How Does the Stock Market View Bank Regulatory Capital Forbearance Policies?

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Abstract

During the subprime crisis, the FDIC has shown, once again, laxity in resolving and closing insolvent institutions. Ronn and Verma (1986) call the tolerance level below which a bank closure is triggered the regulatory policy parameter. We derive a model in which we make this parameter stochastic and bank-specific to infer the stock market view of the regulatory capital forbearance value. For 565 U.S. listed banks during 1990 to 2012, the countercyclical forbearance fraction in capital, most substantial in recessions, could represent 17%, on average, of the market valuation of bank equity and could go as high as 100%.

Keywords: Bank regulatory closure rules or policy parameter, bank insolvency, regulatory forbearance, market-based closure rules, financial crises

JEL classification: G17, G21, G28
1 Introduction

Using market information and discipline to improve the design and implementation of bank regulatory policies, bank risk management, and deposit insurance has long been an important topic in the banking literature. Advocates of market discipline note that bank depositors and security holders in the U.S. can access a wealth of information on the institutions financial conditions on a timely basis and face high-powered incentives to act decisively on that information (e.g., Gropp et al. (2006)). However, Bond and Goldstein (2015) and others show that the use of market-based signals for regulation may be subject to the caveats regarding adverse feedback loops between agents’ actions and government policies. Notwithstanding the efficient market hypothesis not necessarily holding (see, e.g., Malkiel, 2003; Rubinstein, 2001) and the emergence of endogenous risk (see, e.g., Danielsson et al., 2011) especially during financial crises, Flannery (1998), Flannery (2001), Gunther et al. (2001), Krainer and Lopez (2004), and others, do confirm that market information is useful for ranking banks and provides incremental information for bank regulators’ supervisory monitoring and assessment.

In line with the literature above and taking advantage of the relatively higher liquidity and efficiency of listed bank equity prices, our study focuses on the use of stock market information to infer the market perception of the regulatory closure rules, known following Ronn and Verma (1986) as the regulatory policy parameter. This parameter is most often driven at least in part by politics, especially in the case of systemically important financial institutions (SIFI) which will be subject to enhanced capital requirements according to the current Basel 3 regulatory framework. This parameter represents a (hypothetical, conjectural, or even real) limit, expressed as a percentage $\rho$ ($0 \leq \rho \leq 1$) of the total debt value $D$ of the bank at the time of supervisory audit, beyond which the dissolution of assets by regulatory bodies would be a reasonable alternative. If the value of the bank falls between $\rho D$
and $D$, the insuring agency forbears (e.g., the Savings and Loans crisis in the 1980s), and in the case of extreme market turmoil, the government intervenes, as witnessed during crises such as the 2007-2009 subprime crisis via the Troubled Asset Relief Program (TARP, see Veronesi and Zingales, 2010), infusing up to $(1 - \rho)D$ and making it equal to $D$. If the bank value falls below $\rho D$, the insuring agency steps in to dissolve the assets of the bank. A compelling reason for not closing an insolvent bank is the loss of significant franchise value (stemming from core deposits, customer relationships, and valued personnel) that occurs after an FDIC seizure. If the market insolvency closure rule is strictly followed, the policy parameter is equal to one, and there is regulatory capital forbearance when the policy parameter is below one. Along with other assumptions, Ronn and Verma (1986) fix $\rho$ at a constant 0.97 to yield an aggregate weighted average premium of $1/12$ percent for the US deposit insurance premium. Since, in Ronn and Verma (1986)'s model, the value of forbearance is assumed to be the value of the capital assistance, many authors call this practice, regulatory capital forbearance.

Under the Basel 3 countercyclical capital buffer framework, bank regulators would try to ensure that banks build up capital levels during good times so that they can run it down in bad times (see, e.g., Drehmann et al., 2010; Hanson et al., 2011; Kowalik et al., 2011; Elliott, 2011). In effect, to conduct countercyclical capital requirements policies, regulators are compelled to adopt a time-varying policy parameter. To the best of our knowledge, only Lai (1996) treats the policy parameter as a stochastic process to reflect the uncertainty of the bank closure rule set.\footnote{Kane (1986) treats safety-net guarantees as a two-part option: a taxpayer put (see, e.g., Eberlein and Madan, 2012) and a knock-in, stop-loss call on the firm’s assets; while Allen and Saunders (1993) model forbearance as forfeiture by the deposit insurer of the value of its call component of the deposit insurance option. However, these authors do not explore the implications of stochastic $\rho$ on the value of forbearance and do not perform empirical studies because of the lack of data.}

In this paper, we extend Lai (1996)'s framework by modeling $\rho$ as a more realistic stochastic process that is mean-reverting and bound by zero and one. To justify this, it is argued that the regulatory forbearance policy can be treated as “reduced form” and described by a state variable. Further, the policy is constrained by economic, legal, political, regulatory competition and bureaucratic considerations that are mean-reverting throughout their respective cycles. Two main sources of uncertainty regarding the variation of
bank regulatory forbearance may be posited: (1) asymmetric information and (2) stochastic state variables. In the first case, because of confidential information obtained from on-site examinations, private information may exist that is known only to the regulator.\(^2\) For actively stock-listed banks in efficient markets with increased disclosure, such a scenario may be considered not as a major source of uncertainty. For closely-held banks, it is more plausible but we discard this possibility by only studying banks with available equity prices. In the second situation, we suppose that while the policy of the regulator is known to all (e.g., the Too Big to Fail (TBTF) doctrine), it is a function of some external, hardly predictable state variables, structural changes, or unforeseeable events. While asymmetric information and stochastic state variables are two possible sources of regulatory uncertainty, only the latter is explicitly modeled in this paper.\(^3\) With this justification, we develop an enhanced Ronn and Verma (1986) model to infer the market-based, bank-specific, and more flexible policy parameter from the market value of bank equity.\(^4\) Then, based on the calibration of our model with U.S. listed banks, by gauging the size of the capital forbearance value and contributing to the ongoing debate on adequate bank capital, we seek answers to the following two research questions: 1) How does the time-varying capital forbearance portion embedded in bank equity depend on various banks’ own risk and business cycle variables? and 2) How do banks’ market-assessed intrinsic (i.e., de-
void of the forbearance subsidy) capital ratios (or inverse leverage ratios) react to various business cycles and the banks’ own risk variables?

Toward this end, we first develop a two-factor model, in which we model $\rho$ as an exponential of a negative Cox-Ingersoll-Ross (CIR) process (Cox et al., 1985) and the value of the bank as a log normal process with a stochastic drift term. We derive a closed-form solution for the equity represented by market capitalization (market cap), which is viewed as a call option on the value of the bank following the structural approach of Merton (1974). The model is then calibrated to 565 U.S. banks’ market capitalization and total debt data from 1990-2012 by means of the Unscented Kalman Filter in conjunction with the Quasi-Maximum Likelihood Estimate (QMLE) to obtain series of $\rho$s and implied asset values. This approach offers a methodological innovation that could give rise to follow-up studies and the calibration results described later provide useful insights in policy making. Further, we derive the Forbearance Fraction in Capital, which is defined as one minus the ratio of the market value of intrinsic equity to total equity. We estimate bank intrinsic equity, Intrinsic Market Cap, indicative of economic capital, as a hypothetical and counter-factual equity assuming zero capital forbearance as if the FDIC adhered to the market closure rule. We also compute the effective policy parameter, which weighs, not only the likelihood of a bank being insolvent, but also the likelihood of the bank not requiring forbearance. These two novel metrics provide us relatively cleaner measures of the market perceptions of the time-varying regulatory agencies’ implementation of the bank closure rule. Using Wells Fargo as a case study, we show that our model effectively captures the market view about the regulatory capital forbearance practice.

To address our two research questions, we estimate a system of two equations using the “Two-step System Generalized Method of Moments (2SGMM)” (Blundell and Bond, 1998), which controls endogeneity between regression variables. In the first equation, the left-hand-side (LHS) variable is the Forbearance Fraction in Capital computed from the filtered policy parameters, and the right-hand-side (RHS) variables include a one-quarter-lagged

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5 For regulatory purposes (e.g., Basel capital requirements), Tier 1 capital, the major measure of a bank’s financial strength, is composed of core capital. This best quality of capital consists primarily of common stock and disclosed reserves (or retained earnings), but may also include non-redeemable non-cumulative preferred stock.
LHS variable, the business cycle proxies (GDP Growth, GDP Output Gap, and S&P 500 index), banks’ risk factors (Idiosyncratic Volatility, Asset Volatility, Systematic Beta, and Systemic Risk measures), and other control variables (log Total Assets, Intrinsic Market Cap to Implied Asset Value Ratio, and Implied Asset Value to Book Total Assets Ratio). Simply, Intrinsic Market Cap / Implied Asset Value is a measure of the equity capital ratio devoid of the regulatory forbearance value, whereas the Implied Asset Value to Book Total Assets Ratio is a proxy for Keeley (1990)’s bank charter value. In the second equation, the LHS variable is the Intrinsic Market Cap to Implied Asset Value ratio. The RHS variables are the same as those in the first equation except that the Intrinsic Market Cap-to-Implied Asset Value ratio replaces the Forbearance Fraction of Capital.

Our regression results show that the market believes that larger banks benefit more from capital forbearance, which suggests that the “Too Big To Fail” (TBTF) doctrine is prevalent. Naturally, the market expects banks with strong performance and higher market-assessed levels of owner-contributed capital to need less forbearance. The market also expects a bank with high market power does not require forbearance, consistent with the competition-stability paradigm, and an institution with high franchise value to cost less to rescue. The stock market holds the view that, ceteris paribus, banks with higher idiosyncratic risk and systemic risk will benefit from more capital forbearance, thereby leading to higher bailout costs to taxpayers. The market expects banks to receive, in a countercyclical fashion, increased forbearance in bad times and less forbearance in boom times. More forbearance lowers the market intrinsic capital ratio (or bank owner’s contributed capital with the forbearance subsidy removed). For our period of study and our bank sample, the estimated annual capital forbearance subsidy amounts to 7.6 billion USD or 13.5% of the FDIC aggregate cost of explicit deposit insurance. With an embedded mean capital forbearance portion amounting to 17% of the market value equity, the largest banks exhibit a mean market capital to implied asset value ratio of 16.4% and a mean book equity to total assets ratio of 8.9%. Our framework may be useful for market assessments and trackings of regulatory capital forbearance and may serve as an additional red flag tool for supervisory bodies and policymakers.

The remainder of the article is organized as follows. Section 2 sets up the two-factor
model to extend the framework of Ronn and Verma (1986) and Lai (1996) and derive a closed-form solution for the model used to extract the market view and the cost of bank regulatory forbearance policies. Section 3 describes the procedure for model calibration in detail. Section 4 presents the empirical results and discusses the findings. Section 5 concludes the paper. The appendices contain technical details and supplemental results.

2 A model of the bank regulatory policy parameter

In this section, we start from the pay-off function of a bank equity holder, then derive closed-form solutions for the equity of a bank and various market-based variables under Merton (1974)'s structural model framework.

2.1 Pay-off function of a bank equity holder

We follow Ronn and Verma (1986)'s seminal model, as it is relatively simple and will be amenable to econometric implementation later. In the model, the equity of a firm is a European call option on the value of the firm, $V$, with the strike price being the debt face (i.e., nominal) value, $D$. To model the FDIC closure rules, Ronn and Verma (1986) modify the model by simply adjusting the strike price, i.e., multiplying the debt level by the regulatory policy parameter $\rho$. At supervisory audit time, if $\rho D \leq V \leq D$, the equity holder’s pay-off is $V - \rho D$. In their model, the equity holder’s pay-off, when $V > D$, is still $V - \rho D$. A rational investor would not include $(1 - \rho)D$ as part of her pay-off when the bank is solvent ($V > D$). Since in this case there is no forbearance, the pay-off, when $V > D$, is assumed to be $V - D$.\(^6\) Let us underscore that we follow the Ronn and Verma (1986)'s European option framework for tractability and acknowledge its drawbacks since as these authors, we ignore the American contingent claims features and other complex

\(^6\) Admittedly, if one considers the sale of the bank’s equity to a third party during a period in which the bank is not in distress and the FDIC is not stepping in to prop up the sale, the value should still reflect the forbearance which could occur as a function of future outcomes. Alternatively, if the pay-off function simply represents the fact that the FDIC is willing to facilitate a make-whole buyout along the lines of Wells Fargo’s acquisition of Wachovia examined later, in our case study, it is not clear that the equity holders actually do receive the difference between the true value and the debt. If these were the cases, we underestimate the forbearance value.
optionalities affecting the value of forbearance.\textsuperscript{7} One may argue that unrealistic model assumptions might affect the empirical results but clearly the Ronn and Verma (1986) model is useful since it has generated numerous empirical studies by academics and practitioners alike.\textsuperscript{8}

2.2 Model setup

Let us define a $2 \times 1$ column vector of state variables

$$X_t = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

which, under the risk-neutral measure $Q$, follows the stochastic differential equations (SDE)

$$dX_t = d\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} \kappa \theta \\ -\kappa \theta \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} dt + \begin{pmatrix} \sigma_1 \sqrt{x_1(t)} \\ 0 \end{pmatrix} \begin{pmatrix} w_1(t) \\ w_2(t) \end{pmatrix}.$$  \hspace{1cm} (1)

If we look at $x_1$ and $x_2$ separately, $x_1$ follows a CIR process, and $x_2$ follows a Wiener process with a stochastic drift term. $\theta$ is the reversion level of $x_1$, $\kappa$ is the reversion speed of $x_1$, $\sigma_1$ is the volatility scaled by the square root of $x_1$, $\mu$ is the mean of $x_2$ and $\sigma_2$ is the volatility of $x_2$. $w_1(t)$ and $w_2(t)$ are independent Wiener processes under the measure $Q$.

\textsuperscript{7} Notice that the payoff is a non-monotonic function of the value of the bank. We argue that due to the reputational concern, shareholders of a bank would stay away from $V$ being less than $D$ instead of intentionally dumping, destroying or paying the excess assets as dividends to shareholders to take advantage of a higher pay-off when $V < D$, since it is extremely prohibitive for insolvent banks to do so when the close monitoring, scrutiny and discipline by regulators and markets are in order. This means that reputational costs restrain adverse selection incentives so that the conditional distribution of $V$ is invariant to $V - D$. Corbett and Mitchell (2000) find that in many cases policy authorities make offers of bank rescue, and banks are reluctant to accept these offers due to reputation concerns. We acknowledge this only explains that banks wish to stay out of the regulatory insolvency state and it is not necessarily related to the payoffs the bank actually receives or expects to receive should it find itself in the regulatory insolvency state. Another reason why this might hold nevertheless is that in the real world the FDIC steps in a lot earlier and does not allow banks certain type of risk-seeking actions.

\textsuperscript{8} As a matter of fact, a large literature follows Ronn and Verma (1986) and employs a constant regulatory policy parameter applied to all banks to estimate the two unobservable values of the bank assets and asset volatility (e.g., Flannery and Sorescu, 1996; Hassan et al., 1994; Cordell and King, 1995, among numerous others). These two inputs have been used for many ends, for instance, to estimate the value of historical government guarantees as in Hovakimian and Kane (2000) and Flannery (2014). A growing literature uses this methodology with constant $\rho$, to estimate banks’ systemic risk; see, e.g., Lehar (2005), Hovakimian et al. (2012). There has also been a great deal of research on the impact of this policy parameter on deposit insurance pricing; e.g., Lai (1996) and Hwang et al. (2009) among others.
The correlation between $x_1$ and $x_2$ is captured by $\varphi$. Note that we model exogenously the stochastic policy parameter as $\rho_t = e^{-x_1(t)}$, so that $\rho_t$ lies between zero and one as $x_1$ is non-negative, and $\rho_t$ is mean reverting. Since we model the value of a bank as $V_t = e^{x_2(t)}$, the dynamic of $V_t$ is given by,

$$dV_t = de^{x_2(t)} = \mu_V V_t dt + \sigma_V V_t dw_2(t)$$

where

$$\mu_V = \mu + \varphi x_1(t) + \frac{\sigma^2}{2}.$$

For the sake of tractability, we model $\rho_t$ and $V_t$ in a reduced-form manner.\(^9\) We impose economic structure ex-post in later empirical sections to see how $\rho_t$ and $V_t$ correlate with various other variables (both bank-specific and macro variables).

At time $t$, the equity value of a bank represented by the market cap, $E_t$, which is a call option on $V_t$ with maturity $T$, has the following pay-offs at time $T$:

$$E_T = \begin{cases} 
V_T - D & \text{if } V_T > D \\
V_T - \rho_D D & \text{if } \rho_D D \leq V_T \leq D \\
0 & \text{if } V_T < \rho_D D.
\end{cases} \tag{2}$$

The stock market assessment of the supervisory forbearance is captured by this pay-off function with a random strike price. Then, with $\mathbb{E}_t$ denoting the expectation, under the

\(^9\) Endogenizing $\rho$ within the model would of course yield a richer framework which might offer interesting results, but we defer this challenge to a further research. Here we rely on our reduced-form model, which is easy to calibrate to market data, to provide empirical insights for our kind of research inquiry.
measure $Q$ conditional on the information up to $t$, we have\(^\text{10}\)

\[
E_t = B_t(T) \mathbb{E}_t^Q (E_T)
\]

\[
= B_t(T) \left[ (V_T - D) \mathbb{1}_{\{V_T > D\}} + (V_T - \rho_T D) \mathbb{1}_{\{\rho_T D \leq V_T \leq D\}} \right]
\]

\[
= B_t(T) \left\{ \mathbb{E}_t^Q \left[ (V_T - \rho_T D) \mathbb{1}_{\{V_T > \rho_T D\}} \right] - D \mathbb{E}_t^Q \left[ (1 - \rho_T) \mathbb{1}_{\{V_T > D\}} \right] \right\}
\]

\[
= B_t(T) \left\{ \mathbb{E}_t^Q \left[ \left(e^{x_2(T)} - e^{-x_1(T)} D\right)^+ \right] - D \mathbb{E}_t^Q \left[ \left(1 - e^{-x_1(T)}\right) \mathbb{1}_{\{x_2(T) > \log D\}} \right] \right\},
\]  

(3)

where $B_t(T) = e^{-r(T-t)}$ is the discount factor, $r$ is the assumed-constant interest rate, and $\mathbb{1}_{\{\cdot\}}$ is an indicator function equal to one when $\{\cdot\}$ holds, and zero otherwise. In the implementation, we set $D = F/B_t(T)$ where $F$ is the book value of the debt level at time $t$.\(^\text{11}\)

In sum, in the spirit of Ronn and Verma (1986), we offer two enhancements: (a) we employ a modified pay-off function (see (2)) for the call option while Ronn and Verma (1986) and Lai (1996) use a pay-off that may overshoot $\rho$,\(^\text{12}\) and (b) in Ronn and Verma (1986), $\rho$ is a constant over time, whereas we model $\rho$ as a mean reverting stochastic process. Moreover, in Lai (1996), the strike price process $\rho D$ as a whole is assumed to be a geometric Brownian motion, which is not a reasonable assumption for either $\rho$ or $D$ given the fact that the positive $\rho$ should be less than one and $D$ is a deterministic process from $t$ to $T$ (assuming constant interest rate).

\(^{10}\) The risk-free interest rate is assumed to be constant. As in Ronn and Verma (1986), it is also assumed that the effects of interest rate changes are captured in the assets value and associated volatility, i.e., the present value of assets are brought about by anticipated changes in both the investment opportunity set and the interest rate. Unanticipated changes in interest rates are accounted for in the asset risk.

\(^{11}\) Albeit for many banks, in particular large banks, non-core deposits and other debt instruments can be non negligible, an implicit assumption behind $D = F/B_t(T)$ is that all debt is issued at the risk-free interest rate. As mentioned in footnote 7 of Ronn and Verma (1986), this is sensible for the insured deposits, which for most banks represent a substantial portion of their total debt. For the small remaining portion of the total debt, this will have a negligible effect on the values of deposit insurance and equity alike. Another implication of $D = F/B_t(T)$ is that $V_t$ denotes the value of the assets net of the deposit insurance fees levied. In other words, $V_t$, on top of other value drivers, includes the net value of deposit insurance provided by the FDIC (i.e., benefits minus premiums paid).

\(^{12}\) The pay-off function in Ronn and Verma (1986) and Lai (1996) adds an extra forbearance value to the bank when the bank is solvent. Therefore, the implied $\rho$ from a calibrated Ronn and Verma (1986) model may have been higher than the $\rho$ resulting from a sensible non-linear pay-off function. However, Acharya and Dreyfus (1989) show that $\rho$ slightly higher than 1 might be optimal. Therefore, we reckon that from a welfare perspective, whether the socially optimal rule will be $\rho$ less than 1 depends on all sorts of imperfections, externalities (real or informational ones) and other factors that we ignore.
2.3 Market evaluation of the capital forbearance

To apply the model developed above, we propose two novel measures of the stock market evaluation of the forbearance: the Forbearance Fraction in Capital denoted by \( \text{FFC} \), and the Effective Policy Parameter denoted by \( \rho^* \). These are defined as

\[
\text{FFC}_t = \frac{E_t - E_t^{\rho=1}}{E_t} \quad \text{and} \quad \rho_t^* = E_t^P \left[ 1_{\{V_T > D\}} + \rho_T 1_{\{V_T \leq D\}} \right],
\]

where \( E_t^{\rho=1} = B_t(T) \mathbb{E}_i^Q (V_T - D)^+ \) which corresponds to the value of the equity when assuming \( \rho_t = 1 \), and \( E_t^P \) is the expectation under the physical measure \( P \). \( E_t^{\rho=1} \) represents a counter-factual valuation of the bank equity if the FDIC adheres strictly to the market closure rule. Whether \( \rho \) is constant or stochastic, we assume the same value for \( V_t \).

By construction, \( \text{FFC} \) represents how forbearance is valued in terms of total equity, and can be used to decompose the market cap into the forbearance subsidy and intrinsic value; whereas \( \rho^* \) represents an effective value of the policy parameter. \( \rho^* \) is better than \( \rho \) as a proxy for the market view of the policy parameter. This is because \( \rho \) is only relevant when the probability of \( \{ V_T \leq D \} \) is significant, while \( \rho^* \) also takes into account the case of no forbearance.

Further, to compare it with the forbearance subsidy, we compute the value of deposit insurance, which is a put option written by the FDIC with a strike price equal to total debt, and then scaled down by the proportion of total (insured) deposits to total debt. This put is given by

\[
DI_t = v_t B_t(T) \mathbb{E}_i^Q \left[ (D - V_T) 1_{\{V_T < D\}} \right], \tag{4}
\]

One may argue that investors need to know the current stance of forbearance policy when applying the model. However, this is not a necessary condition for our model to hold. Applying our model to infer time-varying information about the policy parameter is analogous to applying Heston (1993)'s stochastic volatility option model to infer time-varying volatility. As long as the market prices carry the relevant information, a properly designed and calibrated model will be able to convey the information. Since most investors are price takers, this does not require all investors to price the stock according to the model.

Albeit subject to the Lucas critique, this assumption is intuitively sensible, \( V_t \) can be roughly represented as \( \rho_t F + E_t(\rho = \rho_t) \), it is reasonable to assume \( (1 - \rho_t) F = E_t(\text{with } \rho \text{ stochastic when } \rho = \rho_t) - E_t(\text{with } \rho \text{ nonstochastic and } \rho = 1) \). Here \( F \) is the total debt book value at time \( t \), and includes the value of the deposit insurance put making up the shortfall to guarantee \( F \).
where $v_t$, not explicitly related to $E_t$, is the Total Deposits to Total Liabilities ratio.

### 2.4 Closed-form solutions of $E_t$, FFC, and $\rho^*$

We first derive closed-form solutions for $E_t$ and $\rho^*$. Within the purview of the affine framework, we employ the techniques developed in Duffie et al. (2000) to derive the closed-form solutions. By PROPOSITIONS 1 and 2 in Duffie et al. (2000), we have

$$
\mathbb{E}_t^Q \left[ \left( e^{x_2(T)} - e^{-x_1(T)} \right) D \right] = \mathbb{E}_t^Q \left[ e^{-[1,0]X_T} \left( e^{[1,1]X_T - D} \right) \right]
$$

$$
= G_{[0,1],-[1,1]}(D; X_t, T - t) - DG_{[-1,0],-[1,1]}(D; X_t, T - t),
$$

(5)

$$
\mathbb{E}_t^Q \left[ \left( 1 - e^{-x_1(T)} \right) \mathbb{1}_{\{x_2(T) > \log D\}} \right] = \mathbb{E}_t^Q \left[ \mathbb{1}_{\{[0,1]X_T > \log D\}} \right] - \mathbb{E}_t^Q \left[ e^{-[1,0]X_T} \mathbb{1}_{\{[0,1]X_T > \log D\}} \right]
$$

$$
= G_{[0,0],[0,-1]}(D; X_t, T - t) - G_{[-1,0],[0,-1]}(D; X_t, T - t),
$$

(6)

$$
\mathbb{E}_t^P \left[ \mathbb{1}_{\{V_T > D\}} + \rho_T \mathbb{1}_{\{V_T < D\}} \right] = \mathbb{E}_t^P \left[ \mathbb{1}_{\{[0,1]X_T > \log D\}} \right] + \mathbb{E}_t^P \left[ e^{-[1,0]X_T} \mathbb{1}_{\{[0,1]X_T < \log D\}} \right]
$$

$$
= G_{[0,0],[0,-1]}^P(D; X_t, T - t) + G_{[-1,0],[0,1]}^P(D^{-1}; X_t, T - t),
$$

(7)

where $[\cdot, \cdot] \in \mathbb{R}^2$ denotes a $1 \times 2$ row vector. Eq. (5) and Eq. (6) (multiplied by $D$) represent the first and second elements in the curly brackets of Eq. (3) used to obtain $E_t$, respectively. Eq. (7) is for $\rho_{t}^*$. $G_{b,\delta}(z; X_t, \tau)$ in these equations is defined in Appendix A.

Using the results from Eq. (5) to Eq. (7), we obtain the closed-form solutions for $E_t$, FFC, $\rho^*$, and $DI_t$. Note that $E_t^{\rho=1}$, which is required for the calculation of FFC, is simply given by the Black and Scholes (1973) formula, since with $\rho = 1$, $V_t$ is a geometric Brownian motion. To calibrate the model, Duffee (2002)’s specification for the market price of risk is employed to derive the dynamic of $X_t$ under the physical measure $P$. That is, the

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15 When applying Black and Scholes (1973) formula to $E_t^{\rho=1}$, the volatility of $V$, $\sigma_2$, is set at its estimate from the full sample calibration of $E_t$; the value of $V$ at time $t$ is the filtered value of $V_t$ from the calibration; the strike price is the debt level at time $t$; and the interest rate is the same as the one used to compute $E_t$. 

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relation between the Wiener processes under the two measures is given by

\[
\begin{align*}
    w_1(t) &= w_1^P(t) + \lambda_1 \int_0^t \sqrt{x_1(s)} ds \\
    w_2(t) &= w_2^P(t) + \lambda_2 t,
\end{align*}
\]

where \( \lambda_1 = \frac{x_1 - x_1^P}{\sigma_1^2} \) and \( \lambda_2 = \frac{\mu^P - \mu}{\sigma_2^2} \). The moment conditions for \( X_t \) under the measure \( P \) used to calibrate the model are presented in Appendix B. The time to the next supervisory examination, \( T - t \), is assumed to be one year in our empirical implementation.\(^{16}\)

Note that in our setup, although the volatility of \( V_t, \sigma_2 \), is constant, the volatility of \( E_t \) is stochastic by construction and is closely linked to \( \sqrt{x_1(t)} \). As pointed out by Duan (1994), the widely employed Ronn and Verma (1986) estimation method that assumes constant equity volatilities is incompatible with the Merton (1977) deposit insurance pricing model. Note also that we allow time-variability for both volatility and risk premium but we do not account for liquidity in our model. This is less critical for our equity model since stock markets are relatively much more liquid than debt and credit default swap markets considered for instance by Acharya et al. (2015) and Kelly et al. (2015) among others.

Further, since we account for measurement errors in the observed market cap, the transformed-data maximum likelihood estimation (MLE) method developed in Duan (1994) is improper. Given the nonlinearity of the model and the latency of the factors, to calibrate the model, we employ the QMLE in conjunction with the UKF.\(^{17}\) We describe the methodology in Section 3.2 and Appendix C.

In the following sections, we conduct empirical analyses to estimate bank regulatory capital forbearance policies and assess their drivers.

\(^{16}\) This assumption is common not only in the banking literature, but also in the credit risk literature in which the Black-Scholes-Merton option-pricing model is widely employed to compute “Distance-To-Default”. See for instance, Crosbie and Bohn (2003), Vassalou and Xing (2004), and Duffie et al. (2007). Ronn and Verma (1986) indicate that varying \( T - t \) between a quarter of a year to five years has little impact on the relative ranking of the banks.

\(^{17}\) In a nutshell, since our state variables are unobservable, and the observable series of bank market caps have a nonlinear dependence on the latent state variables, we obtain the QMLE of our model parameters using the UKF procedure. The UKF deals with nonlinearities in the measurement equation, works through deterministic sampling of points (sigma points) in the distribution of the innovations to the state variables, and captures the conditional mean and variance-covariance matrix of the state variables with adequate accuracy.
3 Empirical analysis I: Model calibration

In this section, we first describe the data, then delineate our model calibration procedure. In the rest of the section, we present a case study: Wells Fargo, and some aggregate results of our model-derived variables.

3.1 Data description

From the Bloomberg database, we build a sample of 23 years (from 1990 to 2012) of both market and financial statements accounting data for 706 banks as well as macroeconomic variables. Because many banks merged or went out of business or do not have complete data during this period, banks are not necessarily included in every year of the sample, giving an unbalanced panel. Our resulting sample consists of 706 banking firms that appear in any one of the 23 years. Based on their total assets, after a thorough analysis of bank size percentiles, we categorize, for statistical purpose, the banks into four groups as the following: Large Banks (\(\geq 90\)th percentile), Big Banks (75th - 89th percentile), Medium Banks (20th - 74th percentile), Small Banks (\(\leq 19\)th percentile). Following the empirical evidence on the TBTF implication on diversification and market discipline (see, e.g., Filson and Olfati, 2014; Acharya et al., 2015), we exclude Small Banks group in our analysis to focus on sizable banks which are commonly believed to receive forbearance benefits, and replace it with All but Small Banks, thereafter All Banks (20th - 100th percentile). This leaves us 565 banking firms. The average total assets of these four groups are: 149 billion, 18 billion, 12 billion, and 2 billion.

\[ \text{We do not exclude merger and acquisition events (M&A) in our sample. On the one hand, M&A is a huge part of our data, many banks underwent different sizes of M&A almost every year in recent years. If we stick to an M&A free sample, two third of our sample will be thrown away. On the other hand, our model is robust to M&A. Just as how the model handles fluctuations in the bank's market cap due to other events, the time-varying market-based policy parameter will be identified through combinations of the market cap and total liabilities of the bank at different point in time. For example, a buoyancy in the bank's market cap due to market's anticipation of a takeover bid without a significant change in the total liabilities will not be identified as an increase in the market perceived forbearance. Furthermore, as noted by Bijlsma and Mocking (2013) and Brewer and Jagtiani (2013), many of the M&As do carry important information about the intention to seek for forbearance (TBTF).} \]

\[ \text{The list of the banking firm names are available from the authors upon request.} \]

\[ \text{These dollar-based size thresholds are mere statistical categorizations since the characteristics of U.S. banking companies do not conform neatly to base ten focal points. In 2012, the thresholds between these decile-based groups were in USD 393 billion, 12 billion, and 2 billion.} \]
six billion, one billion, and 18 billion, respectively.\textsuperscript{21}

To filter the regulatory forbearance parameter and implied asset values, in the model calibration we use one-year U.S. Treasury rates (used to calculate the discount factor $B_t(T)$ in Eq. (3)), banks’ total liabilities, and market caps. The market data are monthly (month end observations from daily data), and the total liabilities data are quarterly. The total liabilities data are fitted to the monthly frequency using cubic smoothing splines to match the frequency of the market data.

### 3.2 State space, Quasi-Maximum Likelihood Estimates, and Unscented Kalman Filter

Basically, we set a two-factor model for the market cap of banks. The first factor is the unobservable regulatory policy parameter, and the second is the unobservable bank asset value. Note that the market value of a bank’s assets includes the value of both booked and unbooked assets including the bank’s franchise value. The model is cast into a state space, and then parameters are calibrated to the observed market cap using QMLE together with the UKF method. More details of the UKF are presented in Appendix C.

Our reduced-form model is similar to the interest rate and credit spread the term structure models. Since we do not have the term (maturity) structures of our data, we rely exclusively on equity data. This makes certain parameters hard to pin down, for example, $\theta$ and $\kappa^P$. We therefore set them to be conventional values during the model calibration. From pre-test results, we also find that setting parameter $\varphi$ to zero has little impact on the filtered state variables, but ensures a more robust model calibration since we do not have to deal with trigonometric functions and imaginary numbers in the numerical procedure. Therefore, in our empirical analysis, we assume that $\varphi$ is zero. We relegate the implementation details to Appendix D.

To calibrate the model, we need the dynamics of the state variables under the real measure $P$. We employ the essentially affine specification for the market prices of risks.

\textsuperscript{21} These averages are calculated over the past two decades and across all individual banks within these groups. The cross-sectional averages in USD at the end of 2012 for the groups of Large, Big, Medium, and All Banks are 393 billion, 12 billion, two billion, and 45 billion, respectively.
(Duffee, 2002), which allows for time-varying risk premiums, to derive the dynamics under measure P from the dynamics under the risk-neutral measure Q in Eq. (1). That is, the transition equation in the state space is given by

$$dX_t = \left[ \begin{pmatrix} \kappa \theta \\ \mu^p \end{pmatrix} + \begin{pmatrix} -\kappa & 0 \\ 0 & 0 \end{pmatrix} X_t \right] dt + \begin{pmatrix} \sigma_1 \sqrt{x_1(t)} & 0 \\ 0 & \sigma_2 \end{pmatrix} dW_t^P,$$

where $W_t^P$ consists of two independent Wiener processes under the measure P. The change of measure reflects the risk premia carried by the state variables.\(^{22}\) Although the transition density of the latent factor is non-Gaussian, Duan and Simonato (1999) show that the first two moments of the latent factors can approximate the distribution of the CIR process very well. Therefore, when employing the UKF, we assume a Gaussian transition density and only consider the first two moments of the transition density. These are given by Eq. (A2) and Eq. (A3). The discrete time interval $\Delta t$ is one month ($1/12$).

Since a bank’s market cap enters into the measurement equation in our state-space setup, we assume that the observed market caps denoted by $y_t$ are contaminated by measurement errors,\(^{23}\) and that the noise is iid and normally distributed as expressed by

$$y_t = E_t + \zeta_t \equiv G(X_t; \Phi, D) + \zeta_t,$$

where $\Phi$ is the parameter set, $E_t \equiv G(X_t; \Phi, D)$ is the nonlinear function of $X_t$ given by Eq. (3) to Eq. (6), and

$$\zeta_t \sim \text{IID } N(0, \omega^2),$$

where $\omega$ is a free parameter. The parameters are estimated by maximizing the sum of the following log transition density over the whole sample:

$$L_{t|t-\Delta t} \propto -\frac{\log \left( P_{y_t|t-\Delta t} \right)}{2} - \frac{(y_t|t-\Delta t - y_t)^2}{2P_{y_t|t-\Delta t}},$$

\(^{22}\) The specification of the market price of risk is required to model non-diversifiable risks. The risks of our state variables are indeed non-diversifiable, here, especially the risk of the non-tradable variable $\rho$.

\(^{23}\) For instance, trading noises in Duan and Fulop (2009) or model errors in Simonato (2013) can be assimilated as measurement errors.
where \( P_{y|t-\Delta t} \) and \( y_{t|t-\Delta t} \) are estimates of the variance and mean of the measurement at time \( t-\Delta t \). These are outputs from the UKF procedure outlined in Appendix C.

To sum up, given a set of market caps, total liabilities, and model parameters, the UKF procedure outputs a series of filtered forbearance factors \( \rho_s \), implied asset values \( V_s \) and a likelihood value specific to a bank. The QMLEs of the model parameters are those that maximize the likelihood value, and the corresponding forbearance factors \( \rho_s \) and the bank implied asset values \( V_s \) are regarded as their optimal estimates. Once the model is calibrated, we can readily compute \( \text{FFC} \) and \( \rho^* \) from \( \rho_s \) and \( V_s \).\(^{24}\)

### 3.3 Case study: Wells Fargo

In this section, we take Wells Fargo as an examples to depict the dynamics of the stock market view of the regulatory forbearance factor manifest in an individual bank.

Figure 1 illustrates the time series of the (fitted) market cap, implied firm value, smoothed total liabilities, \( \rho^* \), and \( \text{FFC} \) of Wells Fargo (on a monthly basis) from early 1998 to late 2012.\(^{25}\) Before the financial crisis, \( \text{FFC} \) was relatively lower and stable. Wells Fargo was one of the banks to receive funds from the first round of government bailout money (the TARP) in the fall of 2008. The acquisition of Wachovia at the end of 2008 significantly increased Wells Fargo’s total liabilities in 2009. In early 2009, when there were negative allegations against Wells Fargo in the media, Wells Fargo’s market cap dropped significantly. Although it rebounded quickly, within a month, the total liabilities remained double what they had been. \( \text{FFC} \) surged from below 10% to about 70% within a quarter, then took almost four years to return to 10%. The drop in the market cap and the increase in the total liabilities in 2009 induced a significant decrease in both the Market Cap / Implied Asset Value (from 20% to 7%) and the Intrinsic Market Cap / Implied Asset Value (from 17% to

\(^{24}\) We calibrate the model using the full sample. In other words, we only run in-sample calibration once for all, and do not use rolling windows to update model parameters. The rolling window calibration seemingly offers time-varying property for the model parameters, e.g., \( \sigma_1 \) and \( \sigma_2 \). However, this time-varying property is theoretically inconsistent with the assumption that these parameters are constant. Therefore, the resulting time-varying \( \sigma_1 \) and \( \sigma_2 \) are not comparable to our time-varying \( \rho \) and \( V \) which are embedded in a natural fashion in the model and filtered using a theoretically consistent approach.

\(^{25}\) Wells Fargo in its present form is the result of a merger between San Francisco-based Wells Fargo & Company and Minneapolis-based Norwest Corporation in 1998, so the financial and market data for Wells Fargo in our database only starts in early 1998.
3\%). The drop in Intrinsic Market Cap / Implied Asset Value was even more dramatic. This observation implies that the market was not optimistic about Wells Fargo’s future despite the huge capital injection (with equity warrants/preferred stock holdings) from the government. In contrast, the Book Equity / Total Asset did not change much in 2009, meaning the book ratio were almost silent about the concern from the market.

Another observation worth noting is that our model fits the equity data very well. There is barely any difference between the fitted Market Cap (the line with dots) and the actual Market Cap (the line with circles) (see the upper panel of Figure 1). The great fit of the model implies that the dynamic of our filtered implied asset values is congruent with the distribution of the observed equity values.

[Insert Figure 1 about here]

3.4 Aggregate results for model-derived variables

In this subsection, we look at our model-derived focal variables, FFC and Intrinsic Market Cap / Implied Asset Value for all the banks. The dynamics of the cross-sectional distribution of FFC over time, on a monthly basis, are shown in Figure 2. Note that the light gray area (75th-90th percentile) covers the majority of the distribution, meaning that the distribution of FFC is highly skewed to the left and has a heavy right tail. As confirmed in Figure 3, we observe large FFCs, which are far from the median value of FFC, much more often than we would do in a normal distribution. This pattern is more significant for larger-sized banks. While not necessarily taking these values literally, we find that, in large banks, capital forbearance takes up on average 17% of the market value of bank equity, and can go as high as 100%. As depicted in the first panel in Figure 2, even though the medians of FFC for all four groups started to rise around the same time during the 2008-2009 financial crisis, the FFCs of a few mega banks show a much earlier upward trend than others. This might indicate that the stock market perceived large banks’ troubles long before the crisis became widespread. Another interesting observation is that during 1998 the FFCs of most banks were actually relatively low. Only a few large banks had very high FFCs during this period. These large banks probably had more interna-
tional exposure than the others, and would therefore have been more affected by the 1998 Asian and Russian financial crisis. This observation might also be related to the consequences of the Gramm-Leach-Bliley Act of 1999 that led to deregulation toward universal banking, allowing banks to consolidate and offer one-stop services. The aggregate pattern of the $FFC$ very much resembles the time series of “Government Support” shown in Fig. 1 of Correa et al. (2014).\textsuperscript{26}

[Insert Figures 2 and 3 about here]

To gauge the magnitude of the market-based forbearance fraction in capital in terms of observed market caps, we plot the dynamics of the cross-sectional distribution of the Forbearance (in units) of Capital ($FC$), which is defined as the product of the $FFC$ and the market cap, in Figure 4. If there were no capital forbearance, each year the aggregate owner-contributed capital of the publicly traded banks in our sample would have been 7.6 billion USD higher. In other words, from 1990 through to 2012, the stock market estimated that these banks saved over 100 billion USD of capital due to benefiting from the forbearance subsidization resulting from FDIC non-adherence to the market closure rule.\textsuperscript{27} In Figure 4, we see much cross-sectional variation in the forbearance capital. This is, however, not surprising, given the fact that forbearance capital depends very much on the individual equity value. It is worth noting that much cross-sectional variation in forbearance capital does not necessary imply a very different effective policy parameter $\rho^*$ (see the definition in Section 2.3), which measures the market-based forbearance treatment, cross individual banks. This can be seen from Figure 5 showing that $\rho^*$ has much less cross-sectional variation over time.

[Insert Figures 4 and 5 about here]

\textsuperscript{26}It may be interesting to implement this model on nonbank firms where regulatory forbearance should not be an issue. However, since it is not straightforward to apply the Ronn and Verma (1986) framework tailored to banks which are special and heavily regulated, to the Merton-KMV class of models used in the pricing of corporate securities issued by the much less leveraged non-financial firms with complex capital structure but devoid of deposits, bank services and government safety nets, we defer this experiment to a future project.

\textsuperscript{27}The total $FC$ at the beginning of 1990 was 5.2 billion USD, and by the end of 2012 it was 105.9 billion USD. On average, the total $FC$ increased annually by $(105.9 - 5.2)/23 = 7.6$ billion USD.
For comparison, we also show the cross-sectional distributions of the bank value of deposit insurance in Figure 7, and the per dollar of insured total debt value of deposit insurance in Figure 6. By a similar calculation we find that, for our sample, banks were receiving 56.3 billion USD of coverage each year from the explicit deposit insurance before the fees were levied. Over these years, the total amount of deposit insurance was over one trillion USD and it is generally perceived that the premia paid by the banks were much lower than the deposit insurance protection they received.\textsuperscript{28} Put in relative terms as contingent liabilities to the government, the aggregate forbearance subsidy amounts to 13.5\% of the deposit insurance costs.\textsuperscript{29}

[Insert Figures 6 to 7 about here]

Following Flannery (2014), we also compute the Market Cap / Implied Asset Value ratio, which measures the market equity capital ratio, i.e., the market-based counterpart of the book equity ratio defined as Book Equity / Total Assets. The lower is this ratio, the lower is the owner-contributed equity ratio, and the higher is the leverage level. As indicated above, by deducting FFC from Market Cap / Implied Asset Value, we obtain Intrinsic Market Cap / Implied Asset Value, which gives us a cleaner measure of the market-based equity ratio and a more accurate measure of the leverage risk since it removes the market view of the forbearance fraction in capital from the bank stock market price and leaves in only the intrinsic value. Figure 8 compares the dynamics of the quarterly medians of Market Cap / Implied Asset Value, Intrinsic Market Cap / Implied Asset Value, and Book Equity / Total Assets. Notably, that of the market-based equity ratio exhibits much more systematic variation than that of the book equity ratio, which tracks the reverse of the non-risk-weighted leverage ratio. The latter contains important information about how the market view of bank capital structure and economic capital changes over time. During the sub-prime crisis, unsurprisingly, the book equity ratio exceeded the

\textsuperscript{28} By and large, it is recognized that an explicit deposit insurance scheme with a fixed-rate premium schedule results in deposit insurance being underpriced for ex-ante risky banks and overpriced for ex-ante healthy banks. Pennacchi (2009) states that, because of the Deposit Insurance Fund reserves, a fair deposit insurance premium that would prevent subsidies and distortions to banks’ costs of funding is not feasible.

\textsuperscript{29} Although we use a different methodology, our historical estimates of government guarantees in the banking sector are comparable with those in Cooperstein et al. (1995), Flannery (2014), Hovakimian et al. (2012), Kelly et al. (2015), and Tsiesmelidakis and Merton (2012).
market-valued equity ratios. A similar pattern is also found in Fig. 3 of Flannery (2014).

The dynamics of the cross-sectional distributions of Intrinsic Market Cap / Implied Asset Value and Market Cap / Implied Asset Value are plotted in Figures 9 and 10, respectively. For comparison purposes, we also plot that of Book Equity / Total Assets in Figure 11. The Book Equity / Total Assets ratio increases steadily during our period of study for the Large Banks and Big Banks groups, while it remains more or less the same for the Medium Banks group. From Figures 9 and 10 we find that, although the quarterly medians of Intrinsic Market Cap / Implied Asset Value and Market Cap / Implied Asset Value exhibit similar dynamics over time, the cross-sectional variation of the former is significantly higher than that of the latter, and particularly for the largest banks. This indicates that, given the forbearance policy, the Intrinsic Market Cap / Implied Asset Value ratio of banks is much more heterogeneous than its partially observable counterpart Market Cap / Implied Asset Value and its recorded value Book Equity / Total Assets. The dynamics of the Book Equity to Total Assets ratio surely reflect the banks’ book keeping and window dressing, carried out to meet regulatory capital requirements.

4 Empirical analysis II: Regression results

In this section, we first introduce the variables used in the regression analysis. After developing the hypotheses to be tested, we present the regression setup and then discuss the results.

4.1 Variables

As said earlier, the regulatory policy parameter is driven by complex and interconnected sets of economic, legal and political drivers exacerbated by regulatory competition and arbitrage. Since deriving a comprehensive model and obtaining data (which may be
not observable and available), bring us too far away, for our empirical analysis, we simply resort to the existing literature on bank capital forbearance to provide us with the key determinants of the policy parameter. To save space, we do not dwell on this literature review, see among others, Thomson (1992), Mailath and Mester (1994), Nagarajan and Sealey (1995), Acharya (2003), Acharya and Yorulmazer (2007), Brown and Dinc (2011) and Morrison and White (2013).\textsuperscript{30}

We use the \( \text{FFC} \), the Intrinsic Market Cap / Implied Asset Value, and \( V \) obtained from Section 3, and other focal and control variables described below to run regressions to address our two research questions.\textsuperscript{31}

The LHS variables in the regressions are \( \text{FFC} \) and Intrinsic Market Cap / Implied Asset Value, respectively, the latter defined as \( (1 - \text{FFC}) \times \text{Market Cap / Implied Asset Value} \). Recall that \( \text{FFC} \) denotes the forbearance fraction in capital. Hence, the intrinsic capital ratio is not only market-assessed but also devoid of the value of regulatory forbearance.

On the RHS, we consider the business cycle, bank risks, and some other control variables. For business cycle proxies, we use the U.S. GDP Growth, GDP Output Gap, and S&P 500 Index Returns. The latter two are used for robustness checks. For bank risks, we compute the following variables. We obtain Idiosyncratic Volatility, following Shumway (2001) and Duan et al. (2012), by first regressing the daily returns of the firm’s market cap on the daily returns of the S&P 500 index, within a quarter, then taking the standard deviation of the residuals of this regression. For Asset Volatility, we follow Duan et al. (2012)’s approach to estimate Distance-To-Default (DTD) for financial firms, and the Asset Volatility is then a byproduct of DTD (see the appendix in Duan et al., 2012). Thirdly, Beta is the coefficient of the return of the S&P 500 index in the regression for idiosyncratic volatility. Beta captures the extent to which a firm is sensitive to systematic risk. Investors holding diversified stock portfolios care about systematic risk, while large shareholders, bank managers, and regulators pay attention to idiosyncratic risk. The Asset Volatility and Beta are used for a robustness check. For discussions on these risk metrics in the banking liter-

\textsuperscript{30} The literature points out a number of interesting characteristics which could influence forbearance. However, we defer to future studies the testing of whether or not any of these matter for these publicly traded banks.

\textsuperscript{31} Given that \( p^* \) exhibits much less variation, and mostly clusters around one, we use \( \text{FFC} \), which has more variation, in the regression analysis.
ature, see, for instance, Acharya (2009), DeYoung et al. (2013), and Pathan (2009).

Following Moore and Zhou (2013), Engle et al. (2014), and many others, we also include the following control variables commonly considered in the extant banking literature:

- **Implied Asset Value / Total Assets**, or Market to Book Asset Value ratio: a measure of the charter value, which is related to banks’ risk taking. It also proxies the degree of competition or market power. See Keeley (1990) and Gan (2004) for further discussion.

- **Log Total Assets**: the natural logarithm of total assets captures the size of a banking firm.

- **Total Deposits / Total Liabilities**: a proxy for a bank’s funding structure.

- **(Short Term Borrowing + Other Short Term Liabilities) / Total Assets**: a proxy for a bank’s funding liquidity;

- **Net Income / Total Assets**: the Return on Assets (ROA).

- **Total Loans / Total Assets**: a proxy for the amount of traditional activities that a bank degree engages in.

Table 1 presents summary statistics of these variables (exclusive of GDP Growth and the one-year Treasury rate). By just looking at these statistics, we do not see any significant size effect among these variables. All four groups have similar distributions for all the variables, except for log Total Assets which is used to discriminate between the different groups. However, we find a significant size effect when we conduct panel data regressions, as discussed later.

[Insert Table 1 about here]

### 4.2 Testable hypotheses

In addressing our two research questions: 1) How does the time-varying capital forbearance portion embedded in bank equity depend on various banks’ own risk and business cycle variables? and 2) How do banks’ market-assessed intrinsic (i.e., devoid of the
forbearance subsidy) capital ratios (or inverse leverage ratios) react to various business cycles and the banks’ own risk variables?, we postulate the following hypotheses. It is natural to expect that FFC is correlated with bank risk. In general, the higher the risk, the bigger is FFC, therefore, we hypothesize that there is a positive relation between FFC and bank risk. In light of numerous studies on TBTF, we expect FFC to be positively related to the size of a banking firm. Banks with relatively more deposits are costlier to save, as bank depositors are indemnified by the FDIC. Therefore, we hypothesize that there is a positive relation between FFC and bank reliance on deposit funding proxied by the relative size of bank deposits. Economic intuition leads us to expect FFC to be related to the business cycle and we hypothesize that FFC is bigger in troubled times.

Intuitively, when the charter value increases, bank intrinsic market equity and implied asset value increase by the same amount.\(^{32}\) Therefore, we expect there is a positive relation between Intrinsic Market Cap / Implied Asset Value and the charter value. Strong capital adequacy typically reduces bank systemic risk, hence, we hypothesize that Intrinsic Market Cap / Implied Asset Value is negatively associated with systemic risk.

### 4.3 Regression analysis

In this section, using the market and the financial data of the 565 banks as well as GDP Growth, we investigate our hypotheses above by studying how FFC and Intrinsic Market Cap / Implied Asset Value are affected by bank-specific risk drivers and business cycles, cross-sectionally and over time.

\(^{32}\) Without loss of generality, we assume that the total assets (TA) and the total liabilities (TL) remain constant. The implied asset value (IA) has two components: market value of debt (TL\(\rho^*\)) and market value of equity (MC). The MC can be decomposed into three components: non-charter (nonC), charter (C), and forbearance capital (FC). The intrinsic market cap (IMC) consists of nonC and C. When C increases (by \(\Delta C\)), IMC and IA will both increase by \(\Delta C\), and therefore the ratio IMC/IA will increase.

\[
IA = TL \cdot \rho^* + MC, \quad MC = IMC + FC, \quad IMC = nonC + C
\]

\[
C \uparrow_{\Delta C} \Rightarrow \left\{ \begin{array}{c}
IMC \uparrow_{\Delta C} \\
MC \uparrow_{\Delta C} \Rightarrow IA \uparrow_{\Delta C}
\end{array} \right\} \Rightarrow \frac{IMC}{IA} \uparrow.
\]
4.3.1 A system of two equations and the Generalized Method of Moments (GMM)

FFC has a positive relation with the degree of supervisory leniency, the bigger is FFC, the higher is the expected capital forbearance from the government. As stated earlier, Intrinsic Market Cap / Implied Asset Value gives us a clean measure of the market-based equity ratio or the leverage ratio that excludes the forbearance subsidy. Recall that Implied Asset Value / Total Assets is our enhanced proxy for both Tobin’s Q and the charter value.

Endogeneity exists between FFC and Intrinsic Market Cap / Implied Asset Value, and the other variables, especially Implied Asset Value / Total Assets and Idiosyncratic Volatility since the latter are obtained from stock market data. To take into account the endogeneity between the variables, we use system GMM to estimate the following system of two dynamic panel equations:

\[
\begin{align*}
\text{FFC}_{j,t} &= f_1 \left( \begin{array}{c}
\text{Intrinsic Market Cap}_{j,t}^{-1/4} \\
\text{Implied Asset Value}_{j,t}^{-1/4} \\
\text{Idios. Volatility}_{j,t} \\
\text{GDP Growth}_t \\
\text{Control Variables}
\end{array} \right) + \epsilon_{j,t}, \quad (8a) \\
\frac{\text{Intrinsic Market Cap}_{j,t}}{\text{Implied Asset Value}_{j,t}} &= f_2 \left( \begin{array}{c}
\text{Intrinsic Market Cap}_{j,t}^{-1/4} \\
\text{Implied Asset Value}_{j,t}^{-1/4} \\
\text{Idios. Volatility}_{j,t} \\
\text{GDP Growth}_t \\
\text{Control Variables}
\end{array} \right) + \epsilon_{j,t}, \quad (8b)
\end{align*}
\]

where \( f_1 (\cdot) \) and \( f_2 (\cdot) \) are linear functions, \( \epsilon_{j,t} \) and \( \epsilon_{j,t} \) are residuals, and subscripts \( j \) and \( t \) indicate that the value is for the \( j \)th firm at time \( t \), and the Control Variables include \( \text{Implied Asset Value}_{j,t} / \text{Total Assets}_{j,t} \), \( \log \text{Total Assets}_{j,t} \), and \( \frac{\text{Total Deposits}_{j,t}}{\text{Total Liabilities}_{j,t}} \). The one-quarter-lagged LHS variables are included on the RHS to capture the system dynamic structure. We use quarterly FFC, Implied Asset Value, and Intrinsic Market Cap / Implied Asset Value, i.e., values of these variables are taken from the last month of each quarter, to match the frequency of the other variables. The correlation coefficients of the variables are presented in Table 2. Consistent with intuition, FFC is strongly negatively correlated with Intrinsic Market Cap / Implied Asset Value (-0.72) and Implied Asset Value / Total Assets (-0.70) whereas these
two latter variables are positively correlated with each other (0.78).  

To avoid the “biases in dynamic models with fixed effects” pointed out in Nickell (1981), we estimate Eq. (8a) to Eq. (8b) using the 2SGM developed by Blundell and Bond (1998). Since individual FFC and Intrinsic Market Cap / Implied Asset Value are unlikely to directly affect GDP Growth, we assume that GDP Growth is a strictly exogenous regressor. Naturally, FFC, Intrinsic Market Cap / Implied Asset Value, Idiosyncratic Volatility, and Implied Asset Value / Total Assets are endogenous variables, as they are all determined endogenously in our model. Although log Total Assets and Total Deposits / Total Liabilities are not explicit in our model, their values are likely to be affected by the market view of forbearance. Therefore, these two variables are also potentially endogenous to FFC and Intrinsic Market Cap / Implied Asset Value.

In the GMM estimation, GDP Growth is used to instrument itself. Other than GDP Growth, all the RHS variables in Eq. (8a) to Eq. (8b) are considered to be predetermined (lagged LHS) or endogenous. Therefore, two-quarter and longer lags of these variables are used as GMM instruments. Since the time length of our panel data is not too short (about 36 quarters on average), we cap the lags at 9 and 20 quarters for the Large Banks and Big Banks groups, respectively. To further limit the number of instruments at a reasonable level relative to the number of observations, these GMM instruments are “collapsed” à la Beck and Levine (2004) and Roodman (2009). We compute standard errors with the Windmeijer (2005) correction. The system GMM approach handles firm fixed effects, and we include year dummies in the regression to account for the time fixed effects.

---

33 FFC is marginally and negatively correlated with Book Equity / Total Assets (-0.15), while Intrinsic Market Cap / Implied Asset Value is positively correlated with Book Equity / Total Assets (0.33). However, in unreported results we find that when regressing FFC or Intrinsic Market Cap / Implied Asset Value on Book Equity / Total Assets and other control variables as in Eq. (8), the coefficient of Book Equity / Total Assets is insignificant. This might indicate that the market view of the forbearance is insensitive to the accounting measure of the capital ratio.

34 Since the LHS variables, FFC and Intrinsic Market Cap / Implied Asset Value, are both persistent processes, this bias is considered to be significant if we use the standard fixed-effect regression.

35 Results are robust even with GDP Growth as an endogenous variable. However, to limit the number of instruments, we use GDP Growth as an exogenous variable.

36 We do not include quarter dummies because we have a quarterly macro variable GDP Growth in our regression, which makes coefficients of quarter dummies unidentifiable. Year dummies are, nevertheless, identifiable.
4.3.2 Results and discussion

We report the estimation results from Eq. (8a) in Table 3. The coefficients of the lagged FFC are all significantly positive for all five groups, which confirms that FFC is a persistent process. The coefficients of Intrinsic Market Cap / Implied Asset Value for the All Banks and Medium Banks groups are significantly negative, consistent with the belief that banks with higher intrinsic equity ratios receive less capital forbearance. One standard deviation increase in the Intrinsic Market Cap / Implied Asset Value reduces the capital forbearance by 0.02 for all banks and 0.03 for medium banks. This increase is 11% of the sample capital forbearance mean. Since Intrinsic Market Cap / Implied Asset Value is the reverse of the (market-based) leverage risk, the significantly negative coefficients also reflect a positive relation between FFC and bank leveraging. This confirms our hypothesis that FFC is positively related to the leverage risk. However, the estimated coefficients for the Large and Big banks are statistically zero. This may be interpreted that for these mega banks FFC is not associated with the intrinsic capital fraction since it is tiny compared to the scale of forbearance and bailout contemplated by the market.

[Insert Table 3 about here]

The significantly positive coefficient of Idiosyncratic Volatility indicates that the market believes, ceteris paribus, that banks with higher non-diversifiable risk will be given more forbearance. We find that one standard deviation in Idiosyncratic Volatility leads to 0.022 increase in the Forbearance Fraction in Capital (FFC)(13.75% the sample average capital forbearance). This effect is higher for big banks in absolute terms. This result is robust to the two other bank specific risk measures: Asset Volatility and Beta.\(^\text{37}\) This is in line with the hypothesis that regulatory forbearance rises with higher idiosyncratic risk.

Although we do not observe a significant size effect on FFC from the summary statistics presented in Table 1, we do find formal confirmation of a significant size effect in Table 3. This is confirmed by the significantly positive coefficients of log Total Assets for the All but Small Banks group. This positive size effect on FFC indicates that the market

\(^{37}\) The coefficients of Asset Volatility and Beta are also negative in our robustness check. Unreported results are available upon request from the authors.
believes that larger banks benefit from more forbearance and cost more to rescue. Since we use log Total Assets to proxy for bank size, we also capture to some degree the nonlinearity in the relation between FFC and Total Assets. Judging from the coefficient estimates in Table 3, the nonlinearity implies that the size effect is unnoticeable when banks become large. This feature explains why the coefficients for the Large Banks and Big Banks groups are insignificant. This is consistent with the hypothesis about the positive size effect on the forbearance value.

A high ratio of Total Deposits / Total Liabilities means banks draw mainly from deposits and rely much less on wholesale funding. The significantly positive relation between FFC and Total Deposits / Total Liabilities in Table 3 implies that it is costlier for the government to forbear in the case of banks that depend on deposits. This is again in line with our hypothesis that since deposits are insured, the FDIC has a heavier obligation to fully payoff depositors of banks having relatively more deposits.

The coefficients of GDP Growth reveal the relation between FFC and the business cycle. The coefficients for All but Small Banks and Medium Banks are significantly negative. Obviously, the market expects banks to benefit from more forbearance or more likely to be rescued in bad times, and to be less prone to requiring financial assistance during booms. FFC is countercyclical as we hypothesize. The results are robust to the other two proxies for the business cycle: GDP Output Gap and S&P Index. To save space, their results are not presented.

The estimation results from Eq. (8b) are presented in Table 4. Again, we find that Intrinsic Market Cap / Implied Asset Value, our proxy for the inverse of the bank’s leverage ratio (market-based), is a persistent process as the coefficients of the lagged value of Intrinsic Market Cap / Implied Asset Value are all significantly positive. The significantly negative coefficients of FFC confirm the hypothesized positive relation between FFC and the leverage risk. Given the coefficient estimates of FFC for All but Small Banks group, we can see that a 10 basis point increase in FFC contemporaneously decreases Intrinsic Market Cap / Implied Asset Value by 0.23 basis points. Further, the marginal contribution of forbearance (FFC) to the intrinsic bank net worth ratio (Intrinsic Market Cap / Implied Asset Value) appears almost identical in all three size-based subsamples of banks.
This interesting result suggests that, for the sake of valuating bank adequate capital, the marginal impact of FFC on the intrinsic capital ratio is the same regardless of the bank size. The significantly negative coefficients of Idiosyncratic Volatility indicate a positive relation between bank idiosyncratic volatility and the leverage ratio.

We observe a significantly positive relation between Intrinsic Market Cap / Implied Asset Value and Implied Asset Value / Total Assets. On average, a standard deviation increase in the Implied Asset Value / Total Assets leads to 0.008 increase in the Intrinsic Market Cap / Implied Asset Value. This increase amounts to 6.35% of the average value of the Intrinsic Market Cap / Implied Asset Value. Recall that our proxy for Tobin’s Q, Implied Asset Value / Total Assets, which is based on the market valuation, measures the charter value more accurately so as to underscore the disciplining impact of the charter value on leverage. This result is consistent with our hypothesis about the positive relation between Intrinsic Market Cap / Implied Asset Value and the charter value. This also indicates that the higher is the charter value, the less is the bank’s leverage, but the disciplining effect is smaller for bigger banks. Unlike the case of FFC, we do not observe a significant size effect on Intrinsic Market Cap / Implied Asset Value, as only the coefficient for Large Banks is significant among the five groups. We find a statistically significant negative relationship between Intrinsic Market Cap / Implied Asset Value and Total Deposits / Total Liabilities for the full sample of 565 banks which appears to be driven by the Large Banks subsample.

In Appendix F, we report results obtained using additional control variables such as a funding-liquidity measure (Short-Term Borrowing & Other Short-Term Liabilities to Total Assets Ratio), a performance measure (ROA), and a traditional banking activities measure (Total Loans to Total Assets Ratio). The main messages above remain unchanged. In unreported exercises, we replace Idiosyncratic Volatility (GDP Growth) with Asset Volatility and Beta (GDP Output Gap and S&P 500 Index), then rerun the same regressions. We also exclude data from the crisis period and rerun the same regressions. The results, available from the authors upon request, deliver the same stories as discussed above.
4.3.3 Results with systemic risk

This section provides additional results on how our FFC and Intrinsic Market Cap / Implied Asset Value are related to the systemic risk of a bank. Using the U.S. banks’ daily stock returns data, we calculate the commonly adopted systemic risk proxy, the *Marginal Expected Shortfall* (MES) proposed by Brownlees and Engle (2012), which utilizes the GARCH and Corrected Dynamic Conditional Correlation (CDCC) methods to better capture correlations between the stock returns of individual banks and the stock index returns. The MES, which captures the marginal exposure of a banking firm to a system-wide collapse, is defined as the negative mean net equity return of a bank conditional on the U.S. S&P 500 index experiencing extreme downward movements. We follow the mechanics with all the assumptions summarized in Annex 4 of Laeven et al. (2014) to estimate our two metrics of systemic risk MES and SRISK. Technical details are presented in Appendix E. As discussed in more detail in Moore and Zhou (2013), Laeven et al. (2014), Engle et al. (2014), and many others in this literature, the systemic risk is most relevant for large banks and macro-prudential governance. Therefore, for compactness, here we only consider the Large Banks group consisting of banks with Total Assets of at least 7.4 billion USD and with average Total Assets of 149 billion USD.

We replace Idiosyncratic Volatility in Eq. (8a) and Eq. (8b) with MES, and also exclude log Total Assets from both equations in light of the previous finding on the insignificant size effect of Large Banks on FFC and Intrinsic Market Cap / Implied Asset Value.\(^{38}\)

\(^{38}\) To better identify the coefficients we keep all the instrumental variables (both IV and GMM types) intact while adding MES as a GMM type instrument.
Specifically, we estimate the following two equations

\[
\text{FFC}_{j,t} = f_1 \left( \begin{array}{c}
\text{FFC}_{j,t-\frac{1}{4}}, \\
\text{Intrinsic Market Cap}_{j,t} \\
\text{Implied Asset Value}_{j,t} \\
\text{MES}_{j,t}, \text{GDP Growth}_{t}, \\
\text{Control Variables}
\end{array} \right) + \epsilon_{j,t}, \quad (9a)
\]

\[
\frac{\text{Intrinsic Market Cap}_{j,t}}{\text{Implied Asset Value}_{j,t}} = f_2 \left( \begin{array}{c}
\text{Intrinsic Market Cap}_{j,t-\frac{1}{4}} \\
\text{Implied Asset Value}_{j,t-\frac{1}{4}}, \text{FFC}_{j,t} \\
\text{MES}_{j,t}, \text{GDP Growth}_{t}, \\
\text{Control Variables}
\end{array} \right) + \epsilon_{j,t}, \quad (9b)
\]

where the Control Variables include \( \frac{\text{Implied Asset Value}_{j,t}}{\text{Total Assets}_{j,t}} \) and \( \frac{\text{Total Deposits}_{j,t}}{\text{Total Liabilities}_{j,t}} \). To perform the regressions, we use the 2SGMM as described before. The results are reported in Table 5.

[Insert Table 5 about here]

We find a significantly positive relation between FFC and MES, while there is a significantly negative relation between Intrinsic Market Cap / Implied Asset Value and MES. These results are in accordance with our hypotheses. Bank systemic risk is positively associated with capital forbearance. The lower is the bank capital, the higher is the bank’s exposure to systemic risk. This finding is robust to an alternative systemic risk measure proposed by Acharya et al. (2012), SRISK, the capital shortfall, defined as a bank’s contribution to the deterioration of the capitalization of the whole financial system (the dominant U.S. stock market) during a crisis.\(^{39}\)

\(^{39}\) The results of this robustness test are available from the authors upon request.
5 Conclusion

Ronn and Verma (1986) call the tolerance level below which the closure of a large, complex and interconnected insolvent bank is triggered the regulatory policy parameter. In this paper, we develop a two-factor model with Ronn and Verma (1986)’s bank regulatory policy parameter being stochastic and bank-specific. The model is calibrated using 565 U.S. banks’ market capitalization and total liabilities data to infer the market impression about the regulatory closure rules for the period from 1990 to 2012. In accordance with economic intuition, the resulting forward-looking bank regulatory policy parameters show that the market expectation on the capital forbearance is significantly driven by bank-specific risk variables and business cycles. We find that the capital forbearance subsidy present in the largest banks could amount to 17% of the market value of equity and could go as high as 100% of a bank’s stock value. The market believes in the “Too Big to Fail” paradigm and expects that banks with lower equity capital ratios will receive more capital forbearance or government assistance, congruent with the banking regulatory authority containing rescue costs. In effect, the market expects a strongly performing bank to receive less capital forbearance, and one with a high charter value and enjoying greater market power to cost less to bail out. Regarding idiosyncratic risk, the market believes that banks with higher volatility will benefit from more forbearance. The market expects banks to benefit from increases in capital forbearance during recessions. The market expectation of forbearance is also a positive function of banks’ systemic risk, consistent with the expectation that banks with higher systemic risk will receive more capital forbearance from the government. Applying the model to the setting of fair market deposit insurance premiums would be a natural next step in future research. By means of the enhanced Ronn and Verma (1986) developed in this paper, one may study the interaction between regulatory forbearance and market discipline in terms of equity valuation.
References


Table 1: Summary statistics of the variables

This table reports summary statistics for the Forbearance Fraction in Capital (FFC), Mkt Cap / Implied Asset Value (MC/IA), Intrinsic Mkt Cap / Implied Asset Value (IMC/IA), Book Equity / Total Assets (BE/TA), Idiosyncratic Volatility (Idio-Vol), Implied Asset Value / Total Assets (IA/TA), log Total Assets (log(TA)), Total Deposits / Total Liabilities (TD/TL), Short-Term Borrowing + Short-Term Other Liabilities / Total Assets (STB/TA), Net Income / Total Assets (ROA), and Total Loans / Total Assets (TLoan/TA) for all banks as well as three categories: Large Banks, Big Banks, and Medium Banks. The Intrinsic Market Cap is the market cap net of the forbearance capital value, and the Implied Asset Value is one of the state variables in our model, and represents the market-based asset value of a bank. Each variable is winsorized at 99% and 1%. The Mean, Standard Deviation (Std), Maximum (Max), 90%, 75%, 50%, 25%, 10% and Minimum (Min) of each category are reported. Note: Total Assets are in millions of USD and results for ROA are reported in percentages to avoid rounding imprecision. Summary statistics are calculated on a quarterly basis. For FFC, MC/IA, and IMC/IA, the data of the last months in quarters are used.

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**Large Banks**

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**Big Banks**

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<td>0.12</td>
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<tr>
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<tr>
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<td>0.08</td>
<td>0.42</td>
<td>0.18</td>
<td>0.12</td>
<td>0.07</td>
<td>0.04</td>
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<th>75%</th>
<th>50%</th>
<th>25%</th>
<th>10%</th>
<th>Min</th>
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<tr>
<td>ROA(%)</td>
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**Medium Banks**

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<th>75%</th>
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<tr>
<td>IMC/IA</td>
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<td>0.38</td>
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<tr>
<td>BE/TA</td>
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<td>0.08</td>
<td>0.06</td>
<td>0.04</td>
</tr>
<tr>
<td>Idio-Risk</td>
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<td>0.02</td>
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<tr>
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<tr>
<td>STB/TA</td>
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<td>0.13</td>
<td>0.07</td>
<td>0.03</td>
<td>0.01</td>
<td>0.00</td>
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<tr>
<td>ROA(%)</td>
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<td>0.26</td>
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<tr>
<td>TLoan/TA</td>
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Figure 1: Individual bank regulatory policy parameters time series: The case of Wells Fargo

This figure presents the results of the case study of Wells Fargo. The upper panel in this figure shows the time series of the actual market cap, fitted market cap, implied asset value, and smoothed total liabilities (y-axis is in billions of USD); the middle panel shows the time series of the Policy Parameter ($\rho$), the Effective Policy Parameter ($\rho^*$), and the Forbearance Fraction in Capital (FFC); the lower panel shows the time series of the ratios of Intrinsic Market Cap / Implied Asset Value and Book Equity / Total Assets (i.e., 1 - Smoothed Total Liabilities / Smoothed Total Assets). The Intrinsic Market Cap is the market cap net of the capital forbearance value, and the Implied Asset Value is one of the state variables in our model, and represents the market-based asset value of a bank. The time series are from early 1998 to late 2012, and the data frequency is monthly.
Figure 2: Forbearance Fraction in Capital (FFC) cross-sectional distribution dynamics

This figure shows the dynamics (from 1990 to 2012, monthly data) of the cross-sectional distributions of the Forbearance Fraction in Capital (FFC) for the four categories: Large Banks, Big Banks, Medium Banks, and All but Small Banks. The black line indicates the median over time, and the dark and light gray bands indicate the 25th to 75th and 10th to 90th percentile intervals, respectively.
Figure 3: Normalized histogram of the Forbearance Fraction in Capital (FFC)

This figure shows the normalized histogram of the Forbearance Fraction in Capital (FFC) (the bars). For the sake of comparison, a nonparametric density curve (the solid line marked with stars) is also plotted.
Figure 4: Forbearance Capital Value (FC) cross-sectional distribution dynamics

This figure shows the dynamics (from 1990 to 2012, monthly data) of the cross-sectional distributions of the Forbearance Capital Value (FC) (in billions of USD) for the four categories: Large Banks, Big Banks, Medium Banks, and All Banks. All the y-axes are in billions of USD. The black line indicates the median over time, and the dark and light gray bands indicate the 25th to 75th and 10th to 90th percentile intervals, respectively.
Figure 5: Effective Policy Parameter ($\rho^*$) cross-sectional distribution dynamics

This figure shows the dynamics (from 1990 to 2012, monthly data) of the cross-sectional distributions of the Effective Policy Parameter ($\rho^*$) for the four categories: Large Banks, Big Banks, Medium Banks, and All Banks. The black line indicates the median over time, and the dark and light gray bands indicate the 25th to 75th and 10th to 90th percentile intervals, respectively.
Figure 6: Value of deposit insurance per dollar of bank-insured debt: Cross-sectional distribution dynamics

This figure shows the dynamics (from 1990 to 2012, monthly data) of the cross-sectional distributions of the value of deposit insurance per dollar of bank insured debt for the four categories: Large Banks, Big Banks, Medium Banks, and All Banks. We assume that all liabilities are insured (see footnote 11), so that the insured debt is equal to the total liabilities. The black line indicates the median over time, and the dark and light gray bands indicate the 25th to 75th and 10th to 90th percentile intervals, respectively.
Figure 7: Value of bank deposit insurance: Cross-sectional distribution dynamics

This figure shows the dynamics (from 1990 to 2012, quarterly data) of the cross-sectional distributions of the value of bank deposit insurance (in billions of USD) for the four categories: Large Banks, Big Banks, Medium Banks, and All Banks. All the y-axes are in billions of USD. The black line indicates the median over time, and the dark and light gray bands indicate the 25th to 75th and 10th to 90th percentile intervals, respectively.
The figure shows the dynamics of the cross-sectional mean values of Intrinsic Market Cap / Implied Asset Value and Book Equity / Total Assets for the four different groups from the beginning of 1990 to the end of 2012. The data frequency is quarterly. The Intrinsic Market Cap is the market cap net of the forbearance capital value, and the Implied Asset Value is one of the state variables in our model, and represents the market-based asset value of a bank. The results for Large Banks, Big Banks, Medium Banks, and All Banks are shown in panels (a), (b), (c), (d), and (e) respectively.
Figure 9: Intrinsic Market Cap / Implied Asset Value distribution dynamics

This figure shows the dynamics (from 1990 to 2012, quarterly data) of the cross-sectional distributions of banks’ Intrinsic Market Cap / Implied Asset Value for the four categories: Large Banks, Big Banks, Medium Banks, and All Banks. The Intrinsic Market Cap is the market cap net of the capital forbearance value, and the Implied Asset Value is one of the state variables in our model, and represents the market-based asset value of a bank. The black line indicates the median over time, and the dark and light gray bands indicate the 25th to 75th and 10th to 90th percentile intervals, respectively.
Figure 10: Market Cap / Implied Asset Value distribution dynamics

This figure shows the dynamics (from 1990 to 2012, quarterly data) of the cross-sectional distributions of banks’ Market Cap / Implied Asset Value for the four categories: Large Banks, Big Banks, Medium Banks, and All Banks. The Implied Asset Value is one of the state variables in our model, and represents the market-based asset value of a bank. The black line indicates the median over time, and the dark and light gray bands indicate the 25th to 75th and 10th to 90th percentile intervals, respectively.
Figure 11: Book Equity / Total Assets distribution dynamics

This figure shows the dynamics (from 1990 to 2012, quarterly data) of the cross-sectional distributions of the ratio of Book Equity / Total Assets for the four categories: Large Banks, Big Banks, Medium Banks, and All Banks. The black line indicates the median over time, and the dark and light gray bands indicate the 25th to 75th and 10th to 90th percentile intervals, respectively.
Table 2: Correlation coefficients of the variables

This table reports the correlation coefficient matrix of the following variables: the Forbearance Fraction in Capital (FFC), Intrinsic Market Cap / Implied Asset Value (IMC/IA), Idiosyncratic Volatility (Idio-Risk), Implied Asset Value / Total Assets (IA/TA), log Total Assets (logTA), Total Deposits / Total Liabilities (TD/TL), GDP Growth (GDPG), (Short-Term Borrowing + Other Short-Term Liabilities) / Total Assets (STB/TA), Net Income / Total Assets (ROA), and Total Loans / Total Assets (TLoan/TA). The Intrinsic Market Cap is the market cap net of the forbearance capital value, and the Implied Asset Value is one of the state variables in our model, and represents the market-based asset value of a bank. The correlation coefficients are computed in a pairwise manner using quarterly data.

<table>
<thead>
<tr>
<th></th>
<th>FFC</th>
<th>IMC/IA</th>
<th>Idio Risk</th>
<th>GDPG</th>
<th>IA/TA</th>
<th>logTA</th>
<th>TD/TL</th>
<th>STB/TA</th>
<th>ROA</th>
<th>TLoan/TA</th>
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<td>GDPG</td>
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<td>0.19</td>
<td>-0.23</td>
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<td>-0.21</td>
<td>-0.09</td>
<td>-0.04</td>
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</tr>
<tr>
<td>TD/TL</td>
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<td>0.05</td>
<td>0.08</td>
<td>0.09</td>
<td>0.04</td>
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<td>1</td>
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<tr>
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<td>0.06</td>
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<tr>
<td>TLoan/TA</td>
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<td>-0.01</td>
<td>0.10</td>
<td>-0.13</td>
<td>-0.07</td>
<td>-0.14</td>
<td>0.16</td>
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Table 3: System GMM estimates of different groups of banks for Eq. (8a) with the LHS variable being the Forbearance Fraction in Capital (FFC)

This table reports the regression results for Eq. (8a).

\[
\text{FFC}_{jt} = f_1 \left( \begin{array}{c}
\text{FFC}_{jt}, \\
\text{Intrinsic Market Cap}_{jt}, \\
\text{Implied Asset Value}_{jt}, \\
\text{Idios. Volatility}_{jt}, \text{GDP Growth}_{t}, \\
\text{Control Variables}
\end{array} \right) + \epsilon_{jt},
\]

where the Control Variables include the \text{Implied Asset Value}_{jt}, \log \text{Total Assets}_{jt}, \text{Total Deposits}_{jt}, \text{Total Liabilities}_{jt}. The LHS variable in the regression is the Forbearance Fraction in Capital (FFC). The first column contains all the RHS variable names, the second column reports the coefficients of the variables for all but small banks, and the third to sixth columns report those for the four categories: All but Small Banks, Large Banks, Big Banks, and Medium Banks. Sample sizes and instrument counts are also reported in the last two rows. Quarterly data are used in the regressions. Windmeijer (2005) corrected standard errors are in parentheses; *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively. Time fixed effects are controlled by including year dummies in the regressions.

<table>
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<th>VARIABLES</th>
<th>All (with Small Banks Excluded)</th>
<th>Large banks</th>
<th>Big banks</th>
<th>Medium banks</th>
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<tr>
<td>Lagged FFC</td>
<td>0.780*** (0.017)</td>
<td>0.824*** (0.038)</td>
<td>0.735*** (0.046)</td>
<td>0.778*** (0.023)</td>
</tr>
<tr>
<td>Intrinsic Market Cap</td>
<td>-0.229** (0.100)</td>
<td>-0.080 (0.243)</td>
<td>-0.081 (0.220)</td>
<td>-0.392*** (0.151)</td>
</tr>
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<td>Implied Asset Value</td>
<td>1.111*** (0.190)</td>
<td>0.948*** (0.349)</td>
<td>1.501*** (0.439)</td>
<td>0.834*** (0.241)</td>
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<td>Idios. Volatility</td>
<td>-0.074 (0.045)</td>
<td>-0.050 (0.119)</td>
<td>-0.057 (0.117)</td>
<td>-0.066 (0.069)</td>
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<tr>
<td>GDP Growth</td>
<td>-0.568*** (0.203)</td>
<td>-0.845** (0.406)</td>
<td>-0.819* (0.420)</td>
<td>-0.368 (0.264)</td>
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<td>Instrument Count</td>
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<td>143</td>
<td>533</td>
</tr>
<tr>
<td>Sample Size</td>
<td>23908</td>
<td>3100</td>
<td>4857</td>
<td>15520</td>
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</table>
Table 4: System GMM estimates of different groups of banks for Eq. (8b) with the LHS variable being the Intrinsic Market Cap / Implied Asset Value

This table reports the regression results for Eq. (8b).

\[
\frac{\text{Intrinsic Market Cap}_{j,t}}{\text{Implied Asset Value}_{j,t}} = f_2 \left( \begin{array}{c}
\frac{\text{Intrinsic Market Cap}_{j,t-1}}{\text{Implied Asset Value}_{j,t-1}}, \frac{\text{FFC}_{j,t}}{2}, \text{Idios. Volatility}_{j,t}, \text{GDP Growth}_{t}, \\
\text{Control Variables}
\end{array} \right) + \varepsilon_{j,t},
\]

where the Control Variables include \(\frac{\text{Implied Asset Value}_{j,t}}{\text{Total Assets}_{j,t}}, \log \text{Total Assets}_{j,t},\) and \(\frac{\text{Total Deposits}_{j,t}}{\text{Total Liabilities}_{j,t}}\). The LHS variable in the regression is \(\frac{\text{Intrinsic Mkt Cap}_{j,t}}{\text{Implied Asset Value}_{j,t}}\), where the Intrinsic Market Cap is the market cap net of the forbearance capital value, and the Implied Asset Value is one of the state variables in our model, and represents the market-based asset value of a bank. The first column contains all the RHS variable names, the second column reports the coefficients of the variables for all but small banks, and the third to sixth columns report those for the four categories: All but Small Banks, Large Banks, Big Banks, and Medium Banks. Sample sizes and instrument counts are also reported in the last two rows. Quarterly data are used in the regressions. Windmeijer (2005) corrected standard errors are in parentheses; *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively. Time fixed effects are controlled by including year dummies in the regressions.

<table>
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<tr>
<th>VARIABLES</th>
<th>All</th>
<th>Large banks</th>
<th>Big banks</th>
<th>Medium banks</th>
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</thead>
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<tr>
<td>Lagged Intrinsic Mkt Cap</td>
<td>0.584***</td>
<td>0.682***</td>
<td>0.607***</td>
<td>0.618***</td>
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<td>Implied Asset Value</td>
<td>(0.022)</td>
<td>(0.058)</td>
<td>(0.048)</td>
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</tr>
<tr>
<td>FFC</td>
<td>-0.023***</td>
<td>-0.024**</td>
<td>-0.021***</td>
<td>-0.019***</td>
</tr>
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<td>Idio Risk</td>
<td>-0.397***</td>
<td>-0.350***</td>
<td>-0.571***</td>
<td>-0.420***</td>
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<td>Implied Asset Value / Total Assets</td>
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<td>0.001</td>
<td>-0.003</td>
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<td>Total Deposits / Total Liability</td>
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<td>0.001</td>
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<td>GDP Growth</td>
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<td>0.120</td>
<td>0.081*</td>
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<tr>
<td>Instrument Count</td>
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<td>77</td>
<td>143</td>
<td>533</td>
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<tr>
<td>Sample Size</td>
<td>23908</td>
<td>3100</td>
<td>4857</td>
<td>15520</td>
</tr>
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</table>
Table 5: System GMM estimates of the Marginal Expected Shortfall (MES) regressions Eq. (9a) and Eq. (9b), based on the Large Banks’ data with the LHS variables being the Forbearance Fraction in Capital and Intrinsic Market Cap / Implied Asset Value.

This table reports the regression results of the Marginal Expected Shortfall (MES) regressions Eq. (9a) and Eq. (9b).

\[
\begin{align*}
\text{FFC}_{j,t} &= f_1 \left( \begin{array}{c}
\text{FFC}_{j,t-1}, \\
\text{Intrinsic Market Cap}_{j,t}, \\
\text{Implied Asset Value}_{j,t}, \\
\text{MES}_{j,t}, \\
\text{GDP Growth}_t, \\
\text{Control Variables}
\end{array} \right) + \epsilon_{j,t}, \\
\text{Intrinsic Market Cap}_{j,t} / \text{Implied Asset Value}_{j,t} &= f_2 \left( \begin{array}{c}
\text{Intrinsic Market Cap}_{j,t-1}, \\
\text{Implied Asset Value}_{j,t-1}, \\
\text{FFC}_{j,t}, \\
\text{MES}_{j,t}, \\
\text{GDP Growth}_t, \\
\text{Control Variables}
\end{array} \right) + \epsilon_{j,t},
\end{align*}
\]

where the Control Variables include \( \frac{\text{Implied Asset Value}_{j,t}}{\text{Total Assets}_{j,t}} \) and \( \frac{\text{Total Deposits}_{j,t}}{\text{Total Liabilities}_{j,t}} \). The LHS variables in the regression are the Forbearance Fraction in Capital (FFC) and \( \frac{\text{Intrinsic Mkt Cap}_{j,t}}{\text{Implied Asset Value}_{j,t}} \), where the Intrinsic Market Cap is the market cap net of the forbearance capital value, and the Implied Asset Value is one of the state variables in our model, and represents the market-based asset value of a bank. The first column contains all the RHS variable names, the second and third columns report the results for FFC and \( \frac{\text{Intrinsic Mkt Cap}_{j,t}}{\text{Implied Asset Value}_{j,t}} \), respectively. Sample sizes and instrument counts are also reported in the last two rows. Quarterly data are used in the regressions. Windmeijer (2005) corrected standard errors are in parentheses; *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively. Time fixed effects are controlled by including year dummies in the regressions.

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>FFC</th>
<th>Intrinsic Mkt Cap / Implied Asset Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagged FFC</td>
<td>0.811***</td>
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<tr>
<td>Lagged Intrinsic Mkt Cap / Implied Asset Value</td>
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<td>0.615*** (0.059)</td>
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<td>FFC</td>
<td>–</td>
<td>– (0.100)</td>
</tr>
<tr>
<td>Intrinsic Mkt Cap / Implied Asset Value</td>
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<tr>
<td>MES</td>
<td>0.534** (0.239)</td>
<td>–1.79*** (0.045)</td>
</tr>
<tr>
<td>GDP Growth</td>
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<td>0.208* (0.115)</td>
</tr>
<tr>
<td>Implied Asset Value / Total Assets</td>
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<td>0.091*** (0.020)</td>
</tr>
<tr>
<td>Total Deposits / Total Liabilities</td>
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<td>–0.031 (0.020)</td>
</tr>
<tr>
<td>Instrument Count</td>
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<td>79</td>
</tr>
<tr>
<td>Sample Size</td>
<td>3100</td>
<td>3100</td>
</tr>
</tbody>
</table>
A Definition of $G$ function

$G_{b,d}(z; X_t, \tau)$ is defined as follows:

$$G_{b,d}(z; X_t, \tau) = \frac{\psi(b, X_t, \tau)}{2} - \frac{1}{\pi} \int_0^\infty \text{Im} \left[ \frac{\psi(b + isd, X_t, \tau) zis}{s} \right] ds,$$  \hspace{1cm} (A1)

where $b$ and $d$ are $1 \times 2$ row vectors. $G^P$ has the same functional form as $G$, but all the parameters are under the measure $P$, $\text{Im}(\cdot)$ denotes the imaginary part of $\cdot$, $i = \sqrt{-1}$, and for any $1 \times 2$ row vector $u = [u_1, u_2]$ (where $u_1$ and $u_2$ can be complex numbers), the transform function $\psi$ is $^{40}$

$$\psi(u, X_t, \tau) = \exp(\alpha(\tau) + \beta(\tau) x_1(t) + u_2 x_2(t)),$$

where $\alpha$ and $\beta$ satisfy the ordinary differential Riccati equations (ODEs)

$$\frac{\partial \beta(\tau)}{\partial \tau} = -\kappa \beta(\tau) + \frac{1}{2} \sigma_1^2 \beta(\tau)^2,$$

$$\frac{\partial \alpha(\tau)}{\partial \tau} = \kappa \theta \beta(\tau) + \mu u_2 + \frac{1}{2} \sigma_2^2 u_2^2,$$

$$\beta(0) = u_1,$$

$$\alpha(0) = 0.$$

$^{40}$ The function $\psi$ is defined via the Fourier-Stieltjes transform of $G_{b,d}(\cdot; X_t, \tau)$, i.e.,

$$\psi(b + isd, X_t, \tau) = \int_0^\infty e^{isz} dG_{b,d}(z; X_t, \tau) = \mathbb{E}_t \left(e^{(b + isd)X_{t+\tau}}\right).$$
When \( u_2 \neq 0 \) and \( \varphi \neq 0 \)

\[
\begin{align*}
\beta(\tau) &= \frac{\kappa - \Upsilon \Omega}{\sigma_1^2} \\
\alpha(\tau) &= -\kappa \theta \left( -\kappa \tau + 2 \log \sigma_1 + \log \left( \frac{2 \kappa u_1 - u_1^2 \sigma_1^2 - 2 \varphi u_2}{\kappa^2 - 2 \varphi u_2 \sigma_1^2} \right) \right) + \left( \frac{\sigma_2^2 u_2^2}{2} + u_2 \mu \right) \tau \\
\Upsilon &= \tan \left( -\frac{1}{2} \tau \Omega + \arctan \left( \frac{\kappa - u_1 \sigma_1^2}{\Omega} \right) \right) \\
\Omega &= \sqrt{2 \varphi u_2 \sigma_1^2 - \kappa^2}.
\end{align*}
\]

Otherwise

\[
\begin{align*}
\beta(\tau) &= \frac{2 \kappa u_1}{\sigma_1^2 (1 - e^{\kappa \tau}) u_1 + 2 \kappa e^{\kappa \tau}} \\
\alpha(\tau) &= -\frac{2 \kappa \theta \log \left( 1 + u_1 \sigma_1^2 \frac{e^{-\kappa \tau} - 1}{2 \kappa} \right)}{\sigma_1^2} + \left( \frac{\sigma_2^2 u_2^2}{2} + u_2 \mu \right) \tau.
\end{align*}
\]

B Moment conditions

Assume \( X_t \) follows a system of stochastic differential equations (SDE) under the (real) physical probability measure \( P \) as follows:

\[
dX_t = \begin{bmatrix} \kappa \theta & -\kappa \theta \tau \\ \mu \end{bmatrix} X_t + \begin{bmatrix} 0 & 0 \\ \sigma_1 \sqrt{x_1(t)} & 0 \\ 0 & \sigma_2 \end{bmatrix} dW_t^P.
\]

This assumption implies that the above SDE dynamics are derived from Eq. (1) by employing the essentially affine market prices of risks specification of Duffee (2002) for \( X_t \).

This specification allows compensations for risks of \( X_t \) to vary independently of \( X_t \).

Note that the results hold even when \( 2 \varphi u_2 \sigma_1^2 < \kappa^2 \) if we allow \( \Omega \) to take values that are imaginary numbers. This is due to the fact that

\[
\tan(x) = \frac{e^{2xi} - 1}{i (e^{2xi} + 1)} , \quad \arctan = \frac{1}{2} \log \left( \frac{1 - xi}{1 + xi} \right).
\]

Regardless of whether \( \Omega \) is an imaginary number or not, \( \beta(\tau) \) and \( \alpha(\tau) \) are always real for any \( \tau > 0 \).
B.1 The conditional expectation

The conditional expectation under measure $P$ satisfies a system of ordinary differential equations (ODE) ($s > t$)

$$\frac{d\mathbb{E}_t (X_s)}{ds} = \begin{pmatrix} \kappa \theta \\ \mu^P \end{pmatrix} + \begin{pmatrix} -\kappa^P \\ 0 \end{pmatrix} \mathbb{E}_t (X_s)$$

with the initial condition

$$\mathbb{E}_t (X_t) = X_t.$$ 

Therefore

$$\mathbb{E}_t (x_1 (s)) = x_1 (t) e^{-\kappa^P (s-t)} + \left(1 - e^{-\kappa^P (s-t)}\right) \frac{\kappa \theta}{\kappa^P}$$

$$\mathbb{E}_t (x_2 (s)) = \mu^P (s-t) + x_2 (t)$$

i.e.,

$$\mathbb{E}_t (X_s) = \begin{pmatrix} e^{-\kappa^P (s-t)} & 0 \\ 0 & 1 \end{pmatrix} X_t + \begin{pmatrix} \left(1 - e^{-\kappa^P (s-t)}\right) \frac{\kappa \theta}{\kappa^P} \\ \mu^P (s-t) \end{pmatrix}$$  \(\text{(A2)}\)

B.2 The conditional variance-covariance

Let us consider $\mathbb{E}_t (x_1^2 (s))$, $\mathbb{E}_t (x_2^2 (s))$, and $\mathbb{E}_t (x_1 (s) x_2 (s))$:

$$\frac{d\mathbb{E}_t (x_1^2 (s))}{ds} = -2\kappa^P \mathbb{E}_t (x_1^2 (s)) + \left(2\kappa \theta + \sigma_1^2\right) \mathbb{E}_t (x_1 (s))$$

$$\frac{d\mathbb{E}_t (x_2^2 (s))}{ds} = 2\mathbb{E}_t (x_2 (s)) \mu^P + \sigma_2^2$$

$$\frac{d\mathbb{E}_t (x_1 (s) x_2 (s))}{ds} = \mu^P \mathbb{E}_t (x_1 (s)) + \kappa \theta \mathbb{E}_t (x_2 (s)) - \kappa^P \mathbb{E}_t (x_1 (s) x_2 (s))$$
with initial conditions

\[ E_t \left( x_1^2(t) \right) = x_1^2(t) \]
\[ E_t \left( x_2^2(t) \right) = x_2^2(t) \]
\[ E_t \left( x_1(t) x_2(t) \right) = x_1(t) x_2(t) \]

Then

\[
E_t \left( x_1^2(s) \right) = x_1^2(t) e^{-2\kappa^p(s-t)} + \left(2\kappa\theta + \sigma_1^2\right) \int_t^s \left( E_t \left( x_1(\tau) \right) \right) e^{-2\kappa^p(\tau-s)} d\tau
\]
\[ = x_1^2(t) e^{-2\kappa^p(s-t)} + \left(2\kappa\theta + \sigma_1^2\right) \int_t^s f(x_1(t), \tau) e^{2\kappa^p\tau} d\tau \]
\[ E_t \left( x_2^2(s) \right) = x_2^2(t) + 2\mu^p \int_t^s \left( E_t \left( x_2(\tau) \right) \right) d\tau + \sigma_2^2(s-t) \]
\[ = x_2^2(t) + \int_0^{s-t} g(x_1(t), x_2(t), \tau) d\tau + \sigma_2^2(s-t) \]
\[ E_t \left( x_1(t) x_2(t) \right) = x_1(t) x_2(t) e^{-\kappa^p(s-t)} + \int_t^s \left( \mu^p E_t \left( x_1(\tau) \right) + \kappa \theta E_t \left( x_2(\tau) \right) \right) e^{-\kappa^p(s-\tau)} d\tau \]
\[ = x_1(t) x_2(t) e^{-\kappa^p(s-t)} + e^{-\kappa^p(s-t)} \int_0^{s-t} m \left( x_1^2(t), x_1(t), x_2(t), \tau \right) e^{\kappa^p\tau} d\tau \]

where

\[
f(x_1(t), \tau) = E_t \left( x_1(t + \tau) \right)\]
\[
g(x_1(t), x_2(t), \tau) = 2E_t \left( x_2(t + \tau) \right) \mu^p\]
\[
m \left( x_1^2(t), x_1(t), x_2(t), \tau \right) = \mu^p E_t \left( x_1(t + \tau) \right) + \kappa \theta E_t \left( x_2(t + \tau) \right)\]

Hence

\[
\text{Var}_t \left( X_s \right) = \begin{pmatrix}
E_t \left( x_1^2(s) \right) & E_t \left( x_1(t) x_2(t) \right) \\
E_t \left( x_1(t) x_2(t) \right) & E_t \left( x_2^2(s) \right)
\end{pmatrix} - \begin{pmatrix}
E_t^2 \left( x_1(s) \right) & E_t \left( x_1(t) \right) E_t \left( x_2(s) \right) \\
E_t \left( x_1(t) \right) E_t \left( x_2(s) \right) & E_t^2 \left( x_2(s) \right)
\end{pmatrix}
\]

(A3)
C  Unscented Kalman Filter

The Unscented Kalman Filter (UKF) is a well-developed technique, widely applied in state estimation, neural networks, and nonlinear dynamic systems (see, e.g., Haykin et al., 2001 and Simon, 2006). Since, in this paper, the measurement equations in the state space formulae are highly nonlinear, the UKF is the natural choice for our estimation procedure. The state space ($\omega$-dimensional transitions and $m$-dimensional measurements) is given by the following system (for notational clarity, we normalize the time interval to one):

**Transition equation**

$$X_t = TX_{t-\Delta t} + \Theta + \sqrt{V_t}e_t, \quad e_t \sim N(0_{\omega \times 1}, I_{\omega \times \omega})$$

where $T$ and $\Theta$ are given by Eq. (A2), $V_t$ is given by Eq. (A3), and $I$ is an identity matrix.

**Measurement equation**

$$y_t = G(X_t) + \zeta_t, \quad \zeta_t \sim N(0_{m \times 1}, S_{m \times m})$$

where $S$ is a diagonal covariance matrix with positive and distinct elements on the diagonal.

The essence of the UKF (Chow et al., 2007) used in this paper can be summarized briefly as follows. For each measurement occasion $t$, a set of deterministically selected points, termed *sigma points*, is used to approximate the distribution of the current state estimated at time $t$ using a normal distribution with a mean vector $X_{t|t-\Delta t}$, a covariance matrix that

---

42 In the typical UKF setting, both transition and measurement equations are nonlinear. Hence, to compute the ex ante predictions of the state variables’ mean and variance, sigma points are needed to approximate the distribution of previous state estimates. However, in our paper, the transition equations are linear, so we can directly compute the ex ante predictions as in the classic Kalman Filter, and do not need sigma points at this stage.
is a function in the state covariance matrix, $P_{X,t|t-\Delta t}$, and conditional covariance $V_t$.

Sigma points are specifically selected to capture the dispersion around $X_{t|t-\Delta t}$, and are then projected using the measurement function $g(\cdot)$, weighted, and then used to update the estimates in conjunction with the newly observed measurements at time $t$ to obtain $X_{t|t}$ and $P_{X,t|t}$.

Next we outline the detailed algorithm of UKF:

1. Initialization\footnote{Refer to item 7 in Appendix D.}

   \[ X_{0|0} = \text{Constants} \]
   \[ P_{0|0} = V^* \]

2. Ex ante predictions of states

   \[ X_{t|t-\Delta t} = T X_{t-\Delta t|t-\Delta t} + \Theta \]
   \[ P_{X,t|t-\Delta t} = T P_{X,t-\Delta t|t-\Delta t} T' + V_t \]

3. Selecting sigma points

Given a $\omega \times 1$ vector of \textit{ex ante} predictions of states $X_{t|t-\Delta t}$, a set of $\omega \times (2\omega + 1)$ sigma points are selected as follows:

\[ X_{t|t-\Delta t} = \begin{bmatrix} X_{0,t-\Delta t} & X_{+,t-\Delta t} & X_{-,t-\Delta t} \end{bmatrix} \]

where

\[ X_{0,t-\Delta t} = X_{t|t-\Delta t} \]
\[ X_{+,t-\Delta t} = 1_{\omega} \otimes X_{t|t-\Delta t} + \sqrt{(\omega + \theta)} \left( T \sqrt{P_{X,t-\Delta t|t-\Delta t}} + \sqrt{V_t} \right) \]
\[ X_{-,t-\Delta t} = 1_{\omega} \otimes X_{t|t-\Delta t} - \sqrt{(\omega + \theta)} \left( T \sqrt{P_{X,t-\Delta t|t-\Delta t}} + \sqrt{V_t} \right). \]
The term \( \theta \) is a scaling constant and given by

\[
\theta = \eta^2 (\omega + \varrho) - \omega
\]

where \( \eta \) and \( \varrho \) are user-specified constants in this paper, with \( \eta = 0.001 \), and \( \varrho = 3 - \omega \). Since the values of these constants are not critical in our case, we omit a detailed description for the sake of saving space. Readers are referred to Chow et al. (2007) or Chapter 7 in Haykin et al. (2001) for further details.

4. Transformation of sigma points by way of the measurement function (predictions of measurements)

\( \chi_{t|t-\Delta t} \) is propagated through the nonlinear measurement function \( G(\cdot) \)

\[
Y_{t|t-\Delta t} = G(\chi_{t|t-\Delta t}),
\]

where the dimension of \( Y_{t|t-\Delta t} \) is \( m \times (2\omega + 1) \). Then define the set of weights for covariance estimates as

\[
W^{(c)} = \text{diag} \left[ \frac{\theta}{\omega + \theta} + 1 - \eta^2 + 2, \frac{1}{2(\omega + \theta)}, \ldots, \frac{1}{2(\omega + \theta)} \right]
\]

obtain weights for the mean estimates as follows:

\[
W^{(m)} = \begin{bmatrix}
\frac{\theta}{\omega + \theta} \\
\frac{1}{2(\omega + \theta)} \\
\vdots \\
\frac{1}{2(\omega + \theta)}
\end{bmatrix}_{(2\omega+1) \times 1}
\]

The predicted measurements and associated variance and covariance matrices are
computed as follows:

$$y_{t| t-\Delta t} = Y_{t| t-\Delta t} W^{(m)}$$

$$P_{y_{t| t-\Delta t}} = \left[ Y_{t| t-\Delta t} - 1_{1 \times (2\omega+1)} \otimes y_{t| t-\Delta t} \right] W^{(c)} \left[ Y_{t| t-\Delta t} - 1_{1 \times (2\omega+1)} \otimes y_{t| t-\Delta t} \right]^\prime + S$$

$$P_{X_{t}, y_{t}} = \left[ X_{t| t-\Delta t} - 1_{1 \times (2\omega+1)} \otimes X_{t| t-\Delta t} \right] W^{(c)} \left[ Y_{t| t-\Delta t} - 1_{1 \times (2\omega+1)} \otimes y_{t| t-\Delta t} \right]^\prime$$

5. Kalman gain and ex-post filtering state update

With the output from Step 4, actual observations are finally brought in and the discrepancy between the model’s predictions and the actual observations is weighted by a Kalman gain $\Xi_t$ function to yield ex-post state and covariance estimates as follows:

$$\Xi_t = P_{X_t, y_t} P_{y_{t| t-\Delta t}}^{-1}$$

$$X_{t| t} = X_{t| t-\Delta t} + \Xi_t \left( y_t - y_{t| t-\Delta t} \right)$$

$$P_{X_t| t} = P_{X_t| t-\Delta t} - \Xi_t P_{y_{t| t-\Delta t}} \Xi_t^\prime$$

$$y_{t| t} = g \left( X_{t| t} \right).$$

D Technical details and assumptions of the model calibration

All the assumptions we make below are for purpose of either tractability or model identification. We emphasize that our empirical results are not at all driven by any of the assumptions here. In other words, relaxing or altering the assumptions makes model estimation tougher but has no material impact on our empirical results.

1. $\phi$ is assumed to be zero for the sake of stable calibration.

2. The numerical integration in Eq. (A1) is performed by way of the Gauss-Kronrod quadrature and truncated at 50. Given the data and parameters, it proves to be fast, accurate, and reliable for our calibration.
3. To apply the filtering techniques, we assume under the physical measure (measure \( P \)) that \( X_t \) follows

\[
\begin{align*}
    dx_1(t) &= \left( \kappa \theta - \kappa^P x_1(t) \right) dt + \sigma_1 \sqrt{x_1(t)} dw_1^P(t) \\
    dx_2(t) &= \mu^P dt + \sigma_2 dw_2^P(t).
\end{align*}
\]

This is equivalent to assuming essentially affine market prices of risks (Duffee, 2002) for \( X_t \).

4. The drift term \( \mu_V \) of \( V_t \) under measure \( Q \) is set to 0.04, which is roughly the sample average of the risk-free interest rate. Therefore, \( \mu \) is set at \( 0.04 - \frac{\sigma^2}{2} \). Consistent with Merton (1974), we assume the bank asset to be tradable so that its drift term under measure \( Q \) is the risk-free interest rate.

5. Although the dynamics of \( x_1 \) are different for each individual bank, it is reasonable to assume that the parameters of \( x_1 \), \{\( \kappa \), \( \kappa^P \), \( \theta \), and \( \sigma_1 \)\}, reflect common market views of characteristics of the forbearance provider, the government. Therefore \{\( \kappa \), \( \kappa^P \), \( \theta \), and \( \sigma_1 \)\} are assumed to be common to all banks.

6. \( \theta \) and \( \kappa^P \) are hard to pin down, and are set to \( -\log(0.9) \) and \( \frac{\log(0.9)}{\log(0.97)} \kappa \), respectively. This means that the long term mean of \( x_1(t) \) is \( -\log(0.9) \) under the measure \( Q \) and \( -\log(0.97) \) under the measure \( P \). Roughly speaking (ignoring Jensen’s inequality), this also means that the long-term mean of \( \rho \) is 0.9 under the measure \( Q \) and 0.97 under the measure \( P \). This assumption implies that there is a negative risk premium associated with \( x_1(t) \), meaning bank equity holders regard downward movements in \( \rho \) as unfavorable shocks to the investment opportunity. This is consistent with the risk premium identified for interest rates and default risk factors in the literature (see, e.g., Jarrow et al., 2010; Filipović and Trolle, 2013). Unlike the studies of interest rates and credit risk, we do not have term structure data here. This might be the reason behind the non-identifiability of \( \theta \) and \( \kappa^P \).

7. The initial values for \( x_1 \) and \( x_2 \) used to start up the UKF are, respectively, \( -\log(0.97) \)
and the log of the individual market cap at the first data point. The initial covariance $V^*$ is given by

$$
\begin{bmatrix}
-\log(0.97)\sigma_1^2 \Delta t & 0 \\
0 & \sigma_2^2 \Delta t
\end{bmatrix}
$$

where $\Delta t = 1/12$.

8. $\sigma_1$ and $\kappa$ are calibrated using the average time series of market cap and total liabilities across all banks. When calibrating $\{\mu^P, \sigma_2\}$ of $x_2$ for each individual bank, $\{\kappa, \kappa^P, \theta, \text{ and } \sigma_1\}$ of $x_1$ are fixed.

E Technical details on estimating the systemic risk

We confine the usual notations defined herein to this appendix only. That is, some notations might be used elsewhere in this paper, but these described here apply exclusively to this appendix and should not be confused with the same notations employed elsewhere in the paper.

Following Brownlees and Engle (2012), the Marginal Expected Shortfall, MES, is expressed as a function of volatility, correlation and tail expectations of the standardized innovations distribution

$$
\text{MES}_{i,t} = \sigma_{i,t} \rho_{i,t} \mathbb{E}(\epsilon_{m,t} | \epsilon_{m,t} < p) + \sigma_{i,t} \sqrt{1 - \rho_{i,t}^2} \mathbb{E}(\epsilon_{i,t} | \epsilon_{m,t} < p)
$$

where $\epsilon_{m,t} = \frac{r_{m,t}}{\sigma_{m,t}}$, $\epsilon_{i,t} = \frac{r_{i,t}}{\sigma_{i,t}}$, and $\sigma_{i,t}$ is the conditional volatility of $i$th bank at time $t$; $r_{m,t}$ and $r_{i,t}$ are daily returns of S&P 500 index and $i$th bank’s market cap at time $t$, respectively; $\rho_{i,t}$ is the conditional correlation between the returns of $i$th bank and those of S&P 500 index; $\mathbb{E}(\epsilon_{i,t} | \epsilon_{m,t} < p)$ is the tail expectation of $i$th bank’s standardized innovations of return given the standardized innovation of returns of S&P 500 index is less than $p$; $\mathbb{E}(\epsilon_{m,t} | \epsilon_{m,t} < p)$ is the tail expectation of S&P 500 index’s standardized innovation of returns conditional on it is less than $p$. As in Brownlees and Engle (2012), we set $p$ to the 5th percentile of the empirical unconditional distribution of $\epsilon_{m,t}$ in the whole sample.
\( \sigma_{m,t} \)s and \( \sigma_{i,t} \)s are estimated using the Threshold ARCH (TARCH) specification (Rabemananjara and Zakoïan, 1993). \( \rho_{i,t} \)s are estimated using the Corrected Dynamic Conditional Correlation (CDCC) approach, which is a Dynamic Conditional Correlation (DCC) framework of Engle (2002) enhanced by Aielli (2013). The tail expectations are computed as the averages of the two standardized innovations in all cases that satisfy the condition (\( \epsilon_{m,t} \) is less than the 5th percentile of the empirical unconditional distribution of \( \epsilon_{m,t} \) in the whole sample).

Given the MES, we follow Acharya et al. (2012), and define the SRISK as

\[
\text{SRISK}_{i,t} = k \frac{\text{TL}_{i,t}}{\text{TA}_{i,t}} + (1 - k)e^{-18 \text{MES}_{i,t}} \text{MC}_{i,t},
\]

where \( \frac{\text{TL}_{i,t}}{\text{TA}_{i,t}} \) is \( i \)th bank’s total liabilities to total assets ratio at time \( t \); \( \text{MC}_{i,t} \) is the market value of equity for \( i \)th bank at time \( t \), which is represented by market cap; as in Acharya et al. (2012), we set \( k \) to 8%.

### F Results of regression specifications with additional control variables

In this appendix, in addition to the results presented in the main text, we present results from other regression specifications: Specifications (S2), (S3), and (S4). In (S2) we add Funding Liquidity (STB/TA) to the RHS of Eq. (8a) and Eq. (8b), in (S3) we add STB/TA and ROA, and in (S4) we add STB/TA, ROA, and Total Loans / Total Assets. STB/TA is defined as the ratio (Short-Term Borrowing + Other Short-Term Liabilities) / Total Assets and ROA is the ratio Net Income / Total Assets. The results are reported in Table A1 and Table A2.
Table A1: System GMM estimates of additional regression specifications for the Forbearance Fraction in Capital (FFC)

This table reports the regression results of specifications (S2), (S3), and (S4) for the Forbearance Fraction in Capital (FFC). The first column contains all the variable names; from the second to the last column, the table reports the coefficients of the variables for all five groups. Each group has three regressions, which are labelled (S2), (S3), and (S4). Sample sizes and instrument counts are also reported in the last two rows. Quarterly data are used in the regressions. Windmeijer (2005) corrected standard errors are in parentheses; *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively. Time fixed effects are controlled by including year dummies in the regressions.

<table>
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<th>VARIABLES</th>
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<th>Big banks</th>
<th>Medium banks</th>
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<td></td>
<td>(S2)</td>
<td>(S3)</td>
<td>(S4)</td>
<td>(S2)</td>
</tr>
<tr>
<td>Lagged FFC</td>
<td>0.77***</td>
<td>0.77***</td>
<td>0.77***</td>
<td>0.82***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Intrinsic Mkt Cap</td>
<td>−0.17</td>
<td>−0.10</td>
<td>−0.29**</td>
<td>0.08</td>
</tr>
<tr>
<td>Implied Asset Value</td>
<td>(0.11)</td>
<td>(0.11)</td>
<td>(0.13)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>Idio Risk</td>
<td>0.98***</td>
<td>1.09***</td>
<td>1.36***</td>
<td>0.70</td>
</tr>
<tr>
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<td>(0.21)</td>
<td>(0.24)</td>
<td>(0.24)</td>
<td>(0.46)</td>
</tr>
<tr>
<td>Total Deposits</td>
<td>−0.15***</td>
<td>−0.23***</td>
<td>−0.14*</td>
<td>−0.16</td>
</tr>
<tr>
<td>Total Liabilities</td>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>log Total Assets</td>
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<td>0.02***</td>
<td>0.04***</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Total Loan</td>
<td>0.09***</td>
<td>0.12***</td>
<td>0.13***</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>GDP Growth</td>
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<td>−0.42*</td>
<td>−0.76***</td>
<td>−0.79*</td>
</tr>
<tr>
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<td>(0.21)</td>
<td>(0.22)</td>
<td>(0.25)</td>
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<tr>
<td>Liquidity</td>
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<td>0.31**</td>
<td>0.29**</td>
<td>0.44</td>
</tr>
<tr>
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<td>(0.14)</td>
<td>(0.14)</td>
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</tr>
<tr>
<td>ROA</td>
<td>−</td>
<td>0.74</td>
<td>1.14</td>
<td>−</td>
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<tr>
<td></td>
<td>(1.85)</td>
<td>(1.82)</td>
<td>(9.83)</td>
<td>(10.13)</td>
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<tr>
<td>Total Loan/Total Assets</td>
<td>−</td>
<td>−</td>
<td>−0.29***</td>
<td>−</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.32)</td>
<td>(0.32)</td>
<td>(0.32)</td>
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| Instrument Count | 569 | 536 | 536 | 77 | 77 | 143 | 143 | 143 | 533 | 530 | 530 |
| Sample Size      | 23646 | 20537 | 20527 | 3096 | 2483 | 2483 | 4815 | 4065 | 4065 | 15311 | 13618 | 13608 |
This table reports the regression results of specifications (S2), (S3), and (S4) for the Intrinsic Market Cap, where the Intrinsic Market Cap is the market cap net of the forbearance capital value, and the Implied Asset Value is one of the state variables in our model, and represents the market-based asset value of a bank. The first column contains all the variable names; from the second to the last column, the table reports the coefficients of the variables for all five groups. Each group has three regressions, which are labelled (S2), (S3), and (S4). Sample sizes and instrument counts are also reported in the last two rows. Quarterly data are used in the regressions. Windmeijer (2005) corrected standard errors are in parentheses; *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively. Time fixed effects are controlled by including year dummies in the regressions.

<table>
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<th>VARIABLES</th>
<th>All</th>
<th>Large banks</th>
<th>Big banks</th>
<th>Medium banks</th>
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<td>0.49***</td>
<td>0.68***</td>
<td>0.61***</td>
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<td>(0.03)</td>
<td>(0.06)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Implied Asset Value</td>
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<td>-0.02***</td>
<td>-0.02***</td>
<td>-0.02*</td>
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<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Idio Risk</td>
<td>-0.36***</td>
<td>-0.31***</td>
<td>-0.18***</td>
<td>-0.24**</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.11)</td>
<td>(0.33)</td>
</tr>
<tr>
<td>Total Assets</td>
<td>0.13***</td>
<td>0.15***</td>
<td>0.18***</td>
<td>0.08***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
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<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Total Deposit</td>
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<td>-0.03***</td>
<td>-0.02***</td>
<td>-0.05**</td>
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<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.03)</td>
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<tr>
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<td>(0.05)</td>
<td>(0.05)</td>
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<td>(0.13)</td>
</tr>
<tr>
<td>GDP Growth</td>
<td>-0.13***</td>
<td>-0.17***</td>
<td>-0.16***</td>
<td>-0.09**</td>
</tr>
<tr>
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<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.04)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Liquidity</td>
<td>-1.13***</td>
<td>1.20***</td>
<td>5.46</td>
<td>5.16*</td>
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<td>(0.39)</td>
<td>(0.40)</td>
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<td>(0.02)</td>
<td>(0.07)</td>
<td>(0.04)</td>
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<td>536</td>
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<td>20527</td>
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