A Theory on the Risk-Taking Behavior of Microfinance Borrowers, Creditor Rights, and Social Capital

Michael S. Padhi
Robert H. Smith School of Business
University of Maryland
College Park, MD 20742
240-423-4784
mpadhi@rhsmith.umd.edu.

Abstract:
This paper provides theory concerning the risk-taking incentives of microfinance borrowers in varying cases: individual liability, group liability without social sanctions, and group liability with social sanctions. The results provide insight into how a community’s social capital and a country’s credit rights interact to induce recipients of microfinance programs to take risk. Consistent with recent anecdotal evidence that suggests a “dark side” to microfinance, the results show that communal ties among joint liability borrowing groups do not lead to higher repayment rates and does have worse welfare effects on the recipients by making the poorest group members unwilling to take the risks necessary to grow a business. This paper contributes to the literature on contract design and on financial development and growth, extending them into the realm of finance for the poorest and least able to access formal financial markets.
I. Introduction

Microfinance is the popular economic development program aimed at the world’s poorest entrepreneurs in many developing countries. It normally consists of lending very small amounts, as little as $75, to invest in self-owned enterprises, in order to provide more opportunities beyond wage labor. Microfinance is intriguing because it provides financial services to a very large segment of the world’s population, who otherwise would borrow in the informal financial market from friends, family, and moneylenders. Furthermore, prior to its first implementation by economics professor Mohammed Yunus in Bangladesh in the 1970s, very poor borrowers in developing countries were not expected to be willing or at least able to repay unsecured business loans made by lenders from outside their communities.

It is commonly believed that microfinance programs succeed in leading to high repayment rates because of the strong social ties that exist among their clients. “Social capital” is considered an alternative to physical capital in the developmental economics literature. While borrowers may not have physical collateral to secure a loan, they do live in tight knit cultures where the social repercussions from defaulting on a loan could be as costly as losing material possessions. The first microfinance programs were thought to tap into this social capital by giving loans to groups, where each member of the group was liable for all the others’ share of the loan (Van Bastalaer, 2000).

Because all members of the borrowing group are liable for each other, they have incentives to punish noncontributing members and to encourage each other to succeed. They also have incentives to screen potential members into the group as well as verify that each group member exerts effort in the projects so that they can repay their shares of the loans. Harnessing social capital is particularly important in countries where lenders have little recourse to deal with borrowers that are able to repay their loans but choose not to do so. This inability is often due to poor institutions like creditor rights. (La Porta, et. al., 1998)

Even if borrowers own land, they may not be able to use it as collateral because of laws that restrict the use of land as collateral or make the ability to seize collateral very costly or impossible. Also, record of title may be unavailable due to poor record keeping.

Despite the theoretical support for the value of social capital in providing financial services to developing nations’ poor entrepreneurs, studies of microfinance programs have also yielded some puzzling results. One is the success of microloans made on an individual liability basis.

If individual liability loans have similar repayment rates as joint liability loans, what does this say about either the need for social capital or the necessity of the joint liability contract in harnessing it? The other puzzle deals with the problem of loans not being put to productive use. That is, many of microborrowers seem to hold onto their loans or consume them rather than invest them in a business.
This is one finding that does not have much theoretical explanation. Intuitively, the reasons given is that the members of the group receiving the loan are too afraid of the repercussions from failure from their peers that they prefer to use the loan to smooth their consumption and payoff the loan with income from pre-existing sources such as a wage earning job or even a business that they did not invest the loan in.

In this paper, I develop a theoretical model on the relationship between the repayment strategies of microfinance borrowers and the types of projects they invest their microloans in. Besley and Coate (1995) model how group lending mitigates the moral hazard problem of borrowers being unable to credibly contract with lenders to repay their loans even when they are able. Besley and Coate address the question of how the lending schemes affect the incentives of borrowers to repay their loans, but they assume that the borrowers’ projects are exogenously given. This paper extends their model by endogenizing project selection under various lending schemes. By modeling the choice of what project a borrower will choose, I provide a theory to explain the relationship between financial contract design, a country’s creditor protections, investment choice, entrepreneurship, and poverty.

I present conditions for microfinance programs resulting in optimal project selection. I show that certain entrepreneurs who are jointly liable for their loans will not take the necessary business risks that microfinance is supposed to induce them to take while not improving the repayment rate over individual liability. This result is driven by strong social ties that exist among borrowing group members, which is usually thought of as a positive effect on social welfare.

The conclusions of this study contribute to the theory of microborrower behavior in two main ways. The first theoretical contribution of this study is an extension of Besley and Coate’s (1995) comparison of the repayment behavior of microborrowers under varying assumptions of liability and existence of social sanctions. I show that borrowers under joint liability without the threat of social sanctions are more likely to repay their loans than when under the threat of social sanctions. By introducing project and peer selection into the model, I get this result because the threat of social sanctions stifles risk sharing and encourages free-riding among borrowers with low upside potential to their project options.

The second theoretical contribution is the identification of people who choose particular types of contracts where there is an option between individual and joint liability contracts. If there are no social sanctions, then people with high upside potential to their projects take the joint liability contract because the benefits of risk sharing outweigh the costs of free-riding. People with low upside project potential, however, take the individual liability contracts because they do not share any risk under the joint liability contract by selecting safe projects. The introduction of social sanctions to the model, however, effectively eliminates the use of the joint liability contract by every microborrower.
Ultimately, studying this question is important because microfinance institutions (MFIs), governments, non-governmental organizations (NGOs), and microfinance investors want to know what model of microfinance to follow. Should a MFI offer joint liability contracts to borrowers who are able to sanction each other if one borrower does not contribute to repayment? Should a social planner subsidize MFIs that offer joint liability contracts or those that offer individual liability contracts?

The remainder of the paper is organized as follows. Section two reviews the relevant literature on microfinance and positions this paper in the existing literature. Section three presents a model of individual liability lending. Section four presents a model of joint liability lending without and with social sanctions. Section five discusses the results of the model. Section six concludes with a summary and direction for further research.

II. Literature Review

The literature on group lending argues for social capital to impact the likelihood of repayment through various channels. The most frequently cited categorization of models explaining how social capital impacts group repayment rates is by Ghatak and Guinnane (1999). The first is in a superior screening ability of peers over delegated monitors because most groups select which borrowers can join them (Ghatak and Guinnane 1999). The second is a superior monitoring ability of peers to control ex ante moral hazard (Stiglitz 1990). The third is a superior auditing ability of peers to limit verification costs. The fourth is a superior enforcement mechanism through imposition of social sanctions should a borrower default to control ex post moral hazard (Besley and Coate 1995). As opposed to comparing the effects of each of these four assumptions on what kind of problem social capital addresses in group lending, I compare the impact of various forms of social capital on risk taking and repayment of borrowers.

Besley and Coate (1995) provide a model to predict how the group liability aspect of microfinance affects the repayment behavior of borrowers when repayment of loans is not enforceable. They compare the model’s predictions of repayment rates among three lending systems: individual lending, group lending without social sanctions, and group lending with social sanctions. The individual lending system describes the traditional arrangement in which the bank lends to an individual who is solely liable. The group lending system without social sanctions describes an arrangement in which the bank lends to a group of borrowers who divide the loan among themselves and invest their shares in their own enterprises, which are independent from one another. In this system, the group as a whole is liable, each member of the group decides whether or not she contributes her share to the group’s repayment, and the group cannot penalize noncontributing members. The group lending system with social sanctions describes the same arrangement, except the group can exert peer pressure on the members to contribute to the repayment of the loan.
Their main conclusions are twofold. First, the impact of group lending on repayment rates over that of individual lending are affected by two countervailing forces: risk-sharing and free-riding. On one hand, if one member of the group cannot contribute her share of the loan because of poor project returns, the other group members can cover her share. Therefore, group lending may improve loan repayment rates through sharing risk. On the other hand, a borrower who would have repaid her loan under individual lending might take advantage of the group's incentives to cover her share of the loan. This free-riding incentive leads to a coordination failure whereby the group as a whole will default even though certain individuals would have repaid their own shares if they were individually liable. Without social sanctions, it is unclear as to which effect dominates. The second conclusion is that the free-rider effect can be lessened if the group is able to use social sanctions to pressure the members if they stand to lose a significant amount of social capital. They show that if social sanctions are great enough, then group lending does dominate individual lending in regard to the repayment rate.

An extension of Besley and Coate's model is Che (2002)'s dynamic model of repayment behavior in a repeated game. Che does not include social sanctions other than exclusion from participation in future borrowing opportunities. He shows that while the static model has ambiguous results concerning group lending's repayment rates due to the free-rider problem, the dynamic model shows that group lending weakly dominates individual lending due to the cost of exclusion from future opportunities.

In addition to extending their model to a dynamic setting, among the recommendations that Besley and Coate make for further research is to model the effect of group lending on the type of project that each member chooses. As this paper will show, Besley and Coate's model can be adapted to find interesting implications for selection of project risk by different lending schemes. In addition, I extend their model to include self selection of borrowers into groups. The outcome of endogenizing project and group member selection is domination of the risk sharing effect without social sanctions and suboptimal risk taking with social sanctions.

In addition to Besley and Coate (1995)'s focus on the enforcement problem, other theories focus on the problem of (i) screening out risky borrowers, (ii) monitoring borrower effort (“ex ante” moral hazard), and (iii) verification of project outcomes. Ghatak (1999) and Ghatak and Guinnane (1999) provide the most cited model of how joint liability microloans induce risky borrowers to select other risky borrowers to form a group and safe borrowers select other safe borrowers. By this assortive matching process, lenders are more able to identify risky borrowers from safe borrowers “by the company they keep”. Other theories such as van Tassell (1999) and Laffont and N'Guessan (2000) also conclude that borrowers will match with borrowers of similar riskiness. Guttman (2006, 2007), however, offers a model that predicts the opposite: Borrowers of opposite risk types will match with each other. Guttman makes this prediction because he assumes that borrowers can make side payments to each other to
attract group members. While both risky and safe borrowers prefer to have safe peers, the value of a safe peer to a risky borrower is greater than to another safe borrower. Intuitively, this greater value comes from a diversification benefit of matching a risky borrower with a safe one. This later conclusion is also supported by my model as well as experimental evidence (Gine, Jakiela, Karlan, and Morduch, 2010).

Stiglitz (1990) finds group lending’s advantage in improving repayment rates come from the peer monitoring effect, thus limiting the ex ante moral hazard of how group members will use their loans and exert effort. He focuses on group members being able to observe the effort each applies to her projects and to write enforceable contracts among themselves.

Ghatak and Guinnane (1999) present a model where verification of states is costly in the spirit of Townsend (1979), demonstrating that joint liability contracts induce truth telling by borrowers concerning their projects' payoffs by delegating the auditing function to group members. Therefore, the lender only has to verify the state when the entire group defaults, thus reducing the number of cases the bank has to incur auditing costs.

While most research deals with comparing available microfinance contracts’ impact on welfare, some offers new kinds of contracts that are currently not observed. Bubna and Chowdrylic (2009), for example, offer a new institution they call, “microfinance franchising” in which a single MFI offers a lending franchise to local capitalists who compete with a single moneylender. Their model is currently being experimented in Samoa. Ayi (2007) suggests that MFIs offer a “microequity” contract whereby the MFI has an equity stake in the microenterprises, similar to venture capitalists.

A similarly titled contract is also suggested by Pretes (2002). The newest stage in the microfinance movement is that of “microinsurance,” which offers insurance contracts to the same people that microloans are intended for (Morduch, 2004; McGuinness and Tounytsky, 2006; Leftley and Mapfumo).

While all the theory on microfinance loan performance assumes that social capital is pivotal, the way in which social capital functions differs among the theories. The empirical research also varies in its conclusions regarding social capital's effect on loan repayment and on borrower welfare.

How aspects of borrowers' relationship to each other and their culture add to or subtract from social capital is complicated. Studies on the type of social ties show that certain aspects of relationships among borrowers in a group differ in their effect on borrower behavior. Hermes and Lensink (2007) survey of empirical studies on social capital’s relationship to loan repayment identifies several characteristics of social capital that actually have negative effects on loan repayment: family membership in group, distance between members, strong social ties, group homogeneity, relatedness,
sharing within group, and high level of joint liability. They also find that some of these characteristics also have positive effects on loan repayment in other studies, indicating that the relationship between social capital and loan repayment is actually ambiguous. Similarly another survey concludes, “The results of available empirical studies are contradictory with respect to virtually all potential determinants of repayment performance” (Guttman, 2006).

Studies show that the joint liability contract is not the only means of harnessing social capital. De Aghion and Morduch (1998) find that microfinance borrowers in transition economies who borrow on an individual liability basis still have incentives to repay their loans because of social stigma over default. Also, an individual liability loan can be marketed to, purchased by, and collected in groups, thereby lowering transactions costs, having a good information source, fostering education and training, and increasing individuals’ comfort with banks. In a randomized controlled field experiment, Karlan and Gine (2010) show that repayment rates of borrowers of a Philippine MFI with branches that are randomly converted from offering joint liability to loans to individual liability loans do not differ from unconverted branches. They find that social capital plays an important role in influencing repayment after branches are converted to offering individual liability loans.

Studies on the impact of microfinance programs on repayment and on poverty reduction are not uniformly supportive. However, they find that the impact of microfinance programs is not uniform across borrowers. Hulme and Mosley (1996) and Morduch (1998) find microfinance programs are less effective in increasing income among those below the poverty line. In Hulme and Mosley (1996), the researchers suggest that microfinance programs’ impact decreases with falling income because the borrowers below the poverty line take less risks, invest less in technology, and use their loans to protect their subsistence. In some cases, the loans lower income among the poorest of the poor (Khawari 2004). Many microfinance borrowers take loans to reduce variation in consumption and not to increase expected income.

Pretes (2002) criticizes the use of microfinance for certain borrowers. He cites cases where very poor borrowers become worse-off because of business failure. They attempt to repay their portion of the loan by borrowing from moneylenders, selling their household assets or food, or leaving their home to find wage labor. Taking these drastic steps as opposed to just defaulting as would be the case in a country with developed bankruptcy laws may be due to “the darker side of collective peer pressure.” Social sanctions may be so strong that the borrower may be in fear of becoming an outcast. Also, since most microfinance borrowers are women in countries where they are under a high degree of male control, the husbands of the borrowers sometimes take the money away, leaving the wife to struggle to find a way to repay the loan. So, the informal institutions of the community and the home may not allow for microfinance to succeed in empowering borrowers to take the appropriate risks as an entrepreneur.
Pretes postulates that the high repayment rates of microfinance programs are misleading as to their effectiveness in reaching the very poor, as “financial benefits disproportionately accrue to the middle poor and do not reach the very poor.”

This paper contributes to the literature on microfinance by combining Besley and Coate’s model with other models of microfinance to consider how the group lending system affects peer and project choices. Allowing for projects of varying levels of risk is an important assumption due to the empirical evidence that project selection matters. The model's results provide a theoretical explanation for the finding that group lending can have the opposite effect on investment decisions than is intended. I show how the nature of social sanctions that are levied by the group play a role in this result.

### III. Individual Lending

Consider an entrepreneur's payoff function when liability is individual. In the first period, \( t = 0 \), the borrower receives a loan of \( l \) at an interest rate, \( i \). At \( t = 1 \), the borrower chooses whether to invest in a safe or risky project. At \( t = 2 \), the investment returns are realized and the borrower chooses whether to repay the loan. Assume that without a loan, the entrepreneur would not be able to invest in any project. All loans are for the same amount and require a repayment of \( r \). \( r = l(1+i) \). The project returns a random variable, \( \tilde{\theta} \). If the risky project is taken, then the outcome has either a high or low payoff (\( \theta^L \) or \( \theta^H \)) with equal probability. If the safe project is taken, then the outcome has a payoff of \( \theta^S \) with probability of 1.

\[
\theta^S = \frac{\theta^L + \theta^H}{2}.
\]

After the payoffs are realized, the entrepreneur has the choice of repaying or not repaying the loan. Partial repayment is not a possibility. If she repays the loan, she has a net payoff of \( \tilde{\theta} - r \).

The assumption of either full repayment or total default is made by Besley and Coate (1995). It is a realistic assumption because these are very small loans, where the amount to be voluntarily paid can be thought of as discrete. Alternatively, the consequence to defaulting can be thought of as discrete, where any kind of default disqualifies the borrower from borrowing in the future. Therefore, she would not have any motivation to make a partial repayment. Also, the model can be adapted to there being partial repayment in that the borrower faces a potential penalty for strategically defaulting. The penalty may include collection of a portion of her project's payoffs when she defaults.

If she does not repay the loan, then she incurs costs of various forms. First, the lender may penalize her for defaulting through refusing to make future loans or sharing this information with other potential lenders (a credit bureau). Second, the lender may retrieve a portion of what is owed by litigation. Third, the borrower may face loss of reputation in her community. Fourth, the borrower
may expend resources to hide her profits from the lender and her community. Fifth, the borrower may inflict guilt on herself for not repaying the loan when she has the means to repay it.

Because I am considering lending in a developing country where institutions that allow for contract enforcement are lacking, the bank is limited in how much it can penalize the borrower. The costs of default may differ from those in a developed country. First, the loan officer may not be able to credibly commit to refusing to make another loan and there may not be a credit bureau if there are other lenders in the market. Second, bringing the case to court may be too costly to the lender for the amount that would be recovered, and the laws governing collection of debts in developing countries often favor the debtors over the creditors. Third, though the borrower is individually liable, she may be concerned with what her community thinks of her if she does not repay a debt. Because the borrowers under consideration typically live in tight knit communities, the knowledge and opinions of others concerning one’s own affairs could be quite significant. On the other hand, her community may have greater solidarity with her rather than the bank, thereby causing it to be understanding of a defaulting borrower’s non-repayment. Fourth, the borrower may very easily hide the amount of her project’s payoffs from observation if she lives in a remote village. Fifth, whether borrowers’ average conscience in developing countries differs from other borrowers is unknown.

Assume that the penalty for default is increasing in her project’s payoffs. Assume there is a fixed cost and a variable cost that is increasing in project payoffs. For simplicity, I assume that penalty function is an affine function in project payoffs. Let \( \alpha_f \) represent the fixed cost and \( \alpha \in (0,1) \) represent the variable cost per unit of project payoff. The penalty function is defined as \( p(\tilde{\theta}) = \alpha_f + \alpha \tilde{\theta} \). Assume for now that \( \alpha = 0 \) so that all of the penalty is linear in project payoffs. Making this assumption will not change the analysis as long as \( \alpha_f \) is not greater than \( r \). I assume that penalties increase in project payoffs for the following reasons. First, limitation on obtaining future financing limits the ability of the successful but defaulting entrepreneur from being able to fully utilize her present project’s payoffs. That is, a portion of the value of project payoffs may be in the ability to invest them in future projects. If additional external financing is also necessary for this future investment and there are increasing returns to scale, then the value of these payoffs will be less. Second, the greater the payoff, the greater is the amount that may be retrieved through litigation. Third, the community would probably think worse of a defaulting borrower the more able she is of repayment. Fourth, the more project payoffs there are, the more there is to hide; the more there is to hide, the more costly it is to hide. Fifth, the defaulting borrower would probably think worse of herself the more she is able to repay.
The entrepreneur will repay only when the payoff from doing so is greater than not doing so, which is when $\tilde{\theta} \geq r/\alpha$. See Proof 1.

Therefore, the individual’s payoff function is $P^*(\tilde{\theta}) = \max(\tilde{\theta} - r, (1 - \alpha)\tilde{\theta})$ from implementing the optimal strategy. Figure 1 graphs the individual lending optimal strategy payoff function.

![Figure 1](image)

**Figure 1.** Individual borrower’s net payoff as a function of her project’s payoffs ($\tilde{\theta}$), assuming penalty parameter ($\alpha$) and amount owed ($r$).

Note that this payoff function is weakly convex in $\tilde{\theta}$:

**Proposition 1:** An individual liability borrower’s optimal repayment strategy ($s^*$) implies the utility ($U$) of the individual liability loan to be convex in her project’s payoffs ($\tilde{\theta}$). See Appendix for proof.

As can be seen in Figure 1, if the entrepreneur were to select between the safe and risky projects, then she would certainly choose the risky project if $\theta^H > r/\alpha > \theta^L$.

**Proposition 2:** An individual liability borrower’s optimal investment decision ($p^*$) is to take the risky project (R) rather than the safe project (S). See Appendix for proof.
However, if $\theta S \geq r/\alpha$, then the lender would prefer that the safe project be chosen, in which case its expected repayment would be $r$, rather than the risky project, whose expected repayment is $r/2$:

**Proposition 3:** The expected repayment rate of an individual liability loan is 50 percent. See Appendix for proof.

Typically, in developed countries' markets for loans where there are significant information asymmetries, credit rationing is understood to occur due to the moral hazard at the investment choice stage induced by setting interest rates high enough to compensate for the prior riskiness of the borrower (Stiglitz and Weis 1981). Because this agent’s action in this moral hazard problem occurs before the project payoffs are realized, this type of moral hazard is referred to as *ex ante* moral hazard. This is demonstrated in Proof 2, where the borrower would choose to invest in the risky project that has only a 50 percent chance of her repaying the loan instead of the safe project, which the bank would prefer her to take. The bank is limited in raising $r$ to compensate for this problem because the critical point, $r/\alpha$, for the borrower to choose to repay the loan increases in $r$. However, if $\alpha$ is higher, then the expectation of repayment is higher because the critical point for the repayment decision, $r/\alpha$, is lower and the fraction of payoffs recoverable in default is higher. Therefore, the bank will limit its losses through controlling the quantity of credit supplied rather than in price.

In undeveloped economies with weak institutions, another type of moral hazard is introduced: whether or not to default when the borrower is in fact capable of repaying the loan (Besley and Coate 1995). This other type of moral hazard is referred to as either *ex post* moral hazard, strategic default, or unenforceability. If $\alpha$ is higher, then the expectation of repayment is higher because the critical point for the repayment decision, $r/\alpha$, is lower and the fraction of payoffs recoverable in default is higher. This can be seen in the proof for Proposition 1.

**IV. Joint Liability Lending**

Next, consider an entrepreneur’s payoff function when the loan is made to a group. I model group lending in a two-player (like Besley and Coate), three-period ($t = \{0,1,2\}$) game in which both borrowers choose whether or not to contribute to the repayment of their loan. The two borrowers are identified as “Borrower $j$” where $j = 1$ or 2. In the first period, $t = 0$, the two-member group forms and receives a loan. The loan agreement stipulates that the group must pay a total of $2r$ in principal and interest in the last period, $t = 2$. (Under individual lending, each borrower would have to pay $r$.) At $t = 1$, each borrower invests her share of the loan in a project that yields a random variable, $\tilde{\theta}_j$. She chooses between two investments, safe or risky.
At $t = 2$, each group member decides whether or not to contribute to the repayment of the loan. Following the assumption of Besley and Coate (1995), the group as a whole can either default on the entire loan or repay the entire amount. If each member does contribute, then borrower $j$ has a payoff of $\tilde{\theta}_j - r$. If one contributes nothing, then the other borrower can either decide to also contribute nothing, thereby allowing the group to default, or to cover both members' share and repay the entire loan. If the group defaults, then, the borrowers are penalized by the same amount as under individual liability ($p(\tilde{\theta}_j)$). The penalties are increasing in $\tilde{\theta}_j$. Therefore, if the group defaults, then each member gets a payoff of $\tilde{\theta}_j (1 - \alpha)$.

IV. A. Joint Liability Lending without Social Sanctions

It has already been recognized that simply making borrowers jointly liable for each other’s loan does not have strictly positive net effects on borrower repayment (Besley and Coate 1995). Furthermore, it has been shown empirically in the Philippines that joint liability is not the only method by which social capital can impact the probability of repayment by groups (Karlan, et. al.). Therefore, the first kind of group lending that I consider is one where the group members are jointly liable for each other but cannot impose any kind of penalty for non-contribution to the repayment of the loan. I call this type of harnessing of social capital as joint liability lending without social sanctions. By making this distinction, I can separate out the effects of joint liability itself from other factors on the enforcement mechanism.

**Optimal Strategies at $t=2$**

The two group members have to choose a strategy of either contribute (C) or not contribute (NC). Let $s_j \in \{C, NC\}$ denote the strategy played by borrower $j$. Each borrower’s net payoff depends on the strategy played by the peer. Denote borrower 1’s net payoff as $U^{11}(s_1, \tilde{\theta}) \equiv U^{11}[(s_1, s_2), (\tilde{\theta}, \tilde{\theta})]$. Their payoffs from each possible strategy are denoted in Figure 2.

<table>
<thead>
<tr>
<th>Borrower 1 Strategy ($s_1$)</th>
<th>Borrower 2 Strategy ($s_2$)</th>
<th>C</th>
<th>NC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>$\tilde{\theta}_1 - r, \tilde{\theta}_2 - r$</td>
<td>$\tilde{\theta}_1 - 2r, \tilde{\theta}_2$</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td>$\tilde{\theta}_1, \tilde{\theta}_2 - 2r$</td>
<td>$\tilde{\theta}_1(1 - \alpha), \tilde{\theta}_2(1 - \alpha)$</td>
</tr>
</tbody>
</table>
Figure 2. Joint liability without social sanctions net payoffs 

\[ U^{J1}(s, \tilde{\theta}), U^{J2}(s, \tilde{\theta}) \] 
under the four possible strategy combinations of both borrowers in a group.

Project returns, \( \tilde{\theta}_1 \) and \( \tilde{\theta}_2 \), determine the payoffs of each combination of strategies. Therefore, the Nash equilibria vary by the realizations of \( \tilde{\theta}_1 \) and \( \tilde{\theta}_2 \). The optimal strategies, then, proven in Lemmas 1.1 – 1.4 in the Appendix. Figure 3 presents these optimal strategies in a table.

<table>
<thead>
<tr>
<th>( \tilde{\theta}_1 )</th>
<th>( \tilde{\theta}_2 &lt; r/\alpha )</th>
<th>( r/\alpha \leq \tilde{\theta}_2 &lt; 2r/\alpha )</th>
<th>( \tilde{\theta}_2 \geq 2r/\alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{\theta}_1 &lt; r/\alpha )</td>
<td>(NC, NC)</td>
<td>(NC, NC)</td>
<td>(NC, C)</td>
</tr>
<tr>
<td>( r/\alpha \leq \tilde{\theta}_1 &lt; 2r/\alpha )</td>
<td>(NC, NC)</td>
<td>(C, C)</td>
<td>(NC, C)</td>
</tr>
<tr>
<td>( \tilde{\theta}_1 \geq 2r/\alpha )</td>
<td>(C, NC)</td>
<td>(C, NC)</td>
<td>(C, NC), (NC, C)</td>
</tr>
</tbody>
</table>

Figure 3. Nash equilibrium strategies (s1*, s2*) for various project under joint liability without social sanctions.

The impact of joint liability without any social sanctions on bank repayment appears ambiguous. On one hand, there is a positive effect of joint liability on borrower repayment. Suppose the bank makes two loans on an individual liability basis to the two members in this group. If one’s project payoffs were less than \( r/\alpha \) but the other’s were greater than \( 2r/\alpha \), then only one loan would have been repaid. However, under joint liability, the borrower with the project with the higher payoffs will pay for her peer’s loan. The bank then has both loan repaid under joint liability. This positive effect on the probability of repayment of loans is the risk-sharing effect.

Consider now a different scenario: Borrower 1 has payoffs greater than \( r/\alpha \) but less than \( 2r/\alpha \), and Borrower 2 has payoffs less than \( r/\alpha \). Under individual liability one loan is repaid. Under joint liability, neither loan is repaid because Borrower 1 will not cover for Borrower 2. This negative effect on the probability of repayment of loans is the free-riding effect.

A borrower’s expected payoff function under optimal strategies, then is more complicated than under individual liability because her net payoffs and optimal strategies depend on the payoffs of the other borrower’s project and her strategy. The dominance of the risk sharing effect or the free riding effect
depends on the project payoff. If her peer contributes to the repayment of the loan, Borrower 1, in contrast to the individual liability case, keeps more of her project’s payoffs in low payoff states ($\tilde{\theta}_i < r/\alpha$) due to the risk-sharing effect, has the same net payoff in intermediate payoffs ($r/\alpha \leq \tilde{\theta}_i < 2r/\alpha$), and keeps all of her project’s payoffs in high payoffs ($\tilde{\theta}_i \geq 2r/\alpha$) due to the free-riding effect.

However, if her peer does not contribute to the repayment of the loan, Borrower 1 keeps the same amount of the loan for low payoffs ($\theta_1 < r/\alpha$), has lower net payoffs under intermediate payoffs ($r/\alpha \leq \tilde{\theta}_i < 2r/\alpha$) due to the coordination failure induced by free-riding, and also has lower net payoffs under high payoffs states due to being exploited by free-riding. These payoffs are represented in Figure 4.

![Graph](image)

**Figure 4.** Optimal strategy net payoff functions of Borrower 1. The solid lines represent $P^*(\theta_1 | C)$, the optimal strategy net payoff if Borrower 2 contributes to repayment. The dashed line represents $P^*(\theta_1 | NC)$, the optimal strategy net payoff if Borrower 2 does not contribute to repayment.

The uncertainty surrounding what a borrower’s payoffs does not only stem from the payoffs of her peer’s project, but also on which equilibrium strategy they play. When both projects have high levels of payoffs, they play either $(C, NO)$ or $(NC, C)$. This surprising result that one borrower allows the other to free-ride on her comes from neither borrower credibly being able to commit to the group to default. This leaves a question of which equilibrium strategy will be played when project payoffs are in these ranges. If the game were moved from static to dynamic or a convention were applied to
it, then the Pareto improving strategy may be played more often. For the purposes of a borrower’s prior beliefs about which equilibrium strategy will be played, I will assume that the probability of 0.5 for each strategy when there are two equilibrium strategies.3

**Optimal Strategies at t=1**

Each borrower will use her expectations of her own project’s payoffs and her peers at t=1 when she chooses what kind of project in which to invest. Assume that there are two projects to choose from, as under the individual liability case. For simplicity, add the assumption that the safe investment’s payoffs are strictly less than $2r/\alpha$. Does Borrower 1 choose the risky or the safe investment if Borrower 2 invests in the safe investment? What does Borrower 1 choose if Borrower 2 invests in the risky investment.

To address the first question, assume that Borrower 2 invests in the safe investment. This means that Borrower 2’s project yields $\theta^S$, where $2r/\alpha > \theta^S \geq r/\alpha$. If Borrower 1 invests in the safe project, then her project’s payoff is in the same range, and they would play (C, C). Borrower 1’s expected net payoff, then, is $\theta^S - r$. If Borrower 1 were to invest in the risky project, then the group will default if her project pays $\theta^L$, or it will repay the loan with her paying all of it if her project pays $\theta^H$. Her expected payoff is $\theta^S \cdot a\theta/2 - r$, which is less than $\theta^S - r$, the net payoff from taking the safe project. She, therefore, prefers to take the safe project:

**Proposition 4:** Under a joint liability contract, if one borrower invests in the safe project, then the other borrower will also invest in the safe project $(S)$ rather than the risky project $(R)$. See Appendix for the proof.

Next, consider what Borrower 1’s net payoffs would be under the two investment options if Borrower 2 takes the risky project. There are now five possible net payoffs if she takes the risky project shown in Figure 5.

---

3 One possible rule of the game is for each borrower to flip a coin when they come together at $t=2$ to determine who plays her strategy first. If both have payoffs greater than $2r/\alpha$, then the loser of the coin flip goes second and therefore covers the entire loan.
The expected payoff for Borrower 1 in taking the risky project when Borrower 2 does the same is \( \theta^S \cdot \alpha \frac{\theta^L}{4} - \frac{3r}{4} \). If Borrower 1 takes the safe project when Borrower 2 takes the risky, then Borrower 1 has possible payoffs shown in Figure 6 instead. Her expected net payoff from taking the risky project would be \( \theta^S \cdot \alpha \frac{\theta^L}{2} \).

**Figure 5.** Optimal strategy net payoffs to Borrower 1 when both borrowers take the risky project.

The expected payoff for Borrower 1 in taking the risky project when Borrower 2 does the same is \( \theta^S \cdot \alpha \frac{\theta^L}{4} - \frac{3r}{4} \). If Borrower 1 takes the safe project when Borrower 2 takes the risky, then Borrower 1 has possible payoffs shown in Figure 6 instead. Her expected net payoff from taking the risky project would be \( \theta^S \cdot \alpha \frac{\theta^L}{2} \).

**Figure 6.** Optimal strategy net payoffs to Borrower 1 when Borrower 2 takes the risky project and Borrower 1 takes the safe project.
Her payoff from choosing the safe project over the risky project is $\theta^s - \alpha \theta^s / 2 - [\theta^s - \alpha \theta^l / 4 - 3r / 4] = 3r / 4 - \alpha \theta^l / 4$. Comparison of the expected utilities yields the following result:

**Proposition 5:** Under a joint liability contract, if one borrower invests in the risky project, then the other borrower will invest in the risky project only if her projects’ high state payoff is greater than $\frac{3r}{\alpha}$. Otherwise, she will invest in the safe project. See Appendix for the proof.

Each borrower, therefore, knows that if she takes the safe project, then the other borrower will take the safe project, also. If she takes the risky project, then the other borrower will take the safe project only if $\theta^H \leq 3r / \alpha$ and will take the risky project only if $\theta^H > 3r / \alpha$. The borrowers investing in different projects, however, is not an equilibrium strategy because if Borrower 2 first chooses a risky investment and Borrower 1 responds by choosing a safe investment because $\theta^H \leq 3r / \alpha$, then Borrower 2 would reverse her investment decision to the safe investment as shown in Proof 4. Therefore, if $\theta^H \leq 3r / \alpha$, then both borrowers choose to make the safe investments; and if $\theta^H > 3r / \alpha$, then both borrowers choose to make the risky investments. In other words, if the mean payoffs of the investments are higher, then the borrowers will choose risky strategies.

**Optimal strategies at t=0**

If the payoffs of the possible projects differ between the group members, then borrowers with $\theta^H \leq 3r / \alpha$, who always prefer the safe project select other borrowers who would prefer the safe project, too. Likewise, the borrowers with $\theta^H > 3r / \alpha$ select other borrowers with the same possible high payoff state. Therefore, when there are no social sanctions on noncontributing group members, there is assortative matching of borrowers, consistent with Ghatak and Guinnane (1999) and Ghatak (1999):

**Corollary 1:** Borrowers with high possible project payoffs ($\theta^H > \frac{3r}{\alpha}$) select each other to take joint liability loans and invest in risky projects. Other borrowers ($\theta^H < \frac{3r}{\alpha}$) select each other and invest in safe projects. See Appendix for the proof.

Optimal group project selection under joint liability without social sanctions is peer dependent. That is, even if the project opportunity sets between the two borrowers differ, each borrower’s strategy is dependent on whether at least one borrower always prefers to take the safe investment.

Does the lender choose to make this joint liability loan rather than two individual liability loans? If $\theta^H \leq 3r / \alpha$, then both borrowers play “safe”, which implies a repayment probability of 1. If $\theta^H > 3r / \alpha$,
then both borrowers play risky, which implies a repayment rate of \((2r \times 0.75 + 0 \times 0.25)/(2r) = 0.75\). These repayment rates are improvements over the repayment rate of making two individually liable loans: \((2r \times 0.25 + r \times 0.5 + 0 \times 0.25)/2r = 0.5\) because the individually liable borrower will always choose the risky investment. Since the lender has a higher probability of being repaid under joint liability, borrowers benefit from reduced credit rationing and reduced interest rates. This is shown formally in the proof for the following proposition in the Appendix:

**Proposition 6:** The expected repayment rate of a joint liability loan is between 75 and 100 percent.

Presuming the same availability and terms of credit, does the borrower choose to accept a joint liability loan over an individual liability loan? If \(\theta_H \leq 3r/\alpha\), then Borrower 1 has an expected payoff of \(\theta_S - r\) if she has a joint liability loan because both take the safe projects. This is less than the expected payoff from taking an individual liability loan. See Proof 2. If \(\theta_H > 3r/\alpha\), then Borrower 1 will have an expected payoff of \(\theta_S - a\theta^2/4 - 3r/4\). This net expected payoff is also less than that under individual liability. The difference in net expected payoffs of the individual liability loan over the joint liability loan is \([\theta_S - a\theta^2/2 - r/2] - [\theta_S - a\theta^2/4 - 3r/4] = (r - a\theta^2)/4 > 0\) since \(\theta_L < r/\alpha\). Therefore, if equal in terms and accessibility, microentrepreneurs would prefer the individual liability loan. See Proposition 12 in the Appendix with proof.

The insight that the joint liability contract without social sanctions induces the borrowers to take safe investments when the payoff possibilities are lower implies that the risk sharing effect does dominate the free riding effect. This result contradicts Besley and Coate (1995), who argue that the ability of borrowers to level punishments on one another is a necessary element to guarantee superior repayment rates for joint liability loans. I find this different result because I relax their assumption of project choice being exogenous.

### IV. B. Joint Lending with Social Sanctions

Now, I allow group members to penalize each other if one does not contribute to the repayment of the loan. Following Besley and Coate (1995), social sanctions are a function of payoffs of the noncontributing borrower, and not on observation of effort or project selection. Social sanctions can be in the form of loss of reputation in the community or inability to participate in future loans. “Social sanctions” can also be seen as the group member’s internal sense of obligation or guilt for not contributing to the repayment of the loan even if the community completely forgives her. With an outside, impersonal institution, she may not have such guilt for defaulting. Social sanctions’ sensitivity to realized returns are assumed to be more punitive than the bank penalty functions; this
belief is the basis for arguments that the cultural realities of these borrowers can induce them to repay their loans better than what a lender outside the community has at its disposal. The structure of the social sanctions function, \( s(\theta_j) \), is similar to that of the bank penalty function: \( s(\tilde{\theta}_j) = s_j + \frac{\alpha}{\beta} \tilde{\theta}_j \), where \( s_j \geq a_j \) and \( a \leq \beta \leq 1 \). \( 1/\beta \) represents the increased degree by which the contributing borrower can penalize the noncontributing borrower over that of the bank. A larger \( \beta \) implies relatively greater leniency by the group.

**Optimal Strategies at \( t=2 \)**

A group member’s payoffs decrease by \( s(\tilde{\theta}_j) \) only when the other group member repays the loan and the member being sanctioned does not contribute. If the loan is not repaid (i.e., neither borrower contributes), then they do not sanction each other, but they are both penalized by the bank. Continue to assume that \( a_f = 0 \), and also assume for now that \( s_f = 0 \). The borrowers’ payoffs to each pair strategies are given in Figure 7.

<table>
<thead>
<tr>
<th>Borrower 1 Strategy (s₁)</th>
<th>C</th>
<th>NC</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>( \tilde{\theta}_1 \cdot r, \tilde{\theta}_2 \cdot r )</td>
<td>( \tilde{\theta}_1 \cdot 2r, \tilde{\theta}_2 (1-\alpha/\beta) )</td>
</tr>
<tr>
<td>NC</td>
<td>( \tilde{\theta}_1 (1-\alpha/\beta), \tilde{\theta}_2 \cdot 2r )</td>
<td>( \tilde{\theta}_1 (1-\alpha), \tilde{\theta}_2 (1-\alpha) )</td>
</tr>
</tbody>
</table>

**Figure 7.** Joint liability with social sanctions net payoffs \( U^{JS1}(s, \tilde{\theta}) \), \( P_x(\tilde{\theta}_i | s_i) \) under the four possible strategy combinations of both borrowers in a group.

Project returns, \( \tilde{\theta}_1 \) and \( \tilde{\theta}_2 \), determine the payoffs of each combination of strategies as in the previous considered case. Therefore, the Nash equilibria vary by the realizations of \( \tilde{\theta}_1 \) and \( \tilde{\theta}_2 \). The optimal strategies are in Lemmas 3.1 – 3.4 in the Appendix with proofs and summarized in a table in Figure 8. The optimal strategies when borrowers can sanction each other are less likely to be dominated by free-riding. When both borrowers have payoffs in excess of \( \beta r/\alpha \), then the entire group repays the loan. Furthermore, since \( \beta \leq 1 \), the lender is more likely to be repaid than in the individual liability case, where the critical point for individual repayment is \( r/\alpha \geq \beta r/\alpha \). However, joint liability with social sanctions still suffers from the problem of a borrower with medium level payoffs not contributing her portion when her peer’s project has very low payoffs.
Figure 8. Nash equilibrium strategies \((s_1^*, s_2^*)\) for various project payoffs under joint liability with social sanctions.

Table:

<table>
<thead>
<tr>
<th>(\theta_1)</th>
<th>(\theta_2 &lt; \beta r / \alpha)</th>
<th>(\beta r / \alpha \leq \theta_2 &lt; 2r / \alpha)</th>
<th>(\theta_2 \geq 2r / \alpha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta_2)</td>
<td>(NC, NC)</td>
<td>(NC, NC)</td>
<td>(NC, C)</td>
</tr>
<tr>
<td>(\theta_1 &lt; \beta r / \alpha)</td>
<td>(NC, NC)</td>
<td>(C, C)</td>
<td>(C, C)</td>
</tr>
<tr>
<td>(\theta_1 \geq 2r / \alpha)</td>
<td>(C, NC)</td>
<td>(C, C)</td>
<td>(C, C)</td>
</tr>
</tbody>
</table>

Figure 9 graphs the payoffs of these optimal strategies. The new payoff functions with social sanctions differ from those without. The joint liability loan without social sanctions does have payoffs that exceed that of the social sanctions case and the individual liability case due to the reduction in the free-riding effect. Borrowers are more likely to both contribute to the repayment of the loan for high realizations of project payoffs. However, there are no net payoffs under joint liability with social sanctions that exceed that of the individual liability case.

Figure 9. Optimal strategy net payoff functions of Borrower 1 under joint liability with social sanctions. The solid line represents \(P^*_1(\tilde{\theta}_1 | C)\), the optimal strategy net payoff if Borrower 2 contributes to repayment. The dashed line represents \(P^*_1(\tilde{\theta}_1 | NC)\), the optimal strategy net payoff if Borrower 2 does not contribute to repayment.
Optimal Strategies at \( t=1 \)

As in the other joint liability case, each borrower will use her expectations of her own project’s payoffs and her peers’ at \( t=1 \) when choosing what kind of project in which to invest. Assume that there are two projects to choose from as before and that the safe investment’s payoffs are strictly less than \( 2r/\alpha \). Does Borrower 1 choose the risky or the safe investment if Borrower 2 invests in the safe investment? What does Borrower 1 choose if Borrower 2 invests in the risky investment?

First, consider what possible net payoffs Borrower 1 faces if Borrower 2 chooses the safe investment. If Borrower 1 chooses the safe investment also, then they would play \((C, C)\), and Borrower 1 has a net payoff of \( \theta^S - r \) for sure. If Borrower 1 chooses the risky investment, then she has a net payoff of either \( \theta^H - r \) or \( \theta^L(1-\alpha/\beta) \), depending on whether \( \theta^L \) is greater than or less than \( \beta r/\alpha \), respectively. If \( \theta^L \geq \beta r/\alpha \), then her expected net payoff is \( (\theta^H - r)/2 + (\theta^L - r)/2 = \theta^S - r \), making her indifferent between taking the safe project and the risky project. In either case, both she and her peer contribute to repayment. If \( \theta^L < \beta r/\alpha \), then her expected net payoff is \( (\theta^H - r)/2 + \theta^L(1-\alpha/\beta)/2 = \theta^S - \frac{\alpha \theta^L}{2\beta} - r/2 \). In this case, she will choose the risky investment:

**Proposition 7:** If borrower 2 invests in the safe project, then borrower 1 will invest in the risky project. See Appendix for the proof.

Next, consider what possible net payoffs Borrower 1 faces if Borrower 2 chooses the risky investment. If Borrower 1 also chooses the risky investment, she faces four possible net payoffs. Figure 10 shows the net payoffs for that case. In this case, there are possibilities that Borrower 1 will either depend on her peer to cover her portion of the loan or she will cover her peer’s portion of the loan. The expected net payoff from taking the risky investment when Borrower 2 also takes the risky investment is \( EU_{SR}^{RS} = \theta^S - \frac{\alpha(1-\frac{1}{\beta})\theta^L + 3r}{4} \). If Borrower 1 takes the safe project, however, then she has possible net payoffs shown in Figure 11. The expected net payoff from taking the safe investment when Borrower 2 takes the risky investment is \( EU_{SR}^{JS} = \theta^S - \frac{\alpha \theta^S + r}{2} \).
The difference in taking the risky project over taking the safe project when Borrower 2 takes the risky project is

\[ EU_{RR}^{JS1} - EU_{SR}^{JS1} = \frac{2\alpha \theta_1^s - (\beta + 2)r}{4} \]

which is greater than zero only if

\[ \theta_1^s \geq \frac{(\beta + 2)r}{2\alpha} \]

Therefore, Borrower 1 chooses the risky project when Borrower 2 chooses a risky project if \( \theta_1^s \geq \frac{(\beta + 2)r}{2\alpha} \) and the safe project otherwise:

**Proposition 8:** If \( \theta_1^L < \frac{\beta r}{\alpha} \) and borrower 2 invests in the risky project, then borrower 1 will take the risky project only if \( \theta_1^s \geq \frac{(2 + \beta)r}{2\alpha} \). Otherwise, borrower 1 will take the safe project. See Appendix for the proof.

| Project 1 Payoff | NE Strategies | \( P_1^*(\theta_1|s_2) \) | Probability |
|------------------|---------------|--------------------------|-------------|
|                  | \( \theta_1^H, \theta_2^H \) | (C, C) | \( \theta_1^H - r \) | 0.25 |
|                  | \( \theta_1^H, \theta_2^L \) | (C, NC) | \( \theta_1^H - 2r \) | 0.25 |
|                  | \( \theta_1^L, \theta_2^H \) | (NC, C) | \( \theta_1^L(1 - \alpha / \beta) \) | 0.25 |
|                  | \( \theta_1^L, \theta_2^L \) | (NC, NC) | \( \theta_1^L (1 - \alpha) \) | 0.25 |

**Figure 10.** Optimal strategy net payoffs to Borrower 1 when both borrowers take the risky project, there are social sanctions, and \( \theta_1^L \leq \beta r / \alpha \).
Optimal Strategies at $t=0$

The following lemmas identify which peers each type of borrower prefers according to her expected project payoff:

**Lemma 4.1:** For the borrower who will always choose the risky project, i.e. those who have project expected payoffs greater than $\frac{(\beta + 2)r}{2\alpha}$, it is preferable for her to find a peer who will choose the safe project. See Appendix for the proof.

**Lemma 4.2:** For the borrower who will invest in the safe project when her peer invests in the risky project, it is preferable for her to find a peer with low expected payoffs, i.e. between $\frac{r}{\alpha}$ and $\frac{(\beta + 2)r}{2\alpha}$. See Appendix for the proof.

By consequence of these preferences, borrowers with similar project expected payoffs will match together, as stated in the following proposition:

**Proposition 9:** Borrowers will match with other borrowers with the same expected project payoffs where there is a possibility of social sanctions. See Appendix for the proof.
As a consequence, there are is separation in investment strategies according to the groups’ projects’ expected payoffs:

**Proposition 10:** The only investment strategies that will be played are $P_1=P_2=R$ for groups with high expected project payoffs and $P_1 \neq P_2$ for groups with low expected project payoffs. See Appendix for the proof.

Would we see groups who play opposite investment strategies? Guttman (2006, 2007) show that if group members can select each other and they can make side payments to each other, then the risky borrowers will match with safe borrowers. My model is consistent with this finding because groups of borrowers with low expected payoffs play opposite investment strategies. However, my model does not identify which borrower would invest in which type of project, though the possibility of side payments could resolve such a question as long as each borrower has an equal probability of offering the side payment and receiving it.

In the above scenario, the repayment rate is 0.5:

**Proposition 11:** The expected repayment rate of a joint liability loan with the possibility of social sanctions is between 50 percent and 75 percent. See Appendix for the proof.

Therefore, the repayment rate of joint liability contracts with social sanctions is greater than that of the individual liability contract. However, if all borrowers have average project prospects that are sufficiently low, then the repayment rate is no greater than the individual liability’s. See the proof for Proposition 11. Therefore, we see a case where a joint liability contract might be inferior to an individual liability contract because both the lender will not be any better off, and the borrower will be worse off.

Note that the conditions for the mixed investment strategy of borrowers, for whom $\theta^S \leq (2 + \beta)r(2\alpha)$, implies that the more stringent the social sanctions (lower $\beta$), the less likely it is that the mixed investment strategy will occur because the minimum threshold for taking the (Risky, Risky) strategy is lowered. Therefore, though joint liability with social sanctions leads to lower repayment rates than a joint liability without them, stronger social sanctions do make it less likely that the strategies of (Risky, Safe) will be played.
V. Welfare Comparisons and General Equilibrium Extension

The ultimate question for microfinance institutions, governments, and NGOs is what type of contract maximizes welfare? The microfinance contract that maximizes societal welfare is one that increases the expected net payoff to one borrower without decreasing the expected repayment to the lender. Recent experience with individual liability contracts supports my contention below that the individual liability contract is often superior to the joint liability contract. It has been found in practice as well as in controlled experiments that MFIs that switched from joint liability to individual liability loans did not experience a decrease in loan repayments.

Social capital, therefore, has a "dark side" of inhibiting some borrowers from taking more risk, which does not improve repayment rates because of the enforceability problem that exists in the countries these loans are made in. These results provide some counter predictions to the usual beliefs that group lending programs improve repayment rates because the group is able to penalize its members by taking away some of their social capital. Furthermore, these results give a theoretical explanation for anecdotal evidence of the negative welfare effects of microfinance programs on very poor borrowers (Khawari 2004, Hulme and Mosley 1996, and Pretes 2002). These borrowers in particular seem to pick relatively overly safe projects. The empirical evidence shows it is the very poor for whom microfinance does not work as well as intended. The results presented here show group lending produces the same choice by borrowers facing project opportunity sets with a low expected outcome. If borrowers' incomes are positively correlated with the project opportunity sets available to them, then these results may explain the disparate impact of microfinance programs. Interestingly, social sanctions may work too well by making borrowers too scared to take on an optimal amount of risk.

V.A. Only One Contract Is Offered

Inspection of Figures 1, 4, and 9 reveals that for the same loan availability and terms, borrowers' expected net payoffs are higher under individual liability contracts for all projects than under either joint liability cases.

**Proposition 12:** In terms of borrower expected utility, the individual liability contract weakly dominates the joint liability contracts assuming the same

---

4An additional criterion for maximum societal welfare is the degree that business activity has a positive externality. If the microentrepreneurs invest loans by purchasing capital and hiring labor within their community and the risky projects require greater investment than the safe project, then this externality effect would also suggest that more risk taking by, *Ceteris Paribus*, would increase welfare. I show later that if such an externality exists, then my conclusions are even stronger.
principal and interest across contracts (i.e., same \( r \)) and only one contract is offered. See Appendix for the proof.

However, I also show that the repayment rate under joint liability without social sanctions is higher by 25 to 50 percent than the repayment rate under individual liability. This increase in repayment probability is due to the risk sharing benefit when risky projects are taken or to the incentive to take safe projects when the payoffs to the risky investment are not very high. Because the lender can expect to be repaid with a higher probability, it will make loans available to more borrowers by rationing credit less and/or reducing the interest rate.

If borrowers are able to sanction each other, however, there actually may be no effect on the repayment rate over the individual liability contract if the sanctions are not sufficiently strong and average payoffs are not sufficiently high. This surprising conclusion is due to the lack of risk-sharing benefit by groups forming whereby only one borrower takes the risky project. This is worse than if the borrowers both chose to take the risky project because if only one borrower takes the risky project and the project pays \( \theta L \), then there will be no chance of the borrower taking the safe project being able to cover for her. The costs of being punished by one's peer are what drive this result.

The question has been raised in microfinance circles of whether the joint liability feature of group lending is what harnesses the social capital that leads to high repayment rates. I also demonstrate how joint liability contracts can induce greater repayment without peers being able to sanction each other. The mechanism by which joint liability works to increase loan repayment is not through inducing peers to punish each other, but rather through borrowers sharing risk or through cooperatively avoiding risk. If peers can punish each other, however, joint liability does not necessarily work better than individual liability loans.

The dominance of the individual liability contract with the constraint that the lender can only offer one contract is demonstrated in the proof of the following proposition, which is in the Appendix:

**V.B. A Choice of Contract Is Offered**

The microfinance industry has developed to the point that microborrowers have options of taking an individual liability or joint liability contract. In this section, I allow the amount due, \( r \), to vary across contract types, which is dependent on the types of borrowers who may separate into either type of contract. It also influences which borrower will take a particular contract type.

First, I compare the individual liability loan to the joint liability loan without the possibility of social
sanctions. The following proposition is obtained:

**Proposition 14:** If given the choice between an individual liability and joint liability contract with the possibility of social sanctions, borrowers with \( \theta_i^{II} < \frac{3r}{\alpha} \) choose the individual liability contract and those with \( \theta_i^{II} \geq \frac{3r}{\alpha} \) choose the joint liability contract.

The total principal and interest due on the individual liability contract is 150% of that of the joint liability contract. See Appendix for the proof.

Proposition 14 implies that borrowers with better prospects \( \theta_i^{II} \geq \frac{3r}{\alpha} \) prefer the joint liability contract because the interest rate is lower because the lender knows that peers share risk with them. Borrowers with lesser prospects \( \theta_i^{II} < \frac{3r}{\alpha} \) prefer the individual liability contract because the cost imposed by the lender is less than the cost of taking the safe project.

By offering the individual liability contract in addition to the joint liability contract, the borrowers who otherwise would have invested in the safe projects under joint liability now invest in the individual liability contract. Therefore, everyone invests in the risky project.

Next, I compare the individual liability loan to the joint liability loan where there is a possibility of social sanctions. The following lemma and proposition are obtained:

**Lemma 5:** If given the choice between an individual and joint liability contract where there will be social sanctions, borrowers for whom \( P^*=(R,R) \) will choose the individual liability contract. See Appendix for the proof.

**Proposition 15:** If given the choice between an individual liability and joint liability loan where there is a possibility of social sanctions, no one will take the joint liability contract. See Appendix for the proof.

Therefore, the dominance of the individual liability contract is maintained where there would be social sanctions under a joint liability contract under the general equilibrium assumptions and assumption A8 that expected project payoffs are always greater than \( r/\alpha \). If this assumption is relaxed, however, then the lower interest rate induced by the possibility of higher repayment under
joint liability would cause more people to take these loans who otherwise would not borrow at all.

VI. Empirical Hypotheses
The conclusions of this study imply some empirical hypotheses using data on the MFI level. Using this level of data is newer in microfinance research because these data have only become available within the last decade. One seminal paper is Cull, Demirguc-Kunt, and Morduch (2009), which studies the financial and customer demographic ratios of MFIs as reported by www.mixmarket.org. They study such key questions in microfinance as what is the potential tradeoff between MFI sustainability and reach to the poorest borrowers. Cull, et. al. (2009) find that contract design substantially impacts MFI profitability, loan repayment, and costs. They find that MFIs that make individual liability loans experience increases in portfolio quality and profitability when they raise interest rates. Whereas they compare contract types of individual versus group based lending, the theory presented here predicts different results among group based lenders based on the social ties of their customers. The difference arises from varying strengths and types of informal institutions that impact how group members respond to non-paying peers.

The data used by Cull, et. al. (2009) could be augmented with measures of formal and informal institutions within in each of the MFIs' markets. If these measures can be obtained and MFIs are identified by their mix of making individual and joint liability contracts, then repayment rates across the three kinds of lending discussed here could be compared.

This study also has implications for testing using micro level data, which is what the majority of studies have used. If a measure of the project opportunities available to microborrowers can be collected, then there are several more testable hypotheses. Microfinance borrowers with higher (lower) possible project payoffs are expected to be more like likely to find peers with similarly higher (lower) possible project payoffs. Microfinance borrowers with higher and more varied possible project payoffs are expected to be more likely to match with peers with lower and less varied possible project payoffs. Joint liability borrowers are expected to switch to riskier projects if they switch to receiving individual liability loans, especially if they have project opportunities with low upside payoffs.

The empirical testing of this model entails data collection challenges. First, measures of forbearance by culture would require conducting surveys among every identifiable culture served by the MFIs being studied. Second, measuring project possibilities directly also would require reliance on surveying microborrowers.

VII. Conclusion
Microfinance has popularly been touted as a program that succeeds in improving its borrowers’
incomes by overcoming the moral hazard problem inherent in individual liability loans in countries with poor institutions. This paper contributes to theory as to why microfinance does not always work as intended. Furthermore, it shows that both formal and informal institutions matter, which could lead to certain policy prescriptions according to a country’s institutions.

Given that the whole purpose of microfinance is to promote entrepreneurship, which is inherently a risky project, and if the borrower’s prospects are low, then she will not use the funds to make a business grow. Rather, she will use it for some other purpose, such as income smoothing. She may even forgo her entrepreneurial pursuits in order to be certain to pay back the loan: There have been stories of borrowers taking on jobs in cities just to repay their portions of loans rather than working on their own businesses. If the real need of these individuals is insurance, then they may benefit more by microinsurance programs instead of microlending ones.

On the one hand, the presence of social sanctions in joint liability contracts may inhibit entrepreneurial activity among people who would otherwise take business loans as individuals. On the other hand, joint liability contracts can be made to groups with social capital but low payoff project opportunities would otherwise not borrow money at all.

There are several testable empirical hypotheses implied by this model that may be tested. First, MFIs that make individual liability loans should have lower repayment rates than those that make joint liability loans if the group members are unlikely to punish one another for non-contribution. Second, MFIs that make individual liability loans should have the same repayment rates as those that make joint liability loans where group members are likely to punish one another for non-contribution and project payoffs are sufficiently high. To test these first two predictions, one would regress repayment rate at the MFI level on the composition of individual liability contracts to total contracts, the average measure of social ties among borrowers’ communities, and the average measure of potential project payoffs to the borrowers. Tests using this regression would be most powerful with a sample of MFIs that serve identifiably specific types of borrowers by culture and economic status. Third, microfinance borrowers with higher (lower) possible project payoffs will be more likely to find peers with similarly higher (lower) possible project payoffs. Fourth, microfinance borrowers with higher and more varied possible project payoffs will be more likely to match with peers with lower and less varied possible project payoffs. Fifth, joint liability loan borrowers will be more likely to switch to riskier projects if they switch to receiving individual liability loans. These third, fourth, and fifth predictions would be tested using individual borrower data, which would depend more heavily on specialized surveys than the methodology for testing the first two predictions.
Appendix

Assumptions:

1. Ordering of Payoff Possibilities for Borrower $i$:
   \[ 0 \leq \theta_i^L < \theta_i^S < \theta_i^H \quad (A1) \]

2. Payoff Probabilities Conditional on Investing in the Risky Project:
   \[ pr(\tilde{\theta}_i = \theta_i^L | p_i = R) = pr(\tilde{\theta}_i = \theta_i^C | p_i = R) = \frac{1}{2} \quad (A2) \]

3. Payoff Probability Conditional on Investing in the Safe Project:
   \[ pr(\tilde{\theta}_i = \theta_i^S | p_i = S) = 1 \quad (A3) \]

4. Equivalence of Expected Payoffs of Both Projects:
   \[ E(\tilde{\theta}_i | p_i = R) = \frac{\theta_i^L + \theta_i^H}{2} = \theta_i^S \quad (A4) \]

5. Bank Penalty Parameter Bounds:
   \[ 0 < \alpha \leq 1 \quad (A5) \]

6. Social Sanctions Parameter Bounds:
   \[ \alpha \leq \beta \leq 1 \quad (A6) \]

7. Low State Payoff Bounds:
   \[ 0 \leq \theta_i^L < \frac{\beta r}{\alpha} < \frac{r}{\alpha} \quad (A7) \]

8. Expected Payoff Bounds for Both Projects:
   \[ \frac{r}{\alpha} < \theta_i^S < \frac{2r}{\alpha} \quad (A8) \]

9. High State Payoff Bounds:
Proposition 1: An individual liability borrower’s optimal repayment strategy (s*) implies the utility (U) of the individual liability loan to be convex in her project’s payoffs (\(\tilde{\theta}\)).

Proof:

\[ U_I(s(\tilde{\theta}), \tilde{\theta}) \] is the utility of individual liability borrower from playing \(s \in \{C, NC\}\) and realizing outcome \(\tilde{\theta}\).

\[ U_I(C, \tilde{\theta}) = \tilde{\theta} \cdot r \] \hspace{1cm} (1)

\[ U_I(NC, \tilde{\theta}) = \tilde{\theta}(1-\alpha) \] \hspace{1cm} (2)

If \(\tilde{\theta} \geq \frac{r}{\alpha}\), then \(U_I(C, \tilde{\theta}) - U_I(NC, \tilde{\theta}) \geq 0\). Therefore, \(s^*(\tilde{\theta} \geq \frac{r}{\alpha}) = C\). \hspace{1cm} (3)

If \(\tilde{\theta} < \frac{r}{\alpha}\), then \(U_I(C, \tilde{\theta}) - U_I(NC, \tilde{\theta}) < 0\). Therefore, \(s^*(\tilde{\theta} < \frac{r}{\alpha}) = NC\). \hspace{1cm} (4)

Therefore, individual liability borrower’s utility under her optimal repayment strategy is as follows (combining (1) – (4)):

\[
U^I[s^*(\tilde{\theta}), \tilde{\theta}] = \begin{cases} 
\tilde{\theta}(1-\alpha) & \text{if } \tilde{\theta} < \frac{r}{\alpha} \\
\tilde{\theta} - r & \text{if } \tilde{\theta} \geq \frac{r}{\alpha}
\end{cases}
\] \hspace{1cm} (5)

Since \(0 < \alpha < 1\), (5) is a convex function.

Proposition 2: An individual liability borrower’s optimal investment decision (p*) is to take the risky project (R) rather than the safe project (S).
Proof:

$E[U_p^i(s^*, \tilde{\theta})]$ is the expected utility of individual liability borrower from take project $p \in (R, S)$ according to playing her optimal repayment strategy $(s^*)$.

The expected utilities from investing in the risky $(R)$ and safe $(S)$ project $(p)$:

If $p = R$:

$$E[U_R^i(s^*, \tilde{\theta})] = \frac{1}{2} U^i(s^*, \theta^L) + \frac{1}{2} U^i(s^*, \theta^H)$$

$$= \frac{1}{2} \theta^L (1 - \alpha) + \frac{1}{2} (\theta^H - r) \quad (6)$$

Recognizing that $\frac{1}{2} \theta^L + \frac{1}{2} \theta^H = \theta^S$ and suppressing the arguments of $U^i_R$, (6) reduces to:

$$EU_R^i = \theta^S - \frac{\alpha \theta^L + r}{2} \quad (7)$$

If $p = S$:

$$E[U_S^i(s^*, \tilde{\theta})] = U^i(s^*, \theta^S) \quad (8)$$

$$EU_S^i = \theta^S - r \quad (9)$$

Subtracting (9) from (7) yields the difference in expected utility between the two investment strategies:

$$EU_R^i - EU_S^i = \frac{r - \alpha \theta^L}{2} \quad (10)$$

Since, by assumption, $\theta^L < \frac{r}{\alpha}$:

$$EU_R^i - EU_S^i > \frac{r - \alpha (\frac{r}{\alpha})}{2} = 0 \quad (11)$$

Therefore, the expected utility to the individual liability borrower of investing in the risky project exceeds that of investing in the safe project, i.e., $p^* = R$. 
**Proposition 3:** The expected repayment rate of an individual liability loan is 50 percent.

**Proof:**

$V^I(s^*, \tilde{\theta})$ is the ex post value to the lender from making an individual liability loan conditional on the project payoffs and the borrower’s optimal repayment strategy.

From Proposition 1:

$$V^I(s^*, \tilde{\theta}) = \begin{cases} 0 & \text{if } \tilde{\theta} < \frac{r}{\alpha} \\ r & \text{if } \tilde{\theta} \geq \frac{r}{\alpha} \end{cases}$$  \hspace{1cm} (i)

$EV^I_P(s^*, \tilde{\theta})$ is the expected value to the lender of making an individual liability loan conditional on the borrower taking investment $P$.

$EV^I_P(s^*, \tilde{\theta}) = EV^I_R(s^*, \tilde{\theta})$ by Proposition 2.

$$EV^I_P(s^*, \tilde{\theta}) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot r = \frac{r}{2}$$  \hspace{1cm} (ii)

Therefore, the expected repayment rate is $\frac{EV^I_P(s^*, \tilde{\theta})}{r} = \frac{1}{2} = 50\%$.

**Lemma 1.1:** The unique Nash Equilibrium strategy $(s^*)$ for both borrowers under a joint liability contract is to not contribute (NC) to the loan repayment when both projects’ payoffs are less than $\frac{2r}{a}$.

**Proof:**
\( U^{J1}(s, \tilde{\theta}) \) is the value to borrower 1 under a joint liability contract from playing \( s = (s_1, s_2) \), where \( s_1 \in (C_1, NC_1) \), \( s_2 \in (C_2, NC_2) \), and \( \tilde{\theta} = (\tilde{\theta}_1, \tilde{\theta}_2) \).

\[
U^{J1}[(NC, NC), \tilde{\theta}] = \bar{\theta}_1 (1 - \alpha) \quad (12)
\]

\[
U^{J1}[(C, NC), \tilde{\theta}] = \bar{\theta}_1 - 2r \quad (13)
\]

Subtracting (13) from (12): \( U^{J1}[(NC, NC), \tilde{\theta}] - U^{J1}[(C, NC), \tilde{\theta}] > 0 \) since \( \bar{\theta}_1 < \frac{2r}{\alpha} \). The same applies to \( U^{J2}(s, \tilde{\theta}) \). Therefore, \( s^* = (NC, NC) \) when \( \bar{\theta}_1 < \frac{2r}{\alpha} \) and \( \bar{\theta}_2 < \frac{2r}{\alpha} \).

**Lemma 1.2:** The unique Nash Equilibrium strategy \( (s^*) \) under joint liability contract is for the borrower with the project payoffs greater than or equal to \( \frac{2r}{\alpha} \) to pay for both loans and the borrower with project payoff less than \( \frac{2r}{\alpha} \) to not contribute.

**Proof:**
Let \( \bar{\theta}_1 \geq \frac{2r}{\alpha} \) and \( \bar{\theta}_2 < \frac{2r}{\alpha} \).

From (12) and (13):
\[
U^{J1}[(C, NC), \tilde{\theta}] - U^{J1}[(NC, NC), \tilde{\theta}] = \alpha \bar{\theta}_1 - 2r \quad (14)
\]

(14) is greater than zero since \( \bar{\theta}_1 \geq \frac{2r}{\alpha} \). Therefore, borrower would not deviate from playing \( C \).

Applying (12) and (13) to borrower 2
\[
U^{J2}[(C, NC), \tilde{\theta}] - U^{J1}[(C, C), \tilde{\theta}] = r \quad (15)
\]

(15) is greater than zero. Therefore, borrower 2 will not deviate from playing \( NC \).
Therefore, \( s^* = (C, NC) \) when \( \tilde{\theta}_1 \geq \frac{2r}{\alpha} \) and \( \tilde{\theta}_2 < \frac{2r}{\alpha} \), and by the same reasoning, \( s^* = (NC, C) \) when

\[
\tilde{\theta}_1 < \frac{2r}{\alpha} \quad \text{and} \quad \tilde{\theta}_2 \geq \frac{2r}{\alpha}.
\]

**Lemma 1.3:** The unique Nash Equilibrium strategy \((s^*)\) under a joint liability contract is for both borrowers to contribute \((C, C)\) to repayment of the loan if both projects’ payoffs are greater than \( \frac{r}{\alpha} \) and less than \( \frac{2r}{\alpha} \).

**Proof:**

Let \( \frac{r}{\alpha} \leq \tilde{\theta}_1 < \frac{2r}{\alpha} \) and \( \frac{r}{\alpha} \leq \tilde{\theta}_2 < \frac{2r}{\alpha} \).

\[
U^{J_1}[(C, C), \tilde{\theta}] = \tilde{\theta}_1 - r \tag{16}
\]

\[
U^{J_1}[(NC, C), \tilde{\theta}] = \tilde{\theta}_1 \tag{17}
\]

Subtracting (17) from (16):

\[
U^{J_1}[(NC, NC), \tilde{\theta}] - U^{J_1}[(C, NC), \tilde{\theta}] = -r \tag{18}
\]

Since (18) is less than zero, borrower 1 would like to deviate from playing \( C \) to \( NC \) if borrower 2 does not respond by deviating. Borrower 1 anticipates how borrower 2 will respond by evaluating the following:

\[
U^{J_2}[(NC, NC), \tilde{\theta}] - U^{J_2}[(NC, C), \tilde{\theta}] = 2r - \alpha \tilde{\theta}_2 \tag{19}
\]

Because \( \frac{r}{\alpha} \leq \tilde{\theta}_2 < \frac{2r}{\alpha} \), the right hand side of (19) is strictly greater than zero: \( r \geq 2r - \alpha \tilde{\theta}_2 > 0 \). Therefore, borrower 2 would respond to borrower 1 deviating by also deviating. Because both borrowers know that deviating from \( C \) to \( NC \) induces the peer to doing likewise, neither borrower
will deviate from playing $C$ if the marginal value to each borrower from the group playing $(C,C)$ over $(NC,NC)$ is positive:

From (12) and (16):

$$U^{J_1}[(C,C),\tilde{\theta}] - U^{J_1}[(NC,NC),\tilde{\theta}] = \alpha \tilde{\theta} - r$$

(20) is weakly greater than zero since \( \frac{r}{\alpha} \leq \tilde{\theta}_1 < \frac{2r}{\alpha} \). Therefore, neither borrower will deviate from $C$, and $s^*=(C,C)$ when \( \frac{r}{\alpha} \leq \tilde{\theta}_1 < \frac{2r}{\alpha} \) and \( \frac{r}{\alpha} \leq \tilde{\theta}_2 < \frac{2r}{\alpha} \).

**Lemma 1.4:** The only Nash Equilibrium strategies ($s^*$) under a joint liability contract is for only one borrower to contribute to repayment of the loan for both $[(C,NC)$ or $(NC,C)]$ if both projects’ payoffs are greater than or equal to $\frac{2r}{\alpha}$.

**Proof:**

From (12) and (13):

$$U^{J_1}[(C,NC),\tilde{\theta}] - U^{J_1}[(NC,NC),\tilde{\theta}] = \alpha \tilde{\theta}_2 - 2r$$

(21) Since \( \tilde{\theta}_1 \geq \frac{2r}{\alpha} \), the right hand side of (21) is greater than or equal to zero. Therefore, borrower 1 will not deviate from playing $C$.

From (13) and (16):

$$U^{J_2}[(C,NC),\tilde{\theta}] - U^{J_2}[(C,C),\tilde{\theta}] = r$$

(22) Since the right hand side of (22) is positive, borrower 2 will not deviate from playing $NC$.

However, the same logic applies to show that it is also a Nash Equilibrium for borrower 1 to not contribute and borrower 2 to contribute. Therefore, $s^*=[(C,NC),(NC,C)]$. 


Proposition 4: Under a joint liability contract, if one borrower invests in the safe project, then the other borrower will also invest in the safe project \((S)\) rather than the risky project \((R)\).

Proof:

\(E[U^{j_1}_{P_1} (\tilde{s}^*, \tilde{\theta})]\) is the expected value to borrower 1 from borrower 1 taking project \(P_1\) and borrower 2 taking project \(P_2\) according to both players’ optimal contribution strategies \((\tilde{s}^*)\) and payoff possibilities \((\tilde{\theta})\).

If \(P_1 = R\) and \(P_2 = S\), then by Lemma 1.1 and Lemma 1.2:

\[
\tilde{s}^* (\tilde{\theta}_1^l, \tilde{\theta}_2^s) = (NC, NC) \tag{23}
\]

\[
\tilde{s}^* (\tilde{\theta}_1^u, \tilde{\theta}_2^s) = (C, NC) \tag{24}
\]

Therefore,

\[
EU^{j_1}_{RS} = \frac{1}{2} U^{j_1} [\tilde{s}^* (\tilde{\theta}_1^l, \tilde{\theta}_2^s)] + \frac{1}{2} U^{j_1} [\tilde{s}^* (\tilde{\theta}_1^u, \tilde{\theta}_2^s)]
\]

\[
= \frac{1}{2} \frac{\theta_1^l (1 - \alpha)}{2} + \frac{1}{2} (\theta_1^u - 2 \theta_2^s)
\]

\[
= \frac{\theta_1^l}{2} - \frac{\alpha \theta_1^l}{2} - \theta_2^s \tag{25}
\]

If \(P_1 = R\) and \(P_2 = S\), then by Lemma 1.3:

\[
\tilde{s}^* (\tilde{\theta}_1^l, \tilde{\theta}_2^s) = (C, C) \tag{26}
\]

Therefore,

\[
EU^{j_1}_{SS} = U^{j_1} [\tilde{s}^* (\tilde{\theta}_1^l, \tilde{\theta}_2^s)] = \theta_1^s - r \tag{27}
\]

To compare the marginal value to borrower 1 from taking the safe project over the risky project when borrower 2 takes the safe project, subtract (25) from (27):
Because the right hand side of (28) is greater than or equal to zero, borrower 1 will invest in the safe project if borrower 2 invests in the safe project. I.e., \( P_1^* = S \) if \( P_2 = S \).

**Proposition 5:** Under a joint liability contract, if one borrower invests in the risky project, then the other borrower will invest in the risky project only if her projects’ high state payoff is greater than \( \frac{3r}{\alpha} \). Otherwise, she will invest in the safe project.

**Proof:**
If \( P_1 = P_2 = R \), by Lemmas 1.1, 1.2, and 1.4:

\[
\begin{align*}
\bar{\xi}^*(\theta_1^L, \theta_2^L) &= (NC, NC) \\
\bar{\xi}^*(\theta_1^L, \theta_2^H) &= (NC, C) \\
\bar{\xi}^*(\theta_1^H, \theta_2^L) &= (C, NC) \\
\bar{\xi}^*(\theta_1^H, \theta_2^H) &= [(C, NC), (NC, C)]
\end{align*}
\]

Therefore, the expected value to borrower 1 from taking the risky project when borrower 2 does likewise is

\[
\begin{align*}
EU_{rr}^{J1} = & \frac{1}{4} U^{J1}[\bar{\xi}^*(\theta_1^L, \theta_2^L)] + \frac{1}{4} U^{J1}[\bar{\xi}^*(\theta_1^L, \theta_2^H)] + \frac{1}{4} U^{J1}[\bar{\xi}^*(\theta_1^H, \theta_2^L)] \\
&+ \frac{1}{4} \left( \frac{1}{2} U^{J1}[(C, NC), (\theta_1^H, \theta_2^H)] + \frac{1}{2} U^{J1}[(NC, C), (\theta_1^H, \theta_2^H)] \right) \\
= & \frac{1}{4} \theta_1^L (1 - \alpha) + \frac{1}{4} \theta_1^L + \frac{1}{4} (\theta_1^H - 2r) + \frac{1}{4} \left( \frac{1}{2} (\theta_1^H - 2r) + \frac{1}{2} \theta_1^H \right)
\end{align*}
\]
\[
\theta_i^s - \frac{\alpha \theta_i^l + 3r}{4} \tag{32}
\]

If \( P_1 = S \) and \( P_2 = R \), by Lemmas 1.1 and 1.2:

\[
\xi^* (\theta_i^s, \theta_2^l) = (NC, NC) \tag{33}
\]

\[
\xi^* (\theta_i^s, \theta_2^H) = (NC, C) \tag{34}
\]

Therefore, the expected value to borrower 1 from taking the safe project when borrower 2 takes the risky project is

\[
EU_{SR}^{i1} = \frac{1}{2} U^{i1}[\xi^* (\theta_i^s, \theta_2^l)] + \frac{1}{2} U^{i1}[\xi^* (\theta_i^s, \theta_2^H)]
\]

\[
= \frac{1}{2} \theta_i^s (1 - \alpha) + \frac{1}{2} \theta_i^s
\]

\[
= \theta_i^s - \frac{\alpha \theta_i^s}{2} \tag{35}
\]

To compare the marginal value to borrower 1 from taking the risky project over the safe project when borrower 2 takes the risky project, subtract (35) from (32):

\[
EU_{RR}^{i1} - EU_{SR}^{i1} = \theta_i^l - \frac{\alpha \theta_i^l + 3r}{4} - (\theta_i^s - \frac{\alpha \theta_i^s}{2}) = \frac{\alpha (2 \theta_i^s - \theta_i^l)}{4} - 3r \tag{36}
\]

Using \( \theta_i^s = \frac{\theta_i^s + \theta_i^H}{2} \) in (36):

\[
EU_{RR}^{i1} - EU_{SR}^{i1} = \frac{\alpha \theta_i^H - 3r}{4} \tag{37}
\]
The right hand side of equation (37) is greater than zero if $\frac{3r}{4} < \theta_H$ and less than zero if $\frac{3r}{4} > \theta_H$. Therefore, $P_1^* = R$ if $P_2 = R$ and $\theta_H > \frac{3r}{4}$, and $P_1^* = S$ if $P_2 = R$ and $\theta_H < \frac{3r}{4}$.

**Lemma 2:** The expected value to one borrower of a joint liability contract when both borrowers invest in the risky project exceeds the expected value when they both invest in the safe project.

**Proof:**

Subtract (27) from (32):

$$EU^{J_1}_{RS} - EU^{J_1}_{SS} = \frac{r - a\theta_L}{4}$$

(38)

Because $\theta_L < \frac{r}{a}$, the right hand side of (38) is greater than zero. Therefore, both borrowers prefer to choose the same investment strategies of $P_1 = P_2 = S$ or $P_1 = P_2 = R$.

**Corollary 1:** Borrowers with high possible project payoffs ($\theta_H > \frac{3r}{a}$) select each other to take joint liability loans and invest in risky projects. Other borrowers ($\theta_H < \frac{3r}{a}$) select each other and invest in safe projects.

**Proof:**

Following Proposition 4 there is only a possibility of $P_1 = P_2 = S$ or $P_1 = P_2 = R$ because it is not possible for one borrower to invest in the safe project and the other to invest in the risky project. Lemma 2 shows that both borrowers would prefer that they both invest in the risky project. From Proposition 5, it is only the borrowers who have high state payoffs under the risky investment who can commit to investing in the risky project when her peer does the same. Therefore, borrowers who can commit to not deviating from investing in the risky project when they both agree to do so would prefer to match with each other. The borrowers who cannot make such a commitment will be forced to match with other such borrowers. Therefore, the borrowers with the very high state payoff possibilities will...
match with each other and invest in risky projects; the borrowers with the moderately high state payoff possibilities will match with each other and invest in safe projects.

**Proposition 6:** The expected repayment rate of a joint liability loan is between 75 and 100 percent.

**Proof:**

Let $\psi \in [0,1]$ be the fraction of borrowers with $\theta_H > \frac{3r}{a}$.

$EV_{J_S}^J(s^*, \tilde{\theta})$ is the expected value per borrower to the lender from making a joint liability loan to a group conditional on borrowers’ investment choices ($P_1$ and $P_2$).

By equation (26):

$$EV_{J_S}^J(s^*, \tilde{\theta}) = 2r/2 = r$$  (39)

By equations (29) - (32):

$$EV_{J_R}^J(s^*, \tilde{\theta}) = \frac{1}{4} \cdot 0 + \frac{3}{4} \cdot 2r = \frac{3r}{2} / 2 = \frac{3r}{4}$$  (40)

$EV^J$ is the expected value to the lender per borrower from making a joint liability loan to a group without knowing what projects are available to them.

$$EV^J = \psi EV_{J_S}^J + (1-\psi)EV_{J_R}^J = \psi r + \frac{3}{4}(1-\psi)r$$

$$= \frac{(3+\psi)r}{4}$$  (41)

Therefore, the expected repayment rate is $\frac{(3+\psi)r}{4} / r = \frac{3+\psi}{4}$, which is bounded between $\frac{3}{4}$ and 1 (75% and 100%) because $0 \leq \psi \leq 1$. 


**Lemma 3.1:** The unique Nash Equilibrium strategy \((s^*)\) for both borrowers under a joint liability contract with a possibility of social sanctions is to not contribute to the loan repayment when both projects’ payoffs are less than \(\frac{\beta r}{\alpha}\).

**Proof:**

\[\text{JS} \equiv \text{joint liability contract with a possibility of social sanctions}\]

\[U^{\text{JS}}[(NC, NC)], \tilde{\theta}_1] = \tilde{\theta}_1 (1 - \alpha) \quad (42)\]

\[U^{\text{JS}}[(C, NC)], \tilde{\theta}_1] = \tilde{\theta}_1 - 2r \quad (43)\]

To compare the value of not contributing over contributing, subtract (43) from (42):

\[U^{\text{JS}}[(NC, NC)], \tilde{\theta}_1] - U^{\text{JS}}[(C, NC)], \tilde{\theta}_1] = 2r - \alpha \tilde{\theta}_1 \quad (44)\]

Applying this case, i.e., \(\tilde{\theta}_1 < \frac{\beta r}{\alpha}\), to equation (44):

\[U^{\text{JS}}[(NC, NC)], \tilde{\theta}_1] - U^{\text{JS}}[(C, NC)], \tilde{\theta}_1] > 2r - \beta r > 0\]

The same logic applies to the borrower 2. Therefore, \(s^* = (NC, NC)\) when \(\tilde{\theta}_1 < \frac{\beta r}{\alpha}\) and \(\tilde{\theta}_2 < \frac{\beta r}{\alpha}\).

**Lemma 3.2:** The unique Nash Equilibrium strategy \((s^*)\) under a joint liability contract and possibility of social sanctions is for the borrower with project payoffs greater than \(\frac{2r}{\alpha}\) to contribute to the repayment of the loan for both borrowers and for the borrower with project payoffs less than \(\frac{\beta r}{\alpha}\) to not contribute.

**Proof:**

Assume that \(\tilde{\theta}_1 \geq \frac{2r}{\alpha}\) and \(\tilde{\theta}_2 < \frac{\beta r}{\alpha}\).
To compare the value of contributing over not contributing for borrower 1, subtract (42) from (43):

$$U^{JS1}[(C, NC), \tilde{\theta}] - U^{JS1}[(NC, NC), \tilde{\theta}] = \alpha \tilde{\theta}_1 - 2r \tag{45}$$

Applying this case, i.e., $\tilde{\theta}_1 \geq \frac{2r}{\alpha}$, to equation (45): $U^{JS1}[(C, NC), \tilde{\theta}] - U^{JS1}[(NC, NC), \tilde{\theta}] \geq 0$.

Therefore, borrower 1 will not deviate from contributing for both.

$$U^{JS2}[(C, NC), \tilde{\theta}] = \tilde{\theta}_2 (1 - \frac{\alpha}{\beta}) \tag{46}$$

$$U^{JS2}[(C, C), \tilde{\theta}] = \tilde{\theta}_2 - r \tag{47}$$

To compare the value of not contributing over contributing for borrower 2, subtract (47) from (46):

$$U^{JS2}[(C, NC), \tilde{\theta}] - U^{JS2}[(C, C), \tilde{\theta}] = r - \frac{\alpha \tilde{\theta}_2}{\beta} \tag{48}$$

Apply this case, i.e. $\tilde{\theta}_2 < \frac{\beta r}{\alpha}$, to (48): $U^{JS2}[(C, NC), \tilde{\theta}] - U^{JS2}[(C, C), \tilde{\theta}] > 0$. Therefore, borrower 2 will not deviate from not contributing.

So, when $\tilde{\theta}_1 \geq \frac{2r}{\alpha}$ and $\tilde{\theta}_2 < \frac{\beta r}{\alpha}$, $s^* = (C, NC)$, and vice versa.

**Lemma 3.3:** The unique Nash Equilibrium strategy ($s^*$) under a joint liability contract with possibility of social sanctions is for both borrowers to contribute to the repayment of the loan if one’s project pays off $\frac{2r}{\alpha}$ or more and the other pays off $\frac{\beta r}{\alpha}$ or more.

**Proof:**

Let $\tilde{\theta}_1 \geq \frac{2r}{\alpha}$ and $\tilde{\theta}_2 \geq \frac{\beta r}{\alpha}$.
Apply (43) and (47) to borrower 1:

\[ U^{JS1}[(C,C), \tilde{\theta}] - U^{JS1}[(NC, C), \tilde{\theta}] = \tilde{\theta}_1 - r - (\tilde{\theta}_1 - \alpha \beta) = \frac{\alpha \tilde{\theta}_1}{\beta} - r \] \quad (49)

Applying this case, i.e. \( \tilde{\theta}_1 \geq \frac{2r}{\alpha} \) to (49): \( U^{JS1}[(C, C), \tilde{\theta}] - U^{JS1}[(NC, C), \tilde{\theta}] \geq 0 \). Therefore, borrower 1 will not deviate from contributing to repayment.

Apply (49) to borrower 2:

\[ U^{JS2}[(C, C), \tilde{\theta}] - U^{JS2}[(NC, NC), \tilde{\theta}] = \tilde{\theta}_2 - r - (\tilde{\theta}_2 - \alpha \beta) = \frac{\alpha \tilde{\theta}_2}{\beta} - r \] \quad (50)

Applying this case, i.e. \( \tilde{\theta}_2 \geq \frac{\beta r}{\alpha} \) to (50): \( U^{JS2}[(C, C), \tilde{\theta}] - U^{JS2}[(NC, NC), \tilde{\theta}] \geq 0 \). Therefore, borrower 2 will not deviate from contributing to repayment.

So, when one project pays \( \frac{2r}{\alpha} \) or more and the other project pays \( \frac{\beta r}{\alpha} \) or more, \( \ast = (C, C) \).

**Lemma 3.4:** The unique Nash Equilibrium strategy (\( \ast \)) under a joint liability contract with the possibility of social sanctions is for neither borrower to contribute to the repayment of the loan if one project payoffs less than \( \frac{\beta r}{\alpha} \) and the other pays off less than \( \frac{2r}{\alpha} \).

**Proof:**

Let \( \frac{\beta r}{\alpha} < \tilde{\theta}_1 \leq \frac{2r}{\alpha} \) and \( \tilde{\theta}_2 < \frac{\beta r}{\alpha} \).

Recall (44):

\[ U^{JS1}[(NC, NC), \tilde{\theta}] - U^{JS1}[(C, NC), \tilde{\theta}] = 2r - \alpha \tilde{\theta}_1 \]
Applying this case, i.e., \( \frac{\beta r}{\alpha} < \tilde{\theta}_1 \leq \frac{2r}{\alpha} \) to the above:
\[
(2 - \beta)r \geq U^{JS_1}[(NC, NC)], \tilde{\theta} - U^{JS_1}[(C, NC)], \tilde{\theta}] > 0.
\]

Apply the above to borrower 2 and this case, i.e., \( \tilde{\theta}_2 < \frac{\beta r}{\alpha} \):
\[
U^{JS_2}[(NC, NC)], \tilde{\theta}] - U^{JS_2}[(C, NC)], \tilde{\theta}] > 0.
\]

Therefore, neither borrower deviates from this strategy. So, \( s^* = (NC, NC) \).

**Proposition 7:** If \( \theta_1^L < \frac{\beta r}{\alpha} \) and borrower 2 invests in the safe project, then borrower 1 will invest in the risky project.

**Proof:**

If \( P_1 = R \) and \( P_2 = S \), then by Lemmas 3.4 and 3.3:

\[
\tilde{s}^*(\theta_1^L, \theta_2^S) = (NC, NC) \tag{51}
\]

\[
\tilde{s}^*(\theta_1^H, \theta_2^S) = (C, C) \tag{52}
\]

Therefore, the expected value to borrower 1 from taking the risky project when her peer takes the safe project is:

\[
EU^{JS_1}_{RS} = \frac{1}{2} \left[ \theta_1^L (1 - \alpha) \right] + \frac{1}{2} (\theta_1^H - r) \tag{53}
\]

Use \( \theta_1^S = \frac{\theta_1^L + \theta_1^H}{2} \) in (53):

\[
EU^{JS_1}_{RS} = \theta_1^S - \frac{\alpha \theta_1^L + r}{2} \tag{54}
\]
If $P_1 = P_2 = S$, then by Lemma 3.3:

$$
\mathcal{s}^* (\theta_1^S, \theta_2^S) = (C, C)
$$

(55)

Therefore, the expected value to borrower 1 from taking the safe project when her peer does likewise is:

$$
EU_{ss}^S = \theta_1^S - r
$$

(56)

To compare the marginal value to borrower 1 from taking the risky project over the safe project when borrower 2 takes the safe project, subtract (56) from (54):

$$
EU_{ss}^{JS1} - EU_{ss}^{JS1} = \theta_1^S - \frac{\alpha \theta_1^L + r}{2} - (\theta_1^S - r) = \frac{r - \theta_1^L}{2}
$$

(57)

Apply this case, i.e., $\theta_1^L < \frac{\beta r}{\alpha}$, to (57): $EU_{ss}^{JS1} - EU_{ss}^{JS1} > \frac{r(1 - \beta)}{2} > 0$. Therefore, borrower 1 will invest in the risky project when her peer invests in the safe project and her risky project’s low state payoff is less than $\frac{\beta r}{\alpha}$.

**Proposition 8:** If $\theta_1^L < \frac{\beta r}{\alpha}$ and borrower 2 invests in the risky project, then borrower 1 will take the risky project only if $\theta_1^S \geq \frac{(2 + \beta)r}{2\alpha}$. Otherwise, borrower 1 will take the safe project.

**Proof:**

If $P_1 = P_2 = R$, then by Lemmas 3.1, 3.2, and 3.3:

$$
\mathcal{s}^* (\theta_1^L, \theta_2^L) = (NC, NC)
$$

(58)

$$
\mathcal{s}^* (\theta_1^L, \theta_2^H) = (NC, C)
$$

(59)
\( S^* (\theta_1^H, \theta_2^L) = (C, NC) \)  

\( S^* (\theta_1^L, \theta_2^H) = (C, C) \)

Therefore, the expected value to borrower 1 from taking the risky project when her peer does the same is:

\[
EU_{RR}^{JS1} = \frac{1}{4} U^{JS1}[S^*, (\theta_1^L, \theta_2^L)] + \frac{1}{4} U^{JS1}[S^*, (\theta_1^L, \theta_2^H)] + \frac{1}{4} U^{JS1}[S^*, (\theta_1^H, \theta_2^H)] \\
+ \frac{1}{4} U^{JS1}[S^*, (\theta_1^H, \theta_2^L)] \\
= \frac{1}{4} \theta_1^L(1 - \alpha) + \frac{1}{4} \theta_1^L(1 - \frac{\alpha}{\beta}) + \frac{1}{4} (\theta_1^H - 2r) + \frac{1}{4} (\theta_1^H - r) 
\]

Use \( \theta_1^S = \frac{\theta_1^L + \theta_1^H}{2} \) in (62) and reduce:

\[
EU_{RR}^{JS1} = \theta_1^S - \frac{\alpha(1 - \frac{1}{\beta})\theta_1^L}{4} + 3r 
\]

If \( P_1 = S \) and \( P_2 = R \), then by Lemmas 3.4 and 3.3:

\( S^* (\theta_1^S, \theta_2^L) = (NC, NC) \)  

\( S^* (\theta_1^S, \theta_2^H) = (C, C) \)

Therefore, the expected value to borrower 1 from taking the safe project when her peer takes the risky project is:

\[
EU_{SR}^{JS1} = \frac{1}{2} U^{JS1}[S^*, (\theta_1^S, \theta_2^L)] + \frac{1}{2} U^{JS1}[S^*, (\theta_1^S, \theta_2^H)] 
\]
\[
\frac{1}{2} \theta^s_i (1 - \alpha) + \frac{1}{2} (\theta^s_i - r) \\
= \theta^s_i - \frac{\alpha \theta^s_i + r}{2}
\]  
(66)

To compare the marginal value of borrower 1 taking the risky project over the safe project when her peer takes the risky project, subtract (66) from (62):

\[
EU^R_{jr1} - EU^S_{jr1} = \frac{2\alpha \theta^s_i - (\beta + 2)r}{4}
\]  
(67)

A necessary and sufficient condition for the right hand side of (67) to be greater than or equal to zero is determined by setting (67) greater than or equal to zero, which yields:

\[
\theta^s_i \geq \frac{(\beta + 2)r}{2\alpha}
\]  
(68)

Therefore, if borrower 1’s projects’ mean expected payoff exceeds \( \frac{(\beta + 2)r}{2\alpha} \), then she invests in the risky project when her peer does also. If her mean expected payoff is between \( \frac{r}{\alpha} \) and \( \frac{(\beta + 2)r}{2\alpha} \), then she invests in the safe project.

**Lemma 4.1:** For the borrower who will always choose the risky project, it is preferable for her to find a peer who will choose the safe project.

**Proof:**

If \( \theta^s_i \geq \frac{(\beta + 2)r}{2\alpha} \), then borrower 1 will invest in the risky project when her peer invests in the risky project (Proposition 8) or if her peer invests in the safe project (Proposition 7). The relative value of finding a peer who will invest in the safe risky project when she invests in the risky project over one who will invest in the safe project is determined by subtracting (54) from (62):
\[ EU_{RS}^{JS1} - EU_{SR}^{JS1} = \theta_i^S - \frac{\alpha(1 - \frac{1}{\beta})\theta_i^L + 3r}{4} - \left(\theta_i^S - \frac{a\theta_i^S + r}{2}\right) = \frac{\alpha(\frac{\beta - 1}{\beta})\theta_i^L - r}{4} < 0 \quad (69) \]

Since (69) is negative, a borrower with an expected payoff on her projects greater than \( \frac{(\beta + 2)r}{2\alpha} \) values having a peer who will invest in the safe project.

**Lemma 4.2:** For the borrower who will invest in the safe project when her peer invests in the risky project, it is preferable for her to find a peer with low expected payoffs, i.e. between \( \frac{r}{\alpha} \) and \( \frac{(\beta + 2)r}{2\alpha} \).

**Proof:**

Proposition 7 states that if one borrower invests in the safe project, then the other will invest in the risky project. Proposition 8 states that if the expected project payoffs are low, then a borrower will maintain investment in the safe project when her peer invests in the risky project. If both borrowers have low expected project payoffs, then their optimal investment strategy, \( P^* = (R, S) \) or \( P^* = (S, R) \). There is no a priori reason for either borrower to expect that she will be the one to play the risky investment strategy. Therefore, both will expect to play either strategy equally.

The expected value for a borrower with low expected payoffs (borrower 1) from having a peer with similarly low expected payoffs is derived from (54) and (66):

\[
\frac{1}{2} EU_{RS}^{JS1} + \frac{1}{2} EU_{SR}^{JS1} = \theta_i^S - \frac{2r + \alpha(\theta_i^L + \theta_i^S)}{4} \quad (70)
\]

If the peer has high expected payoffs, however, the borrower with low expected payoffs is assured to always invest in the safe project because she cannot credibly commit to the risky strategy and the high expected payoff borrower will always invest in the risky project (Proposition 8).

The expected value for a borrower with low expected payoffs (borrower 1) from having a peer with high expected payoffs is given in (64).
Proposition 9: Borrowers will match with other borrowers with the same expected project payoffs where there is a possibility of social sanctions.

Proof:
Borrowers with low expected project payoffs prefer to invest in the risky project (Lemma 4.2).

These borrowers cannot credibly commit to invest in the risky project with a peer with high project payoffs because the peer would always invest in the risky project (Proposition 8).

Therefore, the only chance the low expected project payoffs borrower has to play the risky strategy is to match with a borrower with low expected project payoffs, too (Proposition 7).

Proposition 10: The only investment strategies that will be played are \( P_1 = P_2 = R \) for groups with high expected project payoffs and \( P_1 \neq P_2 \) for groups with low expected project payoffs.

Proof:
This follows from Propositions 7, 8, and 9.

Proposition 11: The expected repayment rate of a joint liability loan with the possibility of social sanctions is between 50 percent and 75 percent.

Proof:
Let \( \phi \in [0,1] \) be the fraction of borrowers with \( \frac{r}{\alpha} < \theta_s < \frac{(\beta + 2)r}{2\alpha} \).
$EV_{rs}^{JS}(\tilde{x}^*, \tilde{\theta})$ is the expected value per borrower to the lender from making a joint liability loan to a group conditional on borrowers’ investment choices ($P_1$ and $P_2$).

\[ EV_{rs}^{JS}(\tilde{x}^*, \tilde{\theta}) = \left( \frac{1}{4} \cdot 0 + \frac{3}{4} \cdot 2r \right) / 2 = \frac{3r}{4} \]  
(72)

\[ EV_{rs}^{JS}(\tilde{x}^*, \tilde{\theta}) = EV_{rs}^{JS}(\tilde{x}^*, \tilde{\theta}) = \left( \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 2r \right) / 2 = \frac{r}{2} \]  
(73)

The expected value to the lender is the weighted average of (72) and (73) by the distribution of all borrowers’ expected project payoffs:

\[ EV^{JS} = (1 - \phi)EV_{rs}^{JS} + \phi EV_{rs}^{JS} = \frac{(3 - \phi)r}{4} \]  
(74)

The expected repayment rate is (74) divided by $r$ and is bounded according to the value of $\phi$:

\[ \frac{1}{2} \leq \frac{3 - \phi}{4} \leq \frac{3}{4} \]  
(75)

**Proposition 12:** In terms of borrower expected utility, the individual liability contract weakly dominates the joint liability contracts assuming the same principal and interest across contracts (i.e., same $r$).

**Proof:**

**Case 1:** Borrowers with low expected payoffs

From Proposition 2, Corollary 1, and Proposition 10 for borrower 1 with $\theta^u_i < \frac{3r}{\alpha}$: $EU^{ij}_1 = EU^{ij}_r$, $EU^{j1}_r = EU^{j1}_s$, and $EU^{j1}_s \in \{EU^{j1}_{rs}, EU^{j1}_{sr}\}$. 

50
The value of the individual liability contract over the joint liability contract without the possibility of social sanctions is given by subtracting (27) from (7):

\[ EU_{p*}^{I} - EU_{p*}^{J} = \frac{r - \alpha \theta^L}{2} > 0 \]  

(76)

The value of the individual liability contract over the joint liability contract with the possibility of social sanctions is given by subtracting (64) from (7) or (71) from (7):

\[ EU_{p*}^{I} - EU_{p*}^{JS} = 0 \]  

(77)

\[ EU_{p*}^{I} - EU_{p*}^{JS} = \alpha(\theta^H - \theta^L) > 0 \]  

(78)

Case 2: Borrowers with high expected payoffs

From Proposition 2, Corollary 1, and Proposition 10 for borrower 1 with \( \theta_i^H \geq \frac{3r}{\alpha} \): \( EU_{p*}^{I} = EU_{p*}^{I} \),

\[ EU_{p*}^{J} = EU_{p*}^{J} \,, \text{ and } EU_{p*}^{JS} = EU_{p*}^{JS} \,.

The value of the individual liability contract over the joint liability contract without the possibility of social sanctions is given by subtracting (32) from (7):

\[ EU_{p*}^{I} - EU_{p*}^{J} = \frac{r - \alpha \theta^L}{4} > 0 \]  

(79)

The value of the individual liability contract over the joint liability contract with the possibility of social sanctions is given by subtracting (63) from (7):

\[ EU_{p*}^{I} - EU_{p*}^{JS} = \frac{r - \alpha \theta^L}{4} > 0 \]  

(80)

**Proposition 14:** If given the choice between an individual liability and joint liability contract without the possibility of social sanctions, borrowers with \( \theta_i^H < \frac{3r}{\alpha} \) choose the individual liability
contract and those with $\theta^u \geq \frac{3r}{\alpha}$ choose the joint liability contract. The total principal and interest due on the individual liability contract is 150% of that of the joint liability contract.

**Proof:**

Let $r_I$ be the amount due for the individual contract and $r_J$ be the amount due for the joint liability contract without the possibility of social sanctions.

From Propositions 3 and 6:

\[
EV^{i}_{p^*} = \frac{r_I}{2} \tag{81}
\]

\[
EV^{j}_{p^*} = \frac{(3 + \psi) r_J}{4} \tag{82}
\]

If the lender were to choose $r_I$ relative to $r_J$ such that the expected values to the lender is equal, then set (81) equal to (82), which yields:

\[
r_I = \frac{(3 + \psi) r_J}{2} \tag{83}
\]

The borrower will choose the individual liability loan over the joint liability loan if

\[
(EU^i_{p^*} | r_I) - (EU^j_{p^*} | r_J) > 0.
\]

Otherwise, she will choose the joint liability loan.

**Case 1: $p^*=(R,R)$**

From Propositions 2 and 5:

\[
(EU^i_{p^*} | r_I) - (EU^j_{p^*} | r_J) = \left( \theta^s - \frac{\alpha \theta^L + r_I}{2} \right) - \left( \theta^s - \frac{\alpha \theta^L + 3r_J}{4} \right) \tag{84}
\]

Substitute $r_J$ with expression (83) in (84):

\[
(EU^i_{p^*} | r_I) - (EU^j_{p^*} | r_J) = -\frac{\alpha \theta^L + (3 + 2\psi) r_I}{4} < 0 \tag{85}
\]
Therefore, borrowers for whom $p^* = (R, R)$ (those with $\theta_{II}^R \geq \frac{3r}{\alpha}$) will always select the joint liability contract.

Case 2: $p^* = (S, S)$

From Propositions 2 and 4:

$$
(EU_{I}^{I} | r_I) - (EU_{II}^{I} | r_I) = \left( \theta_{I}^{S} - \frac{\alpha \theta_{I}^{S} + r_I}{2} \right) - \left( \theta_{I}^{S} - r_I \right) \tag{86}
$$

Substitute $r_I$ with expression (83) in (86):

$$
(EU_{I}^{I} | r_I) - (EU_{II}^{I} | r_I) = \frac{(7 + \psi) r_j - 2 \alpha \theta_{I}^{S}}{4} \tag{87}
$$

Use the assumption that $\theta_{I}^{S} < \frac{r_J}{\alpha}$ to show that the right hand side of (87) is greater than zero:

$$
\frac{(7 + \psi) r_j - 2 \alpha \theta_{I}^{S}}{4} > \frac{(7 + \psi) r_j - 2 r_j}{4} = \frac{(5 + \psi) r_j}{4} > 0 \tag{88}
$$

Therefore, borrowers for whom $p^* = (S, S)$ (those with $\theta_{II}^S < \frac{3r}{\alpha}$), will always choose the individual liability contract.

Since the borrowers will separate in this way, the lender believes that $\psi = 0$. Therefore, substituting in (83): $r_I^* = \frac{3r_J^*}{2}$, where $r_I^*$ and $r_J^*$ are the equilibrium amounts due for both the individual and joint liability contracts.
The lender values the joint liability contract at the general equilibrium amount due, \( r_J^* \), at (82) evaluated with \( \psi = 0 \): \( EV^J_\psi = \frac{3r_J^*}{4} \). The repayment rate in general equilibrium, therefore, is 

\[
EV^J_\psi = \frac{3r_J^*}{4} \quad / \quad r = \frac{3}{4} = 75%.
\]

**Comment 1:** Borrowers with better prospects \( (\theta^u \geq \frac{3r}{\alpha}) \) prefer the joint liability contract because the interest rate is lower because the lender knows that their peers share risk with them. Borrowers with lesser prospects \( (\theta^u \leq \frac{3r}{\alpha}) \) prefer the individual liability contract because the cost imposed by the lender is less than the cost of taking the safe project.

**Comment 2:** Everyone invests in the risky project.

**Lemma 5:** If given the choice between an individual and joint liability contract where there will be social sanctions, a borrowers for whom \( p^*=(R,R) \) will choose the individual liability contract.

**Proof:**

Let \( r_{JS} \) be the amount due under a joint liability contract with the possibility of social sanctions.

From Propositions 3 and 11:

\[
EV^I_\psi = \frac{r_I}{2} \tag{89}
\]

\[
EV^J_\psi = \frac{(3-\phi)r_{JS}}{4} \tag{90}
\]

If the lender were to choose \( r_I \) relative to \( r_{JS} \) such that the expected values to the lender is equal, then set (89) equal to (90), which yields:

\[
r_I = \frac{(3-\phi)r_{JS}}{2} \tag{91}
\]
The borrower will choose the individual liability loan over the joint liability loan if
\((EU_{p^*}^I) - (EU_{p^*}^{JS1}) > 0\). Otherwise, she will choose the joint liability loan.

If \(p^*=(R,R)\), then:

\[
(EU_{p^*}^I) - (EU_{p^*}^{JS1}) = \left( \theta_i^s - \frac{\alpha \theta_i^l + r_i}{2} \right) - \left( \theta_i^s - \frac{\alpha(1 + \frac{1}{\beta})\theta_i^l + 3r_{JS}}{2} \right)
\]

Substitute \(r_i\) with expression (91) in (92):

\[
(EU_{p^*}^I \mid r_i) - (EU_{p^*}^{JS1} \mid r_{p^*}) = \frac{\alpha \theta_i^l (1 - \beta) + 4 \beta \phi r_{JS}}{4 \beta} > 0
\]

Therefore, if \(p^*=(R,R)\), then the borrowers prefer the individual liability contract.

**Proposition 15:** If given the choice between an individual liability and joint liability loan where there is a possibility of social sanctions, no one will take the joint liability contract.

**Proof:**

If anyone does take the joint liability contract, it would be the borrowers with lower expected project payoffs because Lemma 5 shows that the high expected project payoff borrowers will definitely choose the individual liability contract. These lower expected project payoff borrowers are those who would play \((R,S)\) or \((S,R)\) investment strategies. Therefore, if the joint liability contract is taken by at least one group, \(\phi = 1\).

Compare the expected values from taking the individual liability loan to the joint liability loan with the possibility of social sanctions by subtracting (70) from (10) and substituting \(r_i\) with expression (91):

\[
EU_{p^*}^I \left( \frac{1}{2} EU_{RS}^{JS1} + \frac{1}{2} EU_{SR}^{JS1} \right) = \frac{\alpha(\theta_i^s - \theta_i^l) - (1 - \phi)r_{JS}}{4}
\]
As explained above, if this contract is accepted by anyone, it is by the lower expected payoff borrowers. Therefore, evaluate (94) with $\phi = 1$:

$$EU_{p^*} - \left( \frac{1}{2} EU_{RS}^{JS1} + \frac{1}{2} EU_{SR}^{JS1} \right) = \frac{\alpha (\theta^s_i - \theta^i_s)}{4} > 0$$

(95)

Therefore, the borrowers with lower expected project payoffs would choose the individual liability loan.
References


