Real Options, Firm Value, and Product Market Competition:

A Model-Based Analysis

Anming Wu

University of Alberta

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Abstract

This study analyzes the effects of product market competition on firm value and its two components: assets in place and growth options. Growth options are more sensitive to competition than assets in place so that their weight in firm value shrinks as competition intensifies. There is a threshold of competition intensity, above which the expected industry-wide increase in product supply has a negative impact on firm value. Further analysis shows that competition weakens the sensitivity of firm value to underlying volatility. This study suggests an empirical test of the effect of real options on the firm-level relationship between stock returns and stock return volatility at the presence of product market competition.

Keywords: Real options; Firm Value; Product market competition

JEL Classification: G1; G31; D43

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Abstract

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1 Introduction

It is generally believed that product market competition has a negative impact on firm value. For example, as the number of competitors grows, price reduction in output may encroach upon profit margins, market share may shrink, and production inputs may become more expensive. This classical conclusion is usually drawn from models\(^1\) not involving the valuation of growth opportunities or other types of real options that include oil extraction rights, flexibility of production facilities, managerial flexibility, and so on\(^2\). However, according to Myers (1977), firm value comes from two components: assets in place — the present value of cash flows generated by already installed capital; and growth options — the value of a firm’s options to install more capital in the future or to manage the firm flexibly in a broader sense. An examination of the effect of product market competition on the two components is necessary because their differential sensitivities may have an implication for other financial factors such as systematic risk.

One stream of real options literature has discussed the impact of competition on real options. For instance, Grenadier (2002) derives the equilibrium investment strategies in a Cournot-Nash framework and finds that competition significantly erodes the value of growth options. As another example, Aguerrevere (2009) shows that firm beta varies with product market competition because the distinguishing effects of competition on firms’ strategic behavior in more competitive industries and in more concentrated industries change systematic risk as the state of economy alters.

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\(^1\) See Shapiro (1989) for a summary of theories of oligopoly behavior.

\(^2\) Myers (1977) first used the term “growth options” for a firm’s options to make future investments. Later, he created the term “real options” for a broader category of opportunities and flexibility held by a firm. In this article, the two terms may be used interchangeably when there is no confusion.
However, several questions remain unanswered in the existing literature. The first question is about the value of growth options at a moderate degree of competition. The previous research (e.g., McDonald and Siegel (1986)) show that the value of a monopolist’s options to invest in a project is significant and could be equal to the investment cost. Then, there must be a threshold of the degree of competition that leads to the value of growth options drops to zero, given the fact, as shown by Grenadier (2002), that competition has a negative impact on a firm’s future cash flow stream. The existence of a threshold implies that real options may still contribute an important part of a firm’s value at a moderate degree of competition, although they become less valuable in a competitive environment than in an industry where the firm has a monopolistic access to them. The second question is about the differential effects of competition on two components of firm value. Since assets in place and growth options have different sensitivities to a change in market structure, their weights in firm value may vary with the intensity of competition. The third question is about the effect of competition on the sensitivity of firm value to volatility. Although it is well established that the value of real options is an increasing function of underlying volatility\(^3\), competition may weaken this effect. A recent study by Grullon, Lyandres, and Zhdanov (2012) suggests that the firm-level positive return-volatility relationship is largely due to real options. However, given the profound effect of product market competition on real options, their finding may be conditional on market structure. Answering these questions may help us more completely understand the role of real options in this relationship\(^4\) that is a focus of the asset pricing

\(^3\) See Dixit and Pindyck (1994: Chapter 5) for an example.

\(^4\) The positive relationship between firm-level stock returns and firm-level stock return volatility is documented by Duffee (1995).
literature. Therefore, a further examination of the effect of product market competition on firm value and its two components is necessary.

This study first presents a model that links market structure to firm value. The real options approach to investment under uncertainty is the framework of the model. This approach has been widely applied in recent finance literature, including the areas of mergers and acquisitions (Hackbarth and Morellec (2008), Lambrecht (2004), and Morellec and Zhdanov (2008)), real options and product market competition (Aguerrevere (2009) and Grenadier (2002)), real options and stock returns (Aguerrevere (2009) and Carlson et al. (2004)), and real options and debt financing (Fries, Miller and Perraudin (1997) and Zhdanov (2007)).

As in Grenadier (2002), He and Pindyck (1992), and Aguerrevere (2009), the current theoretical analysis begins with a problem of production capacity expansion for firms to compete in outputs. This problem in a symmetric industry is solved using two fundamental arguments. First, He and Pindyck (1992) have showed that a monopolist’s capacity choice problem to maximize firm value can be solved if the optimality condition of incremental capital investment is satisfied. Second, Shapiro (1989) elaborates on the determination of the Cournot Nash equilibrium, especially for the case of a symmetric industry with \( n \) identical firms. He and Pindyck’s (1992) approach can be applied to the competitive environment discussed by Shapiro (1989) simply because each marginal capital investment can be thought of as an independent investment project, given the fact that a demand shock is exogenous and stochastic\(^5\). More importantly, this study, instead of focusing on how to solve a game-theoretic problem as Grenadier (2002) does, pays attention to the model’s three implications and the link to empirical research.

\(^5\) See Dixit and Pindyck (1994: Chapter 11) for a discussion.
For the first implication, the model shows that firm value decreases when the number of participants in their industry increases and that there is a threshold of competition intensity, above which the impact of increase in future product supply on firm value is negative. Although both components of firm value decrease when competition intensifies, they are due to different reasons. The value of installed capital reduces because each individual firm in the industry takes a shrinking market share\(^6\); the impact of future capacity expansion is adverse because competition erodes the value of expansion options. This implication shows that at the low or moderate degree of competition the value of real options may still be an important component of firm value.

The second implication is that the importance of real options in their contribution to firm value diminishes faster than that of assets in place when market becomes more competitive. This is because the value of growth options is a convex function of product price\(^7\) while the value of assets in place is linear. As market competition intensifies, growth options quickly lose their importance in firm value so that an increasing part of firm value comes from cash flows generated by installed capital. This implication predicts that in highly competitive industries real options may account for only a small or even negligible part of firm value while in highly concentrated industries the value of real options may be significant.

Further analysis highlights the third implication which is that product market competition reduces the sensitivity of firm value to underlying volatility. While it is a standard conclusion in the real options literature that a higher underlying volatility leads to a higher value of real options, the analysis in this study shows that competition

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\(^6\) Here, we assume that market demand remains unchanged.

\(^7\) It is also a convex function of demand shock, given the inverse demand function in this study.
undermines the value increase induced by a more volatile underlying process. This point motivates an empirical research that extends the work by Grullon, Lyandres, and Zhdanov (2012) to test the effect of product market competition on the contribution of real options to the relationship between stock returns and stock return volatility at the firm level. This implication predicts that their finding is conditional on market structure so that in highly concentrated industries the effect of real options to the positive relationship can be observed while in highly competitive industries it cannot.

The remaining part of this paper is organized as follows. Section 2 presents a real options model and its closed-form solution. Section 3 discusses its three major implications. Section 4 performs a numerical analysis in support of these implications. Section 5 concludes.

2 A Real Options Model Revisited

2.1 Model Settings

Consider a group of $n$ identical firms operating in a symmetric oligopolistic industry that makes a single homogeneous product. At any time $t$, each firm in the industry already has the capacity to produce $q_{it}$ units of the product, which is assumed to be infinitely divisible. Each firm also owns a sequence of options to irreversibly expand its production capacity by investing in an infinitesimal increment of capital each time. In competition for the exercise of capacity expansion options, each investment is optimized to maximize firm value. Since firms make their investment decision simultaneously based on the current market demand and the expected response from competitors, each firm’s

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8 This assumption is made for the purpose of easier mathematical treatment.
optimal choice is a function of its rival’s choices. At the Cournot-Nash equilibrium of 
capital investment strategies, no firm would like to deviate unilaterally. Our purpose is to 
find firm value at equilibrium and discuss its implications for firms operating in a 
competitive environment.

Following the classical real options literature (e.g., McDonald and Siegel (1986)), this study assumes that the inverse demand function is in the form below:

\[ P_t = Y_t Q_t^{-1/\gamma}, \]  

(1)

where \( P_t \) is the price of a unit of product, \( Y_t \) represents the exogenous multiplicative 
demand shock, \( \gamma \) is a constant parameter, and \( Q_t = \sum_{i=1}^{n} q_{it} \) is the total instant industry 
output produced by \( n \) firms. The simple algebraic operation shows that

\[ \frac{\partial \log Q_t}{\partial \log P_t} = -\gamma, \]

which implies that the price elasticity of demand is constant for a 
given demand shock \( Y_t \). As in Aguerrevere (2009) and Carlson et al. (2004), this study 
assumes that \( \gamma > 1 \). The parameter requirement of \( \gamma \) guarantees that the profit is 
increasing in \( q_{it} \) while the marginal profit is increasing in \( Y_t \) \(^9\).

Similar to the assumption made by Black and Scholes (1973) about the stock 
price underlying financial options, the demand shock, \( Y_t \), as the ultimate source of 
uncertainty, follows a geometric Brownian motion:

\[ dY_t = \mu Y_t dt + \sigma Y_t dz_t, \]  

(2)

where \( \mu Y_t \) is the expected instantaneous drift rate of the shock process, \( \sigma Y_t \) is the 
instantaneous variance rate, \( dz_t \) is the increment of a Wiener process, and both \( \mu \) and \( \sigma \)

\(^9\) To see this, let the profit function \( \pi_{it} = q_{it} \cdot P_t = q_{it} Y_t [\sum_{i=1}^{n} q_{it}]^{-1/\gamma} \). As such, both \( \partial \pi_{it}/\ partial q_{it} > 0 \) and 
\( \partial^2 \pi_{it}/\partial q_{it} \partial Y_t > 0 \) can be easily reached if \( \gamma > 1 \).
are constant parameters. This process means that the percentage change in $Y_t$, $dY_t/Y_t$, is normally distributed with mean $\mu dt$ and variance $\sigma^2 dt$ during any infinitely small time interval $dt$. The distribution of the demand shock therefore dictates that the product price $P_t$ evolves as a geometric Brownian motion as well for any given $Q_t > 0$. The product price is an example of various processes on which the value of real options depends. Other examples include output price (Fries, Miller, and Perraudin (1997)), cash flow (McDonald and Siegel (1986)), and oil price (Dixit and Pindyck (1994)).

Each firm in the oligopolistic industry has to decide on its optimal timing to invest into an incremental capacity in order to maximize its firm value. Since the $n$-firm industry is symmetric and all firms are identical, the marginal firm capacity at equilibrium is simply the average of the marginal industry capacity, that is, $dq_{it} = dQ_t/n$. The capital cost of increasing one unit of capacity is assumed to be $K$, which does not change over time. When the market shock $Y_t$ rises to a trigger level, a representative firm $i$ makes an incremental investment in its product capacity $dq_{it}$ at the cost of $K \cdot dq_{it}$. The industry output and product price adjust accordingly to clear the market. As in the standard real options literature (e.g., Dixit and Pindyck (1994)), I assume that the investment is irreversible and the marginal investment cost $K \cdot dq_{it}$ is sunk. Once the investment is made, it yields a flow of $dq_{it}$ units of output permanently. Thus, firm $i$ at each moment $t$ holds an option to increase its profit marginally by making the incremental investment. However, the behavior of other firms within the same industry is not negligible because they face the same decision problem and each firm fears that the others would make a preemptive decision. Grenadier (2002) gives a thorough account of how to solve the game-theoretic problem. Instead of discussing the solution method as in
Grenadier (2002), I go straight to the simplest case: a symmetric industry with $n$ identical firms producing a homogeneous product. Shapiro (1989) shows the existence and uniqueness of Cournot equilibrium in a static model of strategic interactions. He also points out that, if all firms are equal, the equilibrium problem boils down to solving a representative firm’s problem. Furthermore, He and Pindyck (1992) shows that the optimal investment rule that maximizes firm value of a monopolist facing a capacity choice problem can be found by examining the condition of marginal investment. They derive firm value by summing the value of each installed unit of capital and the value of each option to add an additional unit. Since every demand shock in the model discussed in the current study is time-independent, Shapiro’s (1989) conclusion applies to each marginal capacity expansion decision. Therefore, the simplest case of capacity expansion problem can be solved according to Shapiro’s (1989) conclusion about equilibrium and He and Pindyck’s (1992) valuation method. More importantly, the current study focuses on the model’s implications rather than the solution approach. Thus, the fundamental settings not only make the model easy to solve but also suffice to elaborate on the main argument that provides a theoretical support for the further empirical research into the effect of real options on other financial variables.

2.2 Firm Value

As in He and Pindyck (1992), the value of a representative firm is derived in three steps. The first step is to determine the marginal change in the value of installed capital if the total industry capital increases by $dQ_t$ units. The second step is to find the value of the option to invest in the firm’s infinitesimal increment of capital. At each optimal time,
the representative firm exercises the option by paying the capital cost of the marginal unit
and giving up its option on the marginal investment. The third step is to derive firm value
by summing the values of the whole sequence of installed units and the values of the
options to install future units.

The first step is to find how much the value of a representative firm increases as
the whole industry adds $dQ_t$ units to its capacity. In a symmetric industry with $n$ identical
firms, the total industry capacity of $Q_t$ implies that an individual firm at equilibrium is
installed with the capacity of $Q_t/n$. As a result, an increase of $dQ_t$ units of capacity for
the whole industry at equilibrium corresponds to an increment of $dQ_t/n$ units of capacity
for a representative firm. Denote an individual firm’s capacity by $q_{it}$. Then, in the
absence of any variable production cost, the instant profit of firm $i$ at any time $t$ is given
by

$$\pi_{it} = P_t \cdot q_{it} = Y_t Q_t^{-1/\gamma} \cdot \frac{Q_t}{n}.$$  

Therefore, the marginal change in profit with respect to the marginal change in
industry output is

$$\frac{\partial \pi_{it}}{\partial Q_t} = \frac{\gamma - 1}{n\gamma} Y_t Q_t^{-\frac{1}{\gamma}}.$$  

Using the contingent claims valuation approach, one can show that the increase in a
representative firm’s value with respect to $dQ_t$ units of capacity increase in the whole
industry, $V_{lt}$, has to satisfy the following differential equation\(^\text{10}\):

\(^{10}\) See the Appendix for detailed derivations of all equations presented in the section.
\[
\frac{1}{2} \sigma^2 Y_t^2 \frac{\partial^2 V_{it}}{\partial Y_t^2} + (r - \delta)Y_t \frac{\partial V_{it}}{\partial Y_t} - rV_{it} + \frac{\partial \pi_{it}}{\partial Q_t} = 0, \tag{5}
\]

where \(\delta \equiv r - \mu\) represents the convenience yield or capital gain shortfall on \(Y_t\) and it is analogous to the dividend yield in the stock market. Solving Equation (5) yields

\[
V_{it}(Q_t, Y_t) = \frac{\gamma - 1}{n\gamma} \frac{Y_t Q_t^{-1/\gamma}}{\delta}. \tag{6}
\]

By comparing Equations (4) and (6), one can intuitively understand that \(V_{it}\) is just the present value of the marginal profit flows, \(\{\partial \pi_{it}/\partial Q_t, \tau \geq t\}\), with the discount rate \(\delta\), in response to the marginal increase in the industry output \(dQ_t\) at time \(t\).

The second step is to value a representative firm’s option to invest in the marginal unit of capacity when the total industry capacity increases by \(dQ_t\). Obviously, the option is valuable because it grants a firm an opportunity to expand its capacity and thus to earn more cash flows in the future. Denote the option value by \(f_{it}(Q_t, Y_t)\). Then, by the contingent claims argument, the option value has to satisfy the following differential equation:

\[
\frac{1}{2} \sigma^2 Y_t^2 \frac{\partial^2 f_{it}}{\partial Y_t^2} + (r - \delta)Y_t \frac{\partial f_{it}}{\partial Y_t} - rf_{it} = 0. \tag{7}
\]

Simple algebra yields

\[
f_{it}(Q_t, Y_t) = \left[\frac{\gamma - 1}{n\gamma} \frac{v_n}{\delta} - K\right] \left(\frac{Y_t Q_t^{-1/\gamma}}{v_n}\right)^{\frac{\beta}{\gamma}}, \tag{8}
\]
where \( v_n \) is a constant for a given number of firms \( n \) in the industry and \( \beta > 1 \) is a constant that is determined by other parameters. The specific expressions of \( v_n \) and \( \beta \) are given in the Appendix. Equation (8) essentially summarizes the effects of all firms’ options to add one infinitesimal increment on an individual firm’s value.

In the third step, a representative firm’s value, \( F_{it}(Q_t, Y_t) \), can be found by integration:

\[
F_{it}(Q_t, Y_t) = \int_0^{Q_t} V_{it} d\tilde{q}_t + \int_{Q_t}^{\infty} f_{it} d\tilde{q}_t. \tag{9}
\]

The first integral of Equation (9) represents the sum of present values of the expected future profits from selling \( Q_t/n \) units of product permanently if the whole industry has already expanded its capacity from zero to \( Q_t \) units. The second integral accounts for the overall impact of future increase in the industry-wide output on an individual firm’s value if the total industry capacity could grow from the current \( Q_t \) units to infinity. The Appendix shows

\[
F_{it}(Q_t, Y_t) = \frac{Y_t Q_t^{(\gamma-1)/\gamma}}{n\delta} - \left[ \frac{\gamma - 1}{\gamma} \cdot \frac{v_n}{\delta} - K \right] \frac{v_n^{-\beta}}{n} \cdot \frac{\gamma}{\gamma - \beta} \cdot \gamma \beta Q_t^{(\gamma-\beta)/\gamma}. \tag{10}
\]

Equation (10) is the value of a firm with a series of expansion options in the face of product market competition in the industry where the firm operates. The implications of the value function are left for discussion in the next subsection. The model presented here is similar to the one in Grenadier (2002); however, I solve it by adopting He and Pindyck’s (1992) approach. In the setting of a monopolistic economy, He and Pindyck (1992) solve the optimal capacity expansion problem by examining the monopolist’s
incremental investment decision. It turns out that their approach also applies to the symmetric Nash equilibrium investment policy in an oligopolistic framework. In contrast, Grenadier (2002) starts with the general equilibrium conditions that all firms in the \(n\)-firm industry have to meet and further reduces the dimensionality of the equilibrium by adding other constraints to the symmetric cases. The two different approaches lead to the same result. This is not a coincidence, but accurately reflects firms’ interactions in a competitive environment and reveals how an individual firm’s value as an average is related to the whole symmetric industry at equilibrium. Technically speaking, the relationship between the two approaches is analogous to the one between differentiation and integration in calculus.

3 The Model’s Implications

The model presented in the previous section has several implications that are related to the future empirical study. This section pulls out the model’s implications and develops the hypothesis for the empirical analysis.

3.1 Implication 1 — Firm Value and Competition Intensity

The parameter \(n\) in the model determines the competitiveness of product market. When the industry is a monopoly and hence \(n\) in Equation (10) is equal to 1, the value of the monopolist becomes:

\[
F_t^1(Q_t, Y_t) = \frac{Y_t Q_t^{(y-1)/y}}{\delta} - \left( \frac{Y}{y-1} \cdot \frac{\beta}{\beta-1} \cdot \delta K \right)^{-\beta} \frac{K}{\beta - 1} \cdot \frac{Y}{\gamma - \beta} Y_t^\beta Q_t^{(y-\beta)/y}. \tag{11}
\]
Dixit and Pindyck (1994) reach the same value function for a monopolist that makes an incremental investment into its capacity at each optimal instant along a stochastic demand shock process. Similar to Grenadier (2002), the model presented here generalizes Dixit and Pindyck’s (1994, p. 364) model to the case of oligopolistic industry.

By comparing the monopolistic industry producing $Q_t$ units of output by only one firm with the competitive oligopolistic industry producing the same quantity of output instead of by $n$ firms, we can conclude that the competition on the supply side alone could lead the whole industry to shrink value if the demand $Q_t$ remains unchanged and the demand shock $Y_t$ is exogenous. Mathematically, this claim can be presented as

$$F_{it}^1(Q_t, Y_t) > n \cdot F_{it}(Q_t, Y_t),$$

(12)

if $n > (\beta - 1)/(\beta - \gamma)$. The exact threshold value of $n$ depends on the elasticity parameter $\gamma$ that determines how sensitive an investment option premium is to product market competition.

Inequality (12) holds because, if the number of firms, $n$, exceeds a certain level, the impact of industry capacity expansion on a representative firm’s value becomes negative. To see this, let us compare the value of a monopoly, $F_{it}^1(Q_t, Y_t)$, with the total value of an $n$-firm industry, $n \cdot F_{it}(Q_t, Y_t)$. The first term of each value function represents the present value of future cash inflows by selling $Q_t$ units of products permanently from each industry, whereas the second term is the impact of expected capacity expansion on the industry total value. It is easy to show that both $F_{it}^1(Q_t, Y_t)$ and

11 Clearly, $(\beta - 1)/(\beta - \gamma) > 1$ because $\gamma > 1$. Therefore, the number of firms, $n$, takes a reasonable value.
12 The option premium will be discussed later.
\[ n \cdot F_{it}(Q_t, Y_t) \] have the same first term and that the second term of \( F_{it}^1(Q_t, Y_t) \) is positive while the second term of \( n \cdot F_{it}(Q_t, Y_t) \) is negative under the condition of \( n > (\beta - 1)/(\beta - \gamma) \). The positive second term of \( F_{it}^1(Q_t, Y_t) \) shows that a monopolist could bring an additional part to its value. Without worrying about competition, a monopolist can do this by waiting longer to exercise its investment options until the value of the capacity expansion project is high enough. In this case the option to wait is highly valuable.

However, in a highly competitive industry, the preemptive response from competitors pushes them to make an investment earlier so that they would not lose an expansion opportunity. As a result, the value of options to wait diminishes as competition intensifies.

When competition reaches a certain degree, the expected capacity expansion at the industry level has a negative impact on firm value. At an extreme is a perfectly competitive industry where “the urgency of potential entrants results in an option to wait with zero value; the fear of competitors usurping one’s investment opportunity squeezes out all of the potential value from delay [of investment]”, as described in Grenadier (2002). Therefore, according to our discussion, we could define a highly competitive industry as the one where \( n > (\beta - 1)/(\beta - \gamma) \).

The threshold of market competition intensity has a particular implication for empirical research. Since only if \( n > (\beta - 1)/(\beta - \gamma) \) the second term of \( n \cdot F_{it}(Q_t, Y_t) \) turns negative, this implies that if a small number of firms compete for the same product market the negative impact of competition is not strong enough to erode all the value of growth options. Therefore, if the product market competition is not highly intense, the value of growth options might still account for a significant part of firm value. This result is consistent with the sampling method of empirical research and thus makes the data-
based tests legitimate. In the real world the cases of monopoly are rare so that a conclusion drawn from a single-firm model might not be testable. On the contrary, it is common to observe many examples of both highly concentrated industries and highly competitive industries. The observed cases make the test of the model’s implications possible. For example, to measure the degree of industry concentration empirically, the Herfindahl-Hirschman Index (HHI) is broadly used (e.g., Hoberg and Phillips (2010a)). If an industry is dominated by a single firm, its HHI takes a value of one. At the opposite extreme, if a product market is swamped by a large number of firms, its HHI approaches zero. However, neither of the two extreme cases is usually present in real data, but it is straightforward to identify highly competitive industries and highly concentrated industries using the HHI. In practice, the Antitrust Division of the U.S. Department of Justice, for example, considers industries in which the HHI is lower than 0.15 to be competitive and industries with the HHI higher than 0.25 to be highly concentrated\textsuperscript{13}. Therefore, the theoretical conclusion drawn from the model will be in line with real market structures from which samples are collected for empirical analysis.

We can further rigorously analyze how firm value evolves with the degree of competition even if a market is not highly competitive, that is, if \( n < (\beta - 1)/(\beta - \gamma) \). Taking the derivative of \( F_{it}(Q_t, Y_t) \) with respect to \( n \), we can prove that if \( n < (\beta - 1)/(\beta - \gamma) \), then

\[
\frac{\partial F_{it}(Q_t, Y_t)}{\partial n} < 0.
\]

\textsuperscript{13} For more information about this, see https://www.justice.gov/atr/herfindahl-hirschman-index.
Inequality (13) shows that, as the number of firms in the industry grows\(^\text{14}\), each firm is worth less for the given total demand and the given market shock. This is due to two effects of product competition. First, due to a stronger competition, each firm’s market share shrinks if a total demand is given, reducing the future cash flows to the firm and thus their present value. Intuitively, the more firms in an industry, the smaller the market share a firm can take. It is clear from the first term of Equation (10) that a larger \( n \) implies a smaller market share \((Q_t/n)\). Second, the expected increase in future output has an adverse impact on a firm’s future cash flow. The second term of Equation (9) shows that the future supply could expand from the current capacity \( Q_t \) to infinity. As many firms compete in the same industry, they are more likely to exercise their investment option earlier because every firm wants to be the front-runner to avoid losing its expansion opportunity. As competition reaches a certain level, this overall impact is negative. In the extreme case where \( n \) goes to infinity, firm value approaches zero because each firm takes only an infinitesimal share of market while not making any investment for growth. That is,

\[
\lim_{n \to \infty} F_t(Q_t, Y_t) = 0. \tag{14}
\]

3.2 Implication 2 — A Disproportionate Effect of Competition on Assets In Place and Growth Options

So far, we have shown that a firm’s value declines as it competes for a market share with an increasing number of competitors. In this subsection, it can be further

\(^{14}\) Be aware of the condition \( n < (\beta - 1)/(\beta - \gamma) \).
shown that as product market competition intensifies the competitive pressure of capacity expansion exerts a smaller impact on the value of installed capital than on the value of growth options. In other words, the percentage of total firm value in the value of future cash flows from selling $Q_t/n$ units grows while the proportion of the value due to expected future increase in supply shrinks when product market competition becomes more intense. To see this mathematically, rewrite Equation (10) as the sum of the present value of future cash flows by selling $Q_t/n$ units and the overall impact of industry-wide capacity expansion on firm value, that is,

$$F_{it}(Q_t, Y_t) = C_{it}(Q_t, Y_t) + O_{it}(Q_t, Y_t),$$  

(15)

where $C_{it}(Q_t, Y_t)$ is the first term of Equation (10) and $O_{it}(Q_t, Y_t)$ represents the remaining. It would be sufficient to verify the aforementioned claim if we can establish that,

$$\frac{\partial [F_{it}(Q_t, Y_t)/C_{it}(Q_t, Y_t)]}{\partial n} < 0.$$  

(16)

Inequality (16) implies that in highly concentrated industries the value of growth options accounts for a large part of firm value. In highly competitive industries, however, it might become negligible so that the present value of expected cash flows generated by the currently installed capital dominates firm value. Inequality (16) has an implication distinct from Inequality (13). The former states that the total firm value diminishes as competition intensifies, while the latter attributes the value loss mostly to the disappearing value of growth options. Based on this conclusion, the future empirical research could test the effect of product market competition on the positive return-
volatility relationship that is explained from the perspective of real options. According to Grullon, Lyandres, and Zhdanov (2012), the positive return-volatility relationship is significantly due to firms with an abundance of valuable real options. If this is true, it might be conjectured that the loss of total firm value, due to product market competition, may weaken the return-volatility relationship. However, this argument may not necessarily hold because competition erodes both components of firm value. Instead, the disproportionately larger loss in the real options’ value might have an effect on the return-volatility relationship. Therefore, the implication shown by inequality (16) provides a reason why firms doing business in highly concentrated industries and those operating in highly competitive industries may differ in their return-volatility relationship.

Furthermore, it can be shown that the above result is essentially driven by the adverse impact of competition on the investment option value. We can illustrate this point by examining how the option premium of a unit of capacity varies with the number of competitors in an industry. We can do this because a firm at equilibrium can be thought of as an asset comprised of capitals producing $Q_t/n$ units of output and possessing the opportunity of marginal expansion. Denote the equilibrium value of a unit of capacity investment by $U_{it}(Q_t, Y_t)$ and assume that the constant capital cost of a unit of capacity is $K$. It is straightforward to prove that, at the optimal state of option exercise (the time of optimal investment), $Y_t^*$,

$$\frac{\partial [U_{it}(Q_t, Y_t^*) - K]}{\partial n} \frac{U_{it}(Q_t, Y_t^*)}{U_{it}(Q_t, Y_t^*)} < 0.$$  

In Equation (17), the ratio, $(U_{it}(Q_t, Y_t^*) - K)/U_{it}(Q_t, Y_t^*)$, could be defined as the weight of the option value in a unit of capacity at the instant immediately prior to an
option exercise. And then the derivative on the left-hand side of Equation (17) can be thought of as the sensitivity of the weight to product market competition. Equation (17) therefore indicates that, at the instant that a firm makes its investment in an incremental unit of capacity, the option premium as a percentage of the value of unit capacity decreases as the number of rivals grows. This conclusion further establishes that the value of growth options might be an important part of firm value if only a few market players are producing the same product. However, this value loses its importance for firms operating in highly competitive industries because of an intense competition.

3.3 Implication 3 — The Effect of Competition on the Sensitivity of Firm Value to Volatility

Grullon, Lyandres, and Zhdanov (2012) use an indirect method to test the effect of real options on the relationship between stock returns and the stock return volatility at the firm level. They find that the relationship is more positive for firms with abundant real options than for those with few real options. They therefore attribute the relationship to the standard conclusion in the real options literature that the real options’ value increases with the volatility of underlying process. If we can find a theoretical channel showing that the sensitivity of firm value to underlying volatility decreases with growing competition, then we can claim that there is a more positive return-volatility relationship in highly concentrated industries where real options are valuable and that there is a weak or nonexistent positive return-volatility relationship in highly competitive industries.
First of all, it can be shown that

\[
\frac{\partial F_{it}(Q_t)}{\partial \sigma} > 0. \tag{18}
\]

This result is consistent with the standard conclusion of real options literature. That is, the value is an increasing function of the volatility of a process underlying real options. This is because the nature of real options (analogous to financial call options) allows option holders to benefit from the upside movement of underlying processes but to limit its downside risk (see Dixit and Pindyck (1994): Chapter 5 for details). Consequently, a higher volatility brings more valuable growth potentials to firms that have a lot of real options, and therefore these firms are valued higher than those faced with a lower underlying volatility.

However, the relationship shown by Inequality (18) is weakened by product market competition. This is formally explained by the following formula,

\[
\frac{\partial}{\partial n} \left( \frac{\partial F_{it}(Q_t)}{\partial \sigma} \right) < 0. \tag{19}
\]

Inequality (19) indicates that if firm value is positively correlated with underlying volatility, the correlation diminishes as a growing number of firms compete in the same market. In other words, Inequality (19) says that although a higher volatility makes it possible for firms with abundant real options to amplify the effect of favorable demand shocks while mitigating the effect of adverse demand shocks, the capability or the benefit to do so is significantly destroyed by the increasing competition. For example, if observing an increasing demand on a product, many firms are likely to make an
investment into the production capacity expansion. However, the increasing number of participants reduces the value of the investment, undermining the benefit created by the positive demand shock. Therefore, in the empirical study of the effect of real options on the return-volatility relationship, a measure of competition intensity should be introduced into the test. Inequality (19) essentially justifies that the test should take into consideration the effect of product market competition.

Overall, the above discussion shows that the value of real options is an important part of firm value in a less competitive industry. However, product market competition erodes the value. Therefore, as product market competition intensifies, the value of real options accounts for an ever-decreasing part of total firm value. If there is a relationship, such as the one between stock returns and stock return volatility, that is sensitive to real options, as shown by Grullon, Lyandres, and Zhdanov (2012), the sensitivity should be less — or even not — significant for firms with no or few real options. A highly competitive industry is an example where the value of real options may be negligible. As a result, our discussion on the real options model predicts that the effect of real options on the return-volatility relationship should not be observed in a highly competitive industry. Furthermore, it has been explicitly proven that the effect of real options on the firm value-volatility relationship weakens as competition becomes more intense. These points will be illustrated by the numerical analysis in the next section.

4 Numerical Analysis

This section provides a numerical analysis that explains the main conclusions drawn in the previous section. The numerical results presented here are simulated from
the demand shock process: $Y_t = Y_{t-1} + Y_{t-1}(\mu \Delta t + \sigma \sqrt{\Delta t} \varepsilon_t)$, where $\varepsilon_t$ evolves as a normal distribution with a mean of zero and a standard deviation of one, and the time interval $\Delta t$ is assumed to be one. The parameter values in simulation are $\mu = 0.02, r = 0.05, \sigma = 0.175, \gamma = 1.5, K = 1, Q_0 = 100,$ and $Y_0 = 1.74$.

4.1 Firm Value and Product Market Competition

Figure 1 shows how a representative firm’s equilibrium value and its two components evolve with a simulated process of market demand shocks to an industry. In the example, a varied number of firms compete for the same product market. Each panel of this figure demonstrates that the present value of cash flows generated by installed capital and the overall value impact of expected industry capacity expansion differ in their importance in firm value at a different level of market competition. Panel A of Figure 1 illustrates that, when only one firm dominates a product market ($n = 1$), a significant part of firm value $F_{it}(Q_t, Y_t)$ comes from growth options. The value of growth options is much larger than the present value of future cash flows generated by the monopolist’s installed capital. If, however, two firms compete for the same product supply ($n = 2$), as shown by Panel B, the value of growth options is significantly smaller than the value of installed capital even though it still accounts for a large part of firm value. This implies that the product market competition exerts a larger adverse impact on the value of growth options than the present value of future cash flows generated by installed capital. This point is further illustrated by Panel C, where a firm faces three rivals in the market ($n = 4$). In this case, almost the entire firm value is made up by the value of installed capital but the value of growth options is nearly negligible.
stronger contrast can be observed in Panel D, which demonstrates the case where five competitors are operating in the same industry \((n = 5)\). Now, the expected future increase in supply from the entire industry has a significant negative influence on firm value, which offsets a part of the present value of future cash flows contributed by the already installed capacity. Thus, the curve of firm value in Panel D is located slightly below the curve of the value of installed capital.

[Insert Figure 1 here]

Figure 2 illustrates how a representative firm’s equilibrium value and its constituent components change as an increasing number of firms compete for the same product market. Each panel presents a case where those competitors face a different level of demand shock. Panel A shows that, if only one single firm supplies the product at the demand shock \(Y_t = 1.74\), the capacity expansion options contribute nearly 50% of firm value, close to the contribution from the installed capacity. However, the contribution from growth options drops significantly as the product market competition intensifies. In the simulated example, when more than four firms supply the same product, the expected future capacity expansion in the entire industry leads to a negative value impact on each firm in the industry. Panel B and Panel C of Figure 2 illustrate the same point when the demand shocks are \(Y_t = 3.44\) and \(Y_t = 5.22\), respectively. The striking difference between those panels reveals that the stronger demand shocks bring a higher value of growth options to a representative firm so that growth options contribute a larger part of firm value relative to cash flows generated by assets in place (installed capital). This is consistent with one conclusion of real options theory, which states that a firm with real options can amplify the effect of the positive demand shock (Dixit and Pindyck (1994),
Overall, Figure 2 indicates that the value of growth options is more sensitive to product market competition and thus drops much faster than the present value of installed capital.

[Insert Figure 2 here]

4.2 The Weight of Growth Options in Firm Value and Product Market Competition

Figure 3 shows that the weight of each component — the value of installed capital and the value of growth options — in firm value varies with respect to the number of firms that operate in the same product market. Those convex curves represent the ratio of the value impact of expected industry capacity expansion to the total firm value at equilibrium. The ratio is large if a firm has a monopoly, indicating that the value of growth options makes up a significant portion of firm value. However, the ratio decreases with growing competition, and finally drops below zero as the intensity of competition reaches a certain degree, which is four in the simulated case. In contrast, the ratio of the value of cash flows generated by installed capital to the total firm value at equilibrium goes up with the increasing number of firms that compete for the same product market, as shown by the concave curves. This simulated result indicates that market competition significantly squeezes the value of growth options out of firm value. Even more striking is the case in which, as competition reaches a certain level, the impact of capacity expansion offsets part of the value contributed by installed capital.

[Insert Figure 3 here]
Figure 4 further illustrates that the reduced value of growth options could be explained by the decreasing ratio of the option premium to the value of a unit of capacity at equilibrium. The ratio diminishes as the number of firms in their industry grows. A monopolistic firm could have nearly 70% of the value of one unit of its capacity coming from growth options. However, the ratio drops down to 15% if a firm operates in a four-firm industry, and it gets close to zero if a firm has nineteen competitors.

[Insert Figure 4 here]

4.3 Product Market Competition and the Sensitivity of Firm Value to Volatility

Each panel of Figure 5 demonstrates how the equilibrium value of a representative firm in an n-firm industry evolves with a simulated process of market demand shocks. Evidently, firm value varies positively with underlying volatility. Thus, this figure essentially shows the sensitivity of firm value to underlying volatility. Given the number of market participants, a higher underlying volatility is associated with a higher firm value. The rationale behind the relationship is that firms can more likely take advantage of investment opportunities created by a higher possibility of positive demand shocks than by a lower one while they can still mitigate the downside risk by manipulating their managerial flexibility to change their investment decision. As a result, firm value is an increasing function of underlying volatility. For instance, Panel B of Figure 5 shows the case where two firms (n = 2) produce the same product. Each individual firm at the demand shock $Y_t = 42.82$ ($t = 38$) has a value of 156,699 if the underlying volatility is $\sigma = 0.225$, a value of 35,207 if the underlying volatility is
\( \sigma = 0.2 \), and a value of 22,817 if the underlying volatility is \( \sigma = 0.175 \). Therefore, the numerical result shown by Figure 5 agrees with the real options theory, which implies that the value of real options is increasing in the volatility of underlying process (see Dixit and Pindyck (1994): Chapter 5 for the details).

However, the sensitivity of firm value to underlying volatility is significantly weakened by product market competition, as shown by Figure 6. For example, Panel A shows the increase in firm value when the underlying volatility increases by 0.025 from 0.175 to 0.2. Firm value goes up by 35,938 in the case where the market is dominated by only one firm \( (n = 1) \). In contrast, if the market is shared by two firms \( (n = 2) \), each firm adds to its value by 12,390. And if six firms compete in the same market \( (n = 6) \), each firm’s value increases by only 1,427. Obviously, the firm value-volatility relationship is waning as competition intensifies. The same conclusion is drawn from Panel B, which again shows the sensitivity is decreasing in the competition intensity as the underlying volatility increases from 0.2 to 0.225.

Overall, the numerical analysis is in line with the theoretical conclusions drawn from the model’s implications presented in the last section. That is, in highly concentrated industries, a relatively large portion of firm value comes from real options while in highly competitive industries the value of real options significantly declines. The numerical analysis also shows that the weight of the value of real options in firm value decreases with the increasing intensity of product market competition. Furthermore, firm value is less responsive to changes in volatility in a more competitive market than in a
less competitive market. Therefore, if the positive return-volatility relationship is attributed to real options that firms possess, as shown by Grullon, Lyandres, and Zhdanov (2012), we would expect that stock returns respond differently to changes in underlying volatility of firms facing a different market structure.

5 Conclusion

This paper studies the effects of competitive interactions among firms on firm value in a real options framework. Three implications are drawn from the study. First, at a low or moderate degree of competition, growth options to expand production capacity contribute a significant part of firm value. Below a certain threshold of competition intensity, both the value of growth options and the value of assets in place diminish as competition intensifies, whereas above this threshold the expected industry-wide increase in product supply has a negative impact on firm value. Second, the growing number of rivals in an industry has a disproportionate effect on growth options and assets in place because growth options are more sensitive to product market competition than assets in place. Therefore, the importance of growth options in firm value decreases when product market becomes more competitive. Third, although a firm becomes more valuable if its underlying process is more volatile, product market competition weakens the sensitivity of firm value to underlying volatility.

A recent study by Grullon, Lyandres, and Zhdanov (2012) suggests that the firm-level positive return-volatility relationship is largely due to real options. However, the current study predicts that their conclusion is conditional on product market competition. In highly competitive industries, we may not observe the positive effect of real options on
the return-volatility relationship because of the differential effects of competition on the
two components of firm value. In contrast, in highly concentrated industries we should
still observe “the real options effect” found by Grullon et.al because there is little
competition among firms.
Figures

Figure 1. Evolution of Values with a Simulated Process of Demand Shocks

This figure shows how a representative firm’s equilibrium value and its components — the present value of cash flows generated by installed capital and the value impact of expected industry capacity expansion — evolves with a simulated process of market demand shocks to industries where a different number of firms produce a homogenous product. In the figure, \( n \) is the number of firms, \( F_t(Q_t, Y_t) \) represents the total firm value, \( PVC \) is the present value of future cash flows, and \( Expansion \ Impact \) means the impact of the expected industry-wide increase in future product supply on firm value. The time period process marked by \( t = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40\} \) corresponds to the simulated process of demand shocks \( Y_t = \{1.74, 1.61, 1.36, 1.42, 1.45, 1.29, 1.60, 1.31, 1.37, 1.69, 1.81, 1.82, 2.10, 2.20, 3.05, 3.38, 4.32, 3.91, 3.42, 4.47, 4.93, 4.96, 6.06, 7.27, 7.99, 11.37, 11.12, 12.95, 12.22, 13.31, 13.19, 14.95, 18.51, 17.44, 23.31, 29.25, 34.04, 43.47, 42.82, 50.85, 64.04\} \). The parameter values in simulation are \( \mu = 0.02, r = 0.05, \sigma = 0.175, \gamma = 1.5, K = 1, Q_0 = 100, \) and \( Y_0 = 1.74 \).
Figure 2. Values Varying with the Number of Firms in the Same Product Market

This figure shows how a representative firm’s equilibrium value and its components — the present value of cash flows generated by installed capital and the value impact of expected industry capacity expansion — evolves with the increasing number of firms producing a homogenous product when those firms face the various demand shocks. In the figure, $Y_t$ is the demand shock, $F(Q_t, Y_t)$ represents the total firm value, $PVC$ is the present value of future cash flows, and $Expansion Impact$ means the impact of the expected industry-wide increase in future product supply on firm value. The parameter values in simulation are $\mu = 0.02$, $r = 0.05$, $\sigma = 0.175$, $\gamma = 1.5$, $K = 1$, $Q_0 = 100$, and $Y_0 = 1.74$. The figure only presents the evolutions of values at a few demand shocks drawn from the simulated process $Y_t = \{1.74, 1.86, 2.26, 1.91, 1.93, 1.95, 1.60, 1.92, 1.72, 2.16, 2.69, 3.01, 3.44, 3.75, 4.24, 5.22, 6.10, 3.99, 5.23, 4.06, 3.30\}$. 
This figure shows that the ratio of the value impact of expected industry capacity expansion to the total equilibrium firm value decreases while the ratio of the present value of cash flows generated by installed capital to the total equilibrium firm value increases with the increasing number of firms that produce a homogenous product. The figure also illustrates how the ratios are different when those firms face the different demand shocks. In the figure, $X_t$ is the demand shock, the lines marked with Cash Flows represent the ratio of the present value of cash flows generated by installed capital to the total equilibrium firm value, and the lines marked with Expansion Impact represent the ratio of the value impact of expected industry capacity expansion to the total equilibrium firm value. The parameter values in simulation are $\mu = 0.02$, $r = 0.05$, $\sigma = 0.175$, $y = 1.5$, $K = 1$, $Q_0 = 100$, and $Y_0 = 1.74$. The figure only presents the ratios at a few demand shocks drawn from the simulated process $Y_t = \{1.74, 1.86, 2.26, 1.91, 1.93, 1.95, 1.60, 1.92, 1.72, 2.16, 2.69, 3.01, 3.44, 3.75, 4.24, 5.22, 6.10, 3.99, 5.23, 4.06, 3.30\}$. 

Figure 3. Weights of Each Component in Firm Value Varying with the Number of Firms in the Same Product Market
Figure 4. Ratio of Option Premium to the Value of One Unit of Capacity at Equilibrium

This figure shows that the ratio of option premium to the value of a unit of capacity at equilibrium diminishes when the number of firms in an industry increases. The parameter values in simulation are $\mu = 0.02$, $r = 0.05$, $\sigma = 0.175$, $\gamma = 1.5$, $K = 1$, $Q_0 = 100$, and $Y_0 = 1.74$. 
Figure 5. Firm Value and Volatility

This figure shows how a representative firm’s equilibrium value evolves with a simulated shock process to market demand that has a different underlying volatility. No matter how many firms are in an industry, a higher volatility leads to a higher firm value. In the figure, \( n \) represents the number of firms. The time period series marked by \( t = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40\} \) corresponds to the simulated process of demand shocks \( Y_t = \{1.74, 1.61, 1.36, 1.42, 1.45, 1.29, 1.60, 1.31, 1.37, 1.69, 1.81, 1.82, 2.10, 2.20, 3.05, 3.38, 4.32, 3.91, 3.42, 4.47, 4.93, 4.96, 6.06, 7.27, 7.99, 11.37, 11.12, 12.95, 12.22, 13.31, 13.19, 14.95, 18.51, 17.44, 23.31, 29.25, 34.04, 43.47, 42.82, 50.85, 64.04\} \). The other parameter values in simulation are \( \mu = 0.02, r = 0.05, \gamma = 1.5, K = 1, Q_0 = 100, \) and \( Y_0 = 1.74 \).
Figure 6. Product Market Competition and Sensitivity of Firm Value to Volatility

This figure shows how a representative firm’s equilibrium value evolves with a simulated process of market demand shocks to industries where a varied number of firms produce a homogenous product. The sensitivity of firm value to volatility decreases when the number of firms in the industry increases. In the figure, \( n \) represents the number of firms. The time period series marked by \( t = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40\} \) corresponds to the simulated process of demand shocks \( Y_t = \{1.74, 1.61, 1.36, 1.42, 1.45, 1.29, 1.60, 1.31, 1.37, 1.69, 1.81, 1.82, 2.10, 2.20, 3.05, 3.38, 4.32, 3.91, 3.42, 4.47, 4.93, 4.96, 6.06, 7.27, 7.99, 11.37, 11.12, 12.95, 12.22, 13.31, 13.19, 14.95, 18.51, 17.44, 23.31, 29.25, 34.04, 43.47, 42.82, 50.85, 64.04\} \). The other parameter values in simulation are \( \mu = 0.02, r = 0.05, \gamma = 1.5, K = 1, Q_0 = 100, \) and \( Y_0 = 1.74 \).
Appendix

1. The instant profit flow from an incremental unit of capital

The first thing to find is the instant profit flow contributed by the \((q_{it} + dq_{it})th\) unit of capital to a representative firm \(i\) at time \(t\). Given the current capital stock and demand, the firm decides whether it should purchase an incremental unit of capital to increase its production capacity. To obtain the threshold of demand shocks at which the firm optimally makes the incremental investment, it is necessary to know the instant profit flow generated by the additional capacity. Denote the instant profit flow by \(\pi_{it}\), the total industry output by \(Q_t\), and the firm’s capacity by \(q_{it}\). Then,

\[
\pi_{it} = P_t \cdot q_{it} = Y_t \cdot Q_t^{-\frac{1}{\gamma}} \cdot q_{it},
\]

and

\[
Q_t = \sum_{i=1}^{n} q_{it}.
\]

It is easy to show that the marginal change in the instant profit flow with respect to the change in the firm’s capacity is:

\[
\frac{\partial \pi_{it}}{\partial q_{it}} = \frac{n\gamma - 1}{n\gamma} Y_t Q_t^{-\frac{1}{\gamma}}.
\]

2. The investment trigger of market shock

To derive the value of a representative firm at equilibrium, it is necessary to first find the optimal threshold that triggers a marginal investment in the firm’s capital at the first moment market shock hits it. We can solve this problem by constructing a riskless portfolio. Let \(u_{it}(Q_t, Y_t)\) denote the fundamental value of the installed incremental capital, which contributes to the firm the marginal flow of profits, \(\{\partial \pi_{it}/\partial q_{it}, \tau \geq t\}\). An investor could invest in the following portfolio: \(u_{it}(Q_t, Y_t) - m \cdot Y_t\), where the investor
takes a long position in the asset that generates the marginal flow of profits and a short position in \( m \) units of another asset\(^{15} \) that tracks \( Y_t \). The portfolio yields an instant cash inflow from the marginal investment, \( \partial \pi_{it} / \partial q_{it} \), and the instant dividend outflow from the investor’s short position, \(-m \delta Y_t \). Then by Ito’s Lemma, the marginal value change in the portfolio is as follows:

\[
\begin{align*}
d(u_{it}(Q_t, Y_t) - m \cdot Y_t) &= \left[ \mu Y_t \left( \frac{\partial u_{it}(Q_t, Y_t)}{\partial Y_t} \right) + \frac{1}{2} (\sigma Y_t)^2 \left( \frac{\partial^2 u_{it}(Q_t, Y_t)}{\partial Y_t^2} \right) \right] dt + \sigma Y_t \left( \frac{\partial u_{it}(Q_t, Y_t)}{\partial Y_t} \right) dz_t - m(\mu Y_t dt + \sigma Y_t dz_t) \\
&= \left[ \mu Y_t \left( \frac{\partial u_{it}(Q_t, Y_t)}{\partial Y_t} \right) - m \right] + \frac{1}{2} (\sigma Y_t)^2 \left( \frac{\partial^2 u_{it}(Q_t, Y_t)}{\partial Y_t^2} \right) dt + \sigma Y_t \left( \frac{\partial u_{it}(Q_t, Y_t)}{\partial Y_t} - m \right) dz_t.
\end{align*}
\]

\( (A4) \)

Clearly, taking \( \partial u_{it}(Q_t, Y_t) / \partial Y_t = m \) makes the marginal value of the portfolio independent of the increment of the standard Wiener process, \( dz_t \), and as such, the portfolio becomes riskless at any moment in time. Note that a riskless portfolio is expected to earn an instant riskless return, \( r \), which is equal to the sum of the capital gain rate and dividend yield. It follows that

\[
[u_{it}(Q_t, Y_t) - m \cdot Y_t]r
\]

\[
= \left[ \mu Y_t \left( \frac{\partial u_{it}(Q_t, Y_t)}{\partial Y_t} \right) - m \right] + \frac{1}{2} (\sigma Y_t)^2 \left( \frac{\partial^2 u_{it}(Q_t, Y_t)}{\partial Y_t^2} \right) + \frac{\partial \pi_{it}}{\partial q_{it}} - m \delta Y_t.
\]

\( (A5) \)

Plugging \( m = \partial u_{it}(Q_t, Y_t) / \partial Y_t \) into the above equation yields

\[
\frac{1}{2} \sigma^2 Y_t^2 \left( \frac{\partial^2 u_{it}}{\partial Y_t^2} \right) + (r - \delta) Y_t \frac{\partial u_{it}}{\partial Y_t} - ru_{it} + \frac{\partial \pi_{it}}{\partial q_{it}} = 0.
\]

\( (A6) \)

The general solution to Equation (A6) can be written as

\[
u_{it}(Q_t, Y_t) = a_1(Q_t) Y_t^\beta + a_2(Q_t) Y_t^\alpha + \frac{n\gamma - 1}{n\gamma} Y_t Q_t \frac{1}{\gamma}, \quad (A7)\]

where \( a_1(Q_t) \) and \( a_2(Q_t) \) are two coefficients to be determined, and the powers \( \beta \) and \( \alpha \) are the roots of the characteristic equation of \( \omega \) associated with the homogeneous part of Equation (A6),

\(^{15} \) Similar to the standard real options literature, such as Carlson, Fisher, and Giammarino (2004) and Dixit and Pindyck (1994), it is assumed that there exists an asset that tracks \( Y_t \).
\[
\frac{1}{2} \sigma^2 \omega (\omega - 1) + (r - \delta) \omega - r = 0.
\]

(A8)

The two roots of the characteristic equation are expressed as

\[
\beta = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left[ \frac{r - \delta}{\sigma^2} - \frac{1}{2} \right]^2 + 2r/\sigma^2} > 1,
\]

(A9)

and

\[
\alpha = \frac{1}{2} - \frac{r - \delta}{\sigma^2} - \sqrt{\left[ \frac{r - \delta}{\sigma^2} - \frac{1}{2} \right]^2 + 2r/\sigma^2} < 0,
\]

(A10)

where the two inequalities can be established by a simple algebraic manipulation and they hold for any volatility parameter \( \sigma > 0 \), risk neutral interest rate \( r > 0 \), and cash flow discount rate \( \delta > 0 \).

Equation (A6) must satisfy the following two conditions:

1) **Absorbing barrier.** This condition is imposed by the nature of the geometric Brownian motion, \( Y_t \). It means that if \( Y_t \) ever goes to zero it will remain zero permanently. Because \( \alpha < 0 \), this condition then prevents the value of the installed incremental capital from diverging. Intuitively, without a prospect of any profit flow, an asset should have zero value. As a result, a firm at this condition will not make any marginal investment and therefore \( u_t(Q_t, 0) = 0 \). It follows that \( a_2(Q_t) = 0 \).

2) **Speculative bubble condition.** To rule out the possible speculative bubble, set \( a_1(Q_t) = 0 \). A detailed discussion about this condition is given by Dixit and Pindyck (1994).

As such, Equation (A7) reduces to the fundamental term, which, under the assumption of zero variable cost, is the present value of discounted future net cash flows:
Next, let us value the option to invest in the marginal unit of capital, \( dq_{it} \). Let \( g(Q_t, Y_t) \) represent the option value. By the contingent claims argument again, the option value, \( g_{it}(Q_t, Y_t) \), must satisfy the following differential equation,

\[
\frac{1}{2} \sigma^2 Y_t^2 \frac{\partial^2 g_{it}}{\partial Y_t^2} + (r - \delta) Y_t \frac{\partial g_{it}}{\partial Y_t} - rg_{it} = 0,
\]  

(A12)

which has the general solution:

\[
g_{it}(Q_t, Y_t) = b_1(Q_t)Y_t^\beta + b_2(Q_t)Y_t^\alpha.
\]  

(A13)

Both \( b_1(Q_t) \) and \( b_2(Q_t) \) in the above solution are the coefficients to be determined by the following three conditions:

1) Absorbing barrier. This condition implies that \( b_2(Q_t) = 0 \) and \( g_{it} = b_1(Q_t)Y_t^\beta \) because if \( Y_t \) goes to zero, \( g_{it}(Q_t, Y_t) \) must be zero.

2) Value-matching condition. This condition is expressed mathematically as \( g_{it}(Q_t, Y_t^*) = u_{it}(Q_t, Y_t^*) - K \). Namely, at the optimal moment, \( Y_t^* \), when a representative firm makes an investment in the marginal unit of capital, the option value given up by the firm is just the net payoff, \( u(Q_t, Y_t^*) - K \), where \( K \) is the constant unit cost of capital.

3) Smooth-pasting condition. In mathematical notation, the condition is expressed as \( \partial g_{it}(Q_t, Y_t^*)/\partial Y_t = \partial u_{it}(Q_t, Y_t^*)/\partial Y_t \). It says that the capital expansion is optimal for the representative firm.

The above three conditions require us to solve the following system of equations for \( Y_t^* \) and \( b_1(Q_t) \).
\[
\begin{aligned}
\left\{
\begin{array}{l}
b_1(Q_t)Y_t^{\beta} = \frac{n\gamma - 1}{n\gamma\delta} Y_t^{\frac{1}{\gamma}} Q_t^{\frac{1}{\gamma}} - K \\
b_1(Q_t)\beta Y_t^{\beta - 1} = \frac{n\gamma - 1}{n\gamma\delta} Q_t^{\frac{1}{\gamma}}
\end{array}
\right.
\end{aligned}
\]  \quad (A14)

It is straightforward to find the following solution,

\[ Y_t^{\ast} = v_n Q_t^{\frac{1}{\gamma}}, \]  \quad (A15)

where

\[ v_n = \frac{n\gamma - 1}{n\gamma - 1} - 1 \delta K. \]  \quad (A16)

Accordingly, Equation (A13) becomes

\[ g_{it}(Q_t, Y_t) = \frac{n\gamma - 1}{n\gamma - \beta} v_n^{1-\beta} Q_t^{\beta} Y_t^{\beta}. \]  \quad (A17)

3. Derivation of Equation (6)

Following the same argument used for the derivation of Equation (A6), we can have Equation (5). The only difference is that we are here considering how the representative firm’s value changes with respect to the marginal change in the industry’s total capacity. Similar to Equation (A6), Equation (5) has the following general solution,

\[ V_{it}(Q_t, Y_t) = A_1(Q_t)Y_t^{\beta} + A_2(Q_t)Y_t^{\alpha} + \frac{\gamma - 1}{n\gamma} Y_t Q_t^{-1/\gamma}, \]  \quad (A18)

which, if we rule out the speculative bubble and use the absorbing barrier condition, has the solution expressed as Equation (6).
4. Derivation of Equation (8)

Following the same argument used for the derivation of Equation (A6), we can get Equation (7). Similar to Equation (A12), Equation (7) has the general solution below,

$$f_{it}(Q_t, Y_t) = B_1(Q_t)Y_t^\beta + B_2(Q_t)Y_t^\alpha,$$  \hspace{1cm} (A19)

which is subject to the following conditions,

1) *Absorbing barrier*. This condition implies that $B_2(Q_t) = 0$.

2) *Smooth-pasting condition*. Mathematically, the condition is written as

$$f_{it}(Q_t, Y_t^*) = V_{it}(Q_t, Y_t^*) - K/n.$$  

The condition says that at the optimal investment threshold, $Y_t^*$, the difference between the representative firm’s value and the industry-wide average marginal capital cost $K/n$ is equal to the option value, $f_{it}(Q_t, Y_t^*)$.

The second condition leads to the expression of $B_1(Q_t)$ as follows,

$$B_1(Q_t) = \left[ \gamma - 1 \frac{Y_t^* Q_t^{-1/\gamma}}{n \gamma} \frac{K}{n} \right] Y_t^{* - \beta}. \hspace{1cm} (A20)$$

Plugging $B_1(Q_t)$ and $Y_t^*$ into the general solution, (A19), yields the desired result.

5. Derivation of Equation (10)

The firm value can be found by integration:

$$F_{it}(Q_t, Y_t) = \int_0^{Q_t} V_{it} d\tilde{q}_t + \int_{Q_t}^\infty f_{it} d\tilde{q}_t$$

$$= \int_0^{Q_t} \frac{\gamma - 1}{n \gamma} \frac{Y_t^* Q_t^{-1/\gamma}}{\delta} d\tilde{q}_t + \int_{Q_t}^\infty \left[ \frac{\gamma - 1 \nu_t}{n \gamma} \frac{K}{n} \right] (\nu_t \tilde{q}_t^{-1/\gamma})^{-\beta} Y_t^\beta d\tilde{q}_t. \hspace{1cm} (A21)$$

A simple algebraic manipulation of integrals leads to the desired result. Here, the condition $\gamma < \beta$ is required because the value of a representative firm converges only if it holds. A detailed discussion about this is given by Dixit and Pindyck (1994).
6. Derivation of Inequality (12)

Let $E_1$ represent the second term of $F_{it}^1(Q_t, Y_t)$ and $E_2$ the second term of $n \cdot F_{it}(Q_t, Y_t)$. That is,

$$E_1 = - \left( \frac{\gamma}{\gamma - 1} \cdot \frac{\beta}{\beta - 1} \cdot \delta K \right)^{-\beta} \cdot \frac{K}{\beta - 1} \cdot \frac{\gamma}{\gamma - \beta} \cdot Y_t^\beta Q_t^{(\gamma - \beta)/\gamma}, \quad (A22)$$

and

$$E_2 = - \left( \frac{\gamma - 1}{\gamma} \cdot \frac{u_n}{\delta} - K \right) v_n^{\beta} \cdot \frac{\gamma}{\gamma - \beta} \cdot Y_t^\beta Q_t^{(\gamma - \beta)/\gamma}. \quad (A23)$$

Because it is clear that the first term of $F_{it}^1(Q_t, Y_t)$ is the same as that of $n \cdot F_{it}(Q_t, Y_t)$, it suffices to reach the claim if $E_1 > E_2$ at a certain level of $n$. Clearly, $E_1 > 0$ is true due to the condition $\gamma < \beta$. Therefore, if $E_2 < 0$, we would have obviously reached the desired result. This translates into showing

$$\frac{\gamma - 1}{\gamma} \cdot \frac{u_n}{\delta} - K < 0. \quad (A24)$$

Plugging $u_n$ into the inequality in (A24), it is equivalent to show that

$$\frac{n(\gamma - 1)}{n\gamma - 1} \cdot \frac{\beta}{\beta - 1} < 1. \quad (A25)$$

Then, with the assumption that $\gamma > 1$ and the established inequality that $\beta > 1$, this requires us to prove

$$1 > n(\gamma - \beta) + \beta, \quad (A26)$$

which is equivalent to

$$n > \frac{\beta - 1}{\beta - \gamma} > 1. \quad (A27)$$
The first inequality in (A27) must hold in light of the established inequality $\beta > 1$ and the convergence assumption $\gamma < \beta$. The threshold value (i.e., the greatest lower bound) of $n$ is dependent on the elasticity parameter $\gamma$ that determines how the investment option premium at the moment of option exercise is sensitive to product market competition. The second inequality in (A27) results from $\gamma < \beta$ and $\gamma > 1$.

7. Derivation of Inequality (13)

Let

\[ C_{lt}(Q_t, Y_t) = \frac{Y_t Q_t^{(\gamma-1)/\gamma}}{n\delta}, \quad (A28) \]

and

\[ O_{lt}(Q_t, Y_t) = \left[ K - \frac{\gamma - 1}{\gamma} \cdot \frac{v_n}{\delta} \right] v_n^{-\beta} \cdot \frac{\gamma}{\gamma - \beta} \cdot Y_t^\beta Q_t^{(\gamma-\beta)/\gamma}. \quad (A29) \]

Obviously, $\partial C_{lt}(Q_t, Y_t) / \partial n < 0$ and approaches zero as $n$ increases. Reorganizing $O_{lt}(Q_t, Y_t)$ yields

\[ O_{lt}(Q_t, Y_t) = \left[ \frac{1}{n} - \frac{\gamma - 1}{n\gamma - 1} \cdot \frac{\beta}{\beta - 1} \right] v_n^{-\beta} \cdot \frac{\gamma K}{\gamma - \beta} \cdot Y_t^\beta Q_t^{(\gamma-\beta)/\gamma}. \quad (A30) \]

Let

\[ x_1 = v_n^{-\beta}. \quad (A31) \]

Then straightforward algebra yields

\[ \frac{\partial x_1}{\partial n} = -\beta v_n^{-\beta-1} \frac{\partial v_n}{\partial n} = \frac{\beta v_n^{-\beta}}{n(n\gamma - 1)}. \quad (A32) \]
Let

\[ x_2 = \frac{1}{n} \frac{\gamma - 1}{n\gamma - 1} \frac{\beta}{\beta - 1}. \]  \hspace{1cm} \text{(A33)}

Then, from the first inequality in (A27), it is clear that \( x_2 > 0 \), and

\[ \frac{\partial x_2}{\partial n} = \frac{\gamma(\gamma - 1)}{(n\gamma - 1)^2} \frac{\beta}{\beta - 1} - \frac{1}{n^2} = \frac{n^2\gamma(\gamma - \beta) + (2n\gamma - 1)(\beta - 1)}{n^2(n\gamma - 1)^2(\beta - 1)}. \]  \hspace{1cm} \text{(A34)}

It follows that \( \partial O_{lt}(Q_t, Y_t) / \partial n < 0 \) and approaches zero as \( n \) increases.

Accordingly, it can be demonstrated that

\[ \frac{\partial F_{lt}(Q_t, Y_t)}{\partial n} = \frac{\partial C_{lt}(Q_t, Y_t)}{\partial n} + \frac{\partial O_{lt}(Q_t, Y_t)}{\partial n} \]

\[ = \frac{\partial C_{lt}(Q_t, Y_t)}{\partial n} + \frac{\gamma K}{\gamma - \beta} \cdot Y_t^\beta Q_t^{(\gamma - \beta)/\gamma} \left( x_2 \frac{\partial x_1}{\partial n} + x_1 \frac{\partial x_2}{\partial n} \right) < 0, \]  \hspace{1cm} \text{(A35)}

and \( \partial F_{lt}(Q_t, Y_t) / \partial n \) approaches zero as \( n \) increases.

8. Derivation of Equation (14)

It is clear that the present value of future cash flows \( C_{lt}(Q_t, Y_t) \) approaches zero as the number of firms, \( n \), becomes infinitely large. The same is true for the value impact of industry expansion \( O_{lt}(Q_t, Y_t) \) if we notice that the following limit is finite:

\[ \lim_{n \to \infty} \nu_n = \frac{\beta}{\beta - 1} \delta K \]  \hspace{1cm} \text{(A36)}

9. Derivation of Inequality (16)

The ratio of the firm value to the present value of future cash flows is
\[ \frac{F_{lt}(Q_t, Y_t)}{C_{lt}(Q_t, Y_t)} = \frac{C_{lt}(Q_t, Y_t) + O_{lt}(Q_t, Y_t)}{C_{lt}(Q_t, Y_t)} = 1 + \left[ 1 - \frac{n(\gamma - 1)}{n\gamma - 1} \cdot \frac{\beta}{\beta - 1} \right] v_n^{-\beta} \cdot \frac{\gamma \delta K}{\gamma - \beta} \cdot \frac{Y_t Q_t^{\gamma - \beta / \gamma}}{Y_t Q_t^{1/\gamma}}. \] (A37)

Let

\[ w = \left[ 1 - \frac{n(\gamma - 1)}{n\gamma - 1} \cdot \frac{\beta}{\beta - 1} \right] v_n^{-\beta}. \] (A38)

Then it is easy to show

\[ \frac{\partial w}{\partial n} = \frac{(n - 1)\beta v_n^{-\beta}}{n(n\gamma - 1)^2} > 0. \] (A39)

It follows that

\[ \frac{\partial[F_{lt}(Q_t, Y_t)/C_{lt}(Q_t, Y_t)]}{\partial n} = \frac{\gamma \delta K}{\gamma - \beta} \cdot \frac{Y_t Q_t^{\gamma - \beta / \gamma}}{Y_t Q_t^{1/\gamma}} \cdot \frac{\partial w}{\partial n} < 0, \] (A40)

by noting that \( \gamma < \beta \).

10. Derivation of Inequality (17)

To prove the inequality, we first have to find the value of one unit of capacity in the industry. Similar to (A1), the instant profit flow of one unit of capacity can be written as

\[ \pi_t^1 = P_t \cdot 1 = Y_t \cdot Q_t^{1/\gamma}. \] (A41)

The impact of the marginal investment \( dQ_t \) on the profit flow of one unit of capacity, then, is as follows,
\[ \frac{\partial \pi^1_t}{\partial Q_t} = -\frac{1}{\gamma} Y_t Q_t \frac{1}{\frac{1}{\gamma}}. \] (A42)

Similar to the derivation of Equation (A11), we can show that the present value of discounted future cash flows from one unit of capacity, denoted by \( v^1_t (Q_t, Y_t) \), has to satisfy the following differential equation,

\[ \frac{1}{2} \sigma^2 Y_t^2 \frac{\partial^2 v^1_t}{\partial Y_t^2} + (r - \delta) Y_t \frac{\partial v^1_t}{\partial Y_t} - r v^1_t + \frac{\partial \pi^1_t}{\partial Q_t} = 0. \] (A43)

Again, if taking into consideration the absorbing barrier condition and ruling out the speculative bubble, we can find the present value of discounted future cash flows contributed by one unit of capacity as below,

\[ v^1_t (Q_t, Y_t) = -\frac{1}{\gamma^\delta} Y_t Q_t \frac{1}{\frac{1}{\gamma}}. \] (A44)

Similarly, the impact of the marginal change in the industry capacity on the value of a single unit of capacity, denoted by \( h^1_t (Q_t, Y_t) \), can be expressed as

\[ h^1_t (Q_t, Y_t) = D_1 (Q_t) Y_t^\beta + D_2 (Q_t) Y_t^\alpha, \] (A45)

subject to the following two conditions:

1) **Absorbing barrier.** This implies that \( D_2 (Q_t) = 0 \).

2) **Smooth-pasting condition,** \( h^1_t (Q_t, Y_t^*) = v^1_t (Q_t, Y_t^*) - d(K \cdot 1)/dQ_t = v^1_t (Q_t, Y_t^*) \). This says that at the optimal investment threshold, \( Y_t^* \), when the whole industry increases its capital from \( Q_t \) to \( Q_t + dQ_t \), the value increase of one unit of capacity minus the marginal increase in the unit cost of capital is equal to the option value \( h^1_t (Q_t, Y_t^*) \).

A simple algebraic manipulation shows that

\[ h^1_t (Q_t, Y_t) = -\frac{1}{\gamma^\delta} v_n^{1-\beta} Y_t^{\beta} Q_t \frac{\gamma + \beta}{\gamma}. \] (A46)
Then, the equilibrium value of one capacity unit, denoted by $U_{lt}(Q_t, Y_t)$, is expressed as

$$U_{lt}(Q_t, Y_t) = \int_0^{Q_t} v_t^1 d\tilde{q}_t + \int_{Q_t}^\infty h_t^1 d\tilde{q}_t$$

$$= \frac{1}{\delta} Y_t Q_t^{-\frac{1}{\gamma}} - \frac{1}{\beta\delta} v_n^{1-\beta} Y_t^{\beta} Q_t^{-\frac{\beta}{\gamma}}. \tag{A47}$$

Thus, the value of one unit of production capacity at the optimal exercise point, $Y_t^*$, can be shown as follows,

$$U_{lt}(Q_t, Y_t^*) = \frac{n\gamma}{n\gamma - 1} K. \tag{A48}$$

Therefore, at the instant of optimal exercise, $Y_t^*$, the option value included in the value of one unit of capacity is equal to $U_{lt}(Q_t, Y_t^*) - K$. Then, it is easy to demonstrate that the weight of the option value as a percentage of the value of one capacity unit is

$$\frac{U_{lt}(Q_t, Y_t^*) - K}{U_{lt}(Q_t, Y_t^*)} = \frac{1}{n\gamma}. \tag{A49}$$

Taking the derivative of Equation (A49) with respect to $n$ obviously leads to the result shown by Equation (17).

11. Proofs of Inequality (18) and Inequality (19)

Let us consider how the sensitivity of the equilibrium value of firm $i$ to its underlying volatility changes with product market competition. It is clear that the product market supply, $Q_t$, increases marginally if Equation (A15) is satisfied. This means that a representative firm makes a sequence of investments continuously if demand shocks become stronger after the firm’s initial capacity expansion\(^\dagger\). As a result, every positive demand shock, $Y_t$, corresponds to a unique increasing supply level, $Q_t$. Therefore, we can rewrite the value of firm $i$ as

\(^\dagger\) See Dixit and Pindyck (1994) for a discussion of “Barrier Control”.

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Now, let

\[ e_1 = \frac{\nu_n}{n\delta} \]  

and

\[ e_2 = \left( \frac{\gamma - 1}{\gamma} \cdot \frac{\nu_n}{\delta} - K \right) \frac{\gamma}{n(\beta - \gamma)}. \]  

Then, it is straightforward to show that

\[ \frac{\partial e_1}{\partial \beta} = -\frac{\gamma K}{(n\gamma - 1)(\beta - 1)^2} \]  

and

\[ \frac{\partial e_2}{\partial \beta} = \frac{\gamma K}{(n\gamma - 1)(\beta - 1)^2} \left[ 1 - \frac{1}{n} \left( \frac{\beta - 1}{\beta - \gamma} \right)^2 \right]. \]  

It follows that

\[ \frac{\partial F_{it}(q_t)}{\partial \beta} = -\frac{\gamma K q_t}{n(n\gamma - 1)(\beta - \gamma)^2} < 0. \]  

Thus,

\[ \frac{\partial F_{it}(q_t)}{\partial \sigma} = \frac{\partial F_{it}(q_t)}{\partial \beta} \cdot \frac{\partial \beta}{\partial \sigma} > 0, \]  

by noting that \( \partial \beta / \partial \sigma < 0. \)

Finally, it is clear that
\[
\frac{\partial}{\partial n}\left(\frac{\partial F_{lt}(Q_t)}{\partial \sigma}\right) = \frac{\partial}{\partial n}\left(\frac{\partial F_{lt}(Q_t)}{\partial \beta} \cdot \frac{\partial \beta}{\partial \sigma}\right) = \frac{\partial \beta}{\partial \sigma} \cdot \frac{\partial}{\partial n}\left(\frac{\partial F_{lt}(Q_t)}{\partial \beta}\right) < 0, \tag{A57}
\]

by noting that \(\frac{\partial}{\partial n}\left(\frac{\partial F_{lt}(Q_t)}{\partial \beta}\right) > 0\).
References


He, Hua, and Roberts S. Pindyck, 1992, Investments in flexible production capacity, *Journal of Economic Dynamics and Control* 16, 575–599.


