increase and the volume will decrease. A point worth noting is that the effect of a change in the producer’s risk aversion cannot be predicted without knowing whether she is initially underhedged or overhedged. Coming to the speculator, an increase in his risk aversion tends to make him speculate less, regardless of whether the news is good or bad. If speculator is long, he will wish to decrease his long forward position, which will, other things constant, decrease the equilibrium price as well as volume. If the speculator is short, he will decrease his short forward position, which will, other things constant, increase the equilibrium price but decrease the volume. A point to note is that the effect of a change in the speculator’s risk aversion on the equilibrium price cannot be predicted without knowing whether the speculator is long or short.

Shifting focus from risk aversion to ambiguity, in the previous discussions, we noted that the two key determinants of a change in ambiguity parameters on agents’ positions are (i) the nature of the information, whether favorable to the asset or not – that is to say, whether the signal value is higher or lower than the forward price and (ii) the type of change in the ambiguity parameters – whether optimistic or pessimistic – that is to say, whether the change reflects more confidence in the accuracy of the signal or less. This remains true; but more can be said. This current section is about the effect of a change in the ambiguity parameters on the equilibrium volume and forward price, not just on agents’ positions. An important difference to be noted is that unlike previously, the forward price is now endogenous.

Appendix E contains more details and discussion. Here, we will only highlight the main points. It turns out that the effect of a change in the ambiguity parameters on the equilibrium outcome depends crucially on whether the signal, $s$, is below or above a certain critical value, $\kappa$. This critical value is a specific value that is below the prior mean: $\kappa < \mu_p$. We will refer to the signal taking a value in the range (i) below this critical value as “strongly bad news” about the asset and

$$s = \mu_p - \frac{\gamma \sigma^2_p}{4}$$

(80)
(ii) between this critical value and the prior mean as “moderately bad news”. As always, “good news” will refer to a signal value above the prior mean.

Consider the scenario that the news is “good” or only “moderately bad” – that is to say, \( s > \kappa \). Suppose that the producer currently has a short position and the speculator a corresponding long position. The producer may be underhedged, fully hedged or overhedged. Other things constant, a pessimistic change in the producer’s beliefs about signal accuracy would induce her to increase her short position, which would result in an increase in the equilibrium volume and a decrease in the equilibrium forward price. Regarding the speculator, a pessimistic change in his beliefs about signal accuracy would incline him to decrease his long position. Other things constant, this would lead to a decrease in both the equilibrium volume and forward price.

However, if the news is “strongly bad” – that is to say, if \( s < \kappa \) – an increase in the producer’s pessimism regarding signal accuracy would induce her to reduce her position, and, other things constant, lead to a decrease in the equilibrium volume and an increase in the equilibrium forward price. Regarding the speculator, an increase in his pessimism would induce him to increase his long position and, other things constant, lead to an increase in both the equilibrium quantity and forward price.

Observe that the effect on each agent’s optimal position of \( s < \kappa \) (\( s > \kappa \)) is exactly the same as what we previously observed regarding to be the effect of \( s < f \) (\( s > f \)). This is of course no coincidence. Again, Appendix E contains more details. But it is worth noting that the above statement does not mean that the critical value is the forward price. In general, \( f \neq \kappa \). Rather, what is true is the following:

\[
\begin{align*}
  s < \kappa & \quad \text{if and only if} \quad f < \kappa & \quad \text{if and only if} \quad s < f \\
  s = \kappa & \quad \text{if and only if} \quad f = \kappa & \quad \text{if and only if} \quad s = f \\
  s > \kappa & \quad \text{if and only if} \quad f > \kappa & \quad \text{if and only if} \quad s > f
\end{align*}
\]  

(81)

Although not immediately obvious, we arrive at the result that the effect of a change of the ambiguity parameters on the equilibrium volume is entirely consistent with our previous results relating to the effect on agents’ optimal positions.

In summary, there are important differences between the effects of a change in risk aversion as compared to ambiguity. First, an increase in risk aversion always induces an agent to behave more conservatively. However, even though agents are averse to ambiguity, they will not always react to greater ambiguity with increased caution. Second, predicting the effect on equilibrium of an increase in an agent’s risk aversion typically requires knowing the agent’s current position. For example, the producer will respond to increased risk aversion by increasing her short position if she is underhedged and decreasing her short position if she is overhedged. This applies regardless of whether the news was good or bad. However, in the case of a change in ambiguity, whether an agent will go additionally long or short (buy more forwards or sell) does not depend on his or her current position. Also, what matters (at least in the context of our model) is not whether ambiguity (defined as the range of possibilities) has increased or decreased. The key factors that determine an agent’s actions and therefore the equilibrium turn out to be the type of news about the asset - whether strongly bad or otherwise - and type of change in belief about signal accuracy - whether increased optimism or otherwise.
4 CONCLUSION

In the context of a two-agent, single-asset forward market, consisting of a producer (with an underlying long position) and a speculator (with no underlying position), we develop a static model to study the reaction of producers and speculators to ambiguous news relating to the future spot price of an asset. In this setting of ambiguity or Knightian uncertainty, the agents are ambiguity-averse and use the “maxmin” expected utility framework of Gilboa and Schmeidler (1989) to derive their respective optimal hedging/trading positions for an exogenously given forward price. Well known results relating to nonparticipation and portfolio inertia are confirmed.

Further, the paper lays out the conditions under which different types of equilibria – such as speculative, full, partial and no trade equilibria - will obtain. How the presence of a hedging motive modifies a well-known no-trade theorem under ambiguity is illustrated. It is confirmed that equilibria exist in which some amount of trade occurs despite agents possibly holding the same priors – i.e., beliefs regarding the possible accuracy of new information and thus regarding the possible conditional distributions of the future spot price of the asset. However, when both agents share the same beliefs, market equilibrium allows for partial, but not complete risk-transfer. This result holds despite possible differences in risk aversion between the two agents.

Known results about the discontinuity and lack of monotonicity in the relationship between the signal and equilibrium forward price are confirmed. A higher value for the signal does not necessarily imply a higher forward price. At a certain point, an increase in the value of the signal may actually lower the equilibrium forward price. The reason for this is that agents treat favorable news as being of lower accuracy. As the signal rises through a certain critical value, the news may abruptly be reclassified as “good” rather than “bad” and therefore viewed as being less accurate, which would result in a higher risk premium and therefore a lower equilibrium forward price.

An important contribution of the paper is that it throws light on the differences between the effects of changes in risk aversion and ambiguity on agent behavior and the equilibrium outcome. An increase in risk aversion induces more cautious behavior. For example, an increase in the producer’s risk aversion induces her to move closer to a full or one-for-one hedge by increasing her short position if underhedged and decreasing her short position if overhedged. The manner in which agents respond to changes in ambiguity is very different. For one, they may (although not always) respond to an increase in ambiguity by (what would generally be viewed as) adding to their risk. Roughly speaking, their response is based on the change in their beliefs about the future price of the asset viewed in relation to the current forward price. In the context of this study, what matters is not whether there is an increase or decrease in the extent of ambiguity faced by agents – that is to say, in the range of parameter values considered possible by agents. When there is a change in the ambiguity faced by agents, what determines an agent’s response and therefore the effect on the equilibrium are the following: (i) the type of information – whether the signal is “good news” or “moderately bad news” or “strongly bad news” about the future price of the asset; and (ii) the type of change in beliefs - whether the change reflects the belief that the signal may now possibly be more accurate or less accurate than previously thought. An increase in ambiguity may not always be viewed unfavorably by agents.
Appendix A

This appendix contains more details relating to propositions 1, 2, 3 and 4.

First, we state a simple but important result from Convex Analysis to be used in the proof below.

Definition: Given a continuous, concave function, \( f: \mathcal{R} \rightarrow \mathcal{R} \), the subdifferential of \( f \) at a point \( x \in \mathcal{R} \) is defined as follows:

\[
\partial f(x) = \{ y \in \mathcal{R} : f'_+(x) \leq y \leq f'_-(x) \}
\]

Above \( f'_+(x) \) denotes the right derivative of the function at \( x \) and \( f'_-(x) \) the left derivative. Of course, if the function is differentiable at \( x \), then the subdifferential is a singleton consisting of the value of the derivative at \( x \).

Result from Convex Analysis: Given a continuous, concave function, \( f: \mathcal{R} \rightarrow \mathcal{R} \),

\[
\xi \in \operatorname{argmax}_{x \in \mathcal{R}} f(x) \text{ if and only if } 0 \in \partial f(\xi)
\]

Proof: See Bertsekas, Nedic and Ozdagler (2003) or any other standard text in Convex Analysis.

The above result is used in the proof of Propositions 2 and 4.

Proof of Propositions 1 and 3: Refer to Lemma 1 below. Instead of tackling each of Propositions 1 and 3 individually, we will prove a slightly more general statement, Lemma 1, which is stated below. It is easily seen that Proposition 1 as well as Proposition 3 follow as special cases.

Suppose that the agent’s underlying long position is some quantity \( Q \) (not necessarily equal to unity as in Proposition 1). All other parameters and variables are the same as defined previously. In particular, \( x \) is the agent’s short position in the forward contract. Let the profit function of the agent be given by the following:

\[
\pi = p(Q-x) + fx - c
\]  

(A.1)

Lemma 1: Fix the signal \( s \) and the forward price \( f \). Let \((Q-x)\) be given.

The appropriate beta, \( \beta^* \), in order to minimize the certainty equivalent, \( CE \), is determined as follows:

\[
\begin{align*}
\text{If } (Q-x) &\leq \min(0, \bar{\theta}), \text{then } \beta^* = \beta_a \\
\text{If } \min(0, \bar{\theta}) &\leq (Q-x) \leq \max(0, \bar{\theta}), \text{then } \beta^* = \beta_b \\
\text{If } (Q-x) &\geq \max(0, \bar{\theta}), \text{then } \beta^* = \beta_a
\end{align*}
\]  

(A.2)

Proof: \( CE(\beta) = E_{\beta}(\pi|s) - \frac{1}{2} \gamma \text{Var}_{\beta}(\pi|s) \)

\[
= \left( \mu_p + \beta(s - \mu_p) \right)(Q-x) + fx - c - \frac{1}{2} \gamma (Q-x)^2 \sigma_p^2 (1-\beta)
\]

\[
= \text{terms not involving } \beta + \beta(Q-x)\eta(x)
\]
where \( \eta(x) = s - \mu_p + \frac{1}{2}\gamma(Q - x)\sigma^2_p \)

Therefore, minimizing \( CE(\beta) \) is equivalent to minimizing \( \beta(Q - x)\eta(x) \). It is clear that

- If \((Q - x) > 0 \) and \( \eta(x) > 0 \), then \( \beta^\ast = \beta_a \)
- If \((Q - x) > 0 \) and \( \eta(x) < 0 \), then \( \beta^\ast = \beta_b \)
- If \((Q - x) < 0 \) and \( \eta(x) > 0 \), then \( \beta^\ast = \beta_b \)
- If \((Q - x) < 0 \) and \( \eta(x) < 0 \), then \( \beta^\ast = \beta_a \)

Now, if \((Q - x) > 0 \) and \( \eta(x) > 0 \), then \( \beta^\ast = \beta_a \), else \( \beta^\ast = \beta_b \).

This proves Proposition 5. Proposition 1 is of course just a special case of Proposition 5 with the quantity \( Q \) set to unity. Proposition 3 is just a special case of Proposition 5 with \( Q = 0 \) (as the speculator has no underlying position), and the substitution of \( y \) for \(-x\) (so that \( y \), when positive, can be interpreted as the speculator’s long position).

Observe that for a given \( \beta \) the CE function is a quadratic function of the producer’s net position, \((Q - x)\), and that the Minimum Certainty Equivalent (MCE) function is a continuous and concave function of \((Q - x)\). Moreover, assuming that \( s \neq \mu_p \), it is continuously differentiable except at \((Q - x) = 0 \) and \((Q - x) = \hat{\theta} \).

**Proposition 2 (Parts a, b and c):** Let the signal \( s \) and forward price \( f \) be given. (Also, we revert to the assumption that the producer’s underlying long position is unity.)

**Statement of Part (a):** Suppose that \( s > \mu_p \). Depending on the range in which the forward price lies, the producer’s optimal forward position, \( x^\ast \), is given by one of the five following cases:

\[
\begin{align*}
x_a & \equiv 1 - \theta_a & \text{iff } f & \leq \mu_p + \beta_a(s - \mu_p) \\
1 & & \text{iff } \mu_p + \beta_a(s - \mu_p) & \leq f & \leq \mu_p + \beta_b(s - \mu_p) \\
x_b & \equiv 1 - \theta_b & \text{iff } \mu_p + \beta_b(s - \mu_p) & \leq f & \leq \mu_p + (2 - \beta_b)(s - \mu_p) \\
\hat{x} & \equiv 1 - \hat{\theta} & \text{iff } \mu_p + (2 - \beta_b)(s - \mu_p) & \leq f & \leq \mu_p + (2 - \beta_a)(s - \mu_p) \\
x_a & \equiv 1 - \theta_a & \text{iff } f & \geq \mu_p + (2 - \beta_a)(s - \mu_p)
\end{align*}
\]

(A.3)

**Statement of Part (b):** Suppose that \( s < \mu_p \). Depending on the range in which the forward price lies, the producer’s optimal forward position, \( x^\ast \), is given by one of the five following cases:
\( x_a \equiv 1 - \theta_a \quad \text{iff} \quad f \leq \mu_p + (2 - \beta_a)(s - \mu_p) \)
\( \hat{x} \equiv 1 - \hat{\theta} \quad \text{iff} \quad \mu_p + (2 - \beta_a)(s - \mu_p) \leq f \leq \mu_p + (2 - \beta_b)(s - \mu_p) \)
\( x_b \equiv 1 - \theta_b \quad \text{iff} \quad \mu_p + (2 - \beta_b)(s - \mu_p) \leq f \leq \mu_p + \beta_a(s - \mu_p) \)
\( 1 \quad \text{iff} \quad \mu_p + \beta_b(s - \mu_p) \leq f \leq \mu_p + \beta_a(s - \mu_p) \)
\( x_a \equiv 1 - \theta_a \quad \text{iff} \quad f \geq \mu_p + \beta_a(s - \mu_p) \)

**Statement of Part (c):** Suppose that \( s = \mu_p \).

The producer’s optimal position, \( x^* \) will equal \( 1 - \theta_a = 1 - \frac{(\mu_p - f)}{\gamma \sigma_p^2 (1 - \beta_a)} \).

**Proof of Proposition 2:** Recall that the optimal position of the producer, \( x^* \), is that which maximizes the minimum Certainty Equivalent (CE). For any given \( \beta \) (as noted previously), the CE is given by:

\[
CE(1-x; \beta) = E_p(\pi|s) - \frac{1}{2} \gamma \text{Var}_p(\pi|s) = \left( \mu_p + \beta(s - \mu_p) \right)(1-x) + \frac{1}{2}(1-x)\left(\frac{1}{\gamma} \frac{\sigma_p^2}{(1-\beta)} \right).
\]

The above is a quadratic function in \((1-x)\), and is therefore continuous everywhere and globally concave.

From Proposition 1, it is known that for different ranges of \((1-x)\), \( \beta^* \) takes one of two values, \( \beta_a \) or \( \beta_b \). Denote \( CE(1-x; \beta_a) \) by \( CE_a \) and \( CE(1-x; \beta_b) \) by \( CE_b \). It is therefore clear that the minimum CE function, \( MCE \), is

\[
MCE(\cdot) = \text{Min} (CE_a, CE_b).
\]

For convenience, we define \( \theta \equiv (1-x) \) and \( \theta^* \) as the optimal value of \( \theta \). It is easily calculated that \( CE_a \) and \( CE_b \), viewed as functions of \( \theta \), intersect at \( \theta = 0 \) and \( \theta = \hat{\theta} \) (defined in Proposition 1).

**Consider part (a), the case \( s > \mu_p \):** By assumption, \( s > \mu_p \), therefore \( \hat{\theta} < 0 \). Therefore, the first of the two intersections happens at \( \theta = \hat{\theta} \), and the second at \( \theta = 0 \). From Proposition 1, the minimum CE function is given by:

\[
CE_a \quad \text{if} \quad \theta \leq \hat{\theta} \\
CE_b \quad \text{if} \quad \hat{\theta} \leq \theta \leq 0 \\
CE_a \quad \text{if} \quad \theta \geq 0
\]

(A.5)

Further, as each of \( CE(\beta_a) \) and \( CE(\beta_b) \) is individually continuous (in fact, differentiable) and globally concave, the minimum of these two functions is also continuous everywhere and globally concave. Therefore, the Min CE function has a unique maximum. It is also differentiable everywhere except for the “kinks” at the two points of intersection, \( \theta = 0 \) and \( \theta = \hat{\theta} \).
The possible values of \( \theta \) at which the Min CE function can attain a maximum are \( \theta_a, \theta_b \), (both defined in the main body of the paper), 0 and \( \hat{\theta} \). We will now derive the conditions for each of these \( \theta \) values to be optimal. (Note that while the maximum is unique, it is possible that one or more of these \( \theta \) values coincide. For example, it is possible the maximum occurs at \( \theta_a = 0 \).)

Consider \( \theta^* = \theta_a \). This would be an interior maximum and will occur if and only if the MCE function is \( CE_a \) at \( \theta_a \), and the derivative of the MCE function is zero at \( \theta = \theta_a \). In turn (from Proposition 1), these two conditions will hold iff \( \theta_a \leq \hat{\theta} \) or \( \theta_a \geq 0 \). The first of these is equivalent to \( f \geq \mu_p + (2 - \beta_a)(s - \mu_p) \), the fifth case in Part (a) of Proposition 2, and the second of these to \( f \leq \mu_p + \beta_a(s - \mu_p) \), the first case.

Next, consider \( \theta^* = \theta_b \). This would be an interior maximum as well and will occur iff the MCE function is \( CE_b \) at \( \theta_b \), and the derivative of the MCE function is zero at \( \theta = \theta_b \). In turn, these two conditions will hold iff \( \hat{\theta} \leq \theta_b \leq 0 \), which is equivalent to \( \mu_p + \beta_b(s - \mu_p) \leq f \leq \mu_p + (2 - \beta_b)(s - \mu_p) \), the third case in Part (a) of Proposition 2.

Next, \( \theta^* = \hat{\theta} \) iff the subdifferential at \( \hat{\theta} \), \( \partial MCE(\hat{\theta}) \), contains zero. From A.5 above, the left derivative of \( MCE(\cdot) \) at \( \hat{\theta} \) is the derivative of \( CE_a \) at \( \hat{\theta} \), which is easily calculated to be \( \mu_p + (2 - \beta_a)(s - \mu_p) - f \) and the right derivative of \( MCE(\cdot) \) at \( \hat{\theta} \) is the derivative of \( CE_b \) at \( \hat{\theta} \), which is \( \mu_p + (2 - \beta_b)(s - \mu_p) - f \). For the subdifferential to contain zero, the left derivative has to be non-negative and the right non-positive, which is precisely the condition in the fourth case of Part (a) of Proposition 2.

Lastly, \( \theta^* = 0 \) iff the subdifferential at 0, \( \partial MCE(0) \), contains zero. From A.5 above, the left derivative of \( MCE(\cdot) \) at 0 is the derivative of \( CE_b \) at 0, which is easily calculated to be \( \mu_p + \beta_b(s - \mu_p) - f \) and the right derivative of \( MCE(\cdot) \) at 0 is the derivative of \( CE_a \) at 0, which is \( \mu_p + \beta_a(s - \mu_p) - f \). For the subdifferential to contain zero, the left derivative has to be non-negative and the right non-positive, which is precisely the condition in the second case of Part (a) of Proposition 2.

This completes the formal part of the proof. Below is outlined a less formal, but possibly more intuitive proof. Keep in mind the graph of the MCE function as the minimum of the two quadratic functions, \( CE_a \) and \( CE_b \).

Next, note that the slope of \( CE(\beta) \) at \( \theta = 0 \) is

\[
\left( \mu_p + \beta(s - \mu_p) \right) - f.
\]

And the slope at \( \theta = \hat{\theta} \) is

\[
\left( \mu_p + (2 - \beta)(s - \mu_p) \right) - f
\]

The expressions for these slopes are used to state the necessary and sufficient conditions of the proposition.

There are five possibilities for the optimal value of \( \theta \). They are discussed in the same order that they appear in Part (a) of Proposition 2. \( \theta^* \) may be:
(i) an interior optimum at or to the right of 0, which means it would be an interior optimum of \(CE(\beta_a)\), denoted \(\theta_a\). A necessary and sufficient condition for this to be true is that the slope of \(CE(\beta_a)\) must be zero or positive at 0.

(ii) 0, which is the second kink in the minimum CE curve. Necessary and sufficient conditions are that a) the slope of \(CE(\beta_b)\) must be zero or positive at 0 and b) the slope of \(CE(\beta_b)\) must be zero or negative at 0.

(iii) an interior optimum between \(\hat{\theta}\) and 0, which means it would be an interior optimum of \(CE(\beta_b)\), denoted \(\theta_b\). Necessary and sufficient conditions are that a) the slope of \(CE(\beta_b)\) must be non-negative at \(\hat{\theta}\), and b) the slope of \(CE(\beta_b)\) must be non-positive at 0.

(iv) \(\hat{\theta}\), which is the first kink in the minimum CE curve. Necessary and sufficient conditions are that a) the slope of \(CE(\beta_a)\) must be non-negative at \(\hat{\theta}\), and b) the slope of \(CE(\beta_b)\) must be non-positive at \(\hat{\theta}\).

(v) an interior optimum at or to the left of \(\hat{\theta}\), which means it would be an interior optimum of \(CE(\beta_a)\), denoted \(\theta_a\). A necessary and sufficient condition is that the slope of \(CE(\beta_a)\) must be zero or negative at \(\hat{\theta}\).

This completes the proof of part (a).

Consider part (b), the case \(s < \mu_p\): By assumption, \(s < \mu_p\), therefore \(\hat{\theta} > 0\). Therefore, the first of the two intersections happens at \(\theta = 0\), and the second at \(\theta = \hat{\theta}\). From Proposition 1, the minimum CE function is given by:

\[
CE(\beta_a) \quad \text{if } \theta \leq 0 \\
CE(\beta_b) \quad \text{if } 0 \leq \theta \leq \hat{\theta} \\
CE(\beta_a) \quad \text{if } \theta \geq \hat{\theta}
\]  
(A.6)

Further, (as noted previously), each of \(CE(\beta_a)\) and \(CE(\beta_b)\) is individually continuous (in fact, differentiable) and globally concave. Therefore, the minimum of these two functions is also continuous everywhere and globally concave. Therefore, the MCE function has a unique maximum. It is also differentiable everywhere except for the “kinks” at the two points of intersection, \(\theta = 0\) and \(\theta = \hat{\theta}\).

As in the previous case (part 2a), the possible values of \(\theta\) at which the MCE function can attain a maximum are \(\theta_a\), \(\theta_b\), 0 and \(\hat{\theta}\). We will now derive the conditions for each of these \(\theta\) values to be optimal.

Consider \(\theta^* = \theta_a\). This would be an interior maximum and will occur iff the MCE function is \(CE_a\) at \(\theta_a\), and the derivative of the MCE function is zero at \(\theta = \theta_a\). In turn, these two conditions will hold iff \(\theta_a \geq \hat{\theta}\) or \(\theta_a \leq 0\). The first of these is equivalent to \(f \leq \mu_p + (2 - \beta_a)(s - \mu_p)\), the first case in Part (b) of Proposition 2, and the second of these to \(f \geq \mu_p + \beta_a(s - \mu_p)\), the fifth case.

Next, consider \(\theta^* = \theta_b\). This would be an interior maximum as well and will occur iff the MCE function is \(CE_b\) at \(\theta_b\), and the derivative of the MCE function is zero at \(\theta = \theta_b\). In turn, these two
conditions will hold iff \(0 \leq \theta_b \leq \hat{\theta}\), which is equivalent to \(\mu_p + \beta_b(s - \mu_p) \geq f \geq \mu_p + (2 - \beta_b)(s - \mu_p)\), the third case in Part (b) of Proposition 2.

Next, \(\theta^* = \hat{\theta}\) iff the subdifferential at \(\hat{\theta}\), \(\partial MCE(\hat{\theta})\), contains zero. From A.6 above, the left derivative of \(MCE(\cdot)\) at \(\hat{\theta}\) is the derivative of \(CE_b\) at \(\hat{\theta}\), which is easily calculated to be \(\mu_p + (2 - \beta_b)(s - \mu_p) - f\) and the right derivative of \(MCE(\cdot)\) at \(\hat{\theta}\) is the derivative of \(CE_a\) at \(\hat{\theta}\), which is \(\mu_p + (2 - \beta_a)(s - \mu_p) - f\). For the subdifferential to contain zero, the left derivative has to be non-negative and the right derivative non-positive, which is precisely the condition in the second case of Part (b) of Proposition 2.

Lastly, \(\theta^* = 0\) iff the subdifferential at 0, \(\partial MCE(0)\), contains zero. From A.6 above, the left derivative of \(MCE(\cdot)\) at 0 is the derivative of \(CE_a\) at 0, which is easily calculated to be \(\mu_p + \beta_a(s - \mu_p) - f\) and the right derivative of \(MCE(\cdot)\) at 0 is the derivative of \(CE_b\) at 0, which is \(\mu_p + \beta_b(s - \mu_p) - f\). For the subdifferential to contain zero, the left derivative has to be non-negative and the right derivative non-positive, which is precisely the condition in the fourth case of Part (b) of Proposition 2.

This completes the formal part of the proof of part 2b. A less formal, but perhaps more intuitive explanation follows. There are five possibilities for the optimal value of \(\hat{\theta}\). They are discussed in the same order that they appear in Part (b) of Proposition 2. \(\theta^*\) may be:

(i) an interior optimum at or to the right of \(\hat{\theta}\), which means it would be an interior optimum of \(CE(\beta_a)\), denoted \(\theta_a\). A necessary and sufficient condition for this possibility to be true is that the slope of \(CE(\beta_a)\) must be zero or positive at \(\hat{\theta}\).

(ii) \(\hat{\theta}\), which is the second kink in the minimum CE curve. Necessary and sufficient conditions are that a) the slope of \(CE(\beta_b)\) must be zero or positive at \(\hat{\theta}\) and b) the slope of \(CE(\beta_a)\) must be zero or negative at \(\hat{\theta}\).

(iii) an interior optimum between 0 and \(\hat{\theta}\), which means it would be an interior optimum of \(CE(\beta_b)\), denoted \(\theta_b\). Necessary and sufficient conditions are that a) the slope of \(CE(\beta_b)\) must be non-negative at 0, and b) the slope of \(CE(\beta_b)\) must be non-positive at \(\hat{\theta}\).

(iv) 0, which is the first kink in the minimum CE curve. Necessary and sufficient conditions are that a) the slope of \(CE(\beta_b)\) must be non-negative at 0, and b) the slope of \(CE(\beta_b)\) must be non-positive at 0.

(v) an interior optimum at or to the left of 0, which means it would be an interior optimum of \(CE(\beta_a)\), denoted \(\theta_a\). A necessary and sufficient condition is that the slope of \(CE(\beta_a)\) must be zero or negative at 0.

This completes the proof of part (b).

**Consider part (c), the case \(s = \mu_p\):** By assumption \(s = \mu_p\). Therefore, the CE is given by:

\[
CE(1-x;\beta) = E_\beta(\pi|s) - \frac{1}{2} \gamma \text{Var}_\beta(\pi|s) = \left(\mu_p + \beta(s - \mu_p)\right)(1-x) + fx - c - \frac{1}{2} \gamma (1-x)^2 \sigma_p^2 (1-\beta)
\]

\[
= \mu_p(1-x) + fx - c - \frac{1}{2} \gamma (1-x)^2 \sigma_p^2 (1-\beta)
\]

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The posterior mean is not a function of $\beta$, only the variance. So for any value of $x$, the minimum CE is attained when $\beta = \beta_a$. The minimum CE function is

$$Min (CE_a, CE_b) = CE_a.$$  

The optimal value of $x$ can therefore be determined in a straightforward manner using the FOC.

Observe that if $f > \mu_p$, then the net position $(1 - x^*)$ is net long. If $f < \mu_p$, then the net position is net short. If $f = \mu_p$, then the net position is zero indicating a full hedge.

This completes the proof of Proposition 2.

**Proof of Proposition 4 (parts a, b and c):** The proofs of these propositions are very similar to those of Proposition 2 (parts a, b and c) and are therefore omitted.
Appendix B

Equilibrium Conditions: Proposition 2 shows that that the producer’s optimal position can be characterized as \(1 - \theta_a, 1 - \theta_b\) or \(1 - \hat{\theta}\). Similarly, Proposition 4 shows that the speculator’s optimal position can be characterized as \(y_m, 0, y_n\) or \(\hat{y}\). Thus, there are 16 possible equilibrium types or combinations.

The table below contains a list of all the possible equilibrium types and the conditions for each to obtain. Each row of the table below relates to a particular possible equilibrium type. The first two columns of the table below are the optimal positions of the producer and the speculator respectively. The equilibrium forward price is calculated by setting these two equal to each other. The third and fourth columns list the conditions for this particular equilibrium to obtain, with the conditions stated in terms of bounds on the forward price. The conditions in the third column can be looked up from part (a) of Propositions 2 (for the producer) and 4, (for the speculator) and the conditions in the fourth column are from part (b) of Propositions 2 and 4, but included only if they are consistent with one another as well as the equilibrium forward price. If for a particular case the conditions are inconsistent, then that case or type of equilibrium is marked as “Not Possible.” The fifth and sixth columns are equivalent conditions calculated by plugging in the respective equilibrium value (expression) of the forward price so that the conditions are now expressed in terms of the model parameters. Note that conditions regarding permissible values of the parameters (such as \(\gamma, \gamma_s > 0\) and \(0 < \beta_z < 1\) for \(z = a, b, m, n\)) apply throughout.

At the bottom of the table is an example illustrating how these conditions are derived. Also, Appendix C contains a fully worked out example for the special case in which both the agents share the same risk aversion and ambiguity parameters.

For the sake of brevity, here is some notation:

\[
\begin{align*}
\mu_z &\equiv \mu_p + \beta_z(s - \mu_p), \text{for } z = a, b, m, n \\
\gamma_z &\equiv \gamma s^2(1 - \beta_z), \text{for } z = a, b \\
\gamma_z &\equiv \gamma s^2(1 - \beta_z), \text{for } z = m, n
\end{align*}
\]

<table>
<thead>
<tr>
<th><strong>x</strong></th>
<th><strong>y</strong></th>
<th><strong>Bounds on the forward price for the case</strong> (s &gt; \mu_p)</th>
<th><strong>Bounds on the forward price for the case</strong> (s &lt; \mu_p)</th>
<th><strong>Equilibrium conditions for the case</strong> (s &gt; \mu_p)</th>
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</tr>
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<tbody>
<tr>
<td>(1 - \theta_a)</td>
<td>(y_m)</td>
<td>(f \leq \min(\mu_a, \mu_m))</td>
<td>(f \leq \min(\mu_a - \gamma_a \hat{\theta}, \mu_m - y_m \hat{y}))</td>
<td>((\beta_m - \beta_a)(s - \mu_p) \leq y_m)</td>
<td>(-(s - \mu_p)\left[\frac{1 - \beta_m}{1 - \beta_a} + \frac{2\gamma}{\gamma_s}\right] \leq \gamma s^2)</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>$x^*$</th>
<th>$y^*$</th>
<th><strong>Bounds on the forward price for the case $s &gt; \mu_p$</strong></th>
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<tr>
<td>1 − $\bar{\theta}$</td>
<td>$y_m$</td>
<td>$\mu_a - \gamma_a \bar{\theta} \leq f \leq \min(\mu_b - \gamma_b \bar{\theta}, \mu - \gamma_m \bar{y})$</td>
<td>$\mu_b - \gamma_b \bar{\theta} \leq f \leq \min(\mu_m - \gamma_m \bar{y}, \mu_b)$</td>
<td>Not Possible</td>
<td>$-(s - \mu_p) \left[ \frac{1 - \beta_m}{1 - \beta_m} + 1 + \frac{2\gamma_s}{\gamma} \right] \leq \gamma_s \sigma_p^2$</td>
</tr>
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<td>1 − $\bar{\theta}$</td>
<td>$y_m$</td>
<td>$\mu_a \leq f \leq \min(\mu_m, \mu_b - \gamma_b \bar{\theta})$</td>
<td>$\mu_b - \gamma_b \bar{\theta} \leq f \leq \min(\mu_m - \gamma_m \bar{y}, \mu_b)$</td>
<td>Not Possible</td>
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</tr>
<tr>
<td>1</td>
<td>$y_m$</td>
<td>$\mu_a \leq f \leq \min(\mu_b, \mu_m)$</td>
<td>Not Possible</td>
<td>$\beta_m - \beta_a (s - \mu_p) \geq \gamma_m \beta_m - \beta_b (s - \mu_p) \leq \gamma_m$</td>
<td>$-(s - \mu_p) \left[ \frac{1 - \beta_m}{1 - \beta_m} + 1 + \frac{2\gamma_s}{\gamma} \right] \geq \gamma_s \sigma_p^2$</td>
</tr>
<tr>
<td>1 − $\bar{\theta}$</td>
<td>0</td>
<td>$\mu_m \leq f \leq \min(\mu_m, \mu_a)$</td>
<td>Not Possible</td>
<td>$\beta_m - \beta_m (s - \mu_p) \geq \gamma_m$</td>
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Note: The above implies that $\bar{\theta} < 1$
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<td>$1 - \theta_a$</td>
<td>$y_a$</td>
<td>$\mu_n \leq f \leq \min(\mu_a, \mu_n - y_n \bar{\gamma})$</td>
<td>$\max(\mu_n - y_n \bar{\gamma}) \leq f \leq \min(\mu_n, \mu_a - y_a \bar{\theta})$</td>
<td>$(\beta_a - \beta_n)(s - \mu_p) \geq \gamma_a$</td>
<td>$-(s - \mu_p) \left[ \frac{1 - \beta_n}{1 - \beta_a} + 1 + \frac{2\gamma}{y_s} \right] \geq \gamma \sigma_p^2$</td>
</tr>
<tr>
<td>$1 - \theta_a$</td>
<td>$\hat{y}$</td>
<td>$\mu_m - y_m \hat{\gamma} \leq f \leq \min(\mu_n - y_n \bar{\gamma}, \mu_a - y_a \bar{\theta})$</td>
<td>$\mu_m - y_m \hat{\gamma} \leq f \leq \min(\mu_n - y_n \bar{\gamma}, \mu_a - y_a \bar{\theta})$</td>
<td>Not Possible</td>
<td>$-(s - \mu_p) \left[ \frac{1 - \beta_n}{1 - \beta_a} + 1 + \frac{2\gamma}{y_s} \right] \geq \gamma \sigma_p^2$</td>
</tr>
<tr>
<td>$1 - \theta_b$</td>
<td>$y_n$</td>
<td>Not Possible</td>
<td>$\max(\mu_b - y_b \bar{\theta}, \mu_n - y_n \bar{\gamma}) \leq f \leq \min(\mu_b - y_b \bar{\theta}, \mu_n)$</td>
<td>Not Possible</td>
<td>$-(s - \mu_p) \left[ \frac{1 - \beta_n}{1 - \beta_a} + 1 + \frac{2\gamma}{y_s} \right] \geq \gamma \sigma_p^2$</td>
</tr>
<tr>
<td>$1 - \theta_b$</td>
<td>$\hat{y}$</td>
<td>Not Possible</td>
<td>Not Possible</td>
<td>Not Possible</td>
<td>$-(s - \mu_p) \left[ \frac{1 - \beta_n}{1 - \beta_a} + 1 + \frac{2\gamma}{y_s} \right] \geq \gamma \sigma_p^2$</td>
</tr>
<tr>
<td>$1$</td>
<td>$y_n$</td>
<td>Not Possible</td>
<td>Not Possible</td>
<td>Not Possible</td>
<td>$-(s - \mu_p) \left[ \frac{1 - \beta_n}{1 - \beta_a} + 1 + \frac{2\gamma}{y_s} \right] \geq \gamma \sigma_p^2$</td>
</tr>
<tr>
<td>$1 - \theta_a$</td>
<td>$\hat{y}$</td>
<td>Not Possible</td>
<td>Not Possible</td>
<td>Not Possible</td>
<td>$-(s - \mu_p) \left[ \frac{1 - \beta_n}{1 - \beta_a} + 1 + \frac{2\gamma}{y_s} \right] \geq \gamma \sigma_p^2$</td>
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</table>
Below is an example to illustrate how these conditions have been derived:

For future reference, note that $\mu_p + (2 - \beta_a)(s - \mu_p) = \mu_a - \gamma_a \hat{\theta}$, and that $\mu_p + (2 - \beta_m)(s - \mu_p) = \mu_m - \gamma_m \hat{y}$.

Consider the first possible equilibrium: $1 - \theta_a = y_m$, and the case, $s > \mu_p$.

From part (a) of Proposition 2, for $1 - \theta_a$ to be optimal for the producer, $f < \mu_a$ or $f > \mu_a - \gamma_a \hat{\theta}$. From part (a) of Proposition 4, for $y_m$ to be optimal for the speculator, $f < \mu_m$ or $f > \mu_m - \gamma_m \hat{y}$. Therefore, there are apparently four possible pairs of these conditions. The question is which of these, if any, is possible (i.e., can constitute an equilibrium). Note that in order for a certain forward price, $f$, to be an equilibrium, the optimal positions of the producer and the speculator have to be of the same magnitude and opposite signs.

Now $1 - \theta_a = 1 - \frac{\mu_p + \beta_a(s - \mu_p)}{\gamma \sigma_p^2 (1 - \beta_a)}$, and $y_m = \frac{\mu_p + \beta_m(s - \mu_p)}{\gamma \sigma_p^2 (1 - \beta_m)}$. Setting the two expressions equal to each other and solving for $f$, we get...
\[
    f = \frac{ya \mu_m + y_m (\mu_a - ya)}{ya + y_m} = \frac{ym \mu_a + ya (\mu_m - y_m)}{ya + y_m}
\]

The above expression for \( f \) implies that \( f \) has to be between \( \mu_m \) and \( \mu_a - y_a \) and also between \( \mu_a \) and \( \mu_m - y_m \). Or to put it more precisely,

\[
    \mu_m < f < \mu_a - y_a \text{ OR } \mu_m > f > \mu_a - y_a \\
    \text{AND} \\
    \mu_a < f < \mu_m - y_m \text{ OR } \mu_a > f > \mu_m - y_m
\]

(B.1)

Now, consider the pair, \( f < \mu_a \) and \( f < \mu_m \). This pair is clearly a possibility as the bounds on \( f \) are not necessarily inconsistent with the equilibrium value (the above expression) of \( f \) (as is obvious from B.1)

However, it is easily seen that none of the other pairs is feasible (that is to say, they each fail the consistency test).

Consider the pair, \( f < \mu_a \) and \( f > \mu_m - y_m \hat{y} \). This is impossible because, in general, if \( s > \mu_p \), \( \mu_a < \mu_m - y_m \hat{y} \).

Next, consider the pair, \( f > \mu_a - y_a \hat{\theta} \) and \( f < \mu_m \). This is impossible because, in general, if \( s > \mu_p \), \( \mu_m < \mu_a - y_a \hat{\theta} \).

Last, consider the pair, \( f > \mu_a - y_a \hat{\theta} \) and \( f > \mu_m - y_m \hat{y} \). Note that as \( s > \mu_p \), both \( \hat{\theta} \) and \( \hat{y} \) are negative. Therefore, \( \mu_m - y_m \hat{y} > \mu_m > \mu_m - y_m \), and \( \mu_a - y_a \hat{\theta} > \mu_a > \mu_a - y_a \). However, these conditions are incompatible with B.1 above as (for instance) they would require that \( f \) should be greater than both \( \mu_m \) and \( \mu_a - y_a \). Therefore, this pair too is ruled out.

Thus, for the case \( s > \mu_p \), the only feasible pair of bounds (i.e., consistent with the equilibrium value of \( f \)) is the first one - that \( f \) should be less than both \( \mu_a \) and \( \mu_m \), which is the statement in the third column for this particular equilibrium \( 1 - \theta_a = y_m \).

It is now straightforward to derive the corresponding expressions in the fifth column. First, consider the condition that \( f < \mu_a \). Substituting the equilibrium value for \( f \), we get \( (\beta_m - \beta_a)(s - \mu_p) < y_m \). Next, substituting the equilibrium value for \( f \) into \( f < \mu_m \) yields \( (\beta_a - \beta_m)(s - \mu_p) < y_a \). An intuitive interpretation of these conditions is that, if the signal is good news, then in order for this equilibrium to obtain, the difference between \( \beta_a \) and \( \beta_m \) should not be “too large” – that is to say, the producer and the speculator should not disagree too widely on the worst case estimate of signal accuracy.
Appendix C

This appendix derives the equilibrium outcome for the special case in which the two agents hold the same set of beliefs (that is to say, $\beta_a = \beta_m$ and $\beta_b = \beta_n$) and also have the same coefficient of risk aversion ($\gamma = \gamma_s$).

It’s worth noting that in this special case, there are only four equilibrium combinations to consider, unlike sixteen in the general case. (Actually, there are only three to be considered as the equilibrium in which $x^* = 1$ and $y^* = 0$ is impossible.)

Let us check whether $1 - \theta_a = y_m$ is a possible equilibrium and if so, under what conditions.

Now, $1 - \theta_a = 1 - \frac{\mu_p + \beta_a (s - \mu_p) - f}{\gamma \sigma_p^2 (1 - \beta_a)}$, and $y_m = \frac{\mu_p + \beta_m (s - \mu_p) - f}{\gamma \sigma_p^2 (1 - \beta_m)} = \frac{\mu_p + \beta_a (s - \mu_p) - f}{\gamma \sigma_p^2 (1 - \beta_a)}$. Setting the two expressions equal to each other and solving for $f$, we get

$$f = \mu_p + \beta_a (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_a)}{2}$$ (C.1)

This is the value $f$ will take if this equilibrium were to obtain.

First, take the case, $s > \mu_p$. From part (a) of Propositions 2 and 4, and keeping in mind that $\beta_a = \beta_m$ and $\beta_b = \beta_n$, for this equilibrium to obtain,

$$either \ f < \mu_p + \beta_a (s - \mu_p) \ or \ f > \mu_p + (2 - \beta_a) (s - \mu_p)$$

Since our calculated equilibrium value for $f$ satisfies the first of the two alternative conditions above, we have verified that $1 - \theta_a = y_m$ is a possible equilibrium in case of good news $(s > \mu_p)$. Plugging in the equilibrium value of $f$ in either $1 - \theta_a$ or $y_m$, we can calculate the equilibrium volume to be $\frac{1}{2}$. It will shortly be apparent that this is the only possible equilibrium in case of good news.

Next, consider the case, $s < \mu_p$. From part (b) of Propositions 2 and 4, for this equilibrium to obtain

$$f > \mu_p + \beta_a (s - \mu_p) \ or \ f < \mu_p + (2 - \beta_a) (s - \mu_p)$$

Clearly, the equilibrium value of $f$ is incompatible with $f > \mu_p + \beta_a (s - \mu_p)$ and $f < \mu_p + (2 - \beta_a) (s - \mu_p)$ is the only possibility. The question is, under what conditions the equilibrium value of $f$ is consistent with the latter condition.

$$f = \mu_p + \beta_a (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_a)}{2} < \mu_p + (2 - \beta_a) (s - \mu_p)$$

$$=> s - \mu_p > -\frac{\gamma \sigma_p^2}{4}$$

(C.2)

Next consider the equilibrium, $1 - \theta_b = y_n$. 

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\[ f = \mu_p + \beta_b (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_b)}{2} \]  

Supposing that \( s > \mu_p \), this equilibrium requires that

\[ \mu_p + \beta_b (s - \mu_p) < f < \mu_p + (2 - \beta_b) (s - \mu_p) \]  

(C.3)

Clearly, the equilibrium value of \( f \) is not compatible with this condition, and this equilibrium is impossible if \( s > \mu_p \).

Supposing that \( s < \mu_p \), this equilibrium requires that

\[ \mu_p + (2 - \beta_b) (s - \mu_p) < f < \mu_p + \beta_b (s - \mu_p) \]  

(C.4)

In order for \( f \) to satisfy this condition,

\[ \mu_p + (2 - \beta_b) (s - \mu_p) < f = \mu_p + \beta_b (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_b)}{2} \]  

\( \Rightarrow s - \mu_p < -\frac{\gamma \sigma_p^2}{4} \)  

(C.5)

Lastly, consider the equilibrium \( 1 - \theta = \hat{y} \). This implies that

\[ s - \mu_p = -\frac{\gamma \sigma_p^2}{4} \]  

(C.6)

Clearly, this equilibrium is even a possibility only if \( s < \mu_p \). At this point, it is now clear that \( 1 - \theta_a = y_m \) is the only possible equilibrium if \( s > \mu_p \).

Now, from part (b) of Propositions 2 and 4, the condition for this equilibrium to obtain is given by:

\[ \mu_p + (2 - \beta_a) (s - \mu_p) < f < \mu_p + (2 - \beta_b) (s - \mu_p) \]  

(C.7)

Substituting for \( s - \mu_p \), we see that this condition can be restated as follows:

\[ \mu_p + \beta_a (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_a)}{2} < f < \mu_p + \beta_b (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_b)}{2} \]  

(C.8)

Thus, for this special case, the equilibrium forward price is determined as follows:

\[ f = \mu_p + \beta_b (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_b)}{2} \]  

\[ f = \mu_p + \beta_a (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_a)}{2} \]  

\[ \mu_p + \beta_a (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_a)}{2} < f < \mu_p + \beta_b (s - \mu_p) - \frac{\gamma \sigma_p^2 (1 - \beta_b)}{2} \]  

If \( s - \mu_p < -\frac{\gamma \sigma_p^2}{4} \)

(C.9)

\[ \text{iff } s - \mu_p < -\frac{\gamma \sigma_p^2}{4} \]  

It is easily calculated that for this special case, the equilibrium volume is always \( \frac{1}{2} \). Thus, if both the producer and the speculator face the same degree of ambiguity and share identical beliefs
about the future, then only a partial risk-transfer is possible. Of course, the fact that the equilibrium volume is constant regardless of the signal value is specific to this special case in which both the agents have the same ambiguity parameters. Observe that even in this case, the forward price depends on the signal value, and is in general unique. However, in the knife-edge situation in which $s = \mu_p - \gamma \sigma_p^2 / 4$, it is seen that the forward price can take any of a range of values. In general, such will be the case if the signal takes on the specific value such that the equilibrium occurs at a point that is at the kink of the minimum CE curves of both the agents – in other words, if the equilibrium is of the type $1 - \hat{\theta} = \hat{y}$. For convenience, we will refer to this equilibrium as the “kink” equilibrium.

The forward price above can be interpreted as the appropriate expected value of the future spot price of the asset adjusted for a risk premium. Observe that when the signal value is low (the first of the three cases in C.10), the signal is treated as a “high accuracy” signal which is reflected in the use of $\beta_b$ in the calculation of both the expected value and the risk premium in the expression for the equilibrium forward price. However, once the signal crosses a certain critical value (the second of the three cases above), it is abruptly reclassified as a “low accuracy” signal by the agents which is reflected in the use of $\beta_a$ in the determination of the equilibrium forward price. As a result, the relationship between the signal and the equilibrium forward price is not perfectly monotone. In general, higher the signal value, higher the forward price – except if the signal takes on the “kink equilibrium” value. At this point, there is a downward jump – a point of discontinuity – of the forward price. As also discussed in the main body of the paper, this is the point at which the agents start treating the signal as of low accuracy (or high variance), which increases the required risk premium thereby resulting in a lower equilibrium forward price.
Appendix D

In this appendix, we work through an example to illustrate the effects of a change in the risk parameters on the equilibrium outcome. It can be shown that regardless of the equilibrium, the effect of an increase in the producer’s risk aversion parameter, \( \gamma \), is to bring the equilibrium position closer to a full hedge (i.e., equilibrium volume of unity) and the effect of an increase in the speculator’s risk aversion parameter, \( \gamma_s \), is to bring the equilibrium volume closer to zero.

Example 1:

To illustrate this point, consider the equilibrium \( 1 - \theta_a = y_m \). Using the notation introduced in Appendix B, the equilibrium outcomes, \( f \) (the forward price) and \( q \) (the equilibrium volume) are

\[
f = \frac{y_a \mu_m + y_m (\mu_a - y_a)}{y_a + y_m}
\]

\[
q = \frac{\mu_m - f}{y_m} = \frac{\mu_m - \mu_a + y_a}{y_a + y_m} = \frac{(\beta_m - \beta_a)(s - \mu_p) + y_a}{y_a + y_m}
\]

(D.1)

From the table in Appendix B, one of the conditions for this equilibrium to obtain, regardless of whether \( s > \mu_p \) or \( s < \mu_p \), is that

\[
(\beta_m - \beta_a)(s - \mu_p) < y_m
\]

(D.2)

Further, as can be seen from the entries in columns 3 and 4, one of the conditions on \( f \) in this equilibrium is that \( f < \mu_m \).

Therefore, in this equilibrium, the volume is between zero and unity.

\[
0 < q < 1
\]

(D.3)

Now, consider the partial derivative of the equilibrium quantity with respect to the producer’s risk aversion parameter:

\[
\frac{\partial q}{\partial \gamma} = \frac{\sigma_p^2 (1 - \beta_a) \{\gamma_m - (\beta_m - \beta_a)(s - \mu_p)\}}{(y_a + y_m)^2}
\]

(D.4)

Given the equilibrium condition in (E.2), it is clear that the above partial derivative is always positive, which implies that an increase in the producer’s risk aversion parameter will move the equilibrium volume closer to unity. Coming to the partial derivative of the equilibrium forward price,

\[
\frac{\partial f}{\partial \gamma} = -y_m \frac{\partial q}{\partial \gamma}
\]

(D.5)

The intuition is straightforward. If the producer is currently only partially hedged, then an increase in the producer’s risk aversion will induce her to hedge more, and as a result, the equilibrium volume will increase and the equilibrium forward price will decrease.
Next, consider the partial derivative of the equilibrium quantity with respect to the speculator’s risk aversion parameter:

\[
\frac{\partial q}{\partial \gamma_s} = -\frac{\sigma_p^2(1 - \beta_m)(\gamma_a - (\beta_a - \beta_m)(s - \mu_p))}{(\gamma_a + \gamma_m)^2}
\]  

(D.6)

From the table in Appendix B, one of the conditions for this equilibrium to obtain, regardless of whether \(s > \mu_p\) or \(s < \mu_p\), is that

\[(\beta_a - \beta_m)(s - \mu_p) < \gamma_a\]  

(D.7)

It is therefore clear that the above partial derivative is always negative, which implies that an increase in the speculator’s risk aversion parameter will move the equilibrium volume closer to zero. Coming to the partial derivative of the equilibrium forward price,

\[
\frac{\partial f}{\partial \gamma_s} = \gamma_a \frac{\partial q}{\partial \gamma_s}
\]  

(D.8)

The intuition is straightforward. An increase in the speculator’s risk aversion will induce him to reduce his position, and as a result, the equilibrium volume will decrease and the equilibrium forward price will decrease.

Example 2:

Next, let’s examine a case in which the equilibrium volume is initially greater than unity. Consider the equilibrium \(1 - \theta_b = \gamma_m\), with \(s > \mu_p\). As discussed in the main body of the paper (and confirmed below), this is an “over-hedging” equilibrium. Using the notation introduced in Appendix B, the equilibrium outcomes, \(f\) (the forward price) and \(q\) (the equilibrium volume) are

\[
f = \frac{\gamma_b \mu_m + \gamma_m(\mu_b - \gamma_b)}{\gamma_b + \gamma_m}
\]  

(D.9)

\[
q = \frac{\mu_m - f}{\gamma_m} = \frac{\mu_m - \mu_b + \gamma_b}{\gamma_b + \gamma_m} = \frac{(\beta_m - \beta_b)(s - \mu_p) + \gamma_b}{\gamma_b + \gamma_m}
\]

From the table in Appendix B, one of the conditions for this equilibrium to obtain (when \(s > \mu_p\)) is that \(\mu_b < f < \mu_m\). With minimal algebra, it is seen that \(f > \mu_b\) implies that

\[(\beta_m - \beta_b)(s - \mu_p) > \gamma_m\]  

(D.10)

Therefore, in this equilibrium, the volume is greater than unity, confirming that this is an equilibrium in which the producer is over-hedged.

\[q > 1\]  

(D.11)

Now, consider the partial derivative of the equilibrium quantity with respect to the producer’s risk aversion parameter:

\[
\frac{\partial q}{\partial \gamma} = \frac{\sigma_p^2(1 - \beta_a)(\gamma_m - (\beta_m - \beta_b)(s - \mu_p))}{(\gamma_b + \gamma_m)^2}
\]  

(D.12)
Given the equilibrium condition above, it is clear that the above partial derivative is negative, which implies that an increase in the producer’s risk aversion parameter will decrease the equilibrium volume, thereby moving it closer to unity. Coming to the partial derivative of the equilibrium forward price,

\[ \frac{\partial f}{\partial y} = -\gamma_m \frac{\partial q}{\partial y} \]  
(D.13)

The intuition is straightforward. If the producer is currently over-hedged, an increase in the producer’s risk aversion will induce her to decrease her forward position (thus, shifting the supply curve of the forward contract to the left), and as a result, the equilibrium volume will decrease and the equilibrium forward price will increase.

Next, consider the partial derivative of the equilibrium quantity with respect to the speculator’s risk aversion parameter:

\[ \frac{\partial q}{\partial \gamma_s} = -\sigma_p^2 (1 - \beta_m) \{ \gamma_b - (\beta_b - \beta_m)(s - \mu_p) \} \]  
(D.14)

From the table in Appendix B, one of the conditions for this equilibrium to obtain (if \( s > \mu_p \)) is that

\[ (\beta_b - \beta_m)(s - \mu_p) < \gamma_b \]  
(D.15)

Therefore, the above partial derivative is negative. Coming to the partial derivative of the equilibrium forward price,

\[ \frac{\partial f}{\partial \gamma_s} = \gamma_a \frac{\partial q}{\partial \gamma_s} \]  
(D.16)

The intuition is straightforward. An increase in the speculator’s risk aversion will induce him to reduce his position (reflecting a leftward shift in the demand curve), and as a result, both the equilibrium volume and the equilibrium forward price will decrease.

**Example 3:**

Next, let’s examine a case in which the equilibrium volume is negative – i.e., \( x^* = y^* < 0 \). Consider the equilibrium \( 1 - \theta_b = \gamma_m \), with \( s < \mu_p \) and \( \max(\mu_m, \mu_b - \gamma_b \beta) < f < \mu_b \). As discussed in the main body of the paper (and confirmed below), this is a “purely speculative” equilibrium - that is to say, an equilibrium in which the producer has a long position in the forward contract and the speculator a matching short position. Using the notation introduced in Appendix B, the equilibrium outcomes, \( f \) (the forward price) and \( q \) (the equilibrium volume) are

\[ f = \frac{\gamma_b \mu_m + \gamma_m (\mu_b - \gamma_b)}{\gamma_b + \gamma_m} \]  
(D.17)

\[ q = \frac{\mu_m - f}{\gamma_m} = \frac{\mu_m - \mu_b + \gamma_b}{\gamma_b + \gamma_m} = \frac{(\beta_m - \beta_b)(s - \mu_p) + \gamma_b}{\gamma_b + \gamma_m} \]
With minimal algebra, it is seen that $f > \mu_m$ implies that
\[(\beta_b - \beta_m)(s - \mu_p) > \gamma_b\] (D.18)
This confirms that in this equilibrium, the volume is negative.
\[q < 0\] (D.19)
Also, the condition that $f < \mu_b$ implies that
\[(\beta_m - \beta_b)(s - \mu_p) < \gamma_m\] (D.20)
Now, consider the partial derivative of the equilibrium quantity with respect to the producer’s risk aversion parameter:
\[
\frac{\partial q}{\partial \gamma} = \frac{\sigma_p^2 (1 - \beta_a) \{\gamma_m - (\beta_m - \beta_b)(s - \mu_p)\}}{(\gamma_b + \gamma_m)^2}
\] (D.21)
Given the equilibrium condition above, it is clear that the above partial derivative is positive, which implies that an increase in the producer’s risk aversion parameter will increase the equilibrium volume. As the volume is initially negative, an increase implies that it now becomes a smaller negative number reflecting a smaller long position. Coming to the partial derivative of the equilibrium forward price,
\[
\frac{\partial f}{\partial \gamma} = -\gamma_m \frac{\partial q}{\partial \gamma}
\] (D.22)
It is apparent that this partial derivative is negative. The intuition is straightforward. If the producer is currently holding a long position in the forward contract, an increase in her risk aversion will induce her to decrease this long position, and as a result, the equilibrium volume will decrease in absolute terms and the equilibrium forward price will decrease. Thus, this example illustrates that an increase in the producer’s risk aversion has the effect of moving her position closer to unity (or a full hedge).

Next, consider the partial derivative of the equilibrium quantity with respect to the speculator’s risk aversion parameter:
\[
\frac{\partial q}{\partial \gamma_s} = -\frac{\sigma_p^2 (1 - \beta_m) \{\gamma_b - (\beta_b - \beta_m)(s - \mu_p)\}}{(\gamma_b + \gamma_m)^2}
\] (D.23)
From the equilibrium condition in (E.18), the above partial derivative is positive. The partial derivative of the equilibrium forward price is also positive as it is given by
\[
\frac{\partial f}{\partial \gamma_s} = \gamma_a \frac{\partial q}{\partial \gamma_s}
\] (D.24)
The intuition is straightforward. Recall that the speculator’s initial position is a short position in the forward contract and therefore a negative position. An increase in the speculator’s risk aversion will induce him to reduce his short position. Consequently, the equilibrium volume will decrease in absolute terms and the equilibrium forward price will increase. Thus, this example illustrates that an increase in the speculator’s risk aversion has the effect of moving his position closer to zero.
Appendix E

In this appendix, we work through an example to illustrate the effects of a change in the ambiguity parameters on the equilibrium outcome.

Consider the equilibrium \( 1 - \theta_b = \gamma_n \). As is apparent from the table in Appendix B, this equilibrium can obtain only if \( s < \mu_p \). Using the notation introduced in Appendix B, the equilibrium outcomes, \( f \) (the equilibrium forward price) and \( q \) (the equilibrium volume) are

\[
f = \frac{\gamma_b \mu_n + \gamma_n \mu_b - \gamma_b \gamma_n}{\gamma_b + \gamma_n} \tag{E.1}
\]

\[
q = \frac{\mu_n - \mu_b + \gamma_b}{\gamma_b + \gamma_n}
\]

The partial derivative of the equilibrium forward price with respect to the producer’s relevant ambiguity parameter is

\[
\frac{\partial f}{\partial \beta_b} = -\gamma_n \frac{\partial q}{\partial \beta_b} \tag{E.2}
\]

The sign of \( \partial f/\partial \beta_b \) will just be opposite to that of \( \partial q/\partial \beta_b \).

The partial derivative of the equilibrium quantity with respect to the producer’s relevant ambiguity parameter is shown below:

\[
\frac{\partial q}{\partial \beta_b} = \frac{-(s - \mu_p)\gamma_b (1 - \beta_n)(1 + \gamma_s/\gamma) - \gamma_b \gamma_n}{(1 - \beta_b)(\gamma_b + \gamma_n)^2} \tag{E.3}
\]

Whether the partial derivative above is positive or negative depends on whether the signal takes a value less than or greater than a certain critical value:

\[
\frac{\partial q}{\partial \beta_b} > 0 \text{ if and only if } s < \mu_p - \frac{\gamma \gamma_s \sigma_p^2}{\gamma + \gamma_s} \tag{E.4}
\]

\[
\frac{\partial q}{\partial \beta_b} < 0 \text{ if and only if } s > \mu_p - \frac{\gamma \gamma_s \sigma_p^2}{\gamma + \gamma_s} \tag{E.5}
\]

For convenience, let us denote this critical value as \( \kappa \):

\[
\kappa \equiv \mu_p - \frac{\gamma \gamma_s \sigma_p^2}{\gamma + \gamma_s} \tag{E.6}
\]
It is apparent that depending on the value of the signal, \( s \), the partial derivative could be positive or negative. That is to say, neither possibility can be ruled out. Note in this connection that neither possibility is inconsistent with the relevant equilibrium condition for this equilibrium to obtain (which can be looked up from Appendix B):

\[
-(s - \mu_p) \left[ \frac{1 - \beta_b}{1 - \beta_n} + 1 + \frac{2\gamma}{\gamma} \right] > \gamma_s \sigma_p^2
\]  
(E.7)

Keep in mind that \( \kappa < \mu \). For convenience, let us refer to

\[
\begin{align*}
    s < \kappa & \quad \text{as "strongly bad news"} \\
    \kappa \leq s \leq \mu_p & \quad \text{as "moderately bad news"} \\
    s > \mu_p & \quad \text{as "good news"}
\end{align*}
\]  
(E.8)

If \( \partial q / \partial \beta_b \) is positive (negative), that would mean that an optimistic change in the producer’s ambiguity beliefs (i.e., an increase in \( \beta_b \), indicating an increase in the producer’s confidence in signal accuracy) would result in an increase (decrease) in the equilibrium quantity.

As noted earlier, this particular equilibrium type \( (1 - \theta_b = \gamma_n) \), can obtain only if \( s < \mu_p \). Therefore, the only two cases to consider are “strongly bad” and “moderately bad” news.

If the news is “strongly bad,” an optimistic change in the producer’s beliefs about the signal’s accuracy induces the producer to take the news more seriously than previously, and therefore increase her short position (or reduce her long position). If the prevailing equilibrium is one in which the producer has a short forward position and the speculator a correspondingly long position, this change in the producer’s beliefs would result in an increase in the equilibrium volume and a decrease in the equilibrium forward price. On the other hand, if the news is only “moderately bad,” the producer effectively treats it the same as good news. An optimistic change in the producer’s beliefs about the signal’s accuracy induces the producer to again take the news more seriously than previously, but respond by decreasing her short position (or increasing her long position). Assuming that the producer has a short position, this change in her beliefs would results in a decrease in the equilibrium volume and an increase in the equilibrium forward price.

An increase in \( \beta_b \) can be viewed as an increase in the ambiguity (range of possibilities for the variance of the signal) faced by the producer. The above example thus illustrates that an increase in the ambiguity faced by the producer can result in a higher or lower equilibrium volume and correspondingly, a lower or higher equilibrium forward price. This is in contrast to the result in LW where an increase in the ambiguity faced by the producer always results in an increase in the equilibrium volume and a decrease in the forward price.

Coming to the speculator, the partial derivative of the equilibrium quantity with respect to the speculator’s relevant ambiguity parameter is shown below:
\[
\frac{\partial q}{\partial \beta_n} = \frac{(s - \mu_p)\gamma_n(1 - \beta_b)(1 + \gamma/\gamma_s) + \gamma_b\gamma_n}{(1 - \beta_n)(\gamma_b + \gamma_n)^2}
\] (E.9)

Whether the partial derivative above is positive or negative depends on whether the signal takes a value greater than or less than the same critical value \( \kappa \):

\[
\frac{\partial q}{\partial \beta_n} < 0 \text{ if and only if } s < \mu_p - \frac{\gamma\gamma_s\sigma_p^2}{\gamma + \gamma_s} \tag{E.10}
\]

\[
\frac{\partial q}{\partial \beta_n} > 0 \text{ if and only if } s > \mu_p - \frac{\gamma\gamma_s\sigma_p^2}{\gamma + \gamma_s} \tag{E.11}
\]

Depending on the value of the signal, \( s \), the partial derivative could be positive or negative. That is to say, neither possibility can be ruled out. It can be verified that neither possibility is inconsistent with the relevant equilibrium condition for this equilibrium to obtain (which can be looked up from Appendix B):

\[-(s - \mu_p)\left[ \frac{1 - \beta_n}{1 - \beta_b} + \frac{2\gamma}{\gamma_s} \right] > \gamma\sigma_p^2 \tag{E.12}\]

The partial derivative of the equilibrium forward price with respect to the producer’s relevant ambiguity parameter is shown below:

\[
\frac{\partial f}{\partial \beta_n} = \gamma_b \frac{\partial q}{\partial \beta_n} \tag{E.13}
\]

It is clear that the sign of \( \partial f / \partial \beta_n \) will just be the same that of \( \partial q / \partial \beta_n \).

If \( \partial q / \partial \beta_n \) is positive (negative), that means that an optimistic change in the speculator’s ambiguity parameter (i.e., an increase in \( \beta_n \), indicating an increase in the speculator’s confidence in signal accuracy) would result in an increase (decrease) in the equilibrium quantity.

Now, \( \partial q / \partial \beta_n \) would be negative if \( (s - \mu_p) \) is “large enough” in absolute terms (which means that the news is “strongly bad”) and positive if \( (s - \mu_p) \) is “small” in absolute terms (which means that the news is only “moderately bad”). Thus, if the news is “strongly bad,” then an optimistic change in the speculator’s beliefs about signal accuracy induces the speculator to take the news more seriously than previously, and therefore decrease his long position (or increase his short position). Assuming that he is currently long, this change in his beliefs would result in a decrease in the equilibrium volume and a decrease in the equilibrium forward price. On the other hand, if the news is only “moderately bad,” then the speculator effectively treats it the same as good news. An optimistic change in his beliefs about signal accuracy induces him to again take the news more seriously than previously, but respond by increasing his long position (or
decreasing his short position). Assuming he is currently long, this change in his beliefs would result in an increase in the equilibrium volume and an increase in the equilibrium forward price.

An increase in $\beta_n$ can be viewed as an increase in the ambiguity faced by the speculator. The above example thus illustrates that an increase in the ambiguity faced by the speculator can result in a higher or lower equilibrium volume and correspondingly, a higher or lower equilibrium forward price. This is in contrast to the result in LW where an increase in the ambiguity faced by the speculator always results in a decrease in the equilibrium volume and price.

The discussion above begs the question why agents would treat “moderately bad news” the same as “good news.”

Interestingly, it can be shown (after some algebra) that

\[
\begin{align*}
  s < \kappa & \text{ if and only if } f < \kappa & s < f \\
  s = \kappa & \text{ if and only if } f = \kappa & s = f \\
  s > \kappa & \text{ if and only if } f > \kappa & s > f
\end{align*}
\]

Thus,

\[
\frac{\partial q}{\partial \beta_b} > 0, \quad \frac{\partial q}{\partial \beta_n} < 0 \quad \text{if and only if} \quad s < f \tag{E.15}
\]

\[
\frac{\partial q}{\partial \beta_b} < 0, \quad \frac{\partial q}{\partial \beta_n} > 0 \quad \text{if and only if} \quad s > f \tag{E.16}
\]

Thus, the results here regarding the impact of a change in the relevant ambiguity parameter on the equilibrium volume and price are fully consistent with the previously discussed results relating to the effect on the individual optimal positions of the producer and the speculator.

Further, these last equivalencies shed light on why the agents treat “moderately bad news” in the same manner as “good news.” The reason is that both in the case of “moderately bad news” and “good news,” the equilibrium forward price turns out to be less than the signal value, and agents respond to a change in ambiguity based on the change in their beliefs about the future spot price viewed in relation to the current forward price.

This appendix contains details only relating to one particular equilibrium type, namely, $1 - \theta_b = \gamma_n$. However, these results apply to other equilibrium types as well.
References:


Illeditsch Philipp K., 2009, Ambiguous information, risk aversion and asset pricing, Unpublished manuscript, Wharton.


Knight, F., 1921, Risk, uncertainty and profit, Boston: Houghton Mifflin.


