Asset Prices and Business Cycles with Financial Shocks

Mahdi Nezafat and Ctirad Slavík

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Abstract

We develop a production based asset pricing model with financially constrained firms to explain the observed high asset price volatility. Investment opportunities are scarce and firms face two shocks: classic productivity shocks and financial shocks that affect the tightness of the financial constraint. The source of asset price volatility in the model is the interaction between the scarcity of investment opportunities and time variation in the tightness of the financial constraint. We calibrate the model to the U.S. data and find that it generates a volatility in the price of equity comparable to the observed aggregate stock market volatility. The model also fits key aspects of the behavior of aggregate quantities, in particular, the volatility of aggregate consumption and investment.

JEL Codes: E20, E32, G12

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*Mahdi Nezafat, Michigan State University. E-mail: nezafat@broad.msu.edu. Ctirad Slavík, CERGE-EI. Email: ctirad.slavik@cerge-ei.cz.
1 Introduction

The excess volatility puzzle (Shiller 1981; and LeRoy and Porter 1981) and the equity premium puzzle (Mehra and Prescott 1985) are two fundamental challenges to theoretical models that have been developed in the finance and macroeconomics literature. Building a production economy model that would satisfactorily account for both the dynamics of asset prices and business cycle fluctuations has proven to be rather difficult.

In this paper, we build a model with financial frictions and financial shocks and show that financial shocks play an important role in explaining not only business cycle fluctuations but also the high asset price volatility observed in the data. Calibrating the model to the U.S. data, we find that it generates about 70% of the observed aggregate stock market volatility. In addition, the model matches the time-series properties of aggregate macroeconomic quantities observed in the data. In particular, the model matches the volatility of aggregate investment and consumption.

Our model resembles the model developed in Kiyotaki and Moore (2011). It is a dynamic stochastic general equilibrium model with heterogeneous entrepreneurs, financial frictions, and financial shocks. A unit measure of ex ante identical entrepreneurs with Epstein-Zin preferences produce, consume, and trade financial assets. In every period, entrepreneurs face a common productivity shock and a common financial shock. In addition, in every period only a fraction of entrepreneurs find new investment projects. Entrepreneurs who cannot find a new investment project are willing to buy claims to returns of other entrepreneurs’ projects to replace their depreciated capital. We call these claims equity. Markets are incomplete, and equity is the only financial asset that is traded in the economy.

Entrepreneurs face a financial friction. They can pledge only a fraction of returns of the newly produced capital, i.e., sell only a fraction of the new project as equity. On its

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1A comparison of our model and Kiyotaki and Moore’s model is provided in Section 2.3. Other papers that analyze versions of the Kiyotaki and Moore (2011) model are Ajello (2012), Bigio (2012), Del Negro et al. (2011), and Shi (2015). These papers focus mostly on monetary policy and liquidity shocks, whereas the present paper focuses on asset prices and financial shocks, i.e., shocks to the access to outside financing.
own, this friction is standard in the literature. The novel feature of the model is that we assume that this fraction, which we call financial shock, varies stochastically over time. He and Krishnamurthy (2012) provide a micro-founded theory for time-varying financial constraints.

We show theoretically that the tightness of the financial constraint is a major determinant of the price of equity and its volatility. In particular, if the financial constraints does not bind, then the price of equity is constant. In addition, we show that both the scarcity of investment opportunities and the inability of entrepreneurs to fully finance new investment externally are necessary for the model to generate movements in the price of equity.

The model allows for aggregation because entrepreneurs have a homothetic utility function and their budget constraints are linear. We show that the dynamics of aggregate quantities and prices in the model can be described by a system of first order difference equations. Theoretically, we show that as long as the financial constraint binds, both the productivity shock and the financial shock result in asset price movements. This finding is particularly transparent, if the entrepreneurs’ elasticity of intertemporal substitution (EIS) is one. In this case, the model has a closed form solution for the price of equity as a function of the aggregate shocks and states. If, in addition to EIS being one, capital depreciates fully every period, the financial shock is the only determinant of the price of equity.

To assess the quantitative significance of financial and productivity shocks, we solve the model numerically and calibrate it to match several moments of macroeconomic variables and asset returns, including the relative volatility of investment, the correlation between output and investment and the mean (implicit) risk-free rate. By construction, the model matches the relative volatility of aggregate investment, overcoming a shortcoming of previous papers with convex adjustment costs or factor immobility (see, e.g., Jermann 1998; Boldrin et al. 2001; Guvenen 2009; and Campanale et al. 2010). Moreover, because of the random arrival of investment opportunities, the model produces investment spikes at the firm level observed in the data (see, e.g., Favilukis and Lin 2013 and Khan and Thomas 2013 for an
alternative mechanism that generates investment spikes at the firm level).

The model matches the volatility of aggregate consumption. This is an important result, since for a production based model to offer a plausible mechanism to explain asset prices, it should be consistent with the consumption volatility observed in the data. In our model, entrepreneurs bear all the asset market risk. Hence, it is critical that the model also matches the magnitude of consumption risk that entrepreneurs are facing. The literature finds that the volatility of consumption of stock market participants is 1.5 to 5 times larger than that of non-participants (see Guvenen 2009). In our model, the ratio of the standard deviation of entrepreneurs’ consumption to the standard deviation of workers’ consumption is about 3.5 (in the model, workers do not hold equities), which is broadly consistent with the data.

The calibrated benchmark model with both financial and productivity shocks generates over 70% of the observed quarterly volatility in asset prices and over 35% of the observed quarterly Sharpe ratio. When the financial shock is shut down, the model generates less than 10% of the observed quarterly volatility in asset prices and the Sharpe ratio is negligible. This implies that financial frictions on their own do not strongly propagate productivity shocks (see also Gomes et al. 2003; and Cordoba and Ripoll 2004 for a similar finding in environments based on Carlstrom and Fuerst 1997; and Kiyotaki and Moore 1997). In contrast, the benchmark model results suggest that financial constraints strongly propagate financial shocks.

It is important to emphasize that the high asset price volatility generated by the model is not due solely to the size of the financial shock. First, in our model, asset price volatility is a result of the interaction of financial shocks and the random arrival of investment opportunities. If all entrepreneurs had investment opportunities in each time period, the financial constraint would not bind and the volatility of the financial shock would be irrelevant. Second, in the states in which the financial constraint binds, the standard deviation of the financial shock is only about four times larger than the standard deviation of the productivity shock.
The calibrated benchmark model generates predictable (excess) equity returns. Empirical equity return predictability by financial variables as well as macroeconomic variables is well documented in the literature (see, e.g., Campbell and Shiller 1988; Fama and French 1988; Lettau and Ludvigson 2001; Cooper and Priestley 2009, among many others). However, standard business cycle models do not generate economically significant predictability in excess returns (see Kaltenbrunner and Lochstoer 2010). Our model generates predictable (excess) returns because the expected (excess) equity returns vary systematically over the business (financial) cycle. This is due to a systematic variation in the expected consumption growth as well as in the volatility of the stochastic discount factor.

Our results suggest that asset price volatility in our model arises from time variation in the tightness of the financial constraint. To further explore this observation, we develop and solve an alternative specification of the model in which we shut down the financial shocks and introduce shocks to the fraction of entrepreneurs with an investment opportunity. These shocks also result in fluctuations in the tightness of the financial constraint. This specification of our model generates macroeconomic and asset price dynamics that are very similar to the benchmark model in which investors face financial shocks. This finding implies that the constraint that limits the access of firms to external financing can be a strong propagator of shocks that originate in the financial markets as well as of the shocks that affect the availability of investment opportunities in the economy.

The mechanism that generates volatility in our model is related to the mechanism that generates volatility in models with investment-specific technology (IST) shocks (see, for example, Christiano and Fisher 2003; and Papanikolaou 2011), but differs from that mechanism in an important aspect. In these models, IST shocks affect quantities directly by moving the production possibility frontier, and indirectly through the change in asset prices. This paper proposes an alternative mechanism, in which financial shocks do not affect the production possibility frontier and only the general equilibrium price channel transfers financial shocks to aggregate quantities. In this regard, the mechanism proposed in this paper is related to
the mechanisms in Gourio (2012), Gourio (2013) and Chen and Song (2013) in which shocks to disaster probability or future productivity affect aggregate quantities through general equilibrium effects, but do not affect the (current) production possibility frontier.

Various approaches other than introducing financial frictions and financial shocks have been taken to explain the observed high asset price volatility in production economy models (see, e.g., Jermann 1998; Tallarini 2000; Boldrin et al. 2001; Kuehn 2007; Guvenen 2009; Kaltenbrunner and Lochstoer 2010; Campanale et al. 2010; Papanikolaou 2011; Gourio 2012; Croce 2014; Jaccard 2014). This paper distinguishes itself along an important dimension from these papers. In most of the aforementioned papers, asset price volatility is a feature of the first best allocation coming from preferences, technological constraints and shocks. Therefore, there is no role for the government as long as one assumes that the government is not able to change agents’ preferences or overcome the technological constraints. In our model, on the other hand, asset price volatility originates from financial constraints and shocks. The government could try to relax the financial constraint by, for instance, lending to entrepreneurs with investment opportunities. Although analyzing the optimal government policy in this class of models is of interest, the analysis is outside the scope of this paper (see Del Negro et al. 2011 for one such analysis).

2 The Model

Time is discrete and infinite. There are two types of agents: a unit measure of ex ante identical entrepreneurs who consume, produce, and hold financial assets, but do not work, and a unit measure of identical hand-to-mouth workers who work and consume, but do not hold assets. There are two types of goods and two production technologies: a consumption good and a capital good, and a technology to produce the consumption good and a technology to produce the capital good. There is one type of financial asset traded: claims to returns of capital. Each period is divided into two subperiods. In the first subperiod, the consumption good is produced. In the second subperiod, the capital good is produced and consumption
and asset trading take place.

Next, we describe the details of the two production technologies, the asset trading structure, and the financial friction that entrepreneurs face. We then present the entrepreneurs’ and workers’ optimization problems and define the competitive equilibrium.

2.1 Production Technologies

In the first subperiod of each time period $t$, the consumption good production takes place. All entrepreneurs have access to the consumption good production technology. Entrepreneurs face a stochastic productivity shock, denoted by $A_t$, which is common to all of them. Entrepreneur $T$ enters period $t$ with capital $k^T_t$, hires labor $l^T_t$, and produces the consumption good $y^T_t$ with the following technology ($\alpha$ is the capital share parameter):

$$y^T_t = A_t (k^T_t)^\alpha (l^T_t)^{1-\alpha}.$$

Capital depreciates at rate $\delta$ during the consumption good production, i.e., entrepreneur $T$ enters the second subperiod with capital holdings $(1 - \delta)k^T_t$.

In the second subperiod, only a fraction $\pi$ of entrepreneurs have the opportunity to start new projects. This “investment opportunity” is modeled as the entrepreneurs’ ability to access the capital good production technology. This technology enables the entrepreneurs to produce new capital one-to-one from the consumption good, which is standard in the real business cycle literature. In practice, apart from investment in the depreciated capital, firms adjust their capital stock by taking new projects. However, new projects are not always available. This technological constraint implies that an individual entrepreneur’s investment responds also to the entrepreneurs’ specific real opportunities rather than to the aggregate productivity shocks only. This assumption is further motivated by the empirical observation that only a small fraction of firms invest a lot in a given year. In the benchmark model, $\pi$ is assumed to be constant. The model is extended to the case when $\pi$ is stochastic in Section 6.3.
The arrival of the opportunity to access the capital good production technology is assumed to be i.i.d. over time and over entrepreneurs. Entrepreneurs with access to the capital good production technology are called \textit{investing entrepreneurs} and entrepreneurs without this access are called \textit{noninvesting entrepreneurs}.

Although the arrival of the opportunity to access the capital good production technology could be thought of as a version of an investment-specific technology (IST) shock, we should emphasize that this shock is quite different from the IST shock present in, for example, Papanikolaou (2011), as well as in New Keynesian models, see for example, Smets and Wouters (2007) and Christiano et al. (2014). In these models, the IST shock is aggregate and affects the economy’s production possibility frontier, whereas in our model, the investment shock, i.e., the arrival of an investment opportunity, is idiosyncratic and leaves the economy’s production possibility frontier unaffected.

2.2 \textbf{Trading and Financial Frictions}

In the second subperiod, consumption, capital good production, and asset trading take place. There is one type of financial asset traded: claims to capital returns (these claims are referred to simply as assets or equities).

Before we proceed with the discussion of the asset trading structure, it should be emphasized that the return per unit of capital is equal across entrepreneurs, independent of their capital holdings and independent of their opportunity to access the capital good production technology. Therefore, entrepreneurs are indifferent as to whose equity they hold.

To see the above claim, consider entrepreneur \(T\) with capital \(k^T_t\). In the first subperiod, he hires labor on a competitive labor market at wage \(w_t\) to maximize his profit, which can be written as

\[
profit(k^T_t; A_t, w_t) := A_t \left( k^T_t \right)^\alpha \left( l^T_t \right)^{1-\alpha} - w_t l^T_t.
\]

\(^2\)The i.i.d. assumption is made for simplicity and is common in the literature. In an environment similar to ours, Azariadis et al. (2015) show that making this shock persistent does not change the quantitative results.
The optimal behavior of entrepreneur $T$ implies that he hires labor $l_t^T = \left( \frac{(1-\alpha)A_t}{w_t} \right)^{\frac{1}{\alpha}} k_t^T$. This amount of labor equalizes the wage rate with the marginal product of labor

$$w_t = MPL_t = (1 - \alpha)A_t \left( k_t^T \right)^{\alpha} (l_t^T)^{-\alpha}.$$

Therefore, profit $(k_t^T; A_t, w_t) = \alpha A_t \left[ \frac{(1-\alpha)A_t}{w_t} \right]^{\frac{1-\alpha}{\alpha}} \cdot k_t^T = r_t k_t^T$, where $r_t = \alpha A_t \left[ \frac{(1-\alpha)A_t}{w_t} \right]^{\frac{1-\alpha}{\alpha}}$ denotes the return per unit of capital. Since all entrepreneurs face the same stochastic productivity shock, $A_t$, and hire labor at the same wage, $w_t$ (determined by aggregate labor market clearing), the return on capital, $r_t$, is the same for all entrepreneurs.

To explain the trading structure in the economy, we first describe the capital and asset holdings of the entrepreneurs. Entrepreneurs can hold physical capital and equity to other entrepreneurs’ capital returns. Let us define the individual state of entrepreneur $T$ by $(k_t^T, e_t^T, s_t^T)$, where $k_t^T$ is the physical capital held by the entrepreneur, $e_t^T$ is the equity to other entrepreneurs’ capital, and $s_t^T$ is equity to entrepreneur $T$’s capital sold to other entrepreneurs.

Physical capital $k_t^T$ is used by entrepreneur $T$ in the consumption good production and depreciates at rate $\delta$. Physical capital is not traded in the economy. Equity $e_t^T$ entitles entrepreneur $T$ to the stream of returns of $e_t^T$ units of other entrepreneurs’ capital. Since the underlying capital depreciates at rate $\delta$, one can think of $e_t^T$ as depreciating at rate $\delta$. Finally, $s_t^T$ denotes claims to capital returns sold by entrepreneur $T$, and one can think of these claims as depreciating at rate $\delta$ as well. Therefore, an entrepreneur with an individual state $(k_t^T, e_t^T, s_t^T)$ is entitled to returns from $k_t^T - s_t^T + e_t^T$ units of capital.

In the second subperiod, entrepreneurs face a financial friction, which restricts the amount of external financing. An investing entrepreneur that produces $i_t^T$ units of new capital can sell at most a fraction $\phi_t$ of returns from $i_t^T$. This means that an entrepreneur is able to finance only a fraction of his investment externally. This assumption is motivated by the empirical observation that firms do not fully finance their investments externally. We do not
take a stand on the underlying reasons although problems of asymmetric information have a long-standing tradition in the theory of capital structure in corporate finance.

We further assume that $\phi_t$ is a stochastic process common to all entrepreneurs. This assumption is motivated by an empirical observation that firms’ external financing is time varying. Although a theory that endogenizes the time variation in $\phi_t$ is of interest, this paper does not attempt to incorporate such a theory in the model. He and Krishnamurthy (2012) provide a micro-founded theory with a moral hazard problem between households and intermediaries. To solve the moral hazard problem, households offer incentive contracts to the intermediaries in which the intermediaries’ equity capital constraints are functions of their past performance. The time variation in past performance leads to time variation in financial constraints.

The financial shock in our model shares some similarities with the IST shocks in, for example, Justiniano et al. (2010), Justiniano et al. (2011), Papanikolaou (2011), and Kogan et al. (2013). In these models, a positive IST shock increases the marginal rate of transformation between consumption and investment. As the quantity of new investment increases, the price of existing capital falls, leading to increased volatility in returns. The IST shock affects quantities directly (the production possibility frontier for consumption and investment moves), and indirectly through the price channel. In our model, a positive financial shock relaxes the financial constraint making it possible for investing entrepreneurs to sell a larger fraction of their investments. As a result, they invest more and the price of the existing capital falls, leading to increased volatility in returns. However, in contrast to IST shocks, the marginal rate of transformation between consumption and investment (and hence the aggregate production possibility frontier) is not affected and only the price channel transfers the financial shocks to aggregate quantities.\footnote{Justiniano et al. (2011) argue that IST shocks are equivalent to financial shocks in a financial accelerator model of Carlstrom and Fuerst (1997). This equivalence, which is not true for the financial shocks we consider in this paper, appears to be driven by the fact that in Carlstrom and Fuerst (1997), unlike in our paper, financial shocks have direct real consequences (costs of monitoring).}
in line with the notion that the problem of asymmetric information is less severe for already existing projects. These assumptions imply that the aggregate amount of equity sold up until period $t$ (denoted as $s_{t+1}^T$) can be at most the sum of a fraction $\phi_t$ of period $t$ investment $i_t^T$ and the depreciated period $t$ capital holdings $(1 - \delta)k_t^T$, i.e.,

$$s_{t+1}^T \leq \phi_t i_t^T + (1 - \delta)k_t^T. \quad (1)$$

To understand this constraint, define $k_{t+1}^T = (1 - \delta)k_t^T + i_t^T$ and rewrite inequality (1) as

$$k_{t+1}^T - s_{t+1}^T \geq (1 - \phi_t)i_t^T. \quad (2)$$

The left-hand side of inequality (2) captures the net amount of claims to entrepreneur $T$’s own capital returns that he must carry into period $t + 1$. Since he can sell at most $\phi_t i_t^T$ of “new” equity, he must keep at least $(1 - \phi_t)i_t^T$ of the newly produced capital unsold, which is captured in the right-hand side of inequality (2).

### 2.3 Agents’ Optimization Problems

There is a unit measure of ex ante identical entrepreneurs, who hold capital, trade assets, and consume, but do not work. Preferences of the entrepreneurs are of the recursive Epstein-Zin form

$$v_t = \left[ (1 - \beta)c_t^{\frac{1-\gamma}{1-\psi}} + \beta \left( E_t \left[ v_{t+1}^{1-\gamma} \right] \right) \right]^{\frac{1}{1-\gamma}},$$

where $\theta = \frac{1-\gamma}{1-\psi}$ and $\psi$ is the coefficient of the (constant) elasticity of intertemporal substitution and $\gamma$ is the coefficient of the (constant) relative risk aversion.

Ex post, entrepreneurs will differ in their capital and asset holdings. The budget constraint of an entrepreneur with capital and asset holdings $(k_t^T, c_t^T, s_t^T)$ can be written as

$$c_t^T + i_t^T + q_t [k_{t+1}^T - s_{t+1}^T + c_{t+1}^T] \leq r_t [k_t^T - s_t^T + c_t^T] + (1 - \delta)q_t [k_t^T - s_t^T + c_t^T] + q_t i_t^T,$$
where \( r_t \) is the return on capital. The first term on the right-hand side is the return to which entrepreneur \( T \) is entitled. The second term is the market value of his depreciated unsold capital and asset holdings. The third term is the market value of equity to his newly installed capital at the market price \( q_t \). The left-hand side sums up his expenditure. He can consume \( c_t^T \geq 0 \), invest \( i_t^T \) with investment being generated one-to-one from the consumption good, and carry unsold equity to his own capital \( k_{t+1}^T - s_{t+1} \) and outside equity \( e_{t+1}^T \) into period \( t + 1 \). These are traded at market price \( q_t \).

Let \( IO_t^T \) be a random variable in period \( t \) with \( IO_t^T = 0 \) if entrepreneur \( T \) does not have an investment opportunity in period \( t \) and \( IO_t^T = 1 \) if he does have the opportunity. The maximization problem of this entrepreneur can then be written as (dropping the \( T \) superscripts for simplicity):

\[
\max_{\{c_t, i_t, k_{t+1}, s_{t+1}, e_{t+1}\geq 0\}} \left( (1 - \beta)c_t^T \frac{1}{1-\gamma} + \beta \left( Et \left[v_{t+1}^{1-\gamma}\right]\right)^{1-\gamma} \right) s.t.
\]

\begin{align*}
(\text{IC}) & \quad i_t = 0 \quad \text{if} \quad IO_t = 0, \\
(\text{BC}) & \quad c_t + i_t + q_t [k_{t+1} - s_{t+1} + e_{t+1}] \leq [k_t - s_t + e_t] [r_t + (1 - \delta)q_t] + q_t i_t, \\
(\text{FC1}) & \quad k_{t+1} - s_{t+1} \geq (1 - \phi_t) i_t, \\
(\text{FC2}) & \quad e_{t+1} \geq 0.
\end{align*}

In this problem, expectations are taken over the stochastic processes for \( \phi_t \) and \( A_t \), equilibrium processes for prices (taken as given and correctly forecasted by the entrepreneur), and the arrival of the investment opportunity \( IO_t \).

Note that the return on the unsold capital \( k_{t+1} - s_{t+1} \) and the return from claims to other entrepreneurs’ capital \( e_{t+1} \) are the same given the state of the economy. Moreover, trades in these assets in period \( t + 1 \) are not subject to any restriction. Therefore, inside equity \( k_{t+1} - s_{t+1} \) and outside equity \( e_{t+1} \) are perfect substitutes, and (FC1) binding is equivalent to the no-short-sales (FC2) binding, and they can be summed up without loss of generality.

The intuition for why (FC1) and (FC2) bind at the same time is as follows. An en-
entrepreneur who has the investment opportunity and whose (FC1) is binding will sell all his other assets \( e_t \) to take advantage of this profitable opportunity. Therefore, the maximization problem can be simplified by defining net asset holdings \( n_t = k_t - s_t + e_t \) and writing the entrepreneur’s problem as

\[
\max_{\{c_t, i_t, n_{t+1}\geq 0\}_{t=0}^{\infty}} \left[ (1 - \beta) c_t^{1-\gamma} + \beta \left( E_t [v_{t+1}^{1-\gamma}] \right)^{\frac{1}{\gamma}} \right]^{\frac{\gamma}{\theta}} \quad \text{s.t.}
\]

(1C) \quad i_t = 0 \quad \text{if} \quad IO_t = 0,

(BC) \quad c_t + i_t + q_t n_{t+1} \leq n_t [r_t + (1 - \delta) q_t] + q_t i_t,

(FC) \quad n_{t+1} \geq (1 - \phi_t) i_t.

There is also a unit measure of identical infinitely lived workers, i.e., agents who do not have access to consumption good and capital good production technologies. In each period, a worker decides how much to consume and how much labor to provide. For simplicity, workers are assumed not to participate in asset trading. A worker \( j \)'s maximization problem is thus static and can be written as

\[
\max_{\{c^{w,j}_t, l^{w,j}_t\geq 0\}} U \left( c^{w,j}_t - \frac{\omega}{1 + \eta} \left( l^{w,j}_t \right)^{1+\eta} \right) \quad \text{s.t.} \quad c^{w,j}_t \leq w_t l^{w,j}_t,
\]

where \( c^{w,j}_t \) is the consumption of worker \( j \) in period \( t \), \( l^{w,j}_t \) is the labor provided by worker \( j \) in period \( t \), the function \( U(.) \) is increasing and strictly concave, \( \omega > 0 \) and \( \eta > 0 \).

The setup of our model differs from the model presented in Kiyotaki and Moore (2011) (henceforth KM) along several dimensions. In our model, entrepreneurs have Epstein-Zin preferences and can only invest in equity. In KM, entrepreneurs have log utility and can hold both equity and fiat money. In addition, KM assume that the financial shock \( \phi_t \) is constant over time and focus on liquidity shocks: an entrepreneur can sell only a fraction of his existing equity holdings and this fraction is stochastic. In our model entrepreneurs are able to sell all of their existing equity holdings in every period.
2.4 Equilibrium

A competitive equilibrium is quantities for entrepreneurs \(\{c_{ij}^t, i_{ij}^t, n_{ij}^t, n_{ij+1}^t\}_{t=0}^\infty\) \(j \in [0, 1]\), quantities for workers \(\{c_{wij}^t, l_{wij}^t, \}_{t=0}^\infty\) \(j \in [0, 1]\), and prices \(\{q_t, r_t, w_t\}_{t=0}^\infty\), such that quantities solve workers’ and entrepreneurs’ problems given prices, input prices \(w_t, r_t\) are determined competitively, and markets clear.

3 Characterization of the Model

This section first clarifies the role of the financial friction and the scarcity of investment opportunities. Then it characterizes the solution to the workers’ and entrepreneurs’ optimization problems and studies the equilibrium aggregate dynamics. All proofs are provided in Appendix A.

3.1 Investment Opportunity, Financial Friction and Asset Prices

We start by establishing how the financial constraint interacts with the price of equity.

Proposition 1. The financial constraint binds for all investing entrepreneurs if and only if the price of equity \(q_t > 1\).

This proposition implies that the financial constraint does (not) bind for all investing entrepreneurs independent of their asset holdings. Why does the financial constraint bind when the price of equity is greater than one? If not, then a solution to the investing entrepreneur’s problem does not exist. This is because an investing entrepreneur finds it profitable to decrease his consumption by one unit, to increase his investment by one unit, and to sell claims to the newly produced capital at price \(q_t > 1\). He then increases his consumption by one unit back to the original level and ends up with a net profit of \(q_t - 1 > 0\). This behavior is not consistent with an equilibrium. Why is the price of equity greater than one when the financial constraint binds? A binding financial constraint means that investment is profitable for an entrepreneur with an investment opportunity. Investment is profitable only if the price of equity is larger than one.
The next proposition (whose formal proof is omitted) shows that both the scarcity of investment opportunities (i.e., \(\pi < 1\)) and the financial friction (i.e., \(\phi_t < 1\)) are necessary for the price of equity to be larger than one. Therefore, both the scarcity of investment opportunities and the financial friction are essential for generating asset price volatility in our model.

**Proposition 2.** (i) If all entrepreneurs in the economy have access to the capital good production technology in every period, i.e., \(\pi = 1\), then the price of equity is \(q_t = 1\).

(ii) If the investing entrepreneurs can finance all their investment externally, i.e., \(\phi_t = 1\), then, the price of equity is \(q_t = 1\).

The intuition for this proposition is as follows. If all entrepreneurs in the economy had the ability to invest in every period, then the price of equity is \(q_t = 1\). This is because no entrepreneur would be willing to pay more, given that he can produce new capital one-to-one from the consumption good. If the investing entrepreneurs could finance all their new investment externally, then the price of equity is \(q_t = 1\). This is because if \(q_t\) is larger than one, an investing entrepreneur has an arbitrage opportunity. He converts one unit of the consumption good to one unit of capital and sells claims to the returns to this newly produced capital at price \(q_t\). This way, he ends up with additional \(q_t - 1 > 0\) units of the consumption good. Repeating this strategy would lead to unbounded consumption which is not consistent with an equilibrium.

In the presence of aggregate shocks, one cannot determine analytically when the financial constraint binds. However, Lemma 1 shows under which conditions the financial constraint binds in a steady state equilibrium. A steady state equilibrium is defined as one in which the values of the exogenous shocks are constant, i.e., \(A_t = A\) and \(\phi_t = \phi\), and prices and aggregate endogeneous variables are constant as well.

**Lemma 1.** (i) If \(\pi < \delta(1 - \phi)\), then in the steady state equilibrium, the financial constraint binds and the price of equity is greater than one.
(ii) Assume that the entrepreneurs’ elasticity of intertemporal substitution is $\psi \geq 1$. If in the steady state equilibrium, the financial constraint binds and the price of equity is greater than one, then the parameters satisfy $\pi < \delta(1 - \phi)$.

This lemma suggests that $\pi$, $\phi$ and $\delta$ are the parameters that control the tightness of the financial constraint. If the fraction of entrepreneurs with an investment opportunity, i.e., $\pi$, is small, then the financial constraint binds. This is because with a small $\pi$, the supply of equity by investing entrepreneurs is small and the demand for equity by noninvesting entrepreneurs is large. As a result, the equity demand cannot be met unless the financial constraint binds. Similar arguments apply for $\phi$ and $\delta$. In particular, a small $\phi$ implies that a small fraction of investment can be financed externally and, therefore, the supply of equity is small. A large $\delta$ implies a large demand for new equity by noninvesting entrepreneurs, who want to replace their depreciated capital. In these cases, the financial constraint binds as well.

3.2 Characterizing the Solution to the Workers’ Problem

**Lemma 2.** The equilibrium aggregate labor, denoted by $L_t$, the equilibrium wage rate, denoted by $w_t$, and the equilibrium aggregate consumption by workers, denoted by $C^w_t$, are

$$L_t = \left[ \frac{A_t(1 - \alpha)}{\omega} \right]^{\frac{1}{1+\eta}} K_t^{\frac{\alpha}{\alpha + \eta}} \omega^\frac{\alpha}{\alpha + \eta} [(1 - \alpha)A_t]^{\frac{\eta}{\eta + \alpha}} K_t^{\frac{\eta}{\eta + \alpha}}, \quad w_t = \omega^\frac{\alpha}{\alpha + \eta} [(1 - \alpha)A_t]^{\frac{\eta}{\eta + \alpha}} K_t^{\frac{\eta}{\eta + \alpha}}, \quad C^w_t = (1 - \alpha)Y_t,$$

where $K_t$ denotes aggregate capital stock and $Y_t$ denotes aggregate output.

This lemma shows that the aggregate labor is a function of workers’ utility parameters, production function parameters and aggregate states $K_t$ and $A_t$. In particular, aggregate labor in period $t$ does not depend on the financial shock $\phi_t$ in period $t$. Therefore, in period $t$, aggregate output $Y_t = A_tK_t^\alpha L_t^{1-\alpha}$ is not a function of $\phi_t$, i.e. $Y_t = Y(K_t, A_t)$. Similar
reasoning holds for the return on capital $r_t = MPK_t = \frac{\partial Y_t}{\partial K_t}$, i.e., $r_t = r(K_t, A_t)$. Specifically:

$$Y_t = Y(K_t, A_t) = A_t K_t^\alpha \left( \frac{A_t(1 - \alpha)}{\omega} \right)^{\frac{1}{\alpha + \eta}} K_t^{\frac{\alpha}{\alpha + \eta}}^{1 - \alpha}, \quad (3)$$

$$r_t = r(K_t, A_t) = A_t \alpha K_t^{\alpha - 1} \left( \frac{A_t(1 - \alpha)}{\omega} \right)^{\frac{1}{\alpha + \eta}} K_t^{\frac{\alpha}{\alpha + \eta}}^{1 - \alpha}. \quad (4)$$

**Corollary 1.** The joint dynamics of output and labor and output and workers’ consumptions satisfy

$$\rho(\log L_t, \log Y_t) = 1, \quad \text{var}(\log L_t) = \frac{1}{(1 + \eta)^2} \text{var}(\log Y_t),$$

$$\rho(\log C^w_t, \log Y_t) = 1, \quad \text{var}(\log C^w_t) = \text{var}(\log Y_t),$$

where \( \text{var}(x) \) denotes the variance of variable \( x \) and \( \rho(x, y) \) denotes the correlation between variables \( x \) and \( y \).

This corollary shows that the relative variance of labor and output is determined by the labor supply elasticity parameter \( \eta \), whereas the variance of workers’ consumption is the same as the variance of output. Given that workers account for a large fraction of aggregate consumption in the economy (the combined workers’ and entrepreneurs’ consumption), the dynamics of workers’ consumption will significantly affect the dynamics of aggregate consumption relative to output.

### 3.3 Characterizing the Solution to the Entrepreneurs’ Problem

Our model is one with heterogeneous entrepreneurs. Entrepreneurs differ in their wealth depending on their individual sequences of the idiosyncratic investment opportunity shocks. However, one can solve for aggregate dynamics without having to keep track of the whole wealth distribution. This is because homotheticity of the Epstein-Zin utility function and the linearity of the budget constraint imply linear decision rules. We omit the proof of this
well-known result summarized by the following lemma.\footnote{This result was first derived by Samuelson (1969). For an extension to an environment with entrepreneurial investment risk and Epstein-Zin utility similar to ours, see Angeletos (2007).}

**Lemma 3.** The policy functions describing an individual entrepreneur’s optimal decisions are

\[
    c^i_t = \zeta^i_t n_t (r_t + (1 - \delta)q_t),
\]

\[
    q^R_t n^i_{t+1} = (1 - \zeta^i_t) n_t (r_t + (1 - \delta)q_t),
\]

\[
    c^s_t = \zeta^s_t n_t (r_t + (1 - \delta)q_t),
\]

\[
    q_t n^s_{t+1} = (1 - \zeta^s_t) n_t (r_t + (1 - \delta)q_t),
\]

where $\zeta^i_t$ and $\zeta^s_t$ are the period $t$ consumption-to-wealth ratios of the investing and noninvesting entrepreneurs, respectively, and $n_t$ denotes the period $t$ initial asset holdings of an entrepreneur. Superscript $i$ denotes the state in which this entrepreneur has an investment opportunity in period $t$, and superscript $s$ denotes the state in which he does not have an investment opportunity in period $t$.

Equations (7)-(8) summarize the behavior of the noninvesting entrepreneurs as implied by homotheticity of Epstein-Zin preferences. Equations (5)-(6) summarize the behavior of the investing entrepreneurs. These equations follow from the fact that an investing entrepreneurs’ budget constraint can be expressed as

\[
    c^i_t + q^R_t n^i_{t+1} \leq n_t [r_t + (1 - \delta)q_t],
\]

where $q^R_t$ is the replacement cost of capital and is defined as $q^R_t = (1 - \phi_t q_t)/(1 - \phi_t) \leq 1 \leq q_t$. If $q_t > 1$ then the first inequality is strict, i.e., $q^R_t < 1$. The difference between $q^R_t$ and $q_t$ means that investing and noninvesting entrepreneurs face different effective price of next period’s assets $n_{t+1}$, which gives rise to the difference between $\zeta_t^i$ and $\zeta_t^s$ when the elasticity of intertemporal substitution is different from one (see Lemma 4).
Why is $q_t^R \leq q_t$? The investing entrepreneurs can create new capital from consumption at price one. Consider an entrepreneur with investment opportunity who has one unit of consumption. He can take this unit of consumption, convert it to capital and sell fraction $\phi_t$ of the claims to the returns of this capital at price $q_t$. He then has $1 - \phi_t$ units of capital and $\phi_t \times q_t$ units of consumption. He can then convert $\phi_t \times q_t$ units of consumption to capital, sell fraction $\phi_t$ of that capital’s returns and keep the rest. Repeating this strategy implies that, at the end, the investing entrepreneur will have $(1 - \phi_t)/(1 - \phi_t q_t)$ units of capital which he ‘generated’ from one unit of consumption by taking advantage of his investment opportunity. This implies that his effective price of one unit of capital (his next period’s assets $n_{t+1}$) is $(1 - \phi_t q_t)/(1 - \phi_t) \leq 1$, with a strict inequality if $q_t > 1$. 

Lemma 3 implies that all entrepreneurs of the same type $j \in \{i, s\}$ have the same consumption (savings) to wealth ratio $c_t^j/ew_t$, where an entrepreneur’s wealth, denoted by $ew_t$, is defined as $ew_t := n_t(r_t + (1 - \delta)q_t)$. Observe that $\zeta_t^i$ and $\zeta_t^s$ (as well as $q_t$ and $q_t^R$) are in fact time-invariant functions of aggregate states $(N_t, A_t, \phi_t)$, i.e., $\zeta_t^i(N_t, A_t, \phi_t)$ and $\zeta_t^s(N_t, A_t, \phi_t)$, where $N_t$ denotes the aggregate equity holdings by the entrepreneurs, which is equal to the aggregate capital stock, i.e., $N_t = K_t$. The next proposition derives the equations that determine the dynamic behavior of $\zeta_t^i$ and $\zeta_t^s$.

**Proposition 3.** The entrepreneurs’ consumption-to-wealth ratios $\zeta_t^i$ and $\zeta_t^s$ are described recursively by the following system of equations:

\[
(1 - \beta)(\zeta_t^i)^{1 - \gamma - \theta} = (1 - \beta)(\zeta_t^i)^{1 - \gamma} + \beta \left( \pi E \left[ (1 - \beta) \frac{\theta}{\gamma} (\zeta_{t+1}^i)^{1 - \gamma - \theta} \frac{(1 - \zeta_t^i)(q_{t+1}(1 - \delta) + r_{t+1})}{q_t^R} \right] \right) \left( 1 - \gamma - \theta \right), \tag{9}
\]

\[
(1 - \theta)(\zeta_t^s)^{1 - \gamma - \theta} = (1 - \beta)(\zeta_t^s)^{1 - \gamma} + \beta \left( \pi E \left[ (1 - \beta) \frac{\theta}{\gamma} (\zeta_{t+1}^s)^{1 - \gamma - \theta} \frac{(1 - \zeta_t^s)(q_{t+1}(1 - \delta) + r_{t+1})}{q_t} \right] \right) \left( 1 - \gamma - \theta \right), \tag{10}
\]
Equation (9) and equation (10) describe the dynamic behavior of the optimal policies \( \zeta^s_t \) and \( \zeta^i_t \) as a system of first order difference equations. The two equations are important components of our solution method. The following lemma characterizes the consumption-to-wealth ratios \( \zeta^s_t \) and \( \zeta^i_t \).

**Lemma 4.** (i) Suppose that \( q_t = 1 \). Then \( \zeta^s_t = \zeta^i_t \). (ii) Suppose that \( q_t > 1 \). Then (1) if \( \psi < 1 \), then \( \zeta^s_t < \zeta^i_t \), (2) if \( \psi = 1 \), then \( \zeta^s_t = \zeta^i_t \), and (3) if \( \psi > 1 \), then \( \zeta^s_t > \zeta^i_t \).

This lemma suggests that the entrepreneurs’ EIS, i.e., \( \psi \), plays an important role in the dynamic behavior of the model. In particular, the results suggest that the volatility of entrepreneur’s consumption and the volatility of aggregate investment depend on the value of EIS. Why are the consumption-to-wealth ratios of investing and noninvesting entrepreneurs the same when the price of equity is \( q_t = 1 \)? If \( q_t = 1 \), then \( q_t^R = 1 \) and having an investment opportunity is not profitable. Therefore, having an investment opportunity does not affect the behavior of an entrepreneur with such opportunity. As a result, the optimal decisions of both types of entrepreneurs are the same, i.e., \( \zeta^i_t = \zeta^s_t \). Why is the consumption-to-wealth ratio of investing entrepreneurs smaller than that of noninvesting entrepreneurs when the elasticity of intertemporal substitution is \( \psi > 1 \)? With a high elasticity of intertemporal substitution, \( \psi > 1 \), investing entrepreneurs are willing to accept larger fluctuations in their consumption. Therefore, they invest relatively more and consume relatively less than noninvesting entrepreneurs to take advantage of the profitable investment opportunity, i.e., \( \zeta^i_t < \zeta^s_t \). The logic is opposite when \( \psi < 1 \).

### 3.4 Aggregation and Equilibrium Dynamics

The previous section derives optimal entrepreneurs’ policies. To solve for equilibrium, they need to be aggregated over all entrepreneurs and combined with market clearing conditions.

With linear policy rules, prices are functions of aggregate quantities only. We denote aggregate quantities with capital letters. Given the fact that the arrival of the investment opportunity is i.i.d., entrepreneurs with an investment opportunity hold a fraction \( \pi \) of
aggregate assets in the economy at the beginning of period $t$. Investors without an investment opportunity hold a fraction $1 - \pi$ of aggregate assets at the beginning of period $t$. The evolution of aggregate asset holdings is characterized by the following lemma, where the $t+1$ period’s aggregate assets $N_{t+1}$ is a time-invariant function of the period $t$ states $(N_t, A_t, \phi_t)$.

**Lemma 5.** The dynamics of aggregate asset holdings is characterized by the following equation:

$$N_{t+1}(N_t, A_t, \phi_t) = (1 - \delta)N_t + \alpha Y(N_t, A_t) - [\zeta^i(N_t, A_t, \phi_t)\pi + \zeta^s(N_t, A_t, \phi_t)(1 - \pi)]N_t[r(N_t, A_t) + (1 - \delta)q_t]$$  \hspace{1cm} (11)

Equation (11) is a rewrite of the goods market clearing condition, and hence guarantees that the goods market clears. If the states $(N_t, A_t, \phi_t)$ are such that $q_t = 1$, equation (11) still applies, but can be simplified since in this case $\zeta^i(N_t, A_t, \phi_t) = \zeta^s(N_t, A_t, \phi_t)$.

The equilibrium price of equity is determined by a market clearing condition, which equates the demand for equity by noninvesting entrepreneurs with the supply for equity by investing entrepreneurs. The properties of the equilibrium price of equity $q_t$ are summarized by the following proposition (to simplify notation, the expressions below suppress the dependence of $\zeta^i_t, \zeta^s_t$ and $r_t$ on the states).

**Proposition 4.** The equilibrium price of equity is

$$q_t = \max(1, q^*_t),$$  \hspace{1cm} (12)

where $q^*_t$ is the solution to a quadratic equation: $a_2 q_t^2 + a_1 q_t + a_0 = 0$, where

$$a_0 = -(1 - \zeta^i_t)(1 - \pi)r_t, \quad a_1 = (1 - \delta)[1 - (1 - \zeta^i_t)(1 - \pi)] + \phi_t r_t[(1 - \zeta^i_t)\pi + (1 - \zeta^s_t)(1 - \pi)],$$

and

$$a_2 = (1 - \delta)\phi_t [(1 - \zeta^i_t)\pi + (1 - \zeta^s_t)(1 - \pi) - 1].$$

The relevant root is $q^*_t = \frac{-a_1 + \sqrt{a_1^2 - 4a_0a_2}}{2a_2}$.

Proposition 3, Lemma 5 and Proposition 4 imply that the aggregate dynamics of the economy is fully characterized by equations (9), (10), (11) and (12) (along with the definitions...
of $Y$ and $r$ which are functions of the aggregate states $(N_t, A_t)$, see equations (3) and (4)).

Equations (9) and (10) guarantee that the entrepreneurs’ utility maximization problem is solved, equations (11) and (12) guarantee that goods and equity markets clear, and equations (3) and (4) imply that workers utility maximization problem is solved and the labor and capital markets clear. Solving the system of equations (9), (10), (11) and (12) gives the equilibrium policies $\zeta^i(N_t, A_t, \phi_t)$ and $\zeta^s(N_t, A_t, \phi_t)$, the equilibrium equity price $q(N_t, A_t, \phi_t)$ and the law of motion for the evolution of the aggregate capital stock $N_{t+1}(N_t, A_t, \phi_t)$.

Given an initial state $(N_0, A_0, \phi_0)$, these are sufficient to determine the aggregate equilibrium dynamics. The dynamics of the remaining variables can be easily recovered. Appendix B discusses how we solve the system of equations (9), (10), (11) and (12).

Even though there is no closed form solution for the case of the general Epstein-Zin preference specification, Proposition 4 implies that productivity shocks $A_t$ as well as financial shocks $\phi_t$ lead to movements in asset prices (as long as the financial constraint binds). The quantitative importance of these shocks for asset prices is analyzed in Section 5.2.

The following corollary characterizes the equilibrium dynamics for the special case of the elasticity of intertemporal substitution equal to one, for which a closed form solution exists, which makes the link between shocks and asset prices very transparent.

**Corollary 2.** If the entrepreneurs’ elasticity of intertemporal substitution is $\psi = 1$ then the equilibrium price of equity is $q_t = \max(1, q^*_t)$, where $q^*_t$ is the solution to a quadratic equation:

$\alpha_2q_t^2 + \alpha_1q_t + \alpha_0 = 0$, where $\alpha_2 = -(1 - \delta)\phi_t(1 - \beta)$, $\alpha_1 = (1 - \delta)[1 - \beta(1 - \pi)] + \beta\phi_t r_t$, $\alpha_0 = -\beta(1 - \pi)r_t$. The relevant root is $q^*_t = \frac{-\alpha_1 + \sqrt{\alpha_1^2 - 4\alpha_2\alpha_0}}{2\alpha_2}$. The equilibrium capital dynamics is characterized by $N_{t+1}(N_t, A_t, \phi_t) = (1 - \delta)N_t + \alpha Y(N_t, A_t) - (1 - \beta)N_t[r(N_t, A_t) + (1 - \delta)q_t]$.  

This corollary fully describes the aggregate dynamics of the economy, because all other variables can be determined as functions of $q_t$ and the state variables. Although full capital depreciation is not a realistic assumption, the next corollary, which follows directly from Corollary 2, highlights the importance of financial shocks for asset price volatility when capital fully depreciates in every period.
Corollary 3. If the entrepreneurs’ intertemporal elasticity of substitution is \( \psi = 1 \), capital fully depreciates in every period and the financial constraint binds, then the price of equity is \( q_t = (1 - \pi)/\phi_t \).

This result implies that the price of equity depends only on the realization of the financial shock \( \phi_t \). In particular, \( \text{var}(\log q_t) = \text{var}(\log \phi_t) \). Moreover, equation (11) implies that, in this case, investment and other aggregate variables are independent of \( \phi_t \) (they are determined by the productivity shock only). Financial shocks in this version of the model only affect the price of equity and the distribution of consumption and wealth between investing and noninvesting entrepreneurs.

4 Calibration of the Model

This section summarizes the calibration procedure. One period in the model corresponds to one quarter. In the benchmark calibration, the share of capital in output production is \( \alpha = 0.36 \). The quarterly depreciation rate is \( \delta = 2.26\% \) so that the average annual investment to capital ratio is 9.35% as in the data for the period from 1964 to 2013. For the workers’ utility function parameters, the inverse labor supply elasticity parameter is \( \eta = 2 \). The scaling parameter of the workers’ utility function is \( \omega = 21.74 \) so that the labor supply in steady state is \( l_s = 1/3 \) (\( \omega \) is just a scaling parameter, and none of the reported statistics are affected by its value). For the entrepreneurs’ utility function parameters, the quarterly discount factor is \( \beta = 0.99 \). The intertemporal elasticity of substitution parameter is \( \psi = 0.5 \), close to Vissing-Jorgensen (2002)’s empirical estimate for stockholders, and the risk aversion parameter is \( \gamma = 2 \). This implies that in the benchmark parameterization, entrepreneurs have a time separable CRRA utility function.

The literature has documented several aspects of “infrequent” and “large” capital adjustment (see, e.g., Doms and Dunne 1998). Although this type of capital adjustment has typically been taken as evidence of the existence of fixed costs of investment, it can also be thought of as evidence of an infrequent arrival of investment opportunities. Therefore, \( \pi \) is
calibrated by matching it to the fraction of firms with an investment spike in the data. We made this choice because in our model firms with an investment opportunity generally invest a lot relative to their size. Given that the definition of an investment spike in the literature is not unique, similar to Gourio and Kashyap (2007), we use two definitions: investment exceeding 20% and investment exceeding 35% of the beginning of the period capital. The time series for investment is constructed as an increase in “Net property, plant and equipment,” i.e., variable \( ppent \) in the COMPUSTAT database, \( investment_t = ppent_t - ppent_{t-1} \). We then determine the fraction of firms whose investment at time \( t \) exceeds a given fraction of \( ppent_{t-1} \). As in Gourio and Kashyap (2007), firms are weighted by beginning of the period capital \( ppent_{t-1} \). We find that in 1965-2013 on average 4.3% (10.6%) percent of firms’ investment exceeds 35% (20%) of their initial capital. We set the annual \( \pi \) to an intermediate level of 6% in the benchmark (i.e., quarterly \( \pi = 1.5\% \)).

To calibrate the productivity shocks, \( A_t \), and the financial shock, \( \phi_t \), we assume that they follow the following processes:

\[
\begin{align*}
\log A_{t+1} &= \rho_A \log A_t + \varepsilon_{A,t}, \\
\log \phi_{t+1} &= \log \mu_{\phi} + \rho_{\phi} (\log \phi_t - \log \mu_{\phi}) + \varepsilon_{\phi,t},
\end{align*}
\]

where \( \varepsilon_{A,t} \) and \( \varepsilon_{\phi,t} \) are normally distributed random variables with standard deviations \( \sigma_{\varepsilon_A} \) and \( \sigma_{\varepsilon_{\phi}} \), respectively, which are i.i.d. over time. The correlation coefficient between \( \varepsilon_{A,t} \) and \( \varepsilon_{\phi,t} \) is denoted by \( \rho_{A,\phi} \). Mean \( A_t \) is normalized to 1 without loss of generality and \( \rho_A \) and \( \varepsilon_{A,t} \) are estimated outside of the model using the standard procedure. In particular, the time series for productivity shocks, \( A_t \), is constructed using the time series of output, capital, and labor with the assumption of a Cobb-Douglas production technology with the capital output share of \( \alpha = 0.36 \). The two parameters are then estimated using the linearly detrended version of \( \log A_{t+1} = \rho_A \log A_t + \varepsilon_{A,t} \). Similar to previous studies, we find that \( \rho_A \) is close to 0.95. Our estimate of \( \sigma_{\varepsilon_A} \) is approximately 0.006, implying that the productivity
shock $A_t$ varies approximately by 1.5% on a quarterly basis. The persistence of $\phi_t$ is set to $\rho_\phi = 0.95$, which is the same value as we use for the technology shocks.

The remaining parameters $\mu_\phi, \sigma_{\varepsilon_\phi}$ and $\rho_{A,\phi}$ are then calibrated so that the simulated model generates moments consistent with the data. The properties of the financial shock process are well identified by the properties of aggregate investment and therefore, we use the standard deviation of aggregate investment and the correlation of output with investment as targets. The third moment used as target is the average (implicit) risk-free rate, which well identifies the mean of log $\phi_t$, i.e., $\mu_\phi$. Since we use a process in log for $\phi_t$, $\mu_\phi$ is not equal to the average realized $\phi_t$. Given our approximation procedure, the average $\phi_t$ is 0.2357, i.e., an investing entrepreneur can on average sell about a fourth of his project as equity.

Table 1 reports the benchmark parameters and Table 2 reports the calibration moments.

5 Main Quantitative Results

This section studies the quantitative implications of the model. It shows that the model matches well both macroeconomic quantities and asset prices.

5.1 Macroeconomic Quantities

We start by studying the implications of the model for standard business cycle statistics. These results are reported in Table 3. The data column reports the U.S. statistics for the period 1964-2013. Details of the construction of the time series can be found in Appendix C. Column (1) reports the statistics for the benchmark model with the financial constraint

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5To compare model generated statistics with the data, we simulate the model starting from steady state for 100 years (400 periods) and discard the first 50 years (200 periods) so as to eliminate the effect of initial conditions. This way, the model generated data has the same length as the true data. We then repeat this procedure 10,000 times and report the means and standard deviations over the 10,000 repetitions.

6In general, the estimates of these moments in the data vary depending on the time period chosen. The volatility of investment relative to output varies between 2.90 and 3.05, the correlation between output and investment varies between 0.91 and 0.94 and the risk-free rate varies between 0.23 and 0.25. The model generated means are always within these bounds.

7The model implies a fairly high standard deviation for the estimate of the relative volatility of output and investment, and, in particular, for the estimate of the risk-free rate. The standard deviations decrease substantially, if one increases the number of periods the model is simulated for. In those cases, the mean estimates are very close to those reported here.
and both productivity shocks and financial shocks, i.e., both $A_t$ and $\phi_t$ are stochastic. To explain the role of the financial constraint and financial shocks in the model, Table 3 reports the results from two additional versions of the model. Column (2) reports the statistics for a version of the model with the financial constraint and productivity shocks. In this model, there are no financial shocks, i.e., $\phi_t$ is constant at its mean level of 0.2357. This model is very similar to the one presented in Kiyotaki and Moore (2005) in which a productivity shock, amplified by a financial constraint, is the only source of business cycle fluctuations. Column (3) reports the statistics for a version of the model without the financial constraint. This model is closely related to the standard stochastic one sector growth model.

**Consumption Volatility.** Table 3 shows that the benchmark model matches the relative volatility of aggregate consumption, which was not targeted. This is an important result, since for a production based model to offer a plausible mechanism to explain asset prices, it should be consistent with the consumption volatility observed in the data. Moreover, adding financial shocks, i.e., comparing column (1) to column (2), decreases the relative volatility of aggregate consumption and the correlation of aggregate consumption with output, bringing them closer to the data. This is because a positive productivity shock (associated with high output) is likely to be accompanied by a positive financial shock (due to the positive correlation of the two shocks), making it possible for entrepreneurs to invest more. Higher investment, however, comes at the cost of lower entrepreneurs’ consumption, decreasing aggregate consumption volatility and the correlation of aggregate consumption with output.

In our model, workers do not participate in asset markets and hence entrepreneurs bear all the asset market risk. Hence, for the model to be quantitatively plausible, it is critical that it also matches the magnitude of consumption risk that entrepreneurs face. In the benchmark model, the standard deviation of entrepreneurs’ consumption is 3.58% while the standard deviation of workers’ consumption is 1.01%, implying that the ratio $\sigma_{C^e}/\sigma_{C^w}$ is about 3.56. In the data, the consumption of stock markets participants is very volatile as documented by Mankiw and Zeldes (1991), among others. This literature finds that depending on the
particular definitions of participation and consumption, the volatility of consumption of stock market participants is 1.5 to 5 times larger than that of non-participants (see Guvenen 2009). Our benchmark model is thus broadly in line with this empirical evidence. The two models presented in column (2) and column (3) generate lower entrepreneurs’ than workers’ consumption volatility.

**Investment Volatility.** Looking at column (1) of Table 3, one can see that the benchmark model matches the relative volatility of aggregate investment (which was a targeted moment). Comparing columns (1), (2), and (3) also shows that investment is significantly affected by the financial constraint and shocks. In the model without the financial constraint, shown in column (3), investment is significantly less volatile than in the benchmark model with financial shocks. In the model with the financial constraint but no financial shocks, shown in column (2), investment volatility is further decreased by the endogenous changes in the price of equity. These results indicate that in our model, financial shocks play a more important role than productivity shocks in investment fluctuations.

**Output and Labor Volatility.** Comparing columns (1), (2), and (3) of Table 3 sheds more light on the role of the financial shocks and the financial constraint in macroeconomic fluctuations. The table shows that the presence of neither the financial constraint nor the financial shocks affect the properties of aggregate labor and output. In contrast, in a recent paper, Jermann and Quadrini (2012) build a model in which financial shocks significantly affect output but not asset prices. Our results indicate that the process for output (and labor) in our model is determined by the process for the productivity shock (assumed to be the same in all three versions of the model). All three versions of the model generate a lower volatility of aggregate labor (and output) relative to the data.

**Persistence and Cyclicality.** The benchmark model generates high persistence in major macroeconomic variables. Similar to frictionless real business cycle models, the benchmark model comes short of fully accounting for the high persistence observed in the data. This
is true also for the other two models that we analyze. All three models imply high (un-
conditional) correlations between output and investment, consumption and labor, consistent 
with the correlations observed in the data. In addition, all three models generate a high 
positive correlation between consumption and investment. This finding is further discussed 
in Section 6.1.

5.2 Asset Prices

We next study the implications of the model for asset prices. Table 4 reports the asset price 
statistics for the calibrated model. Similar to Table 3, the data column reports the U.S. 
statistics for the period 1964-2013. Details of the construction of the time series can be 
found in Appendix C. Column (1) reports the statistics for the benchmark model with the 
financial constraint and both productivity shocks and financial shocks, column (2) reports the 
statistics for a version of the model with the financial constraint and productivity shocks, 
but no financial shocks, and column (3) reports the statistics for a version of the model 
without the financial constraint.

**Volatility of Equity Price and Return.** To compare the model with the data, the return 
on equity in the model is defined as 
\[ r_t^e = \frac{r_t + (1 - \delta)q_t}{q_{t-1}} - 1. \]

Our most important result 
shown in Table 4 is that financial shocks generate high asset price and return volatility, which 
is comparable to the data. The benchmark model of column (1) generates around 70% of 
the observed volatility in asset prices, asset returns and total market value (defined in the 
model as \(\text{val}_t = q_t N_t\)). This is an important finding given that the model also matches the 
relative volatility of aggregate and entrepreneurs’ consumption.

Why are the asset prices volatile in our model? This is because financial shocks change 
the tightness of the financial constraint which causes movements in asset prices. To see this 
point, observe that by combining the first order conditions with respect to \(n_{t+1}\) and \(i_t\) in 
the maximization problem of an investing entrepreneur, one can derive the following two
where $\mu_t$ denotes the Lagrange multiplier on the financial constraint and $\lambda_t$ denotes the Lagrange multiplier on the period $t$ budget constraint, which equals to the marginal utility of consumption of the investing entrepreneurs. The summation is taken over aggregate states that follow from the current state with positive probability. Equation (13) shows how the tightness of the financial constraint, $\mu_t$, is related to the financial shock, $\phi_t$, and the residual in the entrepreneur’s Euler equation (the right-hand side of equation (13)). Equation (13) suggests that a lower value of the financial shock $\phi_t$ is associated with a larger $\mu_t$, i.e., a tighter financial constraint. Equation (14) relates the price of equity $q_t$ to the expected equity payoff discounted by the ratio of marginal utilities (the first summand on the right-hand side) and the relative tightness of the financial constraint. The first term is standard. The second term suggests that, independently, the price of equity also varies if the tightness of the financial constraint (relative to the tightness of the budget constraint) varies.

Table 4 also shows that the model with the financial constraint, but no financial shocks, column (2), generates a small volatility in equity returns. The standard deviation of the equity return is 0.76%, which is similar to Gomes et al. (2003). They get an equity return volatility of about 1% in a calibrated model with endogenous borrowing constraints and productivity shocks as the only source of uncertainty. This result highlights that in our model (as well as in theirs), the financial constraint is not a strong propagator of productivity shocks (this result is, in fact, common in various environments, see for instance, Kocherlakota 2000; Cordoba and Ripoll 2004; and Khan and Thomas 2013, among others).

A comparison between column (1) and column (3) highlights how the financial constraint amplifies financial shocks. Column (3) shows that the model without the financial constraint
generates a limited volatility of 0.06% in equity returns. Recall that in this version of the model, the price of equity is \( q_t = 1 \) at all times. Therefore, there are no capital gains and the equity return volatility comes solely from the modest volatility in \( r_t \). Since in this model, there is no financial constraint, even if there were financial shocks they would have no effect. Column (1) then shows how financial shocks are amplified by the emergence of a (binding) financial constraint. Indeed, amplification is large. These amplification results are related to Gourio (2013)' model in which standard productivity shocks are not amplified, but other shocks (namely shocks to beliefs about disasters) are amplified substantially through credit frictions.

One might think that the high asset price volatility generated by our benchmark model of column (1) follows simply from the high volatility of the financial shocks. We should emphasize that this is not the case. First, in our model, asset price volatility follows from the interaction of financial shocks and the random arrival of investment opportunities. If all entrepreneurs had investment opportunities in each time period, the financial constraint would not bind, the price of equity would be one, and the volatility of financial shocks would be irrelevant.

Second, when the financial constraint does not bind, the size of the financial shock itself is irrelevant. In the states in which the constraint binds, the standard deviation of the financial shock is approximately 6%, about four times larger than the standard deviation of the productivity shock. In addition, we show in Section 6.2 that changing the volatility of the financial shock around the benchmark value has a small impact on the results. This is because increasing the volatility of the financial shocks primarily changes the values of the financial shock in states in which the financial constraint does not bind.

**Volatility of Risk-Free Rate.** We compare the risk-free rate in the data with the shadow risk-free rate of the noninvesting entrepreneurs. Noninvesting entrepreneurs are not constrained in their asset holdings, whereas investing entrepreneurs would like to sell more assets, but they cannot because of the financial constraint. This approach is similar to the
one taken in Gomes et al. (2003). One could rationalize this choice by assuming that (investing) entrepreneurs are not allowed to issue bonds. In equilibrium, bonds would not be traded and the risk-free rate would be determined by the shadow risk-free rate of the noninvesting entrepreneurs. This is because investing entrepreneurs find investing and selling equity more profitable than buying the risk-free asset.

The (shadow) risk-free return is defined as: $r^f_t = \frac{1}{E_t} \left[ \frac{v'(c^*_t)}{v'(c^*_t)} \right]$. Recall that the model is calibrated so that the average (shadow) risk-free rate matches the data. Table 4 shows that the benchmark model with financial shocks generates a volatility in the quarterly (shadow) risk-free rate which is higher than the one computed for the 1964 - 2013 sample period. The same is true for the volatility of the annualized risk-free rate. Our benchmark model generates an annual risk-free rate volatility of 7.41%, while in the 1964 - 2013 sample it is 2.35%. However, using longer samples, other authors have used a risk-free rate volatility of 5.5% when making model to data comparisons (see, e.g., Jermann 1998; Boldrin et al. 2001; Gomes and Michaelides 2008; and Guvenen 2009). Our model comes close to matching that number and makes an improvement relative to some of the models in the literature (for example, relative to Jermann 1998; Boldrin et al. 2001; Christiano and Fisher 2003), but it falls short of fully explaining the low volatility of the risk-free rate.\footnote{In an endowment economy, Bansal and Yaron (2004) argue that it is necessary to choose an EIS>1 to explain the behavior of asset prices and, in particular, the low risk-free rate volatility. Increasing the EIS above one in our model reduces the volatility of the risk-free rate, but it significantly increases the volatility of aggregate consumption and investment. Croce (2014) obtains a standard deviation of 0.94% for the annual risk-free rate with EIS=2 and a standard deviation of 23.59% with EIS=0.9 in a production based model with long-run risk. However, the empirical evidence suggests that the EIS is well below one (as in our benchmark calibration), see, for example, Vissing-Jorgensen (2002) and Guvenen (2009).}

**Equity Premium and Sharpe Ratio.** Column (1) of Table 4 shows that the benchmark model with a financial constraint and both productivity and financial shocks generates a quarterly equity premium of 0.36% compared to 1.52% in the data.\footnote{In the real world, firms issue both equity and debt, but our model features only equity. In such a case, researchers often adjust the equity return and volatility by a factor of 5/3, see, for instance, Boldrin et al. (2001), Papanikolaou (2011), and Gourio (2012), among others. Applying this adjustment would result in an equity premium of 0.61%, much closer to the data.} The model generates a quarterly Sharpe ratio of 6% compared to 17% in the data. These results imply that the
model is relatively more successful in capturing the amount of risk in equity returns than in capturing the price of risk (a finding similar to Papanikolaou 2011’s model with IST shocks). The equity premia (and the Sharpe ratios) in the other two versions of our model, i.e., in the model with a financial constraint but without financial shocks shown in column (2) of Table 4, and in the model without the financial constraint shown in column (3) of Table 4, are negligible.

The benchmark model with financial shocks explains a significant fraction of the equity premium while matching the (relative) volatility of aggregate consumption. What is the mechanism that generates the equity premium in the model? It is a combination of two features of the model. First, since workers do not participate in asset trading, asset returns are determined by the consumption dynamics of the entrepreneurs who bear all the risk. Entrepreneurs have a low elasticity of intertemporal substitution, implying a strong desire to smooth consumption over time. Second, scarcity of investment opportunities along with limited access to outside financing imply a binding financial constraint. A binding financial constraint implies that a negative financial shock in period $t + 1$ lowers investment and increases consumption of the entrepreneurs in period $t + 1$. At the same time, a negative financial shock makes equity more valuable, increasing its price $q_{t+1}$ and payoff $r_{t+1} + (1 - \delta)q_{t+1}$. Therefore, period $t + 1$ entrepreneurs’ consumption and equity payoff are positively correlated, which makes equity a bad hedge against entrepreneurs’ consumption risk, which is relatively high in the benchmark calibration. As a consequence, entrepreneurs demand a high equity return. Equity is a bad hedge against productivity shocks as well, but productivity shocks are less important for asset return dynamics. This is because the financial constraint strongly propagates financial shocks, but not productivity shocks.

**Asset Return Predictability.** Although controversial, equity return predictability by financial variables as well as macroeconomic variables is documented in the literature (see, e.g., Campbell and Shiller 1988; Fama and French 1988; Lettau and Ludvigson 2001; Cooper and Priestley 2009, among many others). This section shows that equity returns as well as
excess equity returns are predictable in our model and discusses the mechanism that results in asset return predictability. The ability to generate predictable excess returns is important because standard business cycle models do not generate economically significant predictability in excess returns (see Kaltenbrunner and Lochstoer 2010).

We first investigate whether the price-to-capital return ratio predicts the (cumulative) equity return one year, two years, and five years ahead. Frequently, researchers ask whether future equity returns are predictable by the log of the current price-to-dividend ratio. However, our model does not explicitly include dividends. Therefore, we use the marginal product of capital, i.e., the return on capital $r_t$, instead of dividends to investigate the predictability of equity returns. One can also think of price-to-return on capital ratio as a measure of the value-to-earnings ratio (this ratio is also used in the predictability literature). This is because in our model, the firm value is $q_t k_t$ and earnings is $r_t k_t$. The value-to-earnings ratio is therefore equal to the price-to-return on capital ratio.

The results are reported in Table 5. In our benchmark model with both productivity and financial shocks, the price-to-capital return ratio is negatively correlated with the equity return. In addition, the predictive power is increasing with the time horizon. The results are similar for the excess equity return: the correlation between the price-to-capital return ratio and the excess equity return is negative and the predictability of excess returns is increasing with the time horizon. These properties are consistent with what has been found in the data for the price-to-dividend ratio.\textsuperscript{10} In unreported results, we find that the other two versions of our model, the one with a financial constraint, but without financial shocks, and the one without the financial constraint, do not generate predictable returns.

Why are asset returns predictable in our production economy model? The first order conditions for the non-investing entrepreneurs imply the following standard asset pricing

\textsuperscript{10}Consistent with Lettau and Ludvigson (2001), the benchmark model also generates predictability of equity returns and excess equity returns by the entrepreneurs’ consumption-to-wealth ratio.
equation for equity returns:

\[ E_t \left[ r^e_{t+1} \right] = \left( 1 - \text{cov}(m_{t+1}, r^e_{t+1}) \right) r^f_t, \]

where \( m_{t+1} = \nu'(C^s_{t+1})/\nu'(C^s_t) \) is the stochastic discount factor, \( r^f_t = 1/E_t[m_{t+1}] \) is the (shadow) risk-free rate and \( r^e_{t+1} = \frac{r_{t+1} + (1-\delta)q_{t+1}}{q_t} \) is the return on equity. The above equation can be written as

\[ \log E_t \left[ r^e_{t+1} \right] = \log \left[ 1 - \text{cov}(m_{t+1}, r^e_{t+1}) \right] + \log r^f_t. \] (15)

In the model, the expected equity return and the expected excess equity return vary systematically over the business (financial) cycle and this variation is the source of asset return predictability. The economic mechanism that drives the cyclical variation in expected returns is as follows.

Suppose that the financial constraint tightens due to a negative financial shock. Then, investment decreases and entrepreneurs’ consumption increases. As a result, the expected growth rate of consumption as well as the risk-free rate decrease. In addition, the expected equity return decreases. The decrease in expected equity return is larger than the decrease in the risk-free rate, because the first term on the right-hand side of equation (15) decreases. In particular, both the volatility of the stochastic discount factor and the volatility of the equity return decrease when the financial constraint tightens. As a result (recall that consumption growth is positively correlated with the equity return and hence \( \text{cov}(m_{t+1}, r^e_{t+1}) \) is negative), \( [1 - \text{cov}(m_{t+1}, r^e_{t+1})] \) decreases and the expected equity return decreases by more than the risk-free rate. This means that the expected excess return \( E_t \left[ r^e_{t+1} - r^f_t \right] \) decreases as well.

**Asset Prices and Investment.** Our model generates a negative correlation between asset prices and investment. This feature is not specific to our model. In this class of models, changes in the tightness of the financial constraint, driven by changes in \( \phi_t \) in our model, directly affect the amount of investment but do not affect the productivity of the existing capital. A tighter constraint implies less investment and less new capital, which makes old
capital (and new capital as well) more valuable to agents in the economy. Therefore, a tighter constraint implies higher asset prices.\footnote{We have found this to be true in the original Kiyotaki and Moore (2011) model in which the friction takes the form of limited resaleability. In this model an entrepreneur can sell only a fraction of his assets at a point in time to finance new investments. Tightening this constraint implies a decrease in investment and an increase in the asset price by the same logic. Shi (2015) discusses this result in detail in several versions of the Kiyotaki and Moore (2011) model. Cui and Radde (2014) generate a positive correlation between asset prices and liquidity by modelling fluctuations in liquidity endogeneously in an environment with search frictions. The same logic applies to shocks to monitoring costs in models with costly monitoring such as Carlstrom and Fuerst (1997) or to shocks in the value of default in limited commitment models. A similar result appears in models with investment-specific technological shocks: a positive investment-specific technological shock implies higher investment and lower price of capital/equity, as discussed in more detail in Section 2.2.}

In the data the correlation between asset prices and investment is positive over the whole sample period. Investigating the subperiods, we find that the correlation is small and positive in the subsample ending in 1982, negative in the subsample of 1983 - 1997 and large and positive thereafter (the pattern is in fact similar for the correlation between asset prices and output). Therefore, our model accounts better for the eighties and nineties than for the most recent period.

Comparison to Existing Models. The mechanism that generates high asset return volatility and high equity premium proposed in this paper differs from existing production based models with convex adjustment costs or factor immobility (see, e.g., Jermann 1998; Boldrin et al. 2001; Guvenen 2009; and Campanale et al. 2010). High asset return volatility (equity premium) in our model is a result of an interaction between the scarcity of investment opportunities and financial shocks, whereas in convex adjustment costs models, it is a result of an interaction between a technological assumption and productivity shocks. To create sizable asset price volatility and equity premium, the adjustment costs need to be quite severe. This implies a counterfactually low aggregate investment volatility, which has been one of the main criticisms of these models. The present model improves upon the adjustment cost models by matching the relative volatility of aggregate investment. Because of the random arrival of investment opportunities, our model also produces investment spikes at the firm level, which are present in the data.
The mechanism proposed in this paper also differs from existing production based models with investment-specific technology shocks (see, e.g., Christiano and Fisher 2003; Papanikolaou 2011; Kogan and Papanikolaou 2013; and Kogan and Papanikolaou 2014). Similar to the papers discussed above, these models assume convex investment adjustment costs combined with (investment-specific) technology shocks. In these models, the representative investor holds equities of consumption good and investment good producing firms and the market portfolio is volatile because the investment good producing firms are risky due to IST shocks. Our model assumes limited stock market participation and consumption of the stock market participants (entrepreneurs) is more volatile than aggregate consumption. Entrepreneurs price equity and hence asset prices are volatile. Moreover, the origin and nature of the IST shocks are different from the financial shocks in the present model. IST shocks are the result of, for instance, technological innovations (see, e.g., Kogan et al. 2013) and the equity premium arises as a compensation for exposure to technological risk. Financial shocks in our model are (possibly) a result of changes in the ‘bite’ of the moral hazard problem between households, financial intermediaries, and the firms in the economy (see, e.g., He and Krishnamurthy 2012). The equity premium arises as a compensation for exposure to time varying financial constraints that affect firms’ investment and hence their values.

In addition, in most of the papers discussed above, asset price volatility is a feature of the first best allocation coming from preferences, technological constraints and shocks (as a result, the competitive equilibrium allocation can be typically represented as a solution to a social planning problem). Therefore, there is no role for the government as long as one assumes that the government is not able to change agents’ preferences or overcome the technological constraints. In our model, on the other hand, asset price volatility originates from financial constraints and shocks, and, hence, it is not a feature of the first best allocation.

The results in this paper should not be interpreted as saying that one can fully explain the behavior of asset prices through financial frictions and financial shocks. However, the results do suggest that financial frictions and shocks play quantitatively important roles in
the dynamics of asset prices. This is an important finding because, for instance, Gomes et al. (2003) and Cordoba and Ripoll (2004) show that financial constraints are not quantitatively important for asset prices in models presented in Carlstrom and Fuerst (1997) and Kiyotaki and Moore (1997).

6 Analyzing the Mechanisms

This section analyzes the mechanisms through which the productivity and financial shocks operate. It also analyzes the role of various assumptions in generating high asset price volatility.

6.1 Impulse Responses

To shed light on the mechanisms through which the two aggregate shocks operate, Figure 1 and Figure 2 plot the impulse responses of major macroeconomic variables to the two shocks. Figure 1 shows that a productivity shock generates comovement between major macroeconomic variables. The figure also shows that a productivity shock does not generate hump-shaped responses, except in capital. Figure 2 shows that a financial shock generates comovement between output, labor and investment and hump-shaped responses in output, labor and workers’ consumption. This finding is of interest because the model does not include habit persistence, nominal rigidities and other frictions typically used to generate hump-shaped responses.

Figure 2 also shows that aggregate consumption responds negatively to the financial shock. This is because after a positive financial shock, entrepreneurs invest more and hence must consume less, given that their income is unaffected on impact (this mechanism is similar to what happens after a decrease in disaster probability in Gourio 2012). The decrease in entrepreneurs’ consumption dominates the small increase in workers’ consumption, hence resulting in the negative response of aggregate consumption to the financial shock.

Finally, Figure 2 shows that the financial shock generates opposite responses in investment (output) and consumption. Given this finding, why does the model generate a positive
(unconditional) correlation between investment (output) and consumption as documented in Section 5.1? This is because aggregate consumption and investment respond positively to a positive productivity shock (similar to output). Given the low value of the EIS, entrepreneurs’ consumption does not fall very much in response to a positive financial shock. Therefore, the effect of the productivity shock dominates, generating a positive correlation between investment (output) and aggregate consumption.

6.2 Comparative Statics

This section analyzes to what extent the high asset price volatility the model generates depends on certain assumptions on preferences, shocks processes, etc. To analyze the role of a particular parameter, we perform a comparative statics exercise with respect to the parameter. All comparative statistics results are based on equilibrium quantities and prices. This means that the results can be interpreted as comparisons between economies that differ in one parameter of interest.

**Tightness of the Financial Constraint.** We start by analyzing how the tightness of the financial constraint affects the volatility of quantities and asset prices. Lemma 1 suggests that one of the parameters that controls the tightness of the financial constraint is the fraction of investing entrepreneurs, \( \pi \). Increasing \( \pi \) makes the constraint less tight (and might, in fact, expand the region where it is not binding).

Table 6 reports the results for the benchmark parameterizations with quarterly \( \pi = 1.5\% \) and two cases of low \( \pi = 1.2\% \) and high \( \pi = 1.8\% \). This table shows that increasing \( \pi \) decreases the volatility of investment and increases the volatility of consumption, but leaves output unaffected. In addition, increasing \( \pi \) reduces the asset price volatility and equity premium. Note that even for \( \pi = 1.8\% \), the model still generates an asset price volatility which is almost twice as large as the volatility of aggregate investment. The results are similar if one changes the depreciation rate \( \delta \) or the mean of the financial shock \( \phi \), the other parameters affecting the tightness of the financial constraint, as suggested by Lemma 1.
These results imply that financial shocks are amplified more strongly with a tighter financial constraint.

**Size of the Financial Shocks.** We already know that shutting down financial shocks completely implies that the model generates very limited asset price volatility. Therefore, it is true that, in general, changing the volatility of $\phi_t$ affects the volatility of asset prices (and investment). Table 6 reports the results for the benchmark parametrization with $\sigma_{\varepsilon} = 0.8$ and two cases of low $\sigma_{\varepsilon} = 0.7$ and high $\sigma_{\varepsilon} = 0.9$. This table shows that changing the volatility of the financial shock around the benchmark value does not affect the main results. This is because changing the volatility of the financial shock affects particularly the states in which the value of $\phi_t$ is large. In these states, however, the financial constraint does not bind and hence changing the value of $\phi_t$ is irrelevant.

**Elasticity of Intertemporal Substitution.** Table 6 reports the results for the benchmark parameterizations with $\psi = 0.5$ and two cases of low $\psi = 0.25$ and high $\psi = 0.75$. Table 6 shows that changing the EIS does not have a significant impact on output (or labor), but it does affect consumption and investment, which is consistent with the findings in Lemma 4. The reason is that higher EIS means that entrepreneurs are more willing to substitute current consumption for future consumption. This implies that investment becomes more volatile leading to more volatile entrepreneurs’ consumption. Importantly, the table shows that the EIS is not critical in generating high asset prices volatility. The equity premium is not affected either, because increasing EIS decreases the return on equity and the risk-free return in a similar way. The decrease in equity and risk-free returns can be understood as follows. With a higher EIS, entrepreneurs are more willing to substitute current consumption for future consumption and thus assets must offer a higher return in equilibrium.

In our model, entrepreneurs have access to a single asset (equity) and thus do not solve a portfolio allocation problem. As a result, changing the degree of risk aversion does not affect macro and asset price volatilities.
6.3 Time Varying Investment Opportunities

As we already discussed, asset price volatility is a result of time variation in the tightness of the financial constraint. This section further highlights this finding by showing that shocks to \( \pi \) (the fraction of entrepreneurs with an investment opportunity) generate similar macroeconomic and asset price dynamics to the dynamics generated by shocks to \( \phi_t \). An alternative interpretation of this result is that shocks to \( \pi \) are strongly amplified by the financial constraint. Similar to a financial shock in the benchmark model, a shock to \( \pi \) in this alternative specification is different from an IST shock. The shock to \( \pi \) does not affect the economy’s production possibility frontier and hence only the price channel transfers this shock to asset prices and aggregate quantities.

Shocks to \( \pi \) are incorporated in the model as follows. In the model presented in Section 2, suppose that the financial shock \( \phi_t \) is constant at its mean level, but the fraction of entrepreneurs with an investment opportunity varies over time.\(^{12}\) We denote the fraction of entrepreneurs with an investment opportunity in period \( t \) by \( \pi_t \) and solve for the model’s equilibrium dynamics. To study the model’s quantitative implications, we assume that log \( \pi_t \) follows an AR(1) process and calibrate its parameters to match the same targets in the data as in the benchmark quantitative exercise (volatility of investment relative to output, correlation between investment and output and the mean risk-free rate).

Table 7 reports the standard business cycle and asset price statistics for the benchmark model with stochastic \( \phi_t \) and the alternative model with stochastic \( \pi_t \). A comparison between the two right columns of the table shows that the model with stochastic \( \pi_t \) generates similar dynamics as the benchmark model which has a stochastic \( \phi_t \) and a fixed \( \pi_t = \pi \). The only notable difference is that the model with stochastic \( \pi_t \) generates a lower volatility of the

\(^{12}\)In general, we cannot prove that the two setups are isomorphic in the sense that given the same initial conditions and a stochastic process for \( \phi_t \) in the benchmark model, there exists a stochastic process for \( \pi_t \) so that the model with stochastic \( \pi_t \) delivers the same dynamics as the benchmark model. However, there is a special case in which such a result can be proved. Recall that in the special case of our benchmark model with \( \psi = 1 \) and \( \delta = 1 \), the price of equity is \( q_t = (1 - \pi_t)/\phi_t \). This means that, for a suitable process for \( \pi_t \), the model with stochastic \( \pi_t \) generates exactly the same asset price dynamics as the benchmark model.
(shadow) risk-free rate. The finding that the two models generate similar dynamics suggests that what matters for asset price/return dynamics in our model are the fluctuations in the tightness of the financial constraint over time. Therefore, we conjecture that shocks to the depreciation rate, as in for example Gomes and Michaelides (2008), would also generate large asset return volatility. However, unlike shocks to $\pi$ and $\phi$, depreciation rate shocks have a real component (they directly affect the available quantity of capital), similar to IST shocks, as in for example Fisher (2006).

7 Conclusion

This paper studies the role of financial frictions and shocks in business cycle fluctuations and asset price volatility. Specifically, the paper develops a production economy model with heterogeneous entrepreneurs, financial frictions and financial and productivity shocks. The paper first shows that both financial shocks and productivity shocks result in fluctuations in the price of equity. To assess the quantitative importance of financial and productivity shocks, the model is then calibrated to the U.S. data. The calibrated model generates about 70% of the asset price volatility relative to the aggregate stock market and over 35% of the observed quarterly Sharpe ratio. The model also fits key aspects of the behavior of aggregate quantities. In particular, the model matches the volatility of aggregate investment and consumption relative to the volatility of output.

Financial frictions and shocks have been shown to be quantitatively important in explaining the nature of macroeconomic fluctuations (see, for instance, Jermann and Quadrini 2012). This paper shows that financial frictions and shocks also help us better understand the dynamic behavior of asset prices. In particular, the interaction of time-varying financial frictions with the scarcity of investment opportunities can explain a significant fraction of the observed asset price volatility.
References


## A Appendix A: Proofs

**Proof of Proposition 1:** If an entrepreneur does not have an investment opportunity at time $t$, he must set $i_t = 0$. His financial constraint takes the form $n_{t+1} \geq 0$ and it never binds. If an entrepreneur has an investment opportunity at time $t$, one can take the first order condition with respect to $i_t$. Denote the Lagrange multiplier on the budget constraint by $\lambda_t$, and the Lagrange multiplier on the financial constraint by $\mu_t$. The budget constraint always binds, and therefore $\lambda_t > 0$. The necessary first order condition with respect to $i_t$ is

$$
(q_t - 1)\lambda_t = (1 - \phi_t)\mu_t.
$$

If $\phi_t = 1$, then $q_t = 1$ and the financial constraint cannot bind since it would have to bind for all investing and noninvesting entrepreneurs, which is inconsistent with market clearing. If $\phi_t < 1$, then by equation (16) $q_t > 1 \implies \mu_t > 0$ and vice versa. This result does not depend on the initial asset holdings $n_t$ and therefore applies to all investing entrepreneurs. □

**Proof of Lemma 1:** (i) We prove this part by contradiction. Assume that $q_t = 1$. Denote by $J_t$ the set of indexes identifying entrepreneurs with an investment opportunity at time $t$ and use capital letters for aggregate variables. Since the arrival of the investment opportunity is i.i.d., period $t$ investing entrepreneurs will hold a fraction $\pi$ of the aggregate equity holdings at the beginning of period $t$, i.e., $\int_{j \in J_t} n'_t dj = \pi N_t$. Since the price of equity is $q_t = 1$, investing entrepreneurs solve the same problem as noninvesting entrepreneurs. Therefore, period $t$ investing entrepreneurs will hold a fraction $\pi$ of aggregate equity holdings in period $t+1$ as well, i.e., $\int_{j \in J_t} n'^{t+1}_t = \pi N_{t+1}$. In any equilibrium, the financial constraint must be satisfied for all investing entrepreneurs. Integrating over the set $J_t$ implies that

$$
\pi N_{t+1} \geq (1 - \phi)I_t.
$$

In steady state $N_t = N_{t+1} = N$ and $I_t = \delta N$. Therefore, equation (17) can be rewritten as $\pi - (1 - \phi)\delta \geq 0$. This contradicts our assumption, and thus it must be that in steady state $q > 1$. Proposition 1 implies that the financial constraint binds.

(ii) Suppose that in steady state, the financial constraint binds and $q > 1$, but by way of contradiction $\pi \geq \delta(1 - \phi)$. The financial constraint together with a steady state condition $I_t = \delta N_t$ imply that $N_{t+1} = (1 - \phi)\delta N_t$. Since we assume that $\pi \geq \delta(1 - \phi)$ we get that $N_{t+1} \leq \pi N_t$. Since initial equity holdings of investing entrepreneurs are $\pi N_t$, this implies that investing entrepreneurs’ equity holdings are weakly decreasing between period $t$ and period $t+1$. Since aggregate equity/capital is constant in steady state, it must be that noninvesting entrepreneurs’ equity holding are weakly increasing, i.e., $N^s_{t+1} \geq (1 - \pi)N_t$. Aggregating over equations (6) and (8) and using these two inequalities implies that

$$
N^t_{t+1} = \frac{(1 - \zeta^t)}{q^t} \pi N_t(r_t + (1 - \delta)q_t) \leq \pi N_t,
$$

$$
N^s_{t+1} = \frac{(1 - \zeta^s)}{q_t} (1 - \pi)N_t(r_t + (1 - \delta)q_t) \geq (1 - \pi)N_t.
$$

Combining the two inequalities implies $(1 - \zeta^t)/q^t \leq (1 - \zeta^s)/q_t$, which in turn implies
\( q_t/q_t^R \leq (1 - \zeta^t)/(1 - \zeta^\text{t*}) \). If \( q_t > 1 \), then \( q_t^R < 1 \) and thus it must be that \( \zeta^t > \zeta^\text{t*} \) which contradicts Lemma 4 (observe that the results of Lemma 1 are not used to prove Lemma 4) provided that \( \psi \geq 1 \).

**Proof of Lemma 2:** A worker solves (we drop the index \( j \) for simplicity)

\[
\max_{c_t^w, l_t^w} U \left( c_t^w - \frac{\omega}{1 + \eta} \left( l_t^w \right)^{1 + \eta} \right) \quad \text{s.t.} \quad c_t^w \leq w_t l_t^w,
\]

where \( c_t^w \) denotes the consumption of the worker, \( l_t^w \) denotes the labor provided by the worker, and \( w_t \) denotes the wage rate. Therefore \( l_t^w = (w_t/\omega)^{1/\eta} \), which holds for each worker.

Therefore, the aggregate labor supply, denoted by \( L_t^w \), can be written as

\[
L_t^w = (w_t/\omega)^{1/\eta}.
\]

The aggregate labor demand by the entrepreneurs, denoted by \( L_t \), is determined by \( w_t = A_t(1 - \alpha)K_t^{\alpha}L_t^{-\alpha} \). In equilibrium, supply of labor by the workers is equal to demand of labor by the entrepreneurs, i.e., \( L_t^w = L_t \). Therefore, \( w_t = \omega^{\alpha/\eta} [ (1 - \alpha)A_t ]^{\alpha/\eta} \) and \( L_t = \left[ \frac{A_t(1 - \alpha)}{\omega} \right]^{\alpha/\eta} K_t^{\alpha/\eta} \). Given that workers cannot save, their aggregate consumption, denoted by \( C_t^w \), equals the labor’s share in output, i.e., \( C_t^w = (1 - \alpha)Y_t \).

**Proof of Corollary 1:** The joint dynamics of output and labor is determined by equation (18). Using equilibrium conditions, this equation can be rewritten as

\[
L_t = \left( \frac{w_t}{\omega} \right)^{1/\eta} = \left( \frac{MPL_t}{\omega} \right)^{1/\eta} = \left( \frac{(1 - \alpha)Y_t}{\omega L_t} \right)^{1/\eta}.
\]

This implies that \( (1 + \eta) \log L_t = \log Y_t + \log \left( \frac{1 - \alpha}{\omega} \right) \), which implies \( \rho(\log L_t, \log Y_t) = 1 \) and \( (1 + \eta)^2 \text{var}(\log L_t) = \text{var}(\log Y_t) \), which establishes the first two claims made in Corollary 1. In Lemma 2, we established that \( C_t^w = (1 - \alpha)Y_t \). Therefore, \( \rho(\log C_t^w, \log Y_t) = 1 \) and \( \text{var}(\log C_t^w) = \text{var}(\log Y_t) \).

**Proof of Proposition 3:** Let \( v_t^i \) denote the value function of an investing entrepreneur and \( v_t^s \) denote the value function of a noninvesting entrepreneur. As for \( \zeta_t^i \) and \( \zeta_t^s \) these value functions are functions of aggregate state \((A_t, N_t, \phi_t)\). They satisfy

\[
v_t^i = \left( (1 - \beta)(c_t^i) \frac{1 - \gamma}{\beta} + \beta \left( \pi E[(v_{t+1}^i)^{1 - \gamma}] + (1 - \pi)E[(v_{t+1}^s)^{1 - \gamma}] \right) \right)^{1/\gamma},
\]

\[
v_t^s = \left( (1 - \beta)(c_t^i) \frac{1 - \gamma}{\beta} + \beta \left( \pi E[(v_{t+1}^i)^{1 - \gamma}] + (1 - \pi)E[(v_{t+1}^s)^{1 - \gamma}] \right) \right)^{1/\gamma}.
\]

Lemma 3 implies that \( v_t^i \) and \( v_t^s \) are linear in wealth and in particular \( v_t^i = \xi_t^i \cdot ew_t^i \) and \( v_t^s = \xi_t^s \cdot ew_t^s \), where \( \xi_t^i \) and \( \xi_t^s \) are time-invariant functions of the aggregate state \((A_t, N_t, \xi_t)\).
Therefore,

\[(\zeta^i_t)^{1-\gamma} = (1 - \beta)(\zeta^i_t)^{1-\gamma} + \beta \left( \pi E \left[ \left( \frac{\xi^i_t}{\zeta^i_{t+1}} \right)^{1-\gamma} \right] \right) + (1 - \pi) E \left[ \left( \frac{\xi^i_{t+1}}{\zeta^i_{t+1}} \right)^{1-\gamma} \right]^{\frac{1}{\beta}}, \]

\[(\zeta^s_t)^{1-\gamma} = (1 - \beta)(\zeta^s_t)^{1-\gamma} + \beta \left( \pi E \left[ \left( \frac{\xi^s_t}{\zeta^s_{t+1}} \right)^{1-\gamma} \right] \right) + (1 - \pi) E \left[ \left( \frac{\xi^s_{t+1}}{\zeta^s_{t+1}} \right)^{1-\gamma} \right]^{\frac{1}{\beta}}, \]

which can be simplified to

\[(\zeta^i_t)^{1-\gamma} = (1 - \beta)(\zeta^i_t)^{1-\gamma} + \beta \left( \pi E \left[ \left( \frac{\xi^i_t}{\zeta^i_{t+1}} \right)^{1-\gamma} \right] \right) + (1 - \pi) E \left[ \left( \frac{\xi^i_{t+1}}{\zeta^i_{t+1}} \right)^{1-\gamma} \right]^{\frac{1}{\beta}}, \]

\[(\zeta^s_t)^{1-\gamma} = (1 - \beta)(\zeta^s_t)^{1-\gamma} + \beta \left( \pi E \left[ \left( \frac{\xi^s_t}{\zeta^s_{t+1}} \right)^{1-\gamma} \right] \right) + (1 - \pi) E \left[ \left( \frac{\xi^s_{t+1}}{\zeta^s_{t+1}} \right)^{1-\gamma} \right]^{\frac{1}{\beta}}. \]

Next, we show that a simple relationship holds between \(\xi_t\)'s and \(\zeta_t\)'s. Taking first order conditions in equations (19) and (20) with respect to \(c^i_t\) and \(c^s_t\) or equivalently with respect to \(\zeta^i_t\) and \(\zeta^s_t\) implies that

\[(1 - \beta)(\zeta^i_t)^{1-\gamma} - 1 = \beta(1 - \zeta^i_t)^{1-\gamma} - 1 \left( \pi E \left[ \left( \frac{\xi^i_t}{\zeta^i_{t+1}} \right)^{1-\gamma} \right] \right) + (1 - \pi) E \left[ \left( \frac{\xi^i_{t+1}}{\zeta^i_{t+1}} \right)^{1-\gamma} \right]^{\frac{1}{\beta}}, \]

\[(1 - \beta)(\zeta^s_t)^{1-\gamma} - 1 = \beta(1 - \zeta^s_t)^{1-\gamma} - 1 \left( \pi E \left[ \left( \frac{\xi^s_t}{\zeta^s_{t+1}} \right)^{1-\gamma} \right] \right) + (1 - \pi) E \left[ \left( \frac{\xi^s_{t+1}}{\zeta^s_{t+1}} \right)^{1-\gamma} \right]^{\frac{1}{\beta}}. \]

Combining equation (21) with (19) and equation (22) with (20) implies

\[\xi^i_t = (1 - \beta)^{\alpha/(1-\gamma)} (\zeta^i_t)^{\gamma/(1-\gamma)}, \quad \xi^s_t = (1 - \beta)^{\alpha/(1-\gamma)} (\zeta^s_t)^{\gamma/(1-\gamma)}.\]
Using these two expressions, equations (19) and (20) that describe the dynamics of $\zeta_t^i$ and $\zeta_t^s$ can be written as

\[
(1 - \beta)(\zeta_t^i)^{\frac{1-\gamma}{\theta}} = (1 - \beta)(\zeta_t^i)^{\frac{1-\gamma}{\theta}} + \beta \left( \pi E \left[ \left( (1 - \beta) \tau_{t+1}^\theta \left( \zeta_t^i \right)^{\frac{1-\gamma}{\theta}} \left( 1 - \zeta_t^i \right) \left( q_{t+1} (1 - \delta) + r_{t+1} \right) \right)^{1-\gamma} \right] \right),
\]

\[
+ \left( 1 - \pi \right) E \left[ \left( (1 - \beta) \tau_{t+1}^\theta \left( \zeta_t^i \right)^{\frac{1-\gamma}{\theta}} \left( 1 - \zeta_t^i \right) \left( q_{t+1} (1 - \delta) + r_{t+1} \right) \right)^{1-\gamma} \right] \right]^{\frac{1}{\theta}} .
\]

\[
(1 - \beta)(\zeta_t^s)^{\frac{1-\gamma}{\theta}} = (1 - \beta)(\zeta_t^s)^{\frac{1-\gamma}{\theta}} + \beta \left( \pi E \left[ \left( (1 - \beta) \tau_{t+1}^\theta \left( \zeta_t^s \right)^{\frac{1-\gamma}{\theta}} \left( 1 - \zeta_t^s \right) \left( q_{t+1} (1 - \delta) + r_{t+1} \right) \right)^{1-\gamma} \right] \right),
\]

\[
+ \left( 1 - \pi \right) E \left[ \left( (1 - \beta) \tau_{t+1}^\theta \left( \zeta_t^s \right)^{\frac{1-\gamma}{\theta}} \left( 1 - \zeta_t^s \right) \left( q_{t+1} (1 - \delta) + r_{t+1} \right) \right)^{1-\gamma} \right] \right]^{\frac{1}{\theta}} .
\]

**Proof of Lemma 4:** Part (i) follows from equations (9) and (10) and the fact that if $q_t = 1$ then $q_t^R = 1$. For part (ii), observe that if $q_t > 1$ then $q_t^R < 1$. We rewrite equations (9) and (10) as

\[
\beta Z^\frac{1}{\theta} = (1 - \beta)(\zeta_t^i / (1 - \zeta_t^i))^{\frac{1-\gamma}{\theta}} \frac{1}{\theta} (q_t^R)^{\frac{1-\gamma}{\theta}}, \tag{23}
\]

\[
\beta Z^\frac{1}{\theta} = (1 - \beta)(\zeta_t^s / (1 - \zeta_t^s))^{\frac{1-\gamma}{\theta}} \frac{1}{\theta} (q_t) \frac{1}{\theta}, \tag{24}
\]

where $Z$ is defined as

\[
Z = (1 - \beta)^{\theta} \left\{ \pi E \left[ \left( (\zeta_{t+1}^i)^{\frac{1-\gamma}{\theta}} (q_{t+1} (1 - \delta) + r_{t+1}) \right)^{1-\gamma} \right] \right\} + (1 - \pi) E \left[ \left( (\zeta_{t+1}^s)^{\frac{1-\gamma}{\theta}} (q_{t+1} (1 - \delta) + r_{t+1}) \right)^{1-\gamma} \right].
\]

Combining (23) and (24) we get

\[
\frac{\zeta_t^i / (1 - \zeta_t^i)}{\zeta_t^s / (1 - \zeta_t^s)} = \left[ \frac{q_t^R}{q_t} \right]^{1-\psi}, \tag{25}
\]

where we used the fact that $\theta = \frac{1-\gamma}{1-\frac{1}{\theta}}$. If $\psi = 1$ then $\frac{\zeta_t^i / (1 - \zeta_t^i)}{\zeta_t^s / (1 - \zeta_t^s)} = 1$ and hence $\zeta_t^i = \zeta_t^s$. If $\psi < 1$, then $\left[ \frac{q_t^R}{q_t} \right]^{1-\psi} > 1$, and hence $\frac{\zeta_t^i / (1 - \zeta_t^i)}{\zeta_t^s / (1 - \zeta_t^s)} > 1$, and thus $\zeta_t^i > \zeta_t^s$. The opposite is true if $\psi > 1$. \hfill \Box

**Proof of Lemma 5:** Equation (11) is a rewrite of the goods market clearing condition $C_t^e + C_t^w + I_t = Y_t$. Using the fact that worker’s aggregate consumption $C_t^w$ is a fraction $(1 - \alpha)$ of output $Y_t$, and the fact that $N_{t+1} = (1 - \delta)N_t + I_t$, one can write

\[
N_{t+1} = \alpha Y_t + (1 - \delta)N_t - C_t^e. \tag{26}
\]

Aggregate entrepreneurs’ consumption $C_t^e$ can be computed by aggregating the consumption of investing entrepreneurs in equation (5) and the consumption of noninvesting entrepreneurs in equation (7). The fact that the initial asset holdings of investing entrepreneurs are $\pi N_t$...
and of the noninvesting entrepreneurs \((1 - \pi)N_t\) (this follows from the fact that investment opportunity arrival is i.i.d.) implies

\[
C^e_t = \left[\zeta_i^i(N_t, A_t, \phi_t)\pi + \zeta_i^s(N_t, A_t, \phi_t)(1 - \pi)\right]N_t \left[r(N_t, A_t) + (1 - \delta)q_t\right],
\]

and by plugging into equation (26)

\[
N_{t+1}(N_t, A_t, \phi_t) = (1 - \delta)N_t + \alpha Y(N_t, A_t) - \left[\zeta_i^i(N_t, A_t, \phi_t)\pi + \zeta_i^s(N_t, A_t, \phi_t)(1 - \pi)\right]N_t \left[r(N_t, A_t) + (1 - \delta)q_t\right].
\]

**Proof of Proposition 4:** The equilibrium price of equity is defined as one which clears the equity market, i.e., implies that demand and supply of equity are equal. Aggregating over all noninvesting entrepreneurs using equation (8) their net demand for (new) equity can be written as

\[
D_t^e = N_{t+1}^s - (1 - \delta)(1 - \pi)N_t = (1 - \zeta_i^s)(1 - \pi)N_t\left[\frac{\pi}{q_t} + 1 - \delta\right] - (1 - \delta)(1 - \pi)N_t.
\]

Similarly, aggregating over all investing entrepreneurs using equation (6), the net supply of equity by the investing entrepreneurs is given by

\[
S_t^e = \pi(1 - \delta)N_t + I_t - N_{t+1}^i = \pi(1 - \delta)N_t + \frac{\phi_t}{1 - \phi_t}N_{t+1}^i
\]

\[
= \pi(1 - \delta)N_t + \frac{\phi_t}{1 - \phi_t} \left(1 - \zeta_i^s\right)\pi N_t \left[r_t + (1 - \delta)q_t\right],
\]

where \(I_t\) denotes aggregate investment and we use the fact that the (FC) implies that \(I_t = \frac{N_{t+1}^i}{1 - \phi_t}\). Combining the two equations, one gets a quadratic equation in \(q_t\) : \(a_2q_t^2 + a_1q_t + a_0 = 0\), with the following definitions: \(a_2 = (1 - \delta)\phi_t \left[(1 - \zeta_i^s)\pi + (1 - \zeta_i^s)(1 - \pi) - 1\right] + a_1 = (1 - \delta) \left[1 - (1 - \zeta_i^s)(1 - \pi)\right] + \phi_t r_t \left[(1 - \zeta_i^s)\pi + (1 - \zeta_i^s)(1 - \pi)\right], a_0 = -(1 - \zeta_i^s)(1 - \pi)r_t\). To select the correct root of the quadratic equation, note that \(a_2 < 0, a_1 > 0, a_0 < 0\). In addition, there is exactly one solution of the equation \(a_2q_t^2 + a_1q_t + a_0 = 0\) in the interval \(0, \frac{1}{\phi_t}\).\(^{13}\) To see this point, define \(f(q_t) = a_2q_t^2 + a_1q_t + a_0\) and notice that it has the following properties. \(f(0) < 0, f(\frac{1}{\phi_t}) > 0\). These two facts along with \(a_2 < 0\) imply that one root (the smaller one) of the quadratic equation lies in \((0, \frac{1}{\phi_t})\) and the other root is larger than \(\frac{1}{\phi_t}\). Finally, if \(q_t^* < 1\), then the equilibrium price of equity \(q_t = 1\).\(^{14}\)

\(^{13}\) \(q_t\) cannot be larger than \(\frac{1}{\phi_t}\). If it was, then investing entrepreneurs could make their consumption arbitrarily large. Mathematically, the argument comes from the fact that \(q_t^R\) would be negative.

\(^{14}\) If capital can be converted back to consumption \(q_t < 1\) is not an equilibrium outcome, since the value of one unit of capital is bounded below by the value of one unit of consumption, which is normalized to one. With capital irreversibility, i.e., an \(I_t \geq 0\) constraint, \(q_t\) would be smaller than one if and only if this constraint binds. This could happen only if capital levels were very high, which does not happen in our quantitative analysis. We therefore ignore the possibility that \(q_t < 1\) and do not take a stand on capital reversibility.
Proof of Corollary 2: The corollary follows directly from Lemma 5 and Proposition 4 and the fact that with $\psi = 1$ policies are $\zeta^i(N_t, A_t, \phi_t) = \zeta^s(N_t, A_t, \phi_t) = 1 - \beta$ independent of the states. (We omit the proof of this result.)

B Appendix B: Numerical Method

The equilibrium aggregate dynamics is fully characterized by equations (9), (10), (11) and (12). We solve this system using global methods (shocks are relatively large and the equilibrium dynamics are possibly non-linear, see, Brunnermeier and Sannikov 2014) by reducing it to a system of two first-order difference equations in $\zeta^i(A_t, N_t, \phi_t)$, $\zeta^s(A_t, N_t, \phi_t)$ and $\zeta^i(A_{t+1}, N_{t+1}, \phi_{t+1})$, $\zeta^s(A_{t+1}, N_{t+1}, \phi_{t+1})$. To find the policy functions $\zeta^i(A, N, \phi)$ and $\zeta^s(A, N, \phi)$ that solve the system, i.e., to find the fixed point, we proceed as follows.

1. Fix an initial guess for $\zeta^i(A, N, \phi)$ and $\zeta^s(A, N, \phi)$. If no other guess is available, start with $\zeta^i(A, N, \phi) = \zeta^s(A, N, \phi) = 1 - \beta$.
2. Use the initial guesses for $\zeta^i_t, \zeta^s_t$ on the right-hand side of equations (9) and (10).
3. Solve for $\zeta^i_t, \zeta^s_t$ that solve equations (9) and (10) for given $A_t, N_t, \phi_t$.
4. Do this for each $A_t, N_t, \phi_t$ on the grid.
5. Use the obtained $\zeta^i_t, \zeta^s_t$ as the new guess in step (2).
6. Iterate until the system converges.

C Appendix C: Data Construction

We restrict attention to the time period 1964Q1 - 2013Q4. We use the following databases for constructing the macroeconomic variables.

2. FAT-BEA: Fixed Asset Tables published by the Bureau of Economic Analysis.

Hours, denoted by $L$, is constructed from CES-BLS as $L = \text{Average weekly hours} \times \text{Average number of workers}$. Real capital, denoted by $K$, is constructed by generating quarterly data by interpolating the yearly “Fixed assets and consumer durable goods,” in Table 1.2 in FAT-BEA. Output, denoted by $Y$, is the real GDP, in Table 1.1.6 in NIPA-BEA. Productivity series $A_t$ is computed from the series of output, capital and hours series as $A_t = Y_t/(L_t^{0.64} \times K_t^{0.36})$.

To construct a measure for real investment, denoted by $I$, we first compute nominal investment as $NI = \text{Nominal private fixed investment} + \text{Nominal durable consumption good expenditure}$, where nominal private fixed investment is from Table 1.1.5 in NIPA-BEA and nominal durable consumption good expenditure is from Table 1.1.5 in NIPA-BEA.
Real investment is then constructed by deflating NI with the deflator for gross private domestic investment constructed using Tables 1.1.5 and 1.1.6 in NIPA-BEA. The time-series properties of alternative real investment measures are very similar to the one we consider, with a correlation around 0.95. Including government investment, inventories, or excluding durable consumption makes the series slightly more volatile.

To construct a measure for real consumption, denoted by $C$, we first compute nominal consumption as $NC = \text{Nondurable goods} + \text{Services}$, where nondurable goods is from Table 1.1.5 in NIPA-BEA and services is from Table 1.1.5 in NIPA-BEA. The real counterparts of these nominal series are only reported starting in 1995. Therefore, to generate the real series, these nominal series are deflated by a personal consumption expenditure deflator constructed using Tables 1.1.5 and 1.1.6 in NIPA-BEA.

All nominal prices and returns are deflated using the CPI series from the BLS database. Asset price, denoted by $q$, is the S&P500 Composite Price Index from the CRSP database (Center for Research in Security Prices), where quarterly data is generated as the mean of monthly observations. We have computed the relevant statistics for the Wilshire 5000 Total Market Index from the St. Louis FED database. The time-series properties of the HP-filtered logged real versions of these indexes are very similar (the correlation is 0.99). The Wilshire 5000 is slightly more volatile than the S&P500 (11.0% vs. 10.6%). Asset return, denoted by $r^e$, is constructed using the series $vwretd$ from the CRSP database, value-weighted returns including distributions from NYSE, AMEX, and NASDAQ. Quarterly data is constructed as geometric mean of monthly observations. Total market value, denoted by $val$, is constructed using the series $totval$ from the CRSP database. Quarterly data is constructed as average over monthly observations. Real risk-free rate, denoted by $r^f$, is the three-month return on a three month T-bill (Fama risk-free in the CRSP database). Quarterly data is constructed as geometric mean of monthly observations.
Table 1: Benchmark Parameters

This table shows the parameters used in the benchmark quantitative exercise.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share in output</td>
<td>$\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta$</td>
<td>2.26%</td>
</tr>
<tr>
<td>Quarterly discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Entrepreneurs’ risk aversion parameter</td>
<td>$\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>Entrepreneurs’ intertemporal elasticity of substitution</td>
<td>$\psi$</td>
<td>0.5</td>
</tr>
<tr>
<td>Workers’ inverse Frisch elasticity</td>
<td>$\eta$</td>
<td>2</td>
</tr>
<tr>
<td>Disutility of labor</td>
<td>$\omega$</td>
<td>21.74</td>
</tr>
<tr>
<td>Fraction of firms with investment opportunity</td>
<td>$\pi$</td>
<td>0.015</td>
</tr>
<tr>
<td>Standard deviation of log productivity shock</td>
<td>$\sigma_{\varepsilon_A}$</td>
<td>0.006</td>
</tr>
<tr>
<td>Persistence of log productivity shock</td>
<td>$\rho_z$</td>
<td>0.95</td>
</tr>
<tr>
<td>Persistence of log financial shock</td>
<td>$\rho_\phi$</td>
<td>0.95</td>
</tr>
<tr>
<td>Financial shock constant</td>
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</tr>
<tr>
<td>Standard deviation of log financial shock</td>
<td>$\sigma_{\varepsilon_\phi}$</td>
<td>0.80</td>
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<tr>
<td>Correlation of innovations of log $\phi_t$ and log $A_t$</td>
<td>$\rho_{A,\phi}$</td>
<td>0.95</td>
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</table>
Table 2: Calibration Moments

This table reports the calibration moments.

<table>
<thead>
<tr>
<th>Target</th>
<th>Symbol</th>
<th>Data</th>
<th>Model Mean</th>
<th>Model STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative volatility of investment and output</td>
<td>$\frac{\sigma_I}{\sigma_Y}$</td>
<td>2.971</td>
<td>2.921</td>
<td>0.271</td>
</tr>
<tr>
<td>Correlation of output and investment</td>
<td>$\rho(Y, I)$</td>
<td>0.925</td>
<td>0.932</td>
<td>0.017</td>
</tr>
<tr>
<td>Quarterly risk-free rate</td>
<td>$r^f$</td>
<td>0.249%</td>
<td>0.249%</td>
<td>0.516%</td>
</tr>
</tbody>
</table>
Table 3: Standard Business Cycle Statistics

This table reports the business cycle statistics for three versions of our model. Statistics are computed based on 10,000 replications of size 400 when the first 200 observations are discarded. The symbol $\sigma_x$ represents the standard deviation of variable $x$, $\rho_x$ represents the autocorrelation of $x$, and $\rho(x, y)$ represents the correlation between $x$ and $y$. All variables are in log and HP filtered before statistics are computed. Standard deviations are measured in percentage terms. The ‘Data’ column reports statistics for quarterly U.S. data for the period 1964:1-2013:4. Column (1) reports statistics for the calibrated benchmark model with the financial constraint and both financial and productivity shocks. Column (2) reports the statistics for a version of the model with the financial constraint and no financial shocks. Column (3) reports the statistics for a version of the model with no financial constraint.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>(1) With FC</th>
<th>(2) With FC</th>
<th>(3) Without FC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\phi$ stochastic</td>
<td>$\phi$ constant</td>
<td>$A$ stochastic</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>1.54</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>$\sigma_I/\sigma_Y$</td>
<td>2.97</td>
<td>2.92</td>
<td>1.24</td>
<td>1.42</td>
</tr>
<tr>
<td>$\sigma_C/\sigma_Y$</td>
<td>0.57</td>
<td>0.57</td>
<td>0.92</td>
<td>0.86</td>
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<tr>
<td>$\sigma_L/\sigma_Y$</td>
<td>1.23</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>$\sigma_{C^e}/\sigma_{C^w}$</td>
<td>3.56</td>
<td>3.56</td>
<td>0.50</td>
<td>0.11</td>
</tr>
<tr>
<td>$\rho_Y$</td>
<td>0.88</td>
<td>0.68</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>$\rho_I$</td>
<td>0.90</td>
<td>0.67</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>$\rho_C$</td>
<td>0.90</td>
<td>0.66</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>0.93</td>
<td>0.68</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>$\rho(Y, I)$</td>
<td>0.92</td>
<td>0.93</td>
<td>1.00</td>
<td>1.00</td>
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<tr>
<td>$\rho(Y, C)$</td>
<td>0.87</td>
<td>0.77</td>
<td>1.00</td>
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<tr>
<td>$\rho(Y, L)$</td>
<td>0.86</td>
<td>1.00</td>
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</table>
Table 4: Asset Price Statistics

This table reports the business cycle statistics for three versions of our model. Statistics are computed based on 10,000 replications of size 400 when the first 200 observations are discarded. The symbol $\sigma_x$ represents the standard deviation of variable $x$, $\rho_x$ represents the autocorrelation of $x$, and $\rho(x,y)$ represents the correlation between $x$ and $y$. The price of equity, $q$, and market value, $val$, are in log and HP filtered before statistics are computed. Returns are neither logged nor HP filtered. Standard deviations are measured in percentage terms. The ‘Data’ column reports statistics for quarterly U.S. data for the period 1964:1-2013:4. Column (1) reports statistics for the calibrated benchmark model with the financial constraint and both financial and productivity shocks. Column (2) reports the statistics for a version of the model with the financial constraint and no financial shocks. Column (3) reports the statistics for a version of the model with no financial constraint.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>With FC</td>
<td>With FC</td>
<td>Without FC</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\phi$ stochastic</td>
<td>$\phi$ constant</td>
<td>$A$ stochastic</td>
</tr>
<tr>
<td>$\sigma_q$</td>
<td>10.58</td>
<td>7.63</td>
<td>0.93</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma_{re}$</td>
<td>8.68</td>
<td>6.52</td>
<td>0.76</td>
<td>0.06</td>
</tr>
<tr>
<td>$\sigma_{val}$</td>
<td>11.11</td>
<td>7.66</td>
<td>0.92</td>
<td>0.11</td>
</tr>
<tr>
<td>$\sigma_{rf}$</td>
<td>0.73</td>
<td>2.03</td>
<td>0.17</td>
<td>0.05</td>
</tr>
<tr>
<td>$\rho_q$</td>
<td>0.81</td>
<td>0.64</td>
<td>0.68</td>
<td>1</td>
</tr>
<tr>
<td>$E[r_{re}]$</td>
<td>1.76</td>
<td>0.61</td>
<td>0.75</td>
<td>1.01</td>
</tr>
<tr>
<td>$E[r_{rf}]$</td>
<td>0.25</td>
<td>0.25</td>
<td>0.74</td>
<td>1.01</td>
</tr>
<tr>
<td>$E[r_{re}] - E[r_{rf}]$</td>
<td>1.52</td>
<td>0.36</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>$\frac{E[r_{re}] - E[r_{rf}]}{\sigma_{re}}$</td>
<td>0.17</td>
<td>0.06</td>
<td>0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Table 5: Asset Return Predictability

This table reports the asset return predictability for the U.S. data and our benchmark model. The data column is taken from Guvenen (2009) and contains predictability of the log of the cumulative stock returns by the log of the price-to-dividend ratio. Column ‘Benchmark Model’ reports the predictability of the log of the cumulative stock return by the log of the price-to-capital return ratio for a version of our model with the financial constraint and both productivity shocks and financial stochastic. Statistics are computed based on 10,000 replications of size 400 when the first 200 observations are discarded. Model generated data is not HP filtered.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th></th>
<th>Benchmark Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizon</td>
<td>Coefficient</td>
<td>R²</td>
<td>Coefficient</td>
</tr>
<tr>
<td><strong>Return</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-year</td>
<td></td>
<td>-0.21</td>
<td>0.07</td>
<td>-0.46</td>
</tr>
<tr>
<td>2-years</td>
<td></td>
<td>-0.36</td>
<td>0.12</td>
<td>-0.71</td>
</tr>
<tr>
<td>5-years</td>
<td></td>
<td>-0.70</td>
<td>0.23</td>
<td>-1.01</td>
</tr>
<tr>
<td><strong>Excess Return</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-year</td>
<td></td>
<td>-0.22</td>
<td>0.09</td>
<td>-0.09</td>
</tr>
<tr>
<td>2-years</td>
<td></td>
<td>-0.39</td>
<td>0.14</td>
<td>-0.16</td>
</tr>
<tr>
<td>5-years</td>
<td></td>
<td>-0.77</td>
<td>0.26</td>
<td>-0.29</td>
</tr>
</tbody>
</table>
Table 6: Comparative Statics

This table reports the business cycle statistics for several versions of our model. Statistics are computed based on 10,000 replications of size 400 when the first 200 observations are discarded. The symbol $\sigma_x$ represents the standard deviation of variable $x$, $\rho_x$ represents the autocorrelation of $x$, and $\rho(x,y)$ represents the correlation between $x$ and $y$. All variables (with the exception of returns) are in log and HP filtered before statistics are computed. Standard deviations are measured in percentage terms. Column ‘Data’ reports statistics for quarterly U.S. data for the period 1964:1-2013:4. Column ‘Benchmark’ reports statistics for the benchmark version of the model with the financial constraint and both financial and productivity shocks. Column ‘Low $\pi$’ and column ‘High $\pi$’ report the statistics for a version of the model with $\pi = 1.2\%$, and $\pi = 1.8\%$, respectively, while the benchmark value is $\pi = 1.5\%$. Column ‘Low vol.’ and column ‘High vol.’ report the statistics for a version of the model with $\sigma_{\varepsilon\phi} = 0.7$, and $\sigma_{\varepsilon\phi} = 0.9$, respectively, while the benchmark value is $\sigma_{\varepsilon\phi} = 0.8$. Column ‘Low EIS’ reports the results for ‘$\psi = 0.25$’ and column ‘High EIS’ reports the results for ‘$\psi = 0.75$’, the benchmark value is $\psi = 0.5$.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Benchmark</th>
<th>Low $\pi$</th>
<th>High $\pi$</th>
<th>Low vol.</th>
<th>High vol.</th>
<th>Low EIS</th>
<th>High EIS</th>
</tr>
</thead>
<tbody>
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<td><strong>Macro variables</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>1.54</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>$\sigma_I/\sigma_Y$</td>
<td>2.97</td>
<td>2.92</td>
<td>3.61</td>
<td>2.21</td>
<td>2.93</td>
<td>2.95</td>
<td>1.74</td>
<td>3.86</td>
</tr>
<tr>
<td>$\sigma_C/\sigma_Y$</td>
<td>0.57</td>
<td>0.57</td>
<td>0.62</td>
<td>0.66</td>
<td>0.59</td>
<td>0.55</td>
<td>0.75</td>
<td>0.60</td>
</tr>
<tr>
<td>$\sigma_{C^e}/\sigma_{C^w}$</td>
<td>3.58</td>
<td>5.75</td>
<td>1.83</td>
<td>3.60</td>
<td>3.51</td>
<td>1.64</td>
<td>5.27</td>
<td></td>
</tr>
<tr>
<td>$\rho(Y, I)$</td>
<td>0.92</td>
<td>0.93</td>
<td>0.90</td>
<td>0.97</td>
<td>0.94</td>
<td>0.90</td>
<td>0.98</td>
<td>0.90</td>
</tr>
<tr>
<td>$\rho(Y, C)$</td>
<td>0.87</td>
<td>0.77</td>
<td>0.40</td>
<td>0.96</td>
<td>0.76</td>
<td>0.79</td>
<td>0.98</td>
<td>0.37</td>
</tr>
<tr>
<td><strong>Financial variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_q$</td>
<td>10.58</td>
<td>7.63</td>
<td>12.50</td>
<td>3.88</td>
<td>7.72</td>
<td>7.55</td>
<td>7.97</td>
<td>7.35</td>
</tr>
<tr>
<td>$\sigma_{r^e}$</td>
<td>8.68</td>
<td>6.52</td>
<td>11.07</td>
<td>3.23</td>
<td>6.67</td>
<td>6.36</td>
<td>6.74</td>
<td>6.30</td>
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<tr>
<td>$\sigma_{r^f}$</td>
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<td>2.03</td>
<td>3.42</td>
<td>1.02</td>
<td>2.15</td>
<td>1.94</td>
<td>2.01</td>
<td>2.01</td>
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<tr>
<td>$\rho_q$</td>
<td>0.81</td>
<td>0.64</td>
<td>0.63</td>
<td>0.65</td>
<td>0.63</td>
<td>0.65</td>
<td>0.65</td>
<td>0.64</td>
</tr>
<tr>
<td>$E[r^e]$</td>
<td>1.76</td>
<td>0.61</td>
<td>0.58</td>
<td>0.78</td>
<td>0.60</td>
<td>0.61</td>
<td>0.31</td>
<td>0.70</td>
</tr>
<tr>
<td>$E[r^f]$</td>
<td>0.25</td>
<td>0.25</td>
<td>-0.40</td>
<td>0.69</td>
<td>0.22</td>
<td>0.26</td>
<td>-0.08</td>
<td>0.35</td>
</tr>
<tr>
<td>$E[r^e] - E[r^f]$</td>
<td>1.52</td>
<td>0.36</td>
<td>0.99</td>
<td>0.09</td>
<td>0.38</td>
<td>0.35</td>
<td>0.39</td>
<td>0.35</td>
</tr>
<tr>
<td>$\frac{E[r^e] - E[r^f]}{\sigma_{r^e}}$</td>
<td>0.17</td>
<td>0.06</td>
<td>0.09</td>
<td>0.03</td>
<td>0.06</td>
<td>0.05</td>
<td>0.06</td>
<td>0.06</td>
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</tbody>
</table>
This table reports the business cycle statistics for two versions of our model. Statistics are computed based on 10,000 replications of size 400 when the first 200 observations are discarded. The symbol $\sigma_x$ represents the standard deviation of variable $x$, $\rho_x$ represents the autocorrelation of $x$, and $\rho(x, y)$ represents the correlation between $x$ and $y$. All variables (with the exception of returns) are in log and HP filtered before statistics are computed. Standard deviations are measured in percentage terms. Column ‘Data’ reports statistics for quarterly U.S. data for the period 1964:1-2013:4. Column ‘Benchmark / Stochastic $\phi$’ reports statistics for the benchmark version of the model with the financial constraint and both financial and productivity shocks. Column ‘Alternative / Stochastic $\pi$’ reports the statistics for a version of the model with productivity shocks, a financial constraint and a time varying $\pi_t$, but with no financial shocks.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Benchmark Stochastic $\phi$</th>
<th>Alternative Stochastic $\pi$</th>
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</thead>
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<td><strong>Macro variables</strong></td>
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<td></td>
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<tr>
<td>$\sigma_Y$</td>
<td>1.54</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>$\sigma_I/\sigma_Y$</td>
<td>2.97</td>
<td>2.92</td>
<td>2.94</td>
</tr>
<tr>
<td>$\sigma_C/\sigma_Y$</td>
<td>0.57</td>
<td>0.57</td>
<td>0.57</td>
</tr>
<tr>
<td>$\sigma_C^e/\sigma_C^w$</td>
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<td>3.56</td>
<td>3.52</td>
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<tr>
<td>$\rho(Y, I)$</td>
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<td>0.93</td>
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<tr>
<td>$\rho(Y, C)$</td>
<td>0.87</td>
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<td>0.78</td>
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<tr>
<td><strong>Financial variables</strong></td>
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<td>10.58</td>
<td>7.63</td>
<td>7.87</td>
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<tr>
<td>$\sigma_{r^e}$</td>
<td>8.68</td>
<td>6.52</td>
<td>6.35</td>
</tr>
<tr>
<td>$\sigma_{r^f}$</td>
<td>0.73</td>
<td>2.03</td>
<td>1.64</td>
</tr>
<tr>
<td>$\rho_q$</td>
<td>0.81</td>
<td>0.64</td>
<td>0.68</td>
</tr>
<tr>
<td>$E[r^e]$</td>
<td>1.76</td>
<td>0.61</td>
<td>0.59</td>
</tr>
<tr>
<td>$E[r^f]$</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>$E[r^e] - E[r^f]$</td>
<td>1.52</td>
<td>0.36</td>
<td>0.34</td>
</tr>
<tr>
<td>$E[r^e] - E[r^f] / \sigma_{r^e}$</td>
<td>0.17</td>
<td>0.06</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Figure 1: Impulse Responses to a Productivity Shock

This figure shows the impulse responses of major macroeconomic variables to a productivity shock.
Figure 2: Impulse Responses to a Financial Shock

This figure shows the impulse responses of major macroeconomic variables to a financial shock.