Hedging with Regret †

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JEL Classification: G30, D81

Keywords: risk management; hedging; derivatives; regret aversion

†We thank Tim Adam, Christian Gollier, and Bruno Solnik for helpful comments and suggestions and Rene Johannes Minden for capable research assistance. Financial support from the National Centre of Competence in Research “Financial Valuation and Risk Management” (NCCR FINRISK), Project 3, “Evolution and Foundations of Financial Markets”, and from the University Research Priority Program “Finance and Financial Markets” of the University of Zürich is gratefully acknowledged.

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Hedging with Regret

Abstract

This paper investigates corporate hedging under regret aversion. Regret-averse firms try to avoid deviations of their hedging policy from the ex post best policy, an intuitive consideration if one has to justify one’s decisions afterward. The study presents a model of a firm that faces uncertain prices and seeks to hedge both profit risk and regret risk with derivatives. It characterizes optimal hedge positions and shows that regret aversion leads to stronger incentives to hedge downside price risk than standard expected utility theory. In the profit region of the price distribution, however, regret aversion reduces the hedging of price risk to avoid large regret in the case of increasing prices. The results show that regret aversion has a strong effect on the choice of the hedging instrument and provides a preference-based explanation for the use of options in corporate risk management.

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1 Introduction

Regret is an emotional feeling caused by the ex post knowledge that another decision than the one actually taken would have led to better results. Evidence from psychological experiments shows that regret is a widespread phenomenon and plays an important role in decision making under uncertainty (Gilovich and Medvec, 1995; Zeelenberg, 1999; Connolly and Zeelenberg, 2002). In an economic context, regret is likely to be relevant whenever economic agents have to justify previous decisions, such as managers reporting to shareholders or fund managers reporting to clients. A particularly delicate issue with respect to communication and justification is the decision to hedge with derivatives. Because derivatives can be used for both hedging and speculation, it may be difficult to judge whether losses from derivative positions are the result of a sound risk management strategy or indicate risk management failure. It therefore puts risk managers\(^1\) in a much more comfortable position if the outcomes of their hedging strategies do not deviate too much from what would have been the best strategy ex post.

What is the impact of regret on corporate hedging? How does it change a firm’s hedging policy? These are important questions that we address in this paper. As the main contribution, we develop a model of corporate hedging that considers both standard risk aversion (aversion to uncertain profits) and regret risk aversion (aversion to uncertain regret). The model takes into account that many of the price risks a firm may face, such as energy price risk or exchange rate risk, can be managed with a variety of different hedging instruments, including put and call options with different strike prices. Such a set of derivative instruments allows for the design of tailor-made payoff functions. We characterize such payoff functions and investigate how they are affected by regret aversion. With regret aversion, optimal hedging policies are distinctly different from those resulting under the classical expected utility model of a hedging firm (Holthausen, 1979; Feder, Just, and Schmitz, 1980). The classical model predicts complete elimination of profit risk in the

\(^1\)Survey results by Bodnar, Gimbona, Graham, and Harvey (2014) stress the role of individual risk managers for a firm’s risk management strategy. In particular, risk managers’ personal risk aversion plays a significant role in corporate hedging decisions.
absence of risk premiums. In contrast, regret aversion leads to the hedging of downside risk. Some upside potential remains to avoid large deviations from an unhedged position in the case of favorable spot price movements, leading to large regret. Such a hedging policy is unique to the regret model and would not result under popular alternative non-expected utility models, such as loss aversion models inspired by prospect theory or models of disappointment aversion. In those alternative models, the utility still depends solely on the net outcome achieved but is independent of the state (corresponding to the underlying price) in which this outcome is achieved.

Let us illustrate this point with a simple example of a firm whose profits depend only on an uncertain market price $x$. If the firm obtains a perfect hedge against changes in $x$ on the financial market, then its profits become fixed, that is, all risk is hedged. In this case, standard utility models simply evaluate a safe profit and the stochastic component disappears. The regret model, however, still considers the different possible outcomes, since utility depends on what the firm’s profit would have been without a hedge. If that profit would have been large and, in hindsight, the hedge actually reduced profits, utility would be smaller than in cases where the hedge turned out to save the firm from financial losses due to adverse price moves. Thus, we see that utility is indeed state dependent, that is, differs for different values of $x$, even if the net profit is the same across these states.

The hedging of downside risk under regret aversion has consequences for the choice of hedging instruments, as we will show: Higher regret aversion leads to an increasing use of put options, whereas under standard risk aversion alone forward contracts are sufficient to obtain the optimal payoff. In this sense, the relative strengths of standard risk aversion and regret aversion determine the relative usage of forwards and options in a firm’s hedging policy.

This paper is related to other work on corporate hedging that focuses on instrument choice and motivates the use of options. One rationale for the use of nonlinear instruments is the presence of unhedgeable risks in addition to hedgeable price risk. Brown and Toft (2002) and Korn (2010) show that under additional quantity risk customized nonlinear
contracts are preferred to linear contracts, particularly if price and quantity risk are strongly dependent. Specific forms of basic risk provide another rationale for the use of options (Mahul, 2002; Chang and Wong, 2003). Mahul and Cummins (2008) show that the counterparty credit risk of forward contracts leads to a price risk exposure that can be further reduced via option contracts and Adam-Müller and Panaretou (2009) investigate the effect of liquidity risk arising from the marking to market of derivative contracts. They show that liquidity risk alters the optimal hedging policy in a way that requires the use of options in addition to forwards. Apart from unhedgeable risks, financing constraints can create a hedging role for options. Froot, Scharfstein, and Stein (1993) and Adam (2002) develop models that predict the usefulness of options if financially constrained firms try to match their cash inflows with their investment expenditures and expenditures are nonlinearly related to some risk factor. Adam (2009) finds empirical support for the predictions of these models in a sample of gold mining firms. A common feature of all approaches that explain the use of nonlinear hedging instruments by unhedgeable risks or financial constraints is a firm’s non-linear exposure to price risk. We complement these approaches by providing a different explanation: regret aversion. Our regret model shows that, even with a single price risk and linear exposure, there is a distinct role for nonlinear hedging instruments such as put options.

Our paper is also related to conceptual and experimental work on regret. Bell (1982), Loomes and Sugden (1982), and Bell (1983) develop a formal regret theory. The theory can explain many of the observed deviations from standard expected utility theory and several implications of the theory have been tested in experimental work. Aside from its descriptive appeal, regret theory is (at least partly) designed as a normative theory (Loomes and Sugden, 1982) with an axiomatic foundation (Sugden, 1993; Diecidue and Somasundaram, 2015). Other conceptual work focuses on how regret functions can be measured. Bleichrodt, Cillo, and Diecidue (2010) develop a corresponding method and provide strong empirical evidence for one of the main assumptions of regret theory, namely, the presence of regret aversion. The regret model used in our paper encompasses the original regret theory as a special case.

2Starmer (2000), Section 5.1, provides an overview of experimental work on regret theory.
Finally, this work belongs to a group of papers that applies regret theory to different problems in financial economics. Regret aversion has been incorporated into asset pricing models (Gollier and Salanié, 2006; Solnik and Zuo, 2012) and models of portfolio choice (Muermann, Mitchell, and Volkman, 2006; Hazan and Kale, 2015). It has been used to explain stock market participation (Barberis, Huang, and Thaler, 2006), the disposition effect (Shefrin and Statman, 1985; Muermann and Volkman, 2007), the demand for low deductibles in insurance contracts (Braun and Muermann, 2004), and the level of insurance coverage (Huang, Muermann, and Tzeng, 2015). Regret can also help to explain investors’ demand for cash dividends (Shefrin and Statman, 1984) and managers’ dividend decisions (Ghosh, 1993). Closest to our work is the paper by Michenaud and Solnik (2008), because it investigates corporate hedging under regret aversion. The main difference is that Michenaud and Solnik (2008) restrict their analysis to hedging with forward contracts and our paper investigates the impact of regret on the choice of hedging instruments. Our results show that regret aversion does not necessarily lead to underhedging, as in the case when only forwards are available. Instead, regret aversion has a crucial impact on instrument choice, supporting the use of options instead of forward contracts. Therefore, our analysis extends and complements the results of Michenaud and Solnik (2008) in important ways, particularly with respect to the predictions derived from the regret model.

The remainder of the paper is organized as follows. Section 2 starts with the model setup in Section 2.1 and derives optimal hedges in the absence of risk premiums in Section 2.2. Section 2.3 illustrates the results with different examples and Section 2.4 analyzes the impact of risk premiums on hedging policies under regret aversion. Section 3 discusses some aspects of the implementation of a risk management strategy under regret aversion and some empirical implications of our model. Section 4 concludes the paper.
2 Corporate Hedging Policies under Regret Aversion

2.1 Model Setup

Our model describes the hedging policies of a firm whose profits depend on an uncertain price. This price could refer to a product price or an exchange rate. At the beginning of a period (time 0), the firm has already fixed its production quantity $Q$, decides on its hedging policy, and implements the hedge. At the end of the period (time 1), the firm sells all $Q$ units at a price $x$, which is a random variable from the perspective of time 0. The price is exogenous with probability distribution $p(x)$. In addition to uncertain sales revenues, deterministic production costs $C(Q)$ arise at the end of the period. In total, the firm’s profits from operations $g(x)$ equal

$$g(x) = Qx - C(Q).$$  \hspace{1cm} (1)

As hedging instruments, the firm can use derivative contracts written on $x$ that mature at the end of the period. Denote the payoff function of these derivatives by $h(x)$. Because derivatives with optimal payoff functions are sought, we do not restrict the possible payoffs a priori, for example, by considering only forward contracts or plain vanilla options, but allow for every piecewise continuously differentiable function. Then the question arises how the firm prices these derivative contracts. In our base case, we assume that the expected profit from derivative contracts is generally zero, i.e., the firm does not consider any risk premiums when assessing its derivative positions.\footnote{In this respect, we follow Brown and Toft (2002).} Later, in Section 2.4, this assumption is dropped and the general case with state-dependent risk premiums is studied.

For simplicity, derivatives are also assumed to be deferred payment contracts, that is, no payments occur at time 0. In this setting, the firm’s total profits $y(x)$ at the end of the period become

$$y(x) = g(x) + h(x).$$  \hspace{1cm} (2)

The firm selects its optimal hedging policy by maximizing expected utility according to
a modified utility function that considers both profits and regret, with higher profits
leading to higher utility and more regret leading to lower utility. The concept of regret
requires the determination of what would have been the best strategy ex post among
all feasible strategies. In our setting, we define the borderline cases of feasible strategies
via two natural benchmarks, as suggested by Michenaud and Solnik (2008): (i) The first
benchmark is to fully hedge with forwards, that is, lock in the forward price, denoted
by $f$, and eliminate profit uncertainty completely. (ii) The second benchmark is to do
nothing, that is, use no derivatives. Our choice of benchmarks rules out both over-hedging
and a further increase of the spot position’s price exposure as infeasible. Even if such
strategies could be followed in principle, they clearly have some element of speculation.
Because this paper studies hedging policies, we believe that the chosen restrictions on
feasible strategies are reasonable. Given our choice of benchmark strategies, regret occurs
at the end of the period if the chosen hedging policy leads to a lower profit than the better
of the two benchmarks. Since the level of regret depends on the realization of $x$ and is
uncertain a priori, regret is risky.

Under our assumptions, the firm’s expected utility is expressed by the following modified
expected utility function:

$$U_E(y) = \int_0^\infty u(y(x)) p(x) \, dx + \int_0^f \phi[v(y(x)) - v(g(f))] p(x) \, dx$$

$$+ \int_f^\infty \phi[v(y(x)) - v(g(x))] p(x) \, dx,$$

where $u(.)$, $\phi(.)$, and $v(.)$ are strictly increasing and at least three times differentiable
functions. Moreover, $\phi(0) = 0$, indicating that there is no regret if the chosen
hedging policy is the best choice ex post.

The expected (modified) utility $U_E(y)$ consists of three terms: The first term gives the
(standard) expected utility of profits and the other two the expected (dis)utility of regret.
Regret is measured as the difference between the value that the firm assigns to the actual
profit, $v(y(x))$, and the value that it assigns to the profit it would obtain under the
benchmark strategies, $v(g(f))$ or $v(g(x))$.\footnote{Studies on regret often assume that $v(.) = u(.)$. However, we first study the more general case and...} The first benchmark is relevant for all $x < f$, ...
since full hedging is better than doing nothing in this case, and refers to the second term in equation (3). The third term in equation (3) corresponds to the benchmark of doing nothing, which is relevant for all realizations of \( x \) greater than \( f \). The firm is profit risk averse if \( u'' < 0 \) and \( v'' < 0 \) and regret risk averse if \( \phi'' < 0 \). \(^5\)

The problem to be solved is to find the payoff function \( h(x) \) that maximizes \( U_E(g + h) \) from equation (3) subject to the constraint that all derivatives have zero expected profits. This constraint can be expressed as

\[
\int_0^\infty y(x) p(x) \, dx = R, \tag{4}
\]

where \( R \) denotes the expected profit from operations. Given the formulation of the maximization problem, the firm’s optimal hedging policy can now be derived.

### 2.2 Optimal Hedges

To find an optimal payoff profile for the firm’s derivative contracts, we use the Lagrange function

\[
\Lambda(y, \lambda) = U_E(y) + \lambda \left( \int_0^\infty y(x) p(x) \, dx - R \right)
\]

and apply the variational method. Therefore, \( y(x) \) is replaced with \( y(x) + \varepsilon \psi(x) \) for an arbitrary test function \( \psi \in C_0^\infty \) and the derivative with respect to \( \varepsilon \) is taken at \( \varepsilon = 0 \). This procedure leads to the following expression:

\[
\frac{d}{d\varepsilon}_{|\varepsilon=0} \Lambda(y, \lambda) = \frac{d}{d\varepsilon}_{|\varepsilon=0} \left[ u \right]
\]

\[
= \int_0^\infty u'\left(y(x)\right)\psi(x)p(x)\,dx + \int_0^f \phi'\left(v(y(x)) - v(g(f))\right)v'\left(y(x)\right)\psi(x)p(x)\,dx
\]

\[
+ \int_f^\infty \phi'\left(v(y(x)) - v(g(x))\right)v'\left(y(x)\right)\psi(x)p(x)\,dx + \lambda \int_0^\infty \psi(x)p(x)\,dx. \tag{5}
\]

\(^5\)The functions \( u \) and \( v \) describe how the firm is affected by profit risk, because the arguments of these function are profits. In contrast, the argument of \( \phi \) is regret, that is, the form of the function \( \phi \) describes how the firm is affected by regret risk.

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consider some special cases in Section 2.3.
The sum in the last two rows of equation (5) has to be zero for any test function \( \psi(x) \). This requirement leads to the optimality conditions

\[
u'(y(x)) + \phi'[v(y(x)) - v(g(f))]v'(y(x)) + \lambda = 0, \quad \text{for } x \in (0, f), \tag{6}
\]
\[
u'(y(x)) + \phi'[v(y(x)) - v(g(x))]v'(y(x)) + \lambda = 0, \quad \text{for } x \in (f, \infty). \tag{7}
\]

To characterize the optimal payoff function, we differentiate with respect to \( x \) and obtain the following differential equations that provide information about the characteristics of the function \( y(x) \):

\[
\left[u''(y(x)) + \phi''[v(y(x)) - v(g(f))]v'(y(x))^2 + \phi'[v(y(x)) - v(g(f))]v''(y(x))\right]y'(x) = 0,
\]
for \( x \in (0, f), \tag{8}
\]
\[
\left[u''(y(x)) + \phi''[v(y(x)) - v(g(x))]v'(y(x))^2 + \phi'[v(y(x)) - v(g(x))]v''(y(x))\right]y'(x) = \phi''[v(y(x)) - v(g(x))]v'(g(x))v'(y(x)).
\]
for \( x \in (f, \infty). \tag{9}
\]

Equations (8) and (9) reflect some fundamental economic results about the relevance or irrelevance of risk management. If the firm is both profit risk neutral \( (u'' = 0, v'' = 0) \) and regret risk neutral \( (\phi'' = 0) \), there is no unique solution for \( y'(x) \) and every payoff function provides the same utility level. Since risk aversion implicitly quantifies the need for hedging, risk neutrality (with respect to both profits and regret) corresponds to a case where risk management creates no value and is therefore irrelevant.

Under the more realistic assumption that \( u'', v'', \phi'' \leq 0 \) and at least either \( u'' < 0, v'' < 0, \) or \( \phi'' < 0 \), the following expressions for \( y'(x) \) are obtained:\textsuperscript{6}

\[
y'(x) = 0, \quad \text{for } x \in (0, f), \tag{10}
\]
\[
y'(x) = \frac{\phi''[v(y(x)) - v(g(x))]v'(g(x))v'(y(x))Q}{u''(y(x)) + \phi''[v(y(x)) - v(g(x))]v'(y(x))^2 + \phi'[v(y(x)) - v(g(x))]v''(y(x))},
\]
for \( x \in (f, \infty). \tag{11}
\]

\textsuperscript{6}Note that \( g'(x) = Q \) according to equation (1).
Equations (10) and (11) together with the profit function (2) provide a characterization of the firm’s optimal hedge, as summarized in the following proposition.

**Proposition 1.** Let \( h(x) \) be the payoff function of the optimal hedge position. If \( u, v, \) and \( \phi \) are strictly increasing and concave functions and at least one of them is strictly concave, then

\[
 h'(x) = -Q, \quad \text{for } x \in (0, f),
\]

\[
 h'(x) = \frac{\phi''[v(y(x)) - v(g(x))] v'(g(x)) v'(y(x)) Q}{u''(y(x)) + \phi''[v(y(x)) - v(g(x))] v'(y(x))^2 + \phi''[v(y(x)) - v(g(x))][v''(y(x)] - Q, \quad \text{for } x \in (f, \infty).
\]

The proposition provides several insights that we discuss in turn. For ease of presentation, we look at some special cases and specific aspects and express them in terms of corollaries.

**Corollary 1.** If the firm has no regret risk aversion, that is, \( \phi'' = 0 \), the optimal hedge position consists of forward contracts and we obtain a full forward hedge.

The corollary follows immediately. Without regret risk aversion, the optimal payoff function is linear and has a negative slope of \(-Q\), that is, \( h'(x) = -Q \) for all \( x \). The firm’s risk exposure is therefore fully hedged with short positions in forward contracts and equation (2) shows that the firm makes a certain profit of \( Qf - C(Q) \) in this case.

Corollary 1 describes the same hedging policy as the one obtained under the classical approach that maximizes the expected utility of profits (Holthausen, 1979; Feder, Just, and Schmitz, 1980). Therefore, it is not the consideration of regret per se that makes a difference for a firm’s hedging policy, but the aversion to regret risk. This point becomes very clear in the following complementary case of no profit risk aversion, which stresses the effects of regret risk aversion.

**Corollary 2.** If the firm has no profit risk aversion, that is, \( u'' = 0 \) and \( v'' = 0 \), the optimal derivative contract is an at-the-money put option on a forward contract (or an at-the-money forward option on the spot price) and we obtain a full put hedge.
To prove this corollary, we must demonstrate that $h'(x) = 0$ or, equivalently, that $y'(x) = Q$ for $x \in (f, \infty)$. With $u'' = 0$ and $v'' = 0$, equation (11) reduces to $y'(x) = [v'(g(x)) Q]/v'(y(x))$. Without risk aversion, $v$ is an affine function; therefore $v'$ is constant, which implies the desired result, that $[v'(g(x)) Q]/v'(y(x)) = Q$.

Corollary 2 shows that regret risk aversion has a fundamentally different effect than profit risk aversion: Under regret risk aversion, options are the desired hedging instruments instead of forward contracts. The reason is that the focus lies on the deviation from the ex post optimal strategy and not on the deviation from the expected outcome. Options provide better protection against the former, while forwards provide better protection against the latter. The result in Corollary 2 also highlights that the focus on linear hedges in Michenaud and Solnik’s (2008) study of hedging policies under regret is quite restrictive. An optimal hedging policy would not rely only on forward contracts. This effect also shows up in the general case with both profit risk aversion and regret risk aversion.

**Corollary 3.** If the firm has both profit risk aversion and regret risk aversion, the optimal hedging contract is a mixture of forwards and options ($-Q < h'(x) < 0$ for $x \in (f, \infty)$) and the firm still hedges its downside risk fully ($h'(x) = -Q$ for $x \in (0, f)$). The mixture of forwards and options is governed by the relative strengths of profit risk aversion and regret risk aversion; that is, the forward component increases with profit risk aversion and the option component increases with regret risk aversion.

We prove the corollary by showing that the slope of $y(x)$ is always between zero and $Q$ and performing a comparative static analysis. Note that, in equation (11), both the numerator and denominator are negative, which means that $y'(x)$ is positive. Therefore, the case with no regret risk aversion provides a lower boundary for the slope. To prove that the slope of $y(x)$ is less than $Q$, it is sufficient to show that $v'(y(x)) \geq v'(g(x))$. Note that equation (11) refers to the case with $x > f$. Since we do not allow for speculative positions, we know that $y(x) \leq g(x)$. Since $v'$ is strictly decreasing for a profit risk-averse firm, the desired result follows. A value of $Q$ is the upper limit for the slope of $y(x)$. The comparative static results follow immediately. If regret risk aversion becomes stronger
(φ" more negative), the slope of y(x) increases. On the contrary, if profit risk aversion becomes stronger (u" or v" more negative), the slope decreases.

We always have a full hedge, in the sense that h'(x) = −Q for x ∈ (0, f). However, this hedge is achieved through a mixture of forward contracts and options (not necessarily only at-the-money forward put options). The concrete form of this mixture depends on the relative strengths of profit risk aversion and regret risk aversion. As the comparative static analysis shows, regret risk aversion works in favor of options and profit risk aversion works in favor of forwards.

A final corollary provides some information about the structure of the options position that characterizes the optimal payoff function.

**Corollary 4.** Regret aversion leads to a kink in the payoff function at the forward price.

Because h'(x) = −Q for x ∈ (0, f) and h'(x) > −Q for x ∈ (f, ∞), the optimal payoff function has a kink at x = f. This result shows that, as long as we have at least some regret risk aversion, we will have at least some at-the-money (at-the-money forward) options. Since the optimal payoff function is differentiable for all x except for x = f, we can conclude that the optimal payoff function places more weight on at-the-money options than on in-the-money options. Because h'(x) = −Q for x ∈ (0, f)), out-of-the-money options are not used, according to our base case model.

### 2.3 Examples

#### 2.3.1 Profit-Based Regret

An important special case of the general regret model is profit-based regret. In the general model, regret is measured in terms of the difference between the assigned value of profits under the chosen action (v(y(x))) and the corresponding one under the ex post optimal action (v(g(f)) or v(g(x))). Such a value, however, is difficult to determine and it is plausible that shareholders or supervisory boards judge the success or failure of a hedging strategy in terms of the resulting profit itself and not in terms of value. Consequently,
regret would be based on profits too.

Accordingly, in profit-based regret, \( v \) is the identity function and regret becomes \( \phi [y(x) - g(f)] \) for \( x \in (0, f) \) and \( \phi [y(x) - g(x)] \) for \( x \in (f, \infty) \). With profit-based regret, the problem of finding optimal hedging policies is solved in the same way as in the general case, leading to the following conditions on \( h'(x) \):

\[
h'(x) = -Q, \quad \text{for } x \in (0, f),
\]

\[
h'(x) = \frac{\phi''[y(x) - g(x)]Q}{u''(y(x)) + \phi''[y(x) - g(x)]} - Q, \quad \text{for } x \in (f, \infty).
\]

Using the above conditions, we further illustrate the effects of regret by investigating three specific aspects: the relative importance of forwards and options as hedging instruments, the concavity or convexity of the optimal payoff function, and the impact of production costs on hedging policy.

Closed-form solutions are useful to gain further intuition and to facilitate the implementation of strategies. A closed-form solution for the optimal payoff function \( y(x) \) exists if both the (standard) utility function \( u \) and the regret function \( \phi \) are assumed to be quadratic.\(^7\) Equation (13) shows that with quadratic functions \( u \) and \( \phi \) the following characterization of the optimal payoff function is obtained:

\[
h'(x) = -\frac{a_u}{a_u + a_\phi} Q, \quad \text{for } x \in (f, \infty),
\]

where \( a_u \) and \( a_\phi \) are the risk aversion coefficients corresponding to \( u \) and \( \phi \), respectively.

The optimal hedging strategy is very simple and easy to implement. It consists of a full hedge using a combination of forward contracts and at-the-money (forward) put options. The optimal proportion of the two instruments depends on the (relative) magnitude of the risk aversion coefficients. The more important regret risk aversion becomes, the more options are taken and the more important profit risk aversion is, the more forward contracts

\(^7\)The reasoning is similar to the classical portfolio problem, where quadratic utility leads to considerable simplification in the sense that expected returns, variances, and covariances are sufficient to find optimal portfolios.
are sold. If the two risk aversion coefficients are equal, the resulting risk management policy hedges 50% of the price exposure with forwards and 50% with options.

Another interesting issue is whether our model leads to an optimal payoff function that is concave or convex. To answer this question, we obtain the second derivative of the function $h(x)$. For $x \in (f, \infty)$, this second derivative equals

$$h''(x) = \frac{\phi'''[y(x) - g(x)](y'(x) - Q)u''(y(x)) - \phi''[y(x) - g(x)] u'''(y(x)) y'(x)}{(u''(y(x)) + \phi''[y(x) - g(x)])^2} Q. \quad (15)$$

Equation (15) shows that the concavity or convexity of the optimal payoff function depends on the sign of the third derivative of the functions $u$ and $\phi$. If both $u'''$ and $\phi'''$ are always positive, the payoff function is convex. This case corresponds to a situation with positive prudence. If both are negative, the payoff function is concave. If $u'''$ and $\phi'''$ have different signs or have positive and negative parts, not much can be said in general about the concavity or convexity of the function $y(x)$. It could well have some concave and some convex parts, showing that, even in the case of profit-based regret, different payoff functions can be optimal, depending on the form of profit risk aversion and regret risk aversion.

An economically important aspect is the effect of higher production costs $C$ on the optimal payoff function. The partial derivative of $h'(x)$ with respect to $C$ is

$$\frac{\partial h'(x)}{\partial C} = \frac{\phi''[y(x) - g(x)] Q u'''(y(x))}{(u''(y(x)) + \phi''[y(x) - g(x)])^2}.$$

(16)

With positive prudence ($u''' > 0$), the slope $h'(x)$ is decreasing (becomes more negative) with increasing costs. This is an interesting observation, meaning that a firm with a lower profit margin would be closer to the forward hedge and a firm with a higher profit margin would be closer to the option hedge. However, we must check whether this result is specific to the profit-based regret model or holds more generally.

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8See Kimball (1990) for an exposition of the concept.
2.3.2 Utility-Based Regret

Another important special case is *utility-based regret*, which assumes that the value function $v$ equals the utility functions $u$, that is, regret is measured in terms of utility differences. This case is typically studied in the literature. As an illustration, we look at a situation with both $u$ and $\phi$ having the same general form, representing constant absolute risk aversion (CARA) with respect to profits and regret, respectively. For this case, we revisit the issues discussed under profit-based regret and check whether we find similar structures. In particular, we want to see how the coefficients of absolute profit risk aversion ($\rho_u$) and regret risk aversion ($\rho_\phi$) affect the form of the optimal payoff function of the hedging instruments, if the payoff function is convex or concave, and how production costs affect optimal payoffs.

Optimal payoff functions must be obtained numerically by solving the differential equations as stated in Proposition 1. As our base case for a comparative static analysis, we consider a random future product price that is uniformly distributed on the interval $[0.5, 1.5]$.\(^9\) There is no risk premium, production costs are zero, and the quantity $Q$ equals one. The coefficient of absolute profit risk aversion is $\rho_u = 1$ and the coefficient of absolute regret risk aversion is $\rho_\phi = 1$. First, we perform a comparative static analysis with respect to the coefficients $\rho_u$ and $\rho_\phi$. Figure 1 shows the payoff functions $h(x)$ for different values of $\rho_u$ and Figure 2 the payoff functions for different values of $\rho_\phi$.

[Insert Figure 1 about here]

[Insert Figure 2 about here]

Figure 1 clearly shows that the slope of the payoff function becomes more negative with increasing $\rho_u$. With a value of $\rho_u = 5$, we essentially reach a full forward hedge. The payoff function appears to be (piecewise) linear, that is, it looks linear even in its part

\(^9\)The price distribution does not affect the general form of the payoff function but determines the expected payoff of the derivative positions and therefore derivative prices. Formally, it determines the boundary conditions for the differential equations.
to the right of the forward price \( f = 1 \). Closer inspection of the numbers, however, shows that the function is actually concave. With this result, the convex functional form under profit-based regret for positive prudence cannot be confirmed.\(^{10}\) Measuring regret in terms of value (utility) instead of profits therefore has an impact on the concavity or convexity of the payoff function. Figure 2 shows the corresponding effects of increasing regret risk aversion. With higher \( \rho_\phi \), the payoff functions flatten for values above the forward price. However, even for absolute regret aversion as great as 20, a full option hedge is not reached. With respect to the functional form, a (slightly) concave function is again found to the right of \( f \).

Finally, we vary the production costs. Figure 3 depicts optimal payoff functions for different values of \( C \), varying from zero to 0.3. Costs clearly have an effect on the form of the payoff function. With higher costs, the payoff function flattens to the right of \( f \), leading to a higher proportion of options compared to forwards. This is just the opposite effect as that observed under profit-based regret (more negative slope and more forward-like payoff function). These results highlight that the way regret is measured can be essential for the properties of the payoff function. Figure 3 also demonstrates more clearly that the payoff is indeed concave for prices higher than the forward price, as opposed to the convex function under profit-based regret.

### 2.4 Effects of Risk Premiums

Our results so far have been derived under the assumption of no risk premiums. Do these results remain valid in the more general case with risk premiums? And how does regret interact with such risk premiums? To deal with this issues, we introduce the likelihood ratio \( \ell(x) := q(x)/p(x) \), where \( q(x) \) denotes the risk-neutral distribution. Note that our previous analysis was a special case with \( \ell(x) = 1 \). To solve for the optimal payoff function

\(^{10}\)The chosen CARA specification has the property of positive prudence.
of the derivative positions, the constraint in equation (4) is now replaced by
\[ \int_0^\infty y(x) \ell(x) p(x) \, dx = R_{adj}, \]
which means that the firm’s risk adjusted expected profit \( R_{adj} \) is the same for all choices of the payoff function \( h(x) \). The problem is solved in the same way as in Section 2.2, leading to the following optimality conditions:

\[ u'(y(x)) + \phi'[v(y(x)) - v(g(f))] v'(y(x)) + \lambda \ell(x) = 0, \quad \text{for } x \in (0, f), \quad (18) \]
\[ u'(y(x)) + \phi'[v(y(x)) - v(g(x))] v'(y(x)) + \lambda \ell(x) = 0, \quad \text{for } x \in (f, \infty). \quad (19) \]

Note that equations (18) and (19) imply that \( \lambda \) is negative. The above conditions lead to the following characterization of the optimal payoff function, as presented in the following proposition.

**Proposition 2.** Let \( h(x) \) be the payoff function of the optimal hedge position and \( \ell(x) \) the likelihood ratio. If \( u, v, \) and \( \phi \) are strictly increasing and concave functions and at least one of them is strictly concave, then

\[ h'(x) = \frac{-\lambda \ell'(x)}{u''(y(x)) + \phi''[v(y(x)) - v(g(f))] v''(y(x)) + \phi'[v(y(x)) - v(g(x))] v''(y(x))} - Q, \quad \text{for } x \in (0, f). \]
\[ h'(x) = \frac{\phi''[v(y(x)) - v(g(x))] v'(g(x)) v'(y(x))}{u''(y(x)) + \phi''[v(y(x)) - v(g(x))] v''(y(x)) + \phi'[v(y(x)) - v(g(x))] v''(y(x))} - Q, \quad \text{for } x \in (f, \infty). \]

Of course, the optimal payoff function and its interpretation depend on the functional form of the likelihood ratio. For a producer who wants to hedge against adverse moves of product prices, it seems appropriate to assume that \( \ell'(x) < 0 \), because a premium must be paid to insure the bad price states. If we think about hedging of foreign exchange risk, the bad states for hedgers could be very low and very high values, depending on one’s home currency. Such a situation could be expressed by \( \ell'(x) < 0 \) for \( x < f \) and \( \ell'(x) > 0 \) for \( x > f \).
According to Proposition 2, the following points can be made about the firm’s optimal hedging policy.

(i) The payoff function has a slope that is generally greater (less negative) than \(-Q\) if \(x < f\) and could vary with \(x\); that is, there is room for out-of-the-money options due to the risk premiums. If \(x > f\), the slope is generally greater than \(-Q\) if \(\ell'(x) \leq 0\) in this region.

(ii) The higher the regret risk aversion (and the profit risk aversion), the lower (more negative) the slope of \(h(x)\) is for \(x < f\). This finding means that high risk aversion works in favor of a full hedge and mitigates the effects of risk premiums, which is quite intuitive.

(iii) Higher regret risk aversion works in favor of options (instead of forwards) if \(\ell'(x) \geq 0\); that is, the slope of the payoff function \(h(x)\) becomes higher (less negative) for \(x > f\). Higher regret risk aversion results in two effects: First, regret risk aversion pushes the payoff function to the benchmark, which increases the slope. Second, the slope-reducing effect of the risk premium is mitigated, because “hedging” becomes relatively more important than “speculation”. If \(\ell'(x) < 0\) for \(x > f\), the second effect leads to a reduction of the slope, which means that the total impact of higher regret risk aversion on the slope is not clear.

(iv) The kink (differences between the slopes to the left and right of \(f\)) is unaffected by the risk premiums as long as \(\ell'(x)\) is a continuous function, which is the case in most standard models.

3 Implementation Issues and Empirical Implications

When our model is viewed from a normative perspective, what should a firm keep in mind about its hedging policy when concerned with regret risk? Our model states that it should mainly concentrate on downside risk and retain some upside potential to avoid large regret in the case of favorable price moves. This statement has an immediate consequence for
hedging instrument choice, since put options lend themselves to such downside protection. The specific design of the hedging policy, however, depends on the way regret is measured, on the relative strengths of regret risk aversion and standard risk aversion, and on the risk premiums associated with the price risk under consideration. We have shown that profit-based regret and utility-based regret can have different implications for the concavity or convexity of the optimal payoff function of a firm’s derivative positions and on the impact of production costs on optimal payoffs. A first step for the implementation of a risk management strategy would therefore be to decide how regret is measured.

In a second step, the form and magnitude of regret risk aversion must be quantified. The higher regret risk aversion, the more options should be used for hedging instead of forward contracts. Moreover, it is important to determine whether regret risk aversion refers to all potential price states or just to very large deviations from the ex post optimal strategy. For the former case, we have shown that mainly at-the-money options should be used. The latter case could provide a rationale for the use of out-of-the-money options. Concerning the question whether options with different strike prices should be used simultaneously, the case of profit-based regret and quadratic regret function provides a nice reference point. For the pure hedge component (no consideration of risk premiums), options with a single strike price are sufficient in this case. Finally, the benefits of hedging must be balanced with its costs in terms of transaction costs and risk premiums. With respect to transaction costs, concentration on a few plain vanilla instruments (forwards, futures, and options) is likely preferable to more complex structures. The size of risk premiums obviously depends on the specific price risk under consideration. It is a general result, however, that regret aversion reduces the impact of risk premiums on the hedging of downside risk, because regret risk aversion compounds the effects of profit risk aversion in the loss region.

When our model is viewed from a descriptive perspective, the following predictions are obtained: If regret risk aversion is important, firms will concentrate on hedging downside risk and use options in addition to forward contracts, even in the absence of risk premiums. These predictions are in sharp contrast to those made by standard expected utility theory and alternative behavioral theories based on loss aversion that all lead to a complete
elimination of risk (no upside potential) and identify a full forward hedge as the optimal policy, as long as there is no risk premium.\footnote{Even if a risk premium exists, the deviations from a perfect hedge are rather limited and not in accordance with the results under regret aversion (see Rieger (2007) for details).} Obviously, the predictions also differ from those of Michenaud and Solnik's (2008) regret model, which restricts potential hedging policies to linear instruments in the first place. Survey evidence shows that options are indeed important hedging instruments for non-financial firms (Bodnar, Hayt, and Marston, 1998; Bartram, Brown, and Fehle, 2009), particularly for the management of commodity price risk. Moreover, based on a large sample of nonfinancial firms from 47 countries, Bartram, Brown, and Conrad (2011) provide evidence that firms indeed hedge downside risk. These findings are consistent with our model, notwithstanding other potential reasons to use options as hedging instruments and to hedge downside risk that can be traced back to nonlinear price exposure instead of preference assumptions, as in our analysis.

Although the analysis was presented as a corporate hedging problem, the results also apply to the investment problem that tries to find an investor’s optimal exposure to a risky asset.\footnote{Michenaud and Solnik (2008) already provide this argument for their analysis, which can be interpreted as the classical portfolio problem seeking the best mixture of a risky asset and a risk-free asset.} The case of no hedging then refers to an investment only in the risky asset and the case of a full forward hedge to an investment only in the risk-free asset. Our case without risk premiums and pure regret aversion leads to an investment in the risky asset protected by an at-the-money put option. In general, it can be concluded that high regret aversion points to investment products with downside protection and capital guarantees. The popularity of such features in retail products and insurance contracts (Rieger and Hens, 2012; Hens and Rieger, 2014) is additional empirical evidence consistent with our model.

Undoubtedly, the most serious problem for the empirical testing of our model is the measurement of individual regret aversion, because our model does not make any statements about equilibrium but, rather, predictions about individual firms or investors. Potential ways to obtain such information on individual regret aversion are survey studies or labo-
ratory experiments. Given such information, it could be confronted with actual hedging or investment decisions, testing whether firms or investors with higher regret aversion actually seek more downside protection and use more options.

4 Conclusions

This paper has presented a model of corporate hedging under regret. In the regret model, a firm cares not only about its profits but also about the performance of its hedging strategy in comparison to what would have been the best choice ex post. We believe that regret will be important for risk management as long as managers must answer questions such as how it is possible to lose so much money with derivatives whenever a firm hedged with derivatives and prices increased afterward or why we did nothing to protect ourselves when a firm did not hedge a price risk and prices happened to fall.

Our model shows that protection against profit risk and regret risk works in the same direction in the loss region of the price distribution. Therefore, the regret model provides even stronger incentives for downside risk protection than the standard expected utility model, dampening the impact of risk premiums. In this sense, the regret model is similar to models assuming loss aversion. The effects in the profit region of the price distribution are very different, however. In this region, there is a trade-off between profit risk reduction and regret risk reduction. If profit risk is low, regret risk is high and vice versa. Therefore, firms should hedge less in the profit region under the regret model than under the standard expected utility model and under models with loss aversion. It is important to note that the effects of regret risk on a firm’s hedging policy strongly depend on which hedging instruments are available to the firm. In Michenaud and Solnik’s (2008) model, firms are restricted to forward contracts. Under this restriction, regret risk-averse firms hedge even less downside risk than under the standard model. The reason is that lower profit risk (and regret risk) in the loss region leads to higher regret risk in the profit region due to the linearity of the hedging instrument and the optimal hedging policy needs to trade off these risks. When more flexible hedging instruments are allowed, as in our model, this link between profit risk in the loss region and regret risk in the profit region is cut.
Therefore, a major result of our analysis is that regret aversion has an important effect on instrument choice and works in favor of option hedges instead of pure forward hedges. There are still many open issues concerning the impact of regret on corporate hedging. First, because firms usually face more than one price risk, the consideration of multiple risks is a natural extension of our model. However, since regret risk builds on the deviations from an ex post optimal strategy, one has to decide whether such an optimal strategy refers to each risk individually or to the overall risk position, a decision that could well depend on the firm’s organizational structure. Second, our model uses a setting where regret risk aversion is the sole reason to use option contracts. In the literature, alternative explanations for the use of options based on market frictions and unhedgeable risks have been developed. How regret risk aversion interacts with these other explanations for nonlinear hedging strategies and whether such an interaction eventually increases or decreases the use of options are interesting issues. Finally, the most pressing issue for the empirical testing of the regret model is to devise methods for the measurement of a firm’s regret aversion to analyze the relation between regret aversion and actual hedging behavior.
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This figure shows the firm’s optimal payoff functions of its derivative positions $h(x)$ for different levels of profit risk aversion ($\rho_u$). The results are obtained in the following setting: Product prices follow a uniform distribution on the interval $[0.5, 1.5]$ and no risk premium exists for the price risk. Production costs are zero and the quantity $Q$ equals one. Both the standard utility function $u$ and the regret function $\phi$ have the form of a CARA utility function. The parameter of absolute regret risk aversion ($\rho_\phi$) equals one.
Figure 2: Effects of a variation of regret risk aversion on the optimal payoff function

This figure shows the firm’s optimal payoff functions of its derivative positions $h(x)$ for different levels of regret risk aversion ($\rho_\phi$). The results are obtained in the following setting: Product prices follow a uniform distribution on the interval $[0.5, 1.5]$ and no risk premium exists for the price risk. Production costs are zero and the quantity $Q$ equals one. Both the standard utility function $u$ and the regret function $\phi$ have the form of a CARA utility function. The parameter of absolute profit risk aversion ($\rho_u$) equals one.
This figure shows the firm’s optimal payoff functions of its derivative positions $h(x)$ for different levels of production costs ($C$). The results are obtained in the following setting: Product prices follow a uniform distribution on the interval $[0.5, 1.5]$ and no risk premium exists for the price risk. The quantity $Q$ equals one. Both the standard utility function $u$ and the regret function $\phi$ have the form of a CARA utility function. The parameters of absolute profit risk aversion ($\rho_u$) and absolute regret risk aversion ($\rho_\phi$) both equal one.