IDIOSYNCRATIC JUMP RISK MATTERS: 
EVIDENCE FROM EQUITY RETURNS AND OPTIONS

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Abstract

The recent literature provides conflicting empirical evidence on the pricing of idiosyncratic risk. This paper sheds new light on the matter by exploiting the richness of option data. First, we find that idiosyncratic risk explains 28% of the variation in the risk premium on a stock. Second, we show that the contribution of idiosyncratic risk to the equity premium arises exclusively from jump risk. Finally, we document that the commonality in idiosyncratic tail risk is much stronger than that in total idiosyncratic risk documented in the literature. Tail risk thus plays a central role in the pricing of idiosyncratic risk.

JEL Classification  C51, C58, G12, G13

Keywords  Risk premiums; Tail risk; Idiosyncratic risk; Systematic risk; Option valuation; GARCH.

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1 Introduction

An investor should be rewarded for bearing systematic risk. One of the key insights of Sharpe (1964) and Lintner’s (1965b) CAPM, building on the seminal work of Markowitz (1952), is that idiosyncratic risk, however, should not carry a risk premium, as it can be diversified away. Since the introduction of the CAPM, numerous asset pricing models have been developed that build on the premise that idiosyncratic risk is not priced.\(^1\) However, the recent literature has strongly challenged this notion.\(^2\) Although the channel through which idiosyncratic risk could be priced remains a matter of debate, it is now widely accepted that, given market incompleteness, idiosyncratic risk can be priced.\(^3\) While previous studies are informative about the relative importance of idiosyncratic risk in explaining expected stock returns, they do not attempt to identify whether the importance of idiosyncratic risk arises from its diffusive or tail components. Thus, little is known about the relative contribution of systematic and idiosyncratic diffusive and tail risk in explaining the equity premium.

Our study departs from existing work by decomposing stocks’ systematic and idiosyncratic shocks into a Gaussian and a jump component. Our approach offers an ideal framework to study the relative importance of each factor in explaining expected excess returns on equity. In particular, our study is the first to uncover the central role of idiosyncratic tail risk in explaining expected stock returns. Indeed, we find that idiosyncratic risk explains 28.2% of the variation of the expected excess returns of an average stock and, more important, that this is exclusively due to the jump risk component. Idiosyncratic Gaussian risk is not priced. This finding is consistent with the idea that investors have difficulty hedging idiosyncratic tail risk and, thus, require a premium to bear exposure to this risk.

We exploit the richness of stock option data to extract the expected risk premium associated with

\(^1\)Notably, Merton’s (1973) ICAPM extends the insights of the CAPM to an intertemporal setup. The arbitrage pricing theory of Ross (1976) shows that any common return factor is a potential asset pricing factor. Fama and French (1992, 1993, 2015) and Carhart (1997), for instance, identify such potential factors, but diversifiable idiosyncratic risk is still assumed not to carry any risk premium.

\(^2\)Concerns about the pricing of idiosyncratic risk date back to Douglas (1969) and Lintner (1965a). Goyal and Santa-Clara (2003) and Ang, Hodrick, Xing, and Zhang (2006) contributed to returning this debate to the forefront of the asset pricing literature by providing empirical evidence that idiosyncratic risk matters.

\(^3\)Goyal and Santa-Clara (2003) highlight that a possible channel is background risk; investors hold nontraded assets (e.g., human capital or private businesses) that add background risk to their traded portfolio decisions. Jacobs and Wang (2004) provide evidence that idiosyncratic consumption risk is a priced factor in the cross-section of stock returns. Hence, the average idiosyncratic stock variance being a proxy for idiosyncratic consumption risk could explain why idiosyncratic risk is priced. Consistent with this insight, Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) provide evidence linking average idiosyncratic volatility to the income risk faced by households. Alternatively, Stambaugh, Yu, and Yuan (2015) argue that the negative relationship between idiosyncratic volatility and stock returns could be driven by arbitrage asymmetry, as buying could be easier than shorting for many equity investors. Kelly and Jiang (2014) suggest that firm-level tail risk is informative about the likelihood of market-wide extreme events, justifying the pricing effects of idiosyncratic tail risk.
each risk factor, thereby avoiding the exclusive use of noisy realizations of historical equity returns. To this end, we develop a GARCH-jump model in which a firm’s systematic and idiosyncratic risk have both a Gaussian and a tail component.\(^4\) Our pricing kernel is such that each risk factor can potentially be priced. The model offers quasi-closed form solutions for European option prices. We estimate the model on 260 firms that are or were part the S&P 500 index between 1996 and 2015, using the equity returns and option prices of the market index and of each individual firm.\(^5\) To the best of our knowledge, this is the most comprehensive joint estimation analysis of option-pricing models conducted in the literature. Moreover, our study is also the first to account for any sort of clearing condition across stocks when pricing options.\(^6\)

Our empirical analysis highlights three new results. First, the magnitude of the contribution of idiosyncratic risk to an average stock’s expected excess return is 2.4% per annum (p.a.) in our sample.\(^7\) As we estimate a market equity risk premium of 6.2% p.a. over the same period, the premium on idiosyncratic risk is economically significant. Consistent with Bates (2008), jump and normal risks are priced differently by investors. At the market level, our results further support this view, adding to those of Christoffersen, Jacobs, and Ornthanalai (2012) and Ornthanalai (2014): we find that systematic jump risk accounts for more than 60% of the total equity risk premium (3.9% p.a. out of the total 6.2% p.a.).

Second, and most important, we find that the premium associated with idiosyncratic risk is due solely to idiosyncratic jump risk. That is, the Gaussian component of idiosyncratic risk, which is easily diversifiable, is not priced after accounting for other sources of risk.\(^8\) When estimating a nested version of the model in which idiosyncratic jump risk is omitted, idiosyncratic normal risk appears to be priced. For the great majority of stocks, the nested variant of the model appears to be misspecified, however, since it offers a significantly worse fit to equity returns and options than the model with idiosyncratic jumps. This result is of significant interest, as most of the literature on idiosyncratic risk assumes conditional normality.

\(^4\)The GARCH model used in this paper can be seen as a discretization of a standard jump diffusion model (cf. Online Appendix OA.A). Using the GARCH model simplifies the particle filter needed to estimate the model and dramatically improves its numerical properties.

\(^5\)We considered all 1,000 stocks that were part of the index during this period; neglected stocks were set aside only because an insufficient number of options were liquidly traded over at least a consecutive 5-year window.

\(^6\)To preclude arbitrage opportunities, the idiosyncratic risk premium on constituents of the market must net out to zero at each point in time. Consequently, on any given day, the idiosyncratic risk premium is positive for some stocks and negative for others.

\(^7\)Whereas the average value of the idiosyncratic jump risk premium (IJRP), weighted by market capitalization, is zero at each point in time, the cap-weighted average of the absolute IJRP is 2.4%.

\(^8\)Note that, while the expected stock return is not affected by idiosyncratic Gaussian risk, an option’s vega is still positive and affected by total volatility.
Our third empirical finding is that idiosyncratic jump risk shares a strong commonality across firms. Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) document that idiosyncratic (total) variances have a strong factor structure. Based on the idiosyncratic volatilities of 20,000 CRSP stocks over 85 years, they document that a single factor explains 35% of the time variation in firm-level idiosyncratic risk. In light of these results, our model of stock variance allows for two sources of commonality: one arising from commonality in idiosyncratic normal risk and the other from commonality in idiosyncratic jump risk. Over the 20 years in our sample, these two sources of commonality explain 36.0% of the variation in total idiosyncratic risk. As both our sample and methodology differ significantly from those of Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016), this is remarkably close to the 35% that they document. More important, our measure of commonality in idiosyncratic jump risk (IJR) explains 50.1% of the time variation in IJR, a novel result in the literature. Kelly and Jiang (2014) devise an approach that captures common variation in the tail risks of individual firms using panels of stock returns. Our result lends further support to their insight that the cross-section of extreme returns can be informative about aggregate tail risk. In our framework, substantial common variation in tail risk arises from the exposure of all stocks to systematic jump risk. The common IJR component supplements this channel and strongly covaries with market risk (see, for instance, Bartram, Brown, and Stulz (2016)). Hence, already being difficult to hedge by its nature, tail risk becomes virtually undiversifiable in times of turmoil, which justifies the risk premium attached to it.

Many studies come to conflicting conclusions regarding the sign of the risk premium on idiosyncratic risk. Ruan, Sun, and Xu (2016) argue that this is due to the very low signal-to-noise ratio of the idiosyncratic risk measures used to proxy for undiversified idiosyncratic risk. Indeed, total risk is commonly divided into a systematic and an idiosyncratic component, and the latter is used to assess the pricing role of idiosyncratic risk. There is however no theoretical justification for using total idiosyncratic risk in asset pricing tests. Merton (1987), for instance, argues that under incomplete information undiversified idiosyncratic risk should carry a risk premium. Merton’s (1987) theory, however, implicitly states that a large part of idiosyncratic risk is effectively diversified. Ruan, Sun, and Xu’s (2016)
analysis illustrates that, when undiversified risk is only a small component of total idiosyncratic risk, any inference drawn from total idiosyncratic risk suffers from a severe error-in-variables problem. Correcting for the resulting attenuation bias, they find that undiversified risk indeed carries a positive risk premium that contributes to the total market equity risk premium.

Our analysis can be seen as a cross-sectional complement to that of Ruan, Sun, and Xu’s (2016). On an average day, the variance associated with IJR accounts for only 9.5% of total idiosyncratic variance. Yet, IJR carries a premium that explains substantial cross-sectional variation in expected excess stock returns. The idiosyncratic Gaussian risk premium, however, is negligible for virtually all firms in our sample. Overall, our results highlight that idiosyncratic jump risk is a key determinant of undiversified idiosyncratic risk. In contrast, idiosyncratic Gaussian risk is effectively diversified. As such, it introduces significant noise when total idiosyncratic variance is used to proxy for undiversified idiosyncratic risk.

Interestingly, the average IJRP on the quintile of firms with the lowest market capitalization appears to be consistently positive and large, while the average IJRP of the largest firms is consistently negative and small in magnitude. This is consistent with the intuition behind Merton’s (1987) main mechanism: small firms tend to have a smaller investor base than large firms, potentially leaving them with a larger share of undiversified idiosyncratic risk. On average, firms with lower book-to-market ratios also seem to exhibit a larger IJRP, but the pattern is starkly reversed during the Great Recession of 2007–2009. We also find that that momentum losers tend to have a larger IJRP than momentum winners in periods of turmoil, but the relationship between momentum and IJRP is not clear outside of these periods. Other firm characteristics such as operating profitability (-) and investment levels (+) also seem to correlate with a stock’s level of IJRP, but the sign and strength of these relationships appear to vary over time.

Our study is the first to conduct a joint estimation, based on equity returns and options, of an option-valuation model to disentangle the four risk premiums associated with systematic and idiosyncratic, normal and tail (jump) risk. It is, however, related to several contemporaneous papers. Martin and Wagner (2016) derive a theoretical relationship between a stock’s excess risk-neutral variance relative to the average stock and the cross-sectional variation in stock returns. Their approach has the distinct advantage of being model-free but does not allow one to separate Gaussian from jump idiosyncratic risk.

Christoffersen, Fournier, and Jacobs (2016) document a strong factor structure in equity options. Consequently, building on Heston (1993), they develop a stochastic volatility model in which a firm’s total variance is decomposed into a systematic and an idiosyncratic component. The authors study the
effect of firm beta and market variance to explain the cross-sectional variations of equity options. Among other results, their model predicts that stocks with higher betas have higher implied volatilities and steeper smiles, which are consistent with the empirical findings of Duan and Wei (2009). Our framework extends that of Christoffersen, Fournier, and Jacobs (2016) in that we allow for a jump component both in market returns and in the idiosyncratic part of stock returns. Moreover, our joint estimation methodology builds on those of Christoffersen, Jacobs, and Ornthanalai (2012) and Ornthanalai (2014) and allows us to quantify how the equity risk premium is affected by the four sources of risk affecting stocks in our setup.

Closer to our study, Xiao and Zhou (2017) use a GARCH model inspired by that of Maheu, McCurdy, and Zhao (2013) to study the historical returns on 15 stocks and find that both sources of idiosyncratic risk, Gaussian and jump, are priced. However, since they do not use option data, they are bound to parsimony and use a pricing kernel requiring that either both sources are priced or both are not. Gourier (2014) estimates a continuous-time jump diffusion model using a procedure based on equity returns, options and intraday data observed on 29 stocks between 2006 and 2012. Gourier’s (2014) framework allows her to study the important role played by total (Gaussian and jump) idiosyncratic risk in the equity and, most important, the variance risk premium. She finds that compensation for idiosyncratic risk represents, on average, 50% of the equity risk premium and 80% of the variance risk premium. Although the models, datasets and estimation methods in our studies differ along several dimensions, the results that are common to our two studies are consistent, and our analyses complement one another. In particular, Gourier (2014) provides strong empirical evidence that idiosyncratic risk is a key determinant of the equity and variance risk premiums; we provide strong empirical evidence that tail risk is actually at the core of the relationship between idiosyncratic risk and the equity risk premium.

This paper is organized as follows. Section 2 presents our model of the market and the individual stocks. Section 3 presents the data and discusses the estimation methodology. Then, Section 4 presents our empirical analysis. Section 5 concludes.

2 The Model

We develop a model in which, in the spirit of the CAPM, stocks are exposed to systematic risk. Unlike the traditional one-factor CAPM, however, market and stock returns are not solely driven by a Gaussian component. The market can crash, or more generally jump, and the stocks in our model are exposed to
this systematic jump risk, as well as to idiosyncratic normal and jump risk. As such, our model falls under the framework of Kraus and Litzenberger (1976), but extends it in various directions.

2.1 Stock Returns

Returns on the market index, $R_{M,t+1} = \frac{M_{t+1}}{M_t}$, and a given stock, $R_{S,t+1} = \frac{S_{t+1}}{S_t}$, are modeled as follows:

\begin{align}
    r_{M,t+1} & \equiv \log(R_{M,t+1}) = \mu_{M,t+1} - \xi^p_{M,t+1} + y_{M,t+1}, \\
    r_{S,t+1} & \equiv \log(R_{S,t+1}) = \mu_{S,t+1} - \xi^p_{S,t+1} + \beta_{S,z}z_{M,t+1} + \beta_{S,y}y_{M,t+1} + y_{S,t+1}
\end{align}

where stock returns are driven by the stock’s exposure to systematic Gaussian and jump risk, $z_{M,t+1}$ and $y_{M,t+1}$, as well as stock-specific innovations $z_{S,t+1}$ and $y_{S,t+1}$, capturing idiosyncratic normal and jump risk, respectively.

For $u \in \{M, S\}$, $\xi^p_{u,t+1}$ is the convexity correction (cf. Appendix A) associated with the Gaussian innovations, $z_{u,t+1}$, and the jumps, $y_{u,t+1}$. Hence,

$$E^p_t[M_{t+1}] = M_t \exp(\mu_{M,t+1}) \quad \text{and} \quad E^p_t[S_{t+1}] = S_t \exp(\mu_{S,t+1}).$$

That is, $\mu_{u,t+1} - r_{f,t+1}$ can be interpreted as the instantaneous equity risk premium on the index and the stock, given the instantaneous risk-free rate $r_{f,t+1}$.

Before discussing the exact form of the instantaneous risk premiums (Section 2.2), we further characterize the distribution of the shock processes. The Gaussian innovations are given by

$$z_{u,t+1} = \sqrt{h_{u,z,t+1}} \varepsilon_{u,t+1}, \quad u \in \{M, S\}$$

\text{11} Our model (and pricing kernel) nest those of Elkamhi and Ornthanalai (2010), who quantify the impact of market jump risk on equity options. They find that firms with a larger return compensation for systematic normal risk have a higher option-implied volatility level, while firms with a larger return compensation for systematic jump risk have steeper option-implied volatility slope. However, stocks in their framework do not exhibit idiosyncratic jump risk, and they do not study the pricing of idiosyncratic risk. Along the same lines, Babaoğlu (2015) further documents that a “jump beta” is needed to adequately explain equity returns, market risk exposures, and equity option prices.

\text{12} The filtration is generated by the market noise terms and the stock noise terms, that is $\mathcal{F}_t^Z = \sigma[z_{M,t}, z_{M,t}, z_{S,t}, \varepsilon_{M,t}]$. $E_T^z[S_{t+1}]$ is a shorthand for $E_T^z[S_{t+1}\mid\mathcal{F}_t^M]$. Since all innovation time series are independent, $E_T^z[M_{t+1}\mid\mathcal{F}_t^Z] = E_T^z[M_{t+1}\mid\mathcal{F}_t^M]$ where $\mathcal{F}_t^M = \sigma[z_{M,t}, y_{M,t}]_{t=1}^T$, and we still use $E_T^z[\cdot]$ to represent both conditional expectations.

\text{13} Over a short period of time, $\mu_{M,t+1}$ and $r_{f,t+1}$ are close to zero, such that

$$E_T^z[M_{t+1}\mid M_t] - E_T^z[M_{t+1}\mid M_t] = \exp(\mu_{M,t+1}) - \exp(r_{f,t+1}) \equiv \mu_{M,t+1} - r_{f,t+1}.$$
where the $\epsilon_{u,t+1}$ are serially independent standard normal random variables, and the conditional variance of $z_{u,t+1}$ follows a GARCH dynamics. Indeed, the market conditional variance is

$$
h_{M,z,t+1} = w_{M,z} + b_{M,z}h_{M,z,t} + \frac{a_{M,z}}{h_{M,z,t}} (z_{M,t} - c_{M,z}h_{M,z,t})^2.
$$

$= \sigma_{M,z}^2 + b'_{M,z}(h_{M,z,t} - \sigma_{M,z}^2) + \frac{a_{M,z}}{h_{M,z,t}}(z_{M,t} - h_{M,z,t} - 2c_{M,z}h_{M,z,t}z_{M,t}), \quad (2.3)
$

where $\sigma_{M,z}^2 = \frac{w_{M,z} + a_{M,z}}{1 - b'_{M,z}}$ is the unconditional level of the market variance, and $b'_{M,z} = b_{M,z} + a_{M,z}c_{M,z}^2$ is the variance persistence. The $c_{M,z}$ parameter allows the model to account for the asymmetric response of volatility to positive and negative returns. A positive $c_{M,z}$ effectively induces a negative correlation between return and variance innovations. This feature, often referred to as the leverage effect (Black and Cox (1976)), is particularly important when considering the option-valuation properties of a model.14

The specification of the stock’s conditional variance is inspired by the literature on component volatility models,15 that is

$$
h_{S,z,t+1} = \kappa_{S,z}h_{M,z,t+1} + b_{S,z}(h_{S,z,t} - \kappa_{S,z}h_{M,z,t}) + \frac{a_{S,z}}{h_{S,z,t}}(z_{S,t}^2 - h_{S,z,t} - 2c_{S,z}h_{S,z,t}z_{S,t}). \quad (2.4)
$$

However, rather than varying around a long-run volatility component of its own, the conditional variance of a particular stock loads on market variance through $\kappa_{S,z}h_{M,z,t+1}$. This is a parsimonious reduced-form approach that allows us to account both for the fact that idiosyncratic variance exhibits strong commonality across stocks (cf. Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016)) and the fact that idiosyncratic risk tends to be high when market risk is high (cf. Bartram, Brown, and Stulz (2016)). The idiosyncratic variance in excess of this central tendency, $h_{S,z,t+1} - \kappa_{S,z}h_{M,z,t+1}$, has a GARCH structure.

The jumps, $y_{u,t+1}$, have a normal-inverse Gaussian (NIG) distribution with location parameter set at 0, a tail heaviness parameter $\alpha_u$ and an asymmetry parameter $\delta_u$.16 Following Ornthanalai (2014), the time-homogeneous scale parameter of the distribution is allowed to vary and is denoted by $h_{u,y,t+1}$. We


16Ornthanalai (2014) finds that NIG jumps offer the best fit to market data when compared to Merton (Poisson) jumps, variance gamma jumps, or CGMY jumps (Carr, Geman, Madan, and Yor 2002). Earlier drafts of this paper featured Poisson rather than NIG jumps. While the main results were qualitatively similar, the estimated jump parameters were much less stable. In particular, the Poisson-jump version of the model had difficulty accommodating the positive jumps during and after the Great Recession. The NIG has the econometric advantage of allowing us to account for large negative and positive jumps in a parsimonious fashion.
In particular, total market and stock variances are given by

$$h_{M,Y,t+1} = w_{M,Y} + b_{M,Y}h_{M,Y,t} + \frac{a_{M,Y}}{h_{M,z,t}}(z_{M,t} - c_{M,Y}h_{M,z,t})^2,$$

$$h_{S,Y,t+1} = \kappa_{S,Y}h_{M,Y,t+1} + b_{S,Y}(h_{S,Y,t} - \kappa_{S,Y}h_{M,Y,t}) + \frac{a_{S,Y}}{h_{S,z,t}}(z_{S,t}^2 - h_{S,z,t} - 2c_{S,Y}h_{S,z,t}z_{S,t}).$$

As for the variance of Gaussian shocks, idiosyncratic jump intensity has a central tendency $\kappa_{S,Y}h_{M,Y,t+1}$, and $h_{S,Y,t+1} - \kappa_{S,Y}h_{M,Y,t+1}$ has a GARCH structure.

Following the literature, we define idiosyncratic variance as the variance of the residuals obtained after accounting for systematic risk factors, here normal and jump market risk. In sum, our model of market returns is essentially the NIG-jump model considered in Ornthanalai (2014). We extend his framework to allow stocks (i) to have systematic normal and jump risk exposure and (ii) to exhibit idiosyncratic normal and jump risk. In particular, our model remains in the affine class of models, which is key to obtaining a closed-form solution for the price of European options on the market index and individual stocks (cf. Section 2.3). This solution generalizes those of Elkamhi and Ornthanalai (2010) and Babaoğlu (2015) by adding idiosyncratic jumps in stock returns.

### 2.2 Pricing Kernel and Risk Premiums

In an incomplete market setup, the pricing kernel, $m_{t+1}$, is potentially affected by untraded sources of risk. As highlighted in the literature on modeling the pricing kernel, in our context, it suffices to work

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17 Strictly speaking, $h_{a,y,t+1}$ is not an intensity, as it does not parameterize the number of jumps observed over a period $\Delta t$. However, the normal-inverse Gaussian distribution is closed under convolution in the sense that, given $\alpha_x$ and $\delta_x$, the sum of two NIG shocks with scale parameters $h_1$ and $h_2$ would have a scale parameter of $h_1 + h_2$. Hence, the NIG jump as specified here is observationally equivalent to a compound Poisson process with independent and identically distributed NIG increments, the intensity of which would be time-varying (cf. Online Appendix OA.A.1).

18 Christoffersen, Jacobs, and Ornthanalai (2012) compare, using market data, a model in which a single factor drives normal variance and jump intensity to a model similar to ours. They find that the model with separate variance and intensity dynamics dominates its counterpart in terms of fitting the data.
with the projection of the pricing kernel on the observed sources of risk. Indeed, if \( p_t \) is the time-\( t \) price of an asset with a time-\( t + 1 \) cash flow \( x_{t+1} \) that depends on the realization of \( \{z_{M,t+1}, y_{M,t+1}, z_{S,t+1}, y_{S,t+1}\} \), then

\[
p_t = E^p_t [m_{t+1} x_{t+1}] = E^p_t \left[ E^q_t \left[ m_{t+1} x_{t+1} \mid z_{M,t+1}, y_{M,t+1}, z_{S,t+1}, y_{S,t+1} \right] \right] = E^p_t [\tilde{m}_{t+1} x_{t+1}]
\]

where \( \tilde{m}_{t+1} = E^p_t \left[ m_{t+1} \mid z_{M,t+1}, y_{M,t+1}, z_{S,t+1}, y_{S,t+1} \right] \). If \( z_{S,t+1} \) and \( y_{S,t+1} \) are orthogonal to the pricing kernel, then they do not matter in the pricing and the projection is simply \( \tilde{m}_{t+1} = E^p_t \left[ m_{t+1} \mid z_{M,t+1}, y_{M,t+1} \right] \).

The recent literature, however, highlights that firm-specific (or idiosyncratic) risk can be correlated with risk factors that do enter the pricing kernel. In line with much of the option-pricing literature, we take a reduced-form approach to modeling the pricing kernel and assume an exponentially affine Radon-Nikodym derivative (RND)

\[
e^{\int_{f_{t+1}}^{t} \text{d} \tau} \tilde{m}_{t+1} = \frac{d\mathbb{Q}^M}{d\mathbb{P}^M} \frac{d\mathbb{Q}^S}{d\mathbb{P}^S} \left[ \text{exp}(-\Lambda_{M,t+1} z_{M,t+1} - \sum_{S \in \mathbb{S}} \Lambda_{S,t+1} z_{S,t+1} - \sum_{S \in \mathbb{S}} \Gamma_{S,t+1} y_{S,t+1}) \right]
\]

where \( \mathbb{S} \) is the set of firms in the economy. All prices of risk (\( \Lambda_{M,t+1}, \Gamma_{M,t+1}, \Lambda_{S,t+1}, \) and \( \Gamma_{S,t+1} \)) must be predictable processes. Implicitly, \( \Lambda_{S,t+1} \) and \( \Gamma_{S,t+1} \) are related to the projection of the pricing kernel on \( z_{S,t+1} \) and \( y_{S,t+1} \). In particular, if market prices of risk are constant (\( \Lambda_{M,t+1} = \Lambda_M, \Gamma_{M,t+1} = \Gamma_M, \forall t \)), and if the firm-specific risk factors are not priced (\( \Lambda_{S,t+1} = \Gamma_{S,t+1} = 0, \forall (S, t) \)), then our RND is equivalent to that used by Christoffersen, Jacobs, and Ornthanalai (2012).\(^{20}\) As they note, their RND is consistent with the pricing kernel studied by Bates (2008).\(^{21}\) Appendix B discusses how the model’s innovations are risk-neutralized using this RND.

As our focus is idiosyncratic risk, we will follow the bulk of the literature and assume that market prices of risk are indeed constant. Making the same assumption for idiosyncratic risk would seem a natural first step. However, as we will discuss in the next section, making this assumption in an affine option valuation model requires, to preclude arbitrage opportunities, stringent cross-sectional restrictions on the idiosyncratic prices of risk. These restrictions are definitively not supported empirically (cf.

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\(^{20}\)Boloorforoosh (2014) consider a similar pricing kernel, without jump risk (systematic or idiosyncratic), but allowing idiosyncratic normal risk to be priced. He finds strong empirical support for the hypothesis that idiosyncratic risk is indeed priced.

\(^{21}\)Similar pricing kernels are studied in continuous-time setups by, among others, Bates (1991, 2006), Liu, Pan, and Wang (2005), and Eraker (2008).
Online Appendix OA.F).

**Risk Premiums**

As in Christoffersen, Jacobs, and Ornthanalai (2012) and Ornthanalai (2014), the pricing kernel in (2.9) yields an equity risk premium, \( \mu_{M,t} - r_{f,t} \), which admits a decomposition in terms of a normal and a jump risk premium, that is

\[
\mu_{M,t} - r_{f,t} = \lambda_M h_{M,1,t} + \gamma_M h_{M,2,t} ,
\]

(2.10)

where the mappings between \( \lambda_M \) and \( \gamma_M \) and their pricing kernel counterparts \( \Lambda_M \) and \( \Gamma_M \) are given in Appendix C. Note that if a market price of risk in the RND is zero (e.g., \( \gamma_M = 0 \)), then the associated risk premium is zero (e.g., \( \gamma_M = 0 \)).

Appendix C further establishes that the equity risk premium on a stock, \( \mu_{S,t} - r_{f,t} \), can be decomposed into four risk premiums: the normal and jump market risk premiums and the idiosyncratic normal and jump premiums:

\[
\mu_{S,t} - r_{f,t} = \beta_S \lambda_M h_{M,1,t} + \gamma_M \beta_S \gamma_M h_{M,2,t}^\gamma + \lambda_S h_{S,1,t} + \gamma_S h_{S,2,t}^\gamma .
\]

(2.11)

Once again, if a market price of risk in the RND is zero (e.g., \( \Gamma_S = 0 \)), then the associated risk premium is zero (e.g., \( \gamma_S = 0 \)). Furthermore, note that although the model is affine, the premium associated with a systematic jump depends non-linearly on the jump beta, \( \beta_S \gamma_M \), and the market price of jump risk, \( \gamma_M \), through function \( \gamma_M(\cdot) \), which has a single root at 0. Further details are provided in Appendix C.

To illustrate the difference between our framework and a standard conditional CAPM framework, consider the familiar

\[
\beta_{S,t+1} = \frac{\text{Cov}_t^P \left( R_{S,t+1} - R_{f,t+1}, R_{M,t+1} - R_{f,t+1} \right)}{\text{Var}_t^P \left( R_{M,t+1} - R_{f,t+1} \right)} = \frac{\text{Cov}_t^P \left( e^{S_{t+1}} - e^{f_{t+1}}, e^{M_{t+1}} - e^{f_{t+1}} \right)}{\text{Var}_t^P \left( e^{M_{t+1}} - e^{f_{t+1}} \right)} .
\]

(2.12)

In the context of our model, a first-order approximation of this total beta yields

\[
\beta_{S,t+1} \simeq \frac{\text{Cov}_t^P \left( \beta_S z_{M,t+1} + \beta_S \gamma_M z_{M,t+1} + \gamma_M z_{M,t+1} \right)}{\text{Var}_t^P \left( z_{M,t+1} + \gamma_M z_{M,t+1} \right)} = \frac{\beta_S h_{M,1,t+1} + \beta_S \gamma_M \sigma_{M,t+1}^\gamma h_{M,1,t+1} + \gamma_M \sigma_{M,t+1}^\gamma h_{M,1,t+1}}{h_{M,1,t+1} + \sigma_{M,t+1}^\gamma h_{M,1,t+1}}
\]

(2.13)
and, in a CAPM setting, the risk premium on the stock would be
\[ \mu_{S,t+1}^\text{capm} - r_{f,t+1} = \beta_{S,t+1}^\text{capm} (\mu_M, t - r_{f,t}) = \beta_{S,t+1}^\text{capm} (\Lambda_M h_{M,z,t+1} + \gamma_M h_{M,y,t+1}). \] (2.14)

Contrasting equations (2.14) and (2.11) highlights two features of our model. First, in our model, stocks can have different sensitivities to normal and jump risk. Second, \( \lambda_{S,t} \) and \( \gamma_{S,t} \) are not assumed to be null but are jointly estimated from past returns and option data.

**Priced Idiosyncratic Risk in the Absence of Arbitrage**

Harrison and Kreps (1979) establish that the existence of an empirical martingale measure (EMM) is strictly equivalent to the absence of arbitrage. Given that equation (2.9) defines an EMM \( Q \), it is tempting to invoke Harrison and Kreps’ (1979) results to assert the absence of arbitrage in our economy. There is, however, an important consideration here: the market and its \(| \mathcal{S} |\) constituents are not \(| \mathcal{S} | + 1\) linearly independent assets here. The market return is, by construction, a weighted average of the returns of its constituents.

**Proposition 1** Assume that the market, \( M \), and stocks, \( S \in \mathcal{S} \), follow the multivariate affine GARCH model proposed in this paper. Further, assume that the market prices of idiosyncratic risk are constant, that is, \( \Lambda_{S,t} = \Lambda_S \) and \( \Gamma_{S,t} = \Gamma_S \) for all \( t \). Then, in all generality, the exponentially affine pricing kernel in equation (2.9) does not preclude arbitrage opportunities.

The proof is in Appendix G, but its intuition is straightforward. Consider the instantaneous expected return of a portfolio replicating the index:

\[
\sum_{S \in \mathcal{S}} \omega_S \mu_{S,t}^P = r_{f,t} + \lambda_M \sum_{S \in \mathcal{S}} \omega_S \beta_{S,z,t} h_{M,z,t} + \sum_{S \in \mathcal{S}} \omega_S \gamma_{M,S} \left( \beta_{S,y,t} \right) h_{M,y,t} + \sum_{S \in \mathcal{S}} \omega_S \lambda_{S,z,t} h_{S,z,t} + \sum_{S \in \mathcal{S}} \omega_S \gamma_{S,y,t} h_{S,y,t} \approx r_{f,t} + \lambda_M h_{M,z,t} + \gamma_M h_{M,y,t},
\] (2.15)

\[ \approx r_{f,t} + \lambda_M h_{M,z,t} + \gamma_M h_{M,y,t}. \] (2.16)

\( \text{\textsuperscript{22}} \)It is worth noting that the beta in equation (2.13) varies dynamically with the relative importance of the stock’s exposure to higher market moments, which vary with the market’s jump intensity. As such, co-skewness is implicitly accounted for (cf. Online Appendix OA.C.3). This is important given the results of Harvey and Siddique (2000), who find a 3.6% risk premium for systematic skewness, and of Schneider, Wagner, and Zechner (2016), who highlight the important role of co-skewness in explaining beta- and volatility-based low-risk anomalies. Our results are thus consistent with those of Conrad, Dittmar, and Ghysels (2013), who find evidence that individual securities’ higher moments (skewness) matter even after controlling for differences in co-moments.
where $\omega_S$ represents the weight of stock $S$ in the market index $M$. Assuming that $\sum_{S \in S} \omega_S \gamma_{MS} (\beta_{S,y}) \approx \gamma_M$, the absence of arbitrage imposes the following constraints:

$$
0 = \sum_{S \in S} \omega_S \lambda_S, \forall t, \quad \text{and} \quad 0 = \sum_{S \in S} \omega_S \gamma_{S,y}, \forall t.
$$

(2.17)

Given the time-varying nature of the variance and jump intensity appearing in these two sums, offsetting constant prices of risk could potentially allow the no-arbitrage constraints to be respected at a given point in time but essentially cannot satisfy constraints in equation (2.17) at all points in time.

The theory of Merton (1987) provides guidance on how to impose the above constraints. Indeed, Merton (1987) demonstrates that, in a two-period model with undiversified idiosyncratic risk, the expected return on stock $S$ has the form

$$
E^P \left[ R_S - R_f \right] = \nu \vartheta^P (R_M) \beta_{S,capm} + \tilde{\lambda}_S \sigma^2_S,
$$

(2.18)

where $\nu$ is a risk aversion parameter and $\sigma^2_S$ is the total idiosyncratic variance of $S$. Then aggregating equation (2.18) at the market level yields

$$
E^P \left[ R_M - R_f \right] = \nu \vartheta^P (R_M) + \sum_{S \in S} \omega_S \tilde{\lambda}_S \sigma^2_S.
$$

(2.19)

In our setup, the aggregate impact of undiversified idiosyncratic risk, $\sum_{S \in S} \omega_S \tilde{\lambda}_S \sigma^2_S$, is subsumed in the systematic normal and jump risk premiums.

Considering the expected return on the stock in excess of that on the market, we obtain

$$
E^P [R_S - R_M] = \nu (\beta_{S,capm} - 1) \vartheta^P (R_M) + \left( \tilde{\lambda}_S \sigma^2_S - \sum_{S \in S} \omega_S \tilde{\lambda}_S \sigma^2_S \right).
$$

(2.20)

The first part of equation (2.20) conveys standard intuition from the CAPM: stocks with $\beta_{S,capm} < 1$ ($\beta_{S,capm} > 1$) will earn, in expectation, less (more) than the market. The second part highlights that, once the aggregate impact of undiversified idiosyncratic risk has been accounted for through the compensation for systematic risk, idiosyncratic risk still has an effect in the cross-section. This effect increases the expected returns on some stocks and decreases it on others. Interestingly, this is reminiscent of the

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23 In the Merton (1987) model, underdiversification arises from the fact that each security appears in the investment set of only a fraction, $q_S$, $0 \leq q_S \leq 1$, of the agents in the market. Hence, $\tilde{\lambda}_S$ can be seen as an underdiversification constant that is related to the market-capitalization of the stock, $\omega_S$, and to the investor base, $q_S$, in the following manner: $\tilde{\lambda}_S = \nu \omega_S \left( \frac{1}{q_S} - 1 \right)$.  

12
theoretical result of Martin and Wagner (2016), who demonstrate that $E_T^Q [R_s - R_M]$ should be proportional to the stock’s idiosyncratic (risk-neutral) variance in excess of the average level of idiosyncratic variance across stocks.

Inspired by this result, we consider the following dynamics for idiosyncratic Gaussian and jump risk premiums:

\[
\lambda_{S,t} h_{S,t} \equiv \lambda_S h_{S,t} - \sum_{S'} \omega_{S'} \lambda_{S'} h_{S',t}, \quad (2.21)
\]

\[
\gamma_{S,t} h_{S,t} \equiv \gamma_S h_{S,t} - \sum_{S'} \omega_{S'} \gamma_{S'} h_{S',t}. \quad (2.22)
\]

That is, we assume that the risk premiums associated with idiosyncratic risk are affine, predictable processes determined not only by the predicted variance of the stock but also by the weighted risk premiums on the other stocks in $S$. This specification trivially satisfies the no-arbitrage conditions in equation (2.17).

2.3 Option Prices

The model, once risk-neutralized, remains within the affine class of models (see Appendix D). Hence, we build on the work of Heston and Nandi (2000) and obtain a closed-form solution for the price of European index and stock options. For $u_t \in \{ M_t, S_t \}$, the price of a European call option is

\[
C_t(u_t, K, T) = u_t P_{1,t,T} - K e^{-r_{f,t,T}(T-t)} P_{2,t,T} \quad (2.23)
\]

where $r_{f,t,T} = \frac{1}{T-t} \sum_{j=1}^{T-1} r_{f,t+j}$, in which $r_{f,t+j}$ is the deterministic risk-free rate at time $t + j$. The conditional probabilities $P_{1,t,T}$ and $P_{2,t,T}$ are given by

\[
P_{j,t,T} = \frac{1}{2} + \frac{1}{\pi} \int_{0}^{\infty} \text{Re} \left[ \frac{1}{\phi i} \exp \left( -i \phi \log \tilde{K}_{t,T} \right) \psi_{1,T}^Q (\phi i + 1_{j=1}) \right] d\phi, \quad j = 1, 2,
\]

where $i$ is the imaginary number, and $1_{j=1}$ is an indicator taking value 1 if $j=1$ and 0 otherwise, $\tilde{K}_{t,T} = (K e^{-r_{f,t,T}(T-t)})/u_t$, and the conditional moment-generating function $\psi_{1,T}^Q (\phi) = E^Q_t \left[ \exp \left( \phi \sum_{j=1}^{T-t} \tilde{r}_{u,t+j} \right) \right]$.

---

24Heston and Nandi (2000) rely on an inversion similar to that of Gil-Pelaez (1951).
of the aggregated excess returns \(\sum_{j=1}^{T-t} r_{u,t+j} = \sum_{j=1}^{T-t} \left( r_{u,t+j} - r_{j,t+j} \right)\) over the period \([t, T]\) satisfies

\[
\varphi^Q_{t,T}(\phi) = \exp \left( \mathcal{A}_{u,T-t}(\phi) + \mathcal{B}_{u,T-t}(\phi) h^*_M z_{t+1} + \mathcal{C}_{u,T-t}(\phi) h^*_S y_{t+1} + \mathcal{D}_{u,T-t}(\phi) h^*_M z_{t+1} + \mathcal{E}_{u,T-t}(\phi) h^*_S y_{t+1} \right),
\]

where the \(h^*_M, h^*_S\) are the risk-neutralized variance processes. The deterministic functions \(\mathcal{A}_{u}, \mathcal{B}_{u}, \mathcal{C}_{u}, \mathcal{D}_{u}, \mathcal{E}_{u}\) are calculated based on the recursion in Appendix E. In particular, \(\mathcal{D}_{M,T-t} = \mathcal{E}_{M,T-t} = 0\).

## 3 Joint Estimation Using Returns and Option Prices

Relying on a joint estimation procedure is of particular importance to our study. Indeed, the risk premium parameters that we seek to study are relatively poorly identified under the physical measure (that is, using only return data). However, these parameters play a crucial role in the pricing kernel and, as such, are key to reconciling the price of the options and the underlying returns.\(^{25}\) Moreover, as highlighted by Merton (1976), in the absence of jumps, a deep-out-of-the-money option would be almost worthless, especially if the option is relatively short-dated. These options will thus improve our ability to estimate the likelihood of jumps. Hence, the richness of stock option data plays a key role in allowing us to extract the expected risk premium associated with each risk factor.

### 3.1 Data

To estimate the model, we use the returns and prices of options on the S&P 500, as a proxy for the market, and on 260 stocks that are or were part of the index since 1996. These stocks were selected based on whether their options had been actively traded over at least a consecutive 5-year window. Daily index and stock returns, from January 1996 to August 2015, are obtained from the Center for Research in Security Prices (CRSP).\(^{26}\) To compute the corresponding daily excess log-returns (henceforth, returns), we use the one-month Treasury bill rate (from Ibbotson Associates) as extracted from Kenneth French’s data library.

Our estimation procedure (Section 3.2) relies on implied volatilities (see Renault (1997) for an interesting discussion on the benefits of using Black-Scholes as bijection to work with implied volatilities


\(^{26}\)In fact, we extract returns starting from January 1986. Returns between January 1986 and December 1995 are used to warm up the variance process; their likelihood, however, does not impact the estimation of the parameters. A similar procedure is used for individual stocks.
rather than prices). The implied volatilities of options on the SPX and the stocks, between January 1996 and August 2015, are obtained from OptionMetrics.\textsuperscript{27} Options on individual stocks are American options. OptionMetrics uses a Cox, Ross, and Rubinstein (1979) binomial tree to derive the option’s implied volatility, accounting for dividends. We restrict our analysis to out-of-the-money monthly options with at least one week and at most one year to maturity. Observations for which the ask price is lower than the bid price are excluded. The price of the option is defined as the mid point between the ask and the bid, and options with a price lower than the bid-ask spread are excluded. Moreover, the open interest must be strictly positive and the volume must be larger than one-tenth of the median volume, for each maturity available. We further remove options that violate the common arbitrage conditions. Finally, among the remaining options, we select the most liquid option of eight preselected moneyness buckets, for each maturity available.\textsuperscript{28} In general, this procedure yields four calls and four puts for each maturity and for each Wednesday in our sample.\textsuperscript{29} In the uncommon case of an empty moneyness bucket, we simply select the most liquid calls or puts among the remaining options, for a given maturity. This leaves us with a total of 44,267 implied volatilities on the SPX, and 2,962,621 on the 260 stocks.\textsuperscript{30} Tables 1 and 2 summarize the option data sets. Figure 1 provides an overview of how implied volatilities vary over time as the S&P 500 evolves. As evidenced in the lower panel of the figure, while implied volatilities on stocks co-move with implied volatilities on S&P 500 options, the former are significantly larger than the latter.

### 3.2 Joint Estimation

Following Christoffersen, Jacobs, and Ornthanalai (2012) and Ornthanalai (2014), the model’s parameters are estimated by maximizing the weighted joint log-likelihood function

\[
L_u (\Theta_u) = \frac{T_u + N_u}{2} \left( \frac{L_u,\text{returns} (\Theta_u)}{T_u} + \frac{L_u,\text{options} (\Theta_u)}{N_u} \right),
\]

\textsuperscript{27}The zero-coupon term structure is also extracted from OptionMetrics and used for option pricing. The rate corresponding to an option’s maturity is obtained through linear interpolation whenever necessary.

\textsuperscript{28}The moneyness buckets range from 0.80 to 1.20, with steps of 0.05.

\textsuperscript{29}We follow the literature and use Wednesday data because Wednesday is the least likely day to be a holiday and the least likely to be affected by weekend effects. For further details on the advantages of using Wednesday data, see Dumas, Fleming, and Whaley (1998). If markets are closed on a given Wednesday (e.g., Christmas, January 1, Independence day or 9/11), we use the previous business day.

\textsuperscript{30}In total, we considered options on the 1,000 different firms that were part of the S&P 500 at any point in our sample. Our selection procedure discarded 738 firms. Two additional firms (tickers BEN and NEE) were further discarded because they experimented very extreme returns that caused numerical problems in the particle filter.
where \( u \in \{M,S\} \), \( T_u \) is the number of returns observed, \( N_u \) is the total number of option observations, and \( \Theta_u \) represents the parameter set of the model.

We opt for a two-stage estimation approach. That is, we first maximize the joint likelihood \( L_M \) with respect to \( \Theta_M \) and then turn to maximizing \( L_S \) for each stock while taking the results for the market as given. Although it has drawbacks, this approach is crucial to keeping the estimation procedure tractable in our setting. Indeed, in opposition to typical GARCH processes in which the noise term is fully determined once we condition on observed returns and the initial variance, the presence of jumps implies, focusing on the market model, that the Gaussian component \( z_{M,t} \) and the jump component \( y_{M,t} \) of time-\( t \) innovation cannot be separated. Consequently, as noted by Durham, Geweke, and Ghosh (2015), the conditional variance \( h_{M,z,t} \) and intensity \( h_{M,y,t} \) remain uncertain, even with the observed returns up to time \( t \).\(^{31}\) However, both \( h_{M,z,t+1} \) and \( h_{M,y,t+1} \) can be fully recovered from the initial conditions \( h_{M,z,1} \), \( h_{M,y,1} \), the returns \( r_{M,t} = \{ r_{M,s} \}_{s=1}^{t} \) and the jump innovations \( y_{M,t} = \{ y_{M,s} \}_{s=1}^{t} \). In this spirit, we propose a particle filter that infers the average (filtered) \( z_{M,t} \), \( y_{M,t} \), \( h_{M,z,t} \) and \( h_{M,y,t} \), while accounting for the uncertainty with respect to the conditional variance and the jump intensity. Appendix F further describes the particle filter used to compute the log-likelihood \( L_M(\Theta_M) \). The same procedure is applied to each stock in a second estimation stage, holding the market parameters and latent variables fixed.

Following the option-pricing literature, the log-likelihood of the option fit, \( L_u,\text{options}(\Theta_u) \), is based on relative implied volatility pricing errors.\(^{32}\) In particular, if \( IV^\text{mkt}_{u,k} \) is the Black and Scholes (1973) implied volatility associated with the market price of option \( k \) on underlying \( u \in \{M,S\} \) and \( IV^\text{mean}_{u,k} \) the implied volatility inverted from the corresponding model price, then the relative implied volatility error is

\[
e_{u,k} = \frac{IV^\text{mean}_{u,k} - IV^\text{mkt}_{u,k}}{IV^\text{mkt}_{u,k}}.
\]

Assuming that the relative implied volatility error is normally distributed, \( e_{u,k} \sim \mathcal{N}(0,\sigma_{e}^2) \), and

\(^{31}\)Technically, \( \mathcal{G}_M = \sigma \{ r_{M,t} \}_{t=1}^{T_u} \) is the \( \sigma \)-field generated by the returns process that is coarser than the \( \sigma \)-field \( \mathcal{F}_t^M = \sigma \{ z_{M,s}, y_{M,s} \}_{s=1}^{t} \) generated by the innovation terms. The conditional variance \( h_{M,z,t} \) and the jump intensity \( h_{M,y,t} \) are both \( \mathcal{F}_{t-1}^M \)-measurable, but they are not \( \mathcal{G}_M \)-measurable.

\(^{32}\)This criterion, or variants thereof, is used by Bakshi, Carr, and Wu (2008), Christoffersen, Jacobs, and Ornthanalai (2012), and Ornthanalai (2014). Alternatively, some authors will consider vega-weighted RMSE (VWRMSE) since VWRMSE and IVRMSE take very similar values, while the former have the advantage of being faster to compute than the latter. See, for instance, Carr and Wu (2007) and Trolle and Schwartz (2009). Note that using relative implied volatility errors has the advantage of not assigning excessive weighting to option prices observed during the financial crisis.
uncorrelated with shocks in returns, we obtain

\[
L_{\text{u, options}} (\Theta_u) = -\frac{1}{2} \sum_{k=1}^{N_u} \left( \log(2\pi\sigma_e^2) + \frac{e_{u,k}^2}{\sigma_e^2} \right).
\]

Note that \(\sigma_e\) is identified using the sample standard deviation of \(\{e_{u,k}\}_{k=1}^{N_u}\).

Conceptually, the particle filter could be extended to cope with a one-stage estimation of the market and the 260 firms; numerically, however, this would be absolutely intractable. Our alternative is computationally efficient, and given the richness of the index option data, we are confident that the two-stage estimation procedure yields parameter estimates for the market model that are more accurate than those that could be obtained from a poorly behaved one-stage procedure. Unfortunately, while a one-stage procedure could, in theory, straightforwardly accommodate the no-arbitrage constraints in equation (2.17) by assuming time-varying prices of risk (equations (2.21) and (2.22)), the two-stage procedure can, a priori, only cope with constant prices of idiosyncratic risk. However, we develop in Appendix G an iterative procedure that can accommodate time-varying prices of risk and that converges relatively quickly. The starting point of the algorithm is to use the two-stage estimation procedure while assuming that idiosyncratic prices of risk are constant. Then, prices of risk are adjusted using equations (2.21) and (2.22), and the stock parameters (including, in particular, the constants in the affine dynamics of the prices of risk) are re-estimated. The procedure iterates until the dynamics of the prices of risk remain unaffected by the latest iteration.

4 Empirical Results

4.1 Market

Although the focus of our study is the pricing of idiosyncratic risk, we first briefly discuss results obtained at the market level. Overall, these results are very close to those in the option-pricing literature. In particular, our results are much in line with those reported by Ornthanalai (2014) for the NIG variant of his model, which is essentially our market model. The parameters, reported in Table 3, are largely similar, except for \(a_{M,y}\), the parameter governing the variance of jump intensity, which is much larger in our results than in those of Ornthanalai. This difference could be due to our sample covering more of the Great Recession and its aftermath.
For each subset of option $O$, Table 4 reports two metrics:

$$
IVRMSE = \sqrt{\frac{1}{N} \sum_{k \in O} \left( IV_{model}^k - IV_{mkt}^k \right)^2 } \quad \text{and} \quad RIVRMSE = \sqrt{\frac{1}{N} \sum_{k \in O} \left( \frac{IV_{model}^k - IV_{mkt}^k}{IV_{mkt}^k} \right)^2 }.
$$

(4.25)

The first, $IVRMSE$, provides an absolute measure of the implied-volatility pricing errors. The latter is a relative measure that is likely more informative when comparing pricing errors over time. According to both measures, our market fit to the option data compares favorably to the results in the option-pricing literature, as detailed in Panel A of Table 4. This is true over time and across maturities and moneyness levels. As documented by Ornthanalai, the NIG jumps in our model allow for particularly large levels of (negative) skewness and excess kurtosis (cf. Table 3). This theoretical feature of the model explains its particularly good fit across maturities and moneyness levels. Moreover, the NIG jumps properly capture, empirically, the nonnormal innovations in returns; consequently, the filtered conditionally standard normal innovations, $\epsilon_{M,t}$, have skewness and excess kurtosis that are close to zero, as they should be. Figure 2 plots the filtered normal innovations $z_{M,t}$ (top panel), jumps (middle panel) and volatility components (bottom panel). Again, the results are qualitatively similar to those of Ornthanalai.

Table 3 also reports risk premiums based on the average and median level of normal, $\lambda_M h_{M,z,t}$, and jump, $\gamma_M h_{M,y,t}$ components of the conditional equity risk premium (ERP). The median levels of the premiums are 1.72% and 3.04%, respectively, for a median ERP of 4.76%. These numbers are comparable to those of Ornthanalai, who reports an annualized normal risk premium of 1.43% and a jump risk premium of 3.22%, for a total of 4.65%, based on the unconditional level of variance and jump intensity. Hence, although Table 3 reports that the jump component of variance, $h_{M,y,t}$, explains on average only 26.0% of total variance, $h_{M,z} + \frac{\gamma^2}{\lambda^2 \epsilon^2} h_{M,y}$, the jump risk premium outweighs its normal counterpart.

The average ERP is higher than the median at 6.18% and decomposes into an average normal premium of 2.33% and an average jump premium of 3.85%. Naturally, the average is more sensitive than the median (or any measure based on unconditional GARCH levels) to extreme values of the premiums observed during periods of turmoil. The top panel of Figure 3 reports how the premium evolves over time. At its peak, in November 2008, the estimated ERP reaches 40.16%. While this number may appear high, Martin’s (2017) measure of the ERP, as extracted from one-month-to-maturity options alone, rises to more than 50% around the same time, while its three-month counterpart flirts with the 40% level.
Using a panel of options with median time-to-expiration of 14 business days, Bollerslev and Todorov (2011) find that the jump component of the ERP rises above 40% during the same period.

The bottom panel of Figure 3 reports, on a daily basis, the ratio of the ERP that is explained by the jump component. This ratio is at its lowest during periods of turmoil, when the normal risk carries a higher than usual premium. When the ERP is particularly low, which coincides with periods of low volatility in the market, the jump risk premium explains up to 80% of the total ERP. Hence, while Figure 2 documents that, as Bates (2008) notes, jump risk is countercyclical, the relative importance of jump risk in the ERP appears to be mildly cyclical.

In sum, our results at the market level are consistent with the literature.

4.2 Idiosyncratic Jump Risk Matters

We now turn to our paper’s main empirical contribution. Namely, while our results are consistent with the literature highlighting that idiosyncratic risk does matter for the equity risk premium, we provide evidence that idiosyncratic jump risk is at the center of this empirical phenomenon.

Table 5 reports summary statistics on the parameters associated with the 260 stocks under consideration. While there is substantial cross-sectional variation, the average value of the parameters of the variance and intensity processes are comparable to the parameters obtained for the market model. Remarkably, more than 75% of the firms exhibit less negative skewness and excess kurtosis than the market. This is consistent with Bakshi, Kapadia, and Madan (2003), who document that the option-implied skewness of individual stocks is typically much less negative than that of the market index.33

Of particular interest, the normal and jump betas are on average 0.931 and 1.111, respectively. The normal beta ranges from 0.320 to 1.752, but 50% of firms under consideration have a normal beta between 0.730 and 1.108. In comparison, the jump beta ranges from 0.206 to 3.928, while 50% of firms under consideration have a normal beta between 0.831 and 1.315. Interestingly, the cross-sectional correlation between firms’ normal and jump betas is only 0.237 (cf. Table 6). That is, there is a positive correlation, but firms with large normal betas do not necessarily have large jump betas, and vice versa.

Table 6 further reports the correlation between $\beta_{S,v}, v \in \{z,y\}$, and the firm-by-firm time series average of the systematic normal risk premium (SNRP), $\beta_{S,z} \lambda_M h_{M,z,t}$, and its jump counterpart (SJRP), $\gamma_{MS}(\beta_{S,y}) h_{M,y,t}$. Unsurprisingly, the correlation between $\beta_{S,v}$ and the corresponding systematic pre-

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33 Albuquerque (2012) develops and empirically supports a model in which conditional asymmetric stock return correlations and negative skewness in aggregate returns are caused by cross-sectional heterogeneity in firm announcement events.
mium is high but imperfect. Consistent with the modest correlation between $\beta_{S,z}$ and $\beta_{S,y}$, the correlation between the normal (jump) beta and the systematic jump (normal) premium is positive but modest at 0.236 (0.230). These results highlight the importance of accounting for separate systematic premiums on both types of risk, as emphasized by Elkamhi and Ornthanalai (2010) and Babaoğlu (2015).

Most important, Table 5 also sheds the first light on our main empirical result: the normal component of idiosyncratic risk, which is easily diversifiable, is not priced after accounting for other sources of risk. Indeed, not only is the average value of $\lambda_{S}$ zero, but its range is narrowly centered around zero, going from $-0.017$ to 0.012. However, the price of jump risk parameter, $\gamma_{S}$, is greater than zero for all firms, averaging 2.021.

These levels per se are difficult to interpret, as the pricing of idiosyncratic risk is relative to the average level of idiosyncratic risk in the economy. Figure 4 further illustrates the stark contrast between the pricing of idiosyncratic normal and jump risk. At a given point in time, the no-arbitrage constraints of equation (2.17) ensure that the cap-weighted cross-sectional averages of both idiosyncratic premiums are zero. However, the total premium on a given stock

$$
\mu_{S,t} - r_{f,t} = \frac{\beta_{S,z} h_{M,z,t}}{\text{SNRP}_{S,t}} + \frac{\gamma_{M,S}(\beta_{S,y}) h_{M,y,t}}{\text{SJRP}_{S,t}} + \left( \lambda_{S} h_{S,z,t} - \sum_{S' \in S} w_{S'} \lambda_{S'} h_{S',z,t} \right) + \left( \gamma_{S} h_{S,y,t} - \sum_{S' \in S} w_{S'} \gamma_{S'} h_{S',y,t} \right)
$$

can be larger (or smaller) than the sum of the systematic premiums, depending on the sign and magnitude of the idiosyncratic risk premiums on day $t$. The upper panel of Figure 4 highlights, once again, that the normal component of idiosyncratic risk is not priced. Conversely, the idiosyncratic jump risk premium appears to be of clear economic significance. Indeed, while the cap-weighted average of the absolute INRP is virtually always zero, the average magnitude of the IJRP is similar to that of the SNRP and SJRP. Interestingly, the distribution of IJRP across firms is very asymmetric and varies over time, as evidenced by the lower panel of Figure 4.

34For the normal premium, the time series average

$$
\frac{1}{T_{S}} \sum_{t \in T_{S}} \beta_{S,z} h_{M,z,t} = \beta_{S,z} h_{M,z,T_{S}}, \quad S \in S,
$$

is linear in $\beta_{S,z}$, which makes the imperfect correlation puzzling at first glance. However, the firm-specific window of available data, $T_{S}$, introduces cross-sectional variation in $h_{M,z,T_{S}}$. 

20
Whereas Figure 4 reports time-series of cross-sectional averages, Figure 5 reports the cross-section of the time-series averages for the four components of the expected return on the 260 stocks in our sample. Again, idiosyncratic normal risk is virtually absent from the picture. Furthermore, the asymmetric nature of the IJRP is remarkable. Over time, the IJRP is on average positive for 194 stocks out of 260. The median IJRP is positive for 164 stocks but lower than the average for 236 stocks. Idiosyncratic jump intensities, just like the variances, tend to spike upward dramatically when a given firm is in turmoil; sharp downward movements are much less frequent. This behavior is inherited by the risk premium dynamics.

Averages across industries

Figure 6 presents, for the eight largest Global Industry Classification Standard (GICS) industries covered by our sample, the evolution over time of the component of the industry average firm’s equity risk premium that is due to exposure to idiosyncratic jumps.\textsuperscript{35} Note that all firms load, through their normal and jump betas, on the systematic risk premiums reported in Figure 3. Hence, the idiosyncratic jump risk premium (solid line) reported in Figure 6 adds to the premium arising from the firms’ exposure to systematic risk factors (grey ‘+’ marks).

For all industries, jump risk premiums increase around both recessions in our sample. In fact, the increase is relatively mild around the first recession for all industries, except Information Technology, which had just been hit by the burst of the dot-com bubble. However, the idiosyncratic jump risk premiums increase markedly for many industries around the Great Recession; Health Care, Consumer Staples and Industrials do not exhibit a significant increase. Interestingly, the crisis peak in the idiosyncratic jump risk premium for Financials is not as high as that experienced by Consumer Discretionary or Materials, for instance. However, as reported in Table 7, firms from the Financial sector are, on average, those exhibiting the second-highest normal beta and the second-highest jump beta. Hence, their total premium increases significantly during the crisis.

4.3 Commonality in Idiosyncratic Jump Risk

Following the literature, we have defined idiosyncratic variance as the variance of the residuals obtained after accounting for systematic risk factors, here normal and jump market risk. In particular, a stock’s

\textsuperscript{35}We do not report results for Telecommunication Services (2 firms) and Utilities (3 firms), as we do not have sufficient firms from these sectors.
idiosyncratic variance and jump intensity are defined in equations (2.4) and (2.6). To account for the observed strong commonality in idiosyncratic variances (Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2014), henceforth HKLV), each process evolves around a dynamic level $\kappa_{S,v}M_{v,t+1}$, $v \in \{z,y\}$. Table 5 reports that the normal $\kappa_{S,z}$ is on average 0.863 and the average jump $\kappa_{S,y}$ is 0.335. All firms have positive $\kappa_{S,z}$ and $\kappa_{S,y}$. The magnitude of these coefficients is difficult to interpret, and most important, $\kappa_{S,z}$ and $\kappa_{S,y}$ cannot be compared to one another; it is necessary to account for the interplay with the other parameters of the processes.

Along this line, Table 3 reports the percentage of the variation in idiosyncratic variance, $\text{Var}_{P_t}(zS_{t+1} + yS_{t+1})$, that is explained by both measures of commonality introduced in equations (2.4) and (2.6). This pseudo-$R^2$ can be obtained as follows:

$$R^2_{S,\text{pseudo}} = 1 - \sum_t (\kappa_{S,z}h_{M_z,t+1} + \alpha^2_{S,z}h_{M_z,t+1}\kappa_{S,y}h_{M_y,t+1})^2 \left(\sum_t \text{Var}_{P_t}(zS_{t+1} + yS_{t+1})\right)^2.$$ 

For an average firm, the two sources of commonality explain 36.0% of the variation in total idiosyncratic risk. This is in line with the 35% documented in HKLV.

Our methodology allows us to separate the contributions of the two sources of commonality. Table 3 reports that, on average, the commonality in idiosyncratic jump risk (IJR) explains an impressive 50.09% of the time variation in IJR. Not only is this much greater than the analogous average of 33.57% obtained for idiosyncratic normal risk (INR), but the whole distribution of $R^2$s associated with IJR is shifted to the right compared to the distribution for INR. That is, commonality plays a greater role in tail risk than in normal risk for each of the 260 stocks in our sample. Interestingly, the $R^2$s for the total measure of idiosyncratic risk are only marginally improved compared to the $R^2$s for INR. This is not surprising since INR accounts for 90.5% of total idiosyncratic risk of an average firm. Hence, when considering total idiosyncratic risk, the very strong commonality in IJR is masked by the very volatile INR.

### 4.4 On the Importance of Accounting for Equity-Specific Jumps

Financial theory tells us that diversifiable risk should not be priced. In most models, idiosyncratic risk is simply normal risk. As this normal risk should be easily diversified away, idiosyncratic risk should, in theory, not be priced in conditionally normal models. Our results confirm this intuition. However, a number of empirical studies find results that are at odds with ours and with theory.

When one estimates a conditionally normal model on actual returns, the filtered “normal” innov-
tions are all but normal. They typically have a very large kurtosis; in a misspecified normal model, the supposedly normal innovations are also capturing jumps. Given the importance of the premium on these idiosyncratic jumps in our model, we conjecture that, in conditionally normal models, the risk premium on idiosyncratic normal risk originates from the model’s misspecification.

To validate this conjecture, we estimate a nested version of our model in which the market model remains unchanged, but the stock model does not exhibit idiosyncratic jump risk. That is,

\[
\begin{align*}
    r_{S,t+1} & = \mu_{S,t+1} - \xi_{P}^{\varepsilon_{S,t}} + \beta_{S,z} z_{M,t+1} + \beta_{S,y} y_{M,t+1} + z_{S,t+1}, \\
    \mu_{S,t+1} - r_{f,t+1} & = \beta_{S,z} \lambda_{M} h_{M,z,t+1} + \gamma_{M,S} (\beta_{S,y}) h_{M,y,t+1} + \lambda_{S} h_{S,z,t+1}, \\
    \xi_{S,t+1}^{\varepsilon} & = \xi_{\varepsilon_{M}}^{\varepsilon} (\beta_{S,z}) + \xi_{\varepsilon_{M}}^{\varepsilon} (\beta_{S,y}) + \xi_{S,t}^{\varepsilon} (1),
\end{align*}
\]

where market innovations have separate normal variance and jump intensity, as specified in Section 2, and \( z_{S,t+1} \) is simply assumed to be conditionally normal, with GARCH variance as specified in equation (2.4).

Figure 7 shows that the composition of the total risk premium is drastically different once idiosyncratic jumps are neglected. The systematic components are very similar to those reported in Figure 5. The premiums associated with idiosyncratic normal risk, however, are now more extreme than those associated with idiosyncratic jumps in the full model. As illustrated in Figure 8, in the full model, 152 stocks out of 260 have a total equity risk premium (ERP) of between 5% and 10%, and all stocks have a positive ERP. When ignoring idiosyncratic jumps, the dispersion of the ERP increases dramatically, and 12 stocks even appear to have a negative risk premium. Both Christoffersen, Jacobs, and Ornthanalai (2012 – 22.15%, Table 6) and Ornthanalai (2014 – 15.50%, Table 3) document, at the market level, that the equity risk premium levels implied by the conditional normal model are unreasonably high. It appears that ignoring idiosyncratic jump risk also leads to a severe misspecification at the stock level.

Table 8 provides further evidence of this misspecification. In particular, the filtered \( \varepsilon_{S,t} \), which are supposed to be conditionally standard normal innovations under the model of equation (4.27), exhibit levels of excess kurtosis that are much too high. While the theoretical level should be 0, the median level reported in Table 8 is 9.35. In comparison, the corresponding median is 0.93 in Table 5.

In sum, the contrast between the results obtained when considering or neglecting idiosyncratic jumps highlights the importance of accounting for these equity-specific jumps.
4.5 Realized Premium based Portfolios of Stocks

In a typical study of factor models, the market prices associated with the different risk factors are estimated from the panel of returns. Here, they are identified from returns and option prices. A potential issue, if stock and option markets are partly segmented, is that the estimated risk premiums might reflect, for instance, option market makers’ shadow price of equity. While this concern is partly mitigated by the use of stock returns in the joint estimation, it still might affect the estimated magnitude of the premium. This criticism applies to any study using option prices to learn about the equity risk premium or even the physical distribution.36

To assess whether this criticism indeed indicates a weakness of our framework, we analyze portfolios formed according to the model-implied risk premium associated with idiosyncratic jump risk, the IJRP. This analysis will also allow us to alleviate another concern: in our model, we have two systematic factors that are related to the market, but we ignore other popular factors related to, e.g., size, leverage, or momentum. We seek to demonstrate that the returns generated by IJRP-based portfolios are robust to controlling for these factors that are omitted from our option-based analysis.

First, on each day $t$ of the 4,951 days in our sample, we sort available stocks according to the instantaneous expected excess return associated with idiosyncratic jumps, IJRP. We then divide these stocks into five quintile portfolios P1 to P5, sorted from lowest to highest expected return; stocks within a portfolio are weighted according to their market capitalization on day $t$. Finally, we create a long-short portfolio with a long position in the portfolio with the highest expected return, P5, and a short position in that with the lowest expected return, P1. For comparison, in Table 9, the same procedure is used to create long-short portfolios on the basis of idiosyncratic normal risk premiums, INRP.

Table 9 reports regressions of the returns of these long-short portfolios on some of the most prevalent factors in empirical asset pricing. First, the regression labelled FF3 is based on the Fama and French (1993) 3-factor models: market (MKT), small minus big (SMB), and high minus low (HML). The regression labelled FF5 extends the set of regressors to those of the Fama and French (2015) 5-factor model, adding the robust minus weak (RMW) and conservative minus aggressive (CMA) factors. The regression labelled CF4 considers the Carhart (1997) 4-factor model, essentially adding a momentum (MOM) factor to FF3. The regression labelled AHXZ is inspired by the work of Ang, Hodrick, Xing, and Zhang (2006) and extends FF3 by adding the innovation on the CBOE Volatility Index (VIX) to the

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36For instance, Martin (2017) and Martin and Wagner (2016) directly infer the equity risk premium from option prices. Ross (2015) and Jensen, Lando, and Pedersen (2016) infer the $\mathbb{P}$ distribution from the evolution of the $\mathbb{Q}$ distribution.
Finally, we consider a kitchen sink regression, labelled All, in which we control for all the aforementioned factors, that is:

$$R_{it}^{PS-P1} = \alpha + \beta_{MKT}MKT_t + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \beta_{RMW}RMW_t + \beta_{CMA}CMA_t + \beta_{MOM}MOM_t + \beta_{AVIX}AVIX_t + \epsilon_t,$$

where $R_{it}^{PS-P1}$ is the day-$t$ simple excess return on the long-short portfolio formed on the basis of the model-implied idiosyncratic risk premiums, and $AVIX_t = VIX_t - VIX_{t-1}$ is the innovation on the VIX. The alphas of the regressions are reported in annualized percentage terms.

In the first five columns of Table 9, the long-short portfolio is formed according to the quintiles of the INRP. Regression coefficients are starred when they are significant (**: 5% level; *: 10% level); $t$-stats are based on robust Newey and West (1987) standard errors. None of the alphas in these five regressions are significant at the 10% level. This is not surprising, as the INRP is negligible for virtually all stocks (cf. Figure 4 and 5).

In the last five columns of Table 9, the long-short portfolios are based on quintiles of the IJRP. The alphas of the five regressions are highly significant, both statistically and economically. They vary between 8.4% and 17.0% annually, depending on the regression considered. It is worth noting that, while the metric used to sort stocks into portfolios is inferred in part from option prices, the alphas reported here are obtained from trading only stocks. These results thus confirm that our main result, that idiosyncratic jump risk is priced, is not merely an artefact of using options.

Characteristics of the Quintile Portfolios and Double-Sort Portfolios

In our model, the only systematic risk factors are the Gaussian and jump innovations in the market. Accounting for more factors, such as the Fama and French (2015) or Carhart (1997) factors, in the option-pricing model would have required postulating dynamics for each of the factors, entailing the introduction many more parameters. Moreover, as options are not traded on these factors, we would have been forced to estimate these dynamics exclusively from returns, defeating a central objective of this paper.

However, in Table 9, the loadings on these factors are almost all significant. One can thus conjecture that neglecting these factors in the model led to idiosyncratic risk proxies that are partly driven by these
factors. Consistent with this intuition, Table 10 shows that the quintile portfolios obtained based on the idiosyncratic jump risk premium display near monotonic market betas, market capitalizations (ME), investment levels, and trailing 12-month returns excluding the last month’s return (on the basis of which the momentum factor is usually constructed). See Appendix H for further details on these variables.

The alphas in the IJRP regressions reported in Table 9 are significant even after linearly controlling for the factors corresponding to these variables. The patterns observed in Table 10 could nonetheless raise concerns that the alphas are somehow nonlinearly related to the fundamentals of the stocks in the top and bottom quintile portfolios used in the long-short strategy. To alleviate these concerns, we perform a double sort. On each day $t$, we first sort stocks into quintiles (Q1 to Q5) based on their market betas over the past year. Then, within each market-beta quintile, we sort stocks into terciles based on their IJRP. We then take a long position in a cap-weighted portfolio of the top-tercile stocks and a short position in a cap-weighted portfolio of the bottom-tercile stocks. This leaves us with five long-short portfolios, each of which is composed of stocks with homogenous market betas. The first row of Table 11 reports the alphas of regression (4.28) for each of these long-short portfolios. The procedure is repeated for the other six other variables in Table 10.

The alphas on the long-short portfolios constructed based on idiosyncratic jump risk premiums are positive and significant, both statistically and economically, in 32 out of the 35 regressions. When they are not statistically significant, they are still positive and non-trivial; the lack of significance is mainly due to the large standard errors on some of these portfolios. There are no clear patterns in the alphas across the quintiles of most variables, the exception potentially being volatility betas. In sum, no single one of these seven variables appears to be driving the main result of our paper: idiosyncratic jump risk carries a positive risk premium.

5 Conclusion

In this study, we shed new light on the relationship between idiosyncratic risk and equity returns. We develop a model allowing us to disentangle the contributions of four different risk factors to the equity risk premium: systematic and idiosyncratic risk are both decomposed into their normal and jump com-

38 The market and volatility betas here are obtained by performing the following regression:

$$R_{S,t-k} = \alpha + \beta_{MKT,t-k} + \beta_{\Delta VIX,t-k} + \epsilon_{t-k}, \quad k = 0, \ldots, 252.$$ 

This regression corresponds to the pre-formation regression of Ang, Hodrick, Xing, and Zhang (2006) over the past year (rather than month, as in AHXZ) of data.
ponents. Using 20 years of returns and options on the S&P 500 and 260 stocks, we find that normal and jump risk have drastically different impacts on the expected return on individual stocks.

While our pricing kernel is such that each risk factor can potentially be priced, we find that the normal component of idiosyncratic risk, which is easily diversifiable, is not priced after accounting for other sources of risk. Firm-specific jump risk, however, is priced; it explains nearly 30% of the variation in expected excess returns on an average stock. We find the equity risk premium on an average stock to be 6.2%; the magnitude of the IJRP is on average 2.4%. Given the recent conflicting empirical evidence regarding how idiosyncratic risk affects expected returns, these findings might provide new guidance for future studies.

Our focus in this paper is on the relationship between jump risk and the equity risk premium. Given the strong links between the equity risk premium and the variance risk premium, it is natural to wonder whether our findings extend to the variance risk premium; the results of Gourier (2014) certainly suggest that they do. Hence, it appears that properly accounting for jump risk is crucial in any attempt to study the risk premiums associated with idiosyncratic risk.
References


Figure 1: S&P 500 and ATM Implied Volatilities

The upper panel of this figure presents the level of the S&P 500 index between January 1996 and August 2015; gray-shaded regions highlight NBER-dated recessions. The middle panel reports S&P 500 index excess returns over the same period. The lower panel reports the weekly at-the-money implied volatility from the nearest-to-maturity SPX options as extracted from OptionMetrics, along with the average of the weekly at-the-money implied volatility across the firms.
Figure 2: Filtered Innovations and Variances for the Market Model

This figure presents the states for the market model, filtered using the parameters in Table 3. The top panel reports the filtered Gaussian innovations. The middle panel reports the filtered jump components (superposed on returns). The dashed line in the bottom panel reports, in annualized terms, the contribution of the jump component to the returns’ conditional volatility, $\sqrt{252 \frac{\alpha^2 M - \delta^2 M}{\alpha^2 M - \delta^2 M}} h_{M,y,t}$; the solid line reports the total volatility, $\sqrt{252(h_{M,z,t} + \frac{\alpha^2 M - \delta^2 M}{\alpha^2 M - \delta^2 M} h_{M,y,t})}$. 

![Diagram of filtered innovations and variances for the market model.](image)
Figure 3: Annualized Normal and Jump Risk Premiums for the Market Model
The top panel of this figure reports the annualized equity risk premium, $252(\lambda M_{hM} + \gamma M_{hM,J})$ and the component due to jump risk, $252\gamma M_{hM,J}$. The lower panel reports, on a daily basis, the proportion of the total premium explained by the jump component.

Figure 4: Decomposition of the Equity Risk Premium on Stocks: Time-Series of Cross-Sectional Averages
The top panel of this figure presents, at each point in time, the cap-weighted average of (i) systematic risk premiums and (ii) the absolute value of the idiosyncratic risk premiums. The bottom panel describes the distribution of idiosyncratic jump risk premiums (IJRP) over time. The average IJRP is cap-weighted.
Figure 5: Decomposition of the Equity Risk Premium on Stocks: Cross-Section of Time-Series Averages

This figure presents, for each of the 260 stocks, the decomposition of the equity risk premium in terms of the premiums associated with the four different risk factors in the model: (i) systematic normal, (ii) systematic jump, (iii) idiosyncratic normal, and (iv) idiosyncratic jump. On average, the premium on (i) systematic normal risk is 2.0%, (ii) systematic jump risk, 3.9%, and (iii) the idiosyncratic jump risk premium as an average magnitude of 2.4%. Firms are grouped by industry, based on the Global Industry Classification Standard (GICS). Results for telecommunication services and utilities are not reported since they concern only five firms.
Figure 6: Time Series Decomposition of the Average Equity Risk Premium by Industry

This figure presents, for each of the eight largest GICS industries covered by our sample, the evolution over time of (i) the component of the industry’s average equity risk premium that is due to exposure to idiosyncratic jumps (solid line) and (ii) the premium arising from the firms’ exposure to systematic risk factors (grey ‘+’ marks).
This figure presents, for each of the 260 stocks, the decomposition of the equity risk premium in terms of the premiums associated with the three different risk factors in the nested model of equation (4.27): (i) systematic normal, (ii) systematic jump, and (iii) idiosyncratic normal. On average, the premium on (i) systematic normal risk is 1.8%, (ii) systematic jump risk, 4.5%, and (iii) the idiosyncratic normal risk premium as an average magnitude of 5.8%. Firms are grouped by industry, based on the Global Industry Classification Standard (GICS). All x-axes report premiums in annualized percentage terms. Results for telecommunication services and utilities are not reported since they concern only five firms. The results of 11 firms are truncated for the sake of clarity: Boeing (BA), -10.9; Biogen (BIIB), -14.6; Chipotle Mexican Grill (CMG), -12.2; Compaq Computer (CPQ), -30.0; DuPont (DD), -14.5; Danaher (DHR), -24.0; Fannie Mae (FNMA), 43.0; Freddie Mac (FRE), 33.5; Keuring Green Mountain (GMCR), 82.4; Jacobs Engineering Group (JEC), -14.0; Ralph Lauren (RL), 38.1;
Figure 8: Comparison of the Equity Risk Premium by Firm: With and Without Idiosyncratic Jumps

This figure presents the distribution of the equity risk premium on the 260 in our sample for the full model (on the left) and the nested model in which there are no idiosyncratic jumps (on the right).
Figure 9: Stock Characteristics and the Idiosyncratic Jump Risk Premium

For each graph in this figure, stocks are divided into quintiles based on a given characteristic. Each graph then presents, in its top part, the value-weighted cross-sectional average, over time, of the idiosyncratic jump risk premium for the lowest and highest quintile. The bottom part of each graph reports the high-minus-low difference.
Table 1: Description of the SPX Index Option Data (1996-2015).

Moneyness is defined as $K/F$, where $F$ is the forward price of the index and $K$ is the option’s strike price. DTM stands for days to maturity. Our final option dataset contains 44,267 observations.

Panel A: Number of option contracts.

<table>
<thead>
<tr>
<th></th>
<th>DTM $\leq$ 30</th>
<th>30 $&lt;$ DTM $\leq$ 90</th>
<th>90 $&lt;$ DTM $\leq$ 180</th>
<th>180 $&lt;$ DTM $\leq$ 250</th>
<th>DTM $&gt; 250$</th>
<th>All</th>
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</thead>
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<td>$0.80 &lt; K/F \leq 0.85$</td>
<td>256</td>
<td>1,666</td>
<td>1,046</td>
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<td>487</td>
<td>4,170</td>
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<tr>
<td>$0.85 &lt; K/F \leq 0.90$</td>
<td>505</td>
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<td>1,410</td>
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<td>828</td>
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<td>$0.95 &lt; K/F \leq 1.00$</td>
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<td>$1.15 &lt; K/F \leq 1.20$</td>
<td>18</td>
<td>288</td>
<td>380</td>
<td>373</td>
<td>287</td>
<td>1,346</td>
</tr>
<tr>
<td>All</td>
<td>5,647</td>
<td>16,461</td>
<td>10,020</td>
<td>6,806</td>
<td>5,333</td>
<td>44,267</td>
</tr>
</tbody>
</table>

Panel B: Average option prices.

<table>
<thead>
<tr>
<th></th>
<th>DTM $\leq$ 30</th>
<th>30 $&lt;$ DTM $\leq$ 90</th>
<th>90 $&lt;$ DTM $\leq$ 180</th>
<th>180 $&lt;$ DTM $\leq$ 250</th>
<th>DTM $&gt; 250$</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.80 &lt; K/F \leq 0.85$</td>
<td>1.19</td>
<td>4.18</td>
<td>12.00</td>
<td>21.45</td>
<td>29.92</td>
<td>11.92</td>
</tr>
<tr>
<td>$0.85 &lt; K/F \leq 0.90$</td>
<td>1.61</td>
<td>7.06</td>
<td>18.45</td>
<td>43.18</td>
<td>55.69</td>
<td>17.26</td>
</tr>
<tr>
<td>$0.90 &lt; K/F \leq 0.95$</td>
<td>2.93</td>
<td>13.05</td>
<td>27.94</td>
<td>62.61</td>
<td>78.04</td>
<td>25.69</td>
</tr>
<tr>
<td>$0.95 &lt; K/F \leq 1.00$</td>
<td>8.71</td>
<td>26.05</td>
<td>45.48</td>
<td>80.04</td>
<td>10.695</td>
<td>39.19</td>
</tr>
<tr>
<td>$1.00 &lt; K/F \leq 1.05$</td>
<td>7.29</td>
<td>19.61</td>
<td>38.88</td>
<td>60.88</td>
<td>77.47</td>
<td>30.17</td>
</tr>
<tr>
<td>$1.05 &lt; K/F \leq 1.10$</td>
<td>1.92</td>
<td>6.78</td>
<td>16.99</td>
<td>31.11</td>
<td>44.74</td>
<td>17.12</td>
</tr>
<tr>
<td>$1.10 &lt; K/F \leq 1.15$</td>
<td>1.34</td>
<td>3.68</td>
<td>9.16</td>
<td>18.44</td>
<td>28.45</td>
<td>12.32</td>
</tr>
<tr>
<td>$1.15 &lt; K/F \leq 1.20$</td>
<td>1.22</td>
<td>3.17</td>
<td>6.17</td>
<td>10.57</td>
<td>17.08</td>
<td>9.00</td>
</tr>
<tr>
<td>All</td>
<td>5.65</td>
<td>13.50</td>
<td>25.84</td>
<td>39.90</td>
<td>53.11</td>
<td>24.12</td>
</tr>
</tbody>
</table>

Panel C: Average implied volatility.

<table>
<thead>
<tr>
<th></th>
<th>DTM $\leq$ 30</th>
<th>30 $&lt;$ DTM $\leq$ 90</th>
<th>90 $&lt;$ DTM $\leq$ 180</th>
<th>180 $&lt;$ DTM $\leq$ 250</th>
<th>DTM $&gt; 250$</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.80 &lt; K/F \leq 0.85$</td>
<td>0.4123</td>
<td>0.3094</td>
<td>0.2781</td>
<td>0.2586</td>
<td>0.2478</td>
<td>0.2920</td>
</tr>
<tr>
<td>$0.85 &lt; K/F \leq 0.90$</td>
<td>0.3252</td>
<td>0.2676</td>
<td>0.2509</td>
<td>0.2364</td>
<td>0.2294</td>
<td>0.2594</td>
</tr>
<tr>
<td>$0.90 &lt; K/F \leq 0.95$</td>
<td>0.2515</td>
<td>0.2320</td>
<td>0.2117</td>
<td>0.2186</td>
<td>0.2172</td>
<td>0.2290</td>
</tr>
<tr>
<td>$0.95 &lt; K/F \leq 1.00$</td>
<td>0.1830</td>
<td>0.1968</td>
<td>0.2049</td>
<td>0.2024</td>
<td>0.2018</td>
<td>0.1976</td>
</tr>
<tr>
<td>$1.00 &lt; K/F \leq 1.05$</td>
<td>0.1482</td>
<td>0.1574</td>
<td>0.1720</td>
<td>0.1844</td>
<td>0.1866</td>
<td>0.1636</td>
</tr>
<tr>
<td>$1.05 &lt; K/F \leq 1.10$</td>
<td>0.1910</td>
<td>0.1595</td>
<td>0.1609</td>
<td>0.1669</td>
<td>0.1701</td>
<td>0.1647</td>
</tr>
<tr>
<td>$1.10 &lt; K/F \leq 1.15$</td>
<td>0.2757</td>
<td>0.1879</td>
<td>0.1691</td>
<td>0.1679</td>
<td>0.1696</td>
<td>0.1778</td>
</tr>
<tr>
<td>$1.15 &lt; K/F \leq 1.20$</td>
<td>0.3725</td>
<td>0.2367</td>
<td>0.1884</td>
<td>0.1687</td>
<td>0.1657</td>
<td>0.1909</td>
</tr>
<tr>
<td>All</td>
<td>0.2046</td>
<td>0.2057</td>
<td>0.2043</td>
<td>0.2024</td>
<td>0.1996</td>
<td>0.2040</td>
</tr>
</tbody>
</table>
Table 2: Description of Firms Option Data (1996-2015).

Moneyness is defined as $K/F$, where $F$ is the forward price of the underlying and $K$ is the option’s strike price. DTM stands for days to maturity. Our final option dataset contains 2,962,621 observations.

Panel A: Number of option contracts.

<table>
<thead>
<tr>
<th></th>
<th>DTM ≤ 30</th>
<th>30 &lt; DTM ≤ 90</th>
<th>90 &lt; DTM ≤ 180</th>
<th>180 &lt; DTM ≤ 250</th>
<th>DTM &gt; 250</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80 &lt; $K/F$ ≤ 0.85</td>
<td>6,791</td>
<td>53,859</td>
<td>75,543</td>
<td>46,305</td>
<td>14,036</td>
<td>196,534</td>
</tr>
<tr>
<td>0.85 &lt; $K/F$ ≤ 0.90</td>
<td>16,416</td>
<td>100,294</td>
<td>101,743</td>
<td>56,700</td>
<td>15,120</td>
<td>290,273</td>
</tr>
<tr>
<td>0.90 &lt; $K/F$ ≤ 0.95</td>
<td>42,747</td>
<td>168,293</td>
<td>120,217</td>
<td>61,422</td>
<td>15,549</td>
<td>408,228</td>
</tr>
<tr>
<td>0.95 &lt; $K/F$ ≤ 1.00</td>
<td>100,267</td>
<td>216,362</td>
<td>123,431</td>
<td>60,425</td>
<td>14,575</td>
<td>515,060</td>
</tr>
<tr>
<td>1.00 &lt; $K/F$ ≤ 1.05</td>
<td>103,111</td>
<td>245,827</td>
<td>160,306</td>
<td>85,404</td>
<td>20,169</td>
<td>614,817</td>
</tr>
<tr>
<td>1.05 &lt; $K/F$ ≤ 1.10</td>
<td>36,508</td>
<td>169,034</td>
<td>146,104</td>
<td>80,884</td>
<td>19,519</td>
<td>452,049</td>
</tr>
<tr>
<td>1.10 &lt; $K/F$ ≤ 1.15</td>
<td>13,119</td>
<td>90,264</td>
<td>110,095</td>
<td>67,035</td>
<td>17,543</td>
<td>298,056</td>
</tr>
<tr>
<td>1.15 &lt; $K/F$ ≤ 1.20</td>
<td>5,390</td>
<td>47,073</td>
<td>73,054</td>
<td>47,754</td>
<td>14,333</td>
<td>187,604</td>
</tr>
<tr>
<td>All</td>
<td>324,349</td>
<td>1,091,006</td>
<td>910,493</td>
<td>505,929</td>
<td>130,844</td>
<td>2,962,621</td>
</tr>
</tbody>
</table>

Panel B: Average option prices.

<table>
<thead>
<tr>
<th></th>
<th>DTM ≤ 30</th>
<th>30 &lt; DTM ≤ 90</th>
<th>90 &lt; DTM ≤ 180</th>
<th>180 &lt; DTM ≤ 250</th>
<th>DTM &gt; 250</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80 &lt; $K/F$ ≤ 0.85</td>
<td>1.04</td>
<td>1.42</td>
<td>2.16</td>
<td>2.96</td>
<td>4.05</td>
<td>2.24</td>
</tr>
<tr>
<td>0.85 &lt; $K/F$ ≤ 0.90</td>
<td>1.06</td>
<td>1.54</td>
<td>2.56</td>
<td>3.55</td>
<td>5.00</td>
<td>2.44</td>
</tr>
<tr>
<td>0.90 &lt; $K/F$ ≤ 0.95</td>
<td>1.12</td>
<td>1.79</td>
<td>3.26</td>
<td>4.52</td>
<td>6.22</td>
<td>2.73</td>
</tr>
<tr>
<td>0.95 &lt; $K/F$ ≤ 1.00</td>
<td>1.45</td>
<td>2.52</td>
<td>4.48</td>
<td>5.98</td>
<td>8.11</td>
<td>3.34</td>
</tr>
<tr>
<td>1.00 &lt; $K/F$ ≤ 1.05</td>
<td>1.47</td>
<td>2.50</td>
<td>4.36</td>
<td>5.85</td>
<td>8.03</td>
<td>3.46</td>
</tr>
<tr>
<td>1.05 &lt; $K/F$ ≤ 1.10</td>
<td>1.16</td>
<td>1.79</td>
<td>3.08</td>
<td>4.21</td>
<td>6.03</td>
<td>2.77</td>
</tr>
<tr>
<td>1.10 &lt; $K/F$ ≤ 1.15</td>
<td>1.14</td>
<td>1.61</td>
<td>2.51</td>
<td>3.34</td>
<td>4.77</td>
<td>2.50</td>
</tr>
<tr>
<td>1.15 &lt; $K/F$ ≤ 1.20</td>
<td>1.16</td>
<td>1.53</td>
<td>2.26</td>
<td>2.98</td>
<td>4.03</td>
<td>2.36</td>
</tr>
<tr>
<td>All</td>
<td>1.33</td>
<td>2.03</td>
<td>3.25</td>
<td>4.31</td>
<td>5.87</td>
<td>2.89</td>
</tr>
</tbody>
</table>

Panel C: Average implied volatility.

<table>
<thead>
<tr>
<th></th>
<th>DTM ≤ 30</th>
<th>30 &lt; DTM ≤ 90</th>
<th>90 &lt; DTM ≤ 180</th>
<th>180 &lt; DTM ≤ 250</th>
<th>DTM &gt; 250</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80 &lt; $K/F$ ≤ 0.85</td>
<td>0.7919</td>
<td>0.5436</td>
<td>0.4274</td>
<td>0.3872</td>
<td>0.3515</td>
<td>0.4569</td>
</tr>
<tr>
<td>0.85 &lt; $K/F$ ≤ 0.90</td>
<td>0.6392</td>
<td>0.4623</td>
<td>0.3851</td>
<td>0.3585</td>
<td>0.3352</td>
<td>0.4183</td>
</tr>
<tr>
<td>0.90 &lt; $K/F$ ≤ 0.95</td>
<td>0.4903</td>
<td>0.3869</td>
<td>0.3537</td>
<td>0.3398</td>
<td>0.3238</td>
<td>0.3785</td>
</tr>
<tr>
<td>0.95 &lt; $K/F$ ≤ 1.00</td>
<td>0.3618</td>
<td>0.3388</td>
<td>0.3386</td>
<td>0.3318</td>
<td>0.3169</td>
<td>0.3418</td>
</tr>
<tr>
<td>1.00 &lt; $K/F$ ≤ 1.05</td>
<td>0.3504</td>
<td>0.3258</td>
<td>0.3214</td>
<td>0.3162</td>
<td>0.3066</td>
<td>0.3268</td>
</tr>
<tr>
<td>1.05 &lt; $K/F$ ≤ 1.10</td>
<td>0.4675</td>
<td>0.3617</td>
<td>0.3217</td>
<td>0.3092</td>
<td>0.3017</td>
<td>0.3453</td>
</tr>
<tr>
<td>1.10 &lt; $K/F$ ≤ 1.15</td>
<td>0.5917</td>
<td>0.4244</td>
<td>0.3435</td>
<td>0.3168</td>
<td>0.2993</td>
<td>0.3703</td>
</tr>
<tr>
<td>1.15 &lt; $K/F$ ≤ 1.20</td>
<td>0.7147</td>
<td>0.4897</td>
<td>0.3802</td>
<td>0.3407</td>
<td>0.3072</td>
<td>0.4017</td>
</tr>
<tr>
<td>All</td>
<td>0.4252</td>
<td>0.3819</td>
<td>0.3514</td>
<td>0.3334</td>
<td>0.3163</td>
<td>0.3661</td>
</tr>
</tbody>
</table>
Table 3: Index Parameters Estimated Using Returns and Option Data

The index parameters are estimated using daily index returns and weekly cross-sections of out-of-the-money options, from January 1996 to August 2015. Parameters are estimated using multiple simplex search method optimizations (fminsearch in Matlab). Robust standard errors are calculated from the outer product of the gradient at the optimal parameter values. The average volatilities of volatilities are annualized and given in percentages. They are computed by multiplying the square root of the average of $\text{Var}_{t-1} [h_{M,t+1}]$ by $100 \times 252^{3/2}$.

<table>
<thead>
<tr>
<th></th>
<th>Normal</th>
<th>Jump</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_M / \gamma_M$</td>
<td>0.824</td>
<td>0.701</td>
</tr>
<tr>
<td></td>
<td>(2.40E-10)</td>
<td>(3.55E-10)</td>
</tr>
<tr>
<td>$w_{M,t} / w_{M,t-1}$</td>
<td>-1.63E-06</td>
<td>-4.05E-07</td>
</tr>
<tr>
<td></td>
<td>(2.50E-08)</td>
<td>(1.88E-08)</td>
</tr>
<tr>
<td>$a_{M,t} / a_{M,t-1}$</td>
<td>2.42E-06</td>
<td>4.44E-06</td>
</tr>
<tr>
<td></td>
<td>(3.36E-09)</td>
<td>(7.83E-09)</td>
</tr>
<tr>
<td>$b_{M,t} / b_{M,t-1}$</td>
<td>0.940</td>
<td>0.934</td>
</tr>
<tr>
<td></td>
<td>(6.94E-10)</td>
<td>(4.58E-09)</td>
</tr>
<tr>
<td>$c_{M,t} / c_{M,t-1}$</td>
<td>144.19</td>
<td>140.50</td>
</tr>
<tr>
<td></td>
<td>(2.86E-11)</td>
<td>(1.30E-11)</td>
</tr>
<tr>
<td>$\alpha_M$</td>
<td>11.856</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.36E-09)</td>
<td></td>
</tr>
<tr>
<td>$\delta_M$</td>
<td>-7.018</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.01E-10)</td>
<td></td>
</tr>
</tbody>
</table>

Skewness of innovations, $e_{M,t}$: -0.12  
Ex. kurtosis of innovations, $e_{M,t}$: -0.07

Percent of annual variance: 74.0 26.0
Average volatility:
Total: 15.6  9.0
Average risk premium:
Total: 2.3  3.9
Median risk premium:
Total: 1.7  3.0

Return log-likelihood: 78395.3
Option log-likelihood: 12787.7
Total log-likelihood: 91183.0
RIVRMSE: 14.4
Table 4: Valuation Errors on the Options Used for the Estimation
We use the joint MLE estimates from Tables 3 and 5 to compute implied volatility root mean squared errors (IVRMSquares) and relative implied volatility root mean squared errors (RIVRMSquares) for various moneyness, maturity, and year bins. We then average IVRMSquares and RIVRMSquares for each moneyness, maturity and year bin across firms. IVRMSquares and RIVRMSquares are given in percentages.

Panel A: Valuation Errors on the Options Used for the Estimation of the Market Model

<table>
<thead>
<tr>
<th>Overall IVRMSquare and RIVRMSquare</th>
<th>Sorting by year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IVRMSquare</td>
</tr>
<tr>
<td>All</td>
<td>3.086</td>
</tr>
<tr>
<td>Sorting by maturity</td>
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</tr>
<tr>
<td>DTM ≤ 30</td>
<td>3.364</td>
</tr>
<tr>
<td>30 &lt; DTM ≤ 90</td>
<td>3.108</td>
</tr>
<tr>
<td>90 &lt; DTM ≤ 180</td>
<td>2.936</td>
</tr>
<tr>
<td>180 &lt; DTM ≤ 270</td>
<td>3.016</td>
</tr>
<tr>
<td>270 &lt; DTM ≤ 365</td>
<td>3.076</td>
</tr>
<tr>
<td>Sorting by moneyness</td>
<td></td>
</tr>
<tr>
<td>0.80 &lt; K/F ≤ 0.85</td>
<td>3.815</td>
</tr>
<tr>
<td>0.85 &lt; K/F ≤ 0.90</td>
<td>3.533</td>
</tr>
<tr>
<td>0.90 &lt; K/F ≤ 0.95</td>
<td>3.334</td>
</tr>
<tr>
<td>0.95 &lt; K/F ≤ 1.00</td>
<td>3.006</td>
</tr>
<tr>
<td>1.00 &lt; K/F ≤ 1.05</td>
<td>2.684</td>
</tr>
<tr>
<td>1.05 &lt; K/F ≤ 1.10</td>
<td>2.729</td>
</tr>
<tr>
<td>1.10 &lt; K/F ≤ 1.15</td>
<td>2.808</td>
</tr>
<tr>
<td>1.15 &lt; K/F ≤ 1.20</td>
<td>3.037</td>
</tr>
</tbody>
</table>

Panel B: Average Valuation Errors on the Options Used for the Estimation of the Firm Model

<table>
<thead>
<tr>
<th>Overall average IVRMSquare and RIVRMSquare</th>
<th>Sorting by year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IVRMSquare</td>
</tr>
<tr>
<td>All</td>
<td>7.355</td>
</tr>
<tr>
<td>Sorting by maturity</td>
<td></td>
</tr>
<tr>
<td>DTM ≤ 30</td>
<td>9.799</td>
</tr>
<tr>
<td>30 &lt; DTM ≤ 90</td>
<td>7.223</td>
</tr>
<tr>
<td>90 &lt; DTM ≤ 180</td>
<td>6.590</td>
</tr>
<tr>
<td>180 &lt; DTM ≤ 270</td>
<td>6.762</td>
</tr>
<tr>
<td>270 &lt; DTM ≤ 365</td>
<td>7.032</td>
</tr>
<tr>
<td>Sorting by moneyness</td>
<td></td>
</tr>
<tr>
<td>0.80 &lt; K/F ≤ 0.85</td>
<td>9.957</td>
</tr>
<tr>
<td>0.85 &lt; K/F ≤ 0.90</td>
<td>8.494</td>
</tr>
<tr>
<td>0.90 &lt; K/F ≤ 0.95</td>
<td>7.179</td>
</tr>
<tr>
<td>0.95 &lt; K/F ≤ 1.00</td>
<td>6.523</td>
</tr>
<tr>
<td>1.00 &lt; K/F ≤ 1.05</td>
<td>6.127</td>
</tr>
<tr>
<td>1.05 &lt; K/F ≤ 1.10</td>
<td>6.667</td>
</tr>
<tr>
<td>1.10 &lt; K/F ≤ 1.15</td>
<td>7.805</td>
</tr>
<tr>
<td>1.15 &lt; K/F ≤ 1.20</td>
<td>9.104</td>
</tr>
</tbody>
</table>
Table 5: Firm Parameters Estimated Using Returns and Option Data

Parameters for each stock are estimated using daily index returns and available weekly cross-sections of out-of-the-money options, from January 1996 to August 2015. This table reports statistics on the cross-section of joint MLE estimates obtained for the 260 stocks in our sample. The table further documents the distribution of model-implied measures of volatility, risk premiums, and fit to the data. Q1 and Q3 report the 25th and 75th percentiles of the estimates.

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>S.D.</th>
<th>Min</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{S,z}$</td>
<td>0.931</td>
<td>0.277</td>
<td>0.320</td>
<td>0.730</td>
<td>0.935</td>
<td>1.108</td>
<td>1.752</td>
</tr>
<tr>
<td>$\beta_{S,y}$</td>
<td>1.111</td>
<td>0.412</td>
<td>0.206</td>
<td>0.831</td>
<td>1.050</td>
<td>1.315</td>
<td>3.928</td>
</tr>
<tr>
<td>$\lambda_y$</td>
<td>0.000</td>
<td>0.002</td>
<td>-0.017</td>
<td>-0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.012</td>
</tr>
<tr>
<td>$\kappa_{z}$</td>
<td>0.863</td>
<td>0.686</td>
<td>0.057</td>
<td>0.402</td>
<td>0.670</td>
<td>1.110</td>
<td>3.939</td>
</tr>
<tr>
<td>$a_{z} \times 10^6$</td>
<td>1.920</td>
<td>1.344</td>
<td>0.113</td>
<td>1.007</td>
<td>1.574</td>
<td>2.473</td>
<td>9.652</td>
</tr>
<tr>
<td>$b_{z}$</td>
<td>0.992</td>
<td>0.003</td>
<td>0.978</td>
<td>0.990</td>
<td>0.992</td>
<td>0.994</td>
<td>1.000</td>
</tr>
<tr>
<td>$c_{z}$</td>
<td>97.56</td>
<td>47.14</td>
<td>-87.97</td>
<td>60.81</td>
<td>104.77</td>
<td>133.80</td>
<td>209.16</td>
</tr>
<tr>
<td>$\kappa_{y}$</td>
<td>0.335</td>
<td>0.321</td>
<td>0.091</td>
<td>0.103</td>
<td>0.217</td>
<td>0.460</td>
<td>2.201</td>
</tr>
<tr>
<td>$a_{y} \times 10^6$</td>
<td>4.301</td>
<td>3.044</td>
<td>0.454</td>
<td>1.995</td>
<td>3.929</td>
<td>5.939</td>
<td>28.221</td>
</tr>
<tr>
<td>$b_{y}$</td>
<td>0.930</td>
<td>0.059</td>
<td>0.292</td>
<td>0.909</td>
<td>0.935</td>
<td>0.965</td>
<td>0.996</td>
</tr>
<tr>
<td>$c_{y}$</td>
<td>119.49</td>
<td>52.56</td>
<td>-262.69</td>
<td>101.52</td>
<td>126.53</td>
<td>141.30</td>
<td>381.37</td>
</tr>
<tr>
<td>$\alpha_{S}$</td>
<td>10.655</td>
<td>4.704</td>
<td>0.764</td>
<td>8.379</td>
<td>10.705</td>
<td>12.398</td>
<td>43.839</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-6.263</td>
<td>2.200</td>
<td>-15.334</td>
<td>-7.528</td>
<td>-6.517</td>
<td>-5.243</td>
<td>-0.332</td>
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</tbody>
</table>

Proportion of variation in $Y$ explain by $X$ ($Y \sim X$):

<table>
<thead>
<tr>
<th>Var</th>
<th>$\varepsilon_{S,z,t+1}$ ~ Common idio. normal risk (%)</th>
<th>33.57</th>
<th>18.47</th>
<th>0.65</th>
<th>18.36</th>
<th>31.75</th>
<th>48.28</th>
<th>73.23</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var</td>
<td>$\varepsilon_{y,S,t+1}$ ~ Common idio. jump risk (%)</td>
<td>50.09</td>
<td>21.31</td>
<td>6.67</td>
<td>32.75</td>
<td>46.08</td>
<td>67.41</td>
<td>99.47</td>
</tr>
<tr>
<td>Var</td>
<td>$\varepsilon_{z,S,t+1} + \varepsilon_{y,S,t+1}$ ~ Common idiosyncratic risk (%)</td>
<td>36.02</td>
<td>17.59</td>
<td>2.07</td>
<td>21.83</td>
<td>34.22</td>
<td>50.19</td>
<td>75.09</td>
</tr>
<tr>
<td>ERP</td>
<td>~ IJRP (%)</td>
<td>28.17</td>
<td>19.93</td>
<td>0.62</td>
<td>12.01</td>
<td>22.79</td>
<td>40.06</td>
<td>84.42</td>
</tr>
</tbody>
</table>

Ex. kurtosis of Gaussian innovations, $\epsilon_{S,z}$

| Ex. kurtosis | $\epsilon_{S,z}$ | 0.11 | 0.28 | -2.48 | 0.07 | 0.13 | 0.19 | 1.44  |

Skewness of Gaussian innovations, $\epsilon_{S,z}$

| Skewness | $\epsilon_{S,z}$ | 1.52 | 4.21 | -0.08 | 0.65 | 0.93 | 1.20 | 58.99 |

RIVRMSE

| RIVRMSE | 17.24 | 5.61 | 9.50 | 13.43 | 16.38 | 19.74 | 59.23  |

Distribution of 4,951 weighted cross-sectional average:

<table>
<thead>
<tr>
<th>Total volatility (%)</th>
<th>31.80</th>
<th>8.48</th>
<th>18.26</th>
<th>24.94</th>
<th>29.58</th>
<th>37.08</th>
<th>62.54</th>
</tr>
</thead>
<tbody>
<tr>
<td>Idio. variance / Total variance (%)</td>
<td>69.23</td>
<td>8.62</td>
<td>47.45</td>
<td>62.98</td>
<td>70.26</td>
<td>75.58</td>
<td>90.39</td>
</tr>
<tr>
<td>Idio. jump variance / Idio. variance (%)</td>
<td>9.50</td>
<td>1.18</td>
<td>6.05</td>
<td>8.62</td>
<td>9.49</td>
<td>10.35</td>
<td>13.66</td>
</tr>
<tr>
<td>Absolute value of INRP (%)</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Absolute value of IJRP (%)</td>
<td>2.44</td>
<td>1.09</td>
<td>0.92</td>
<td>1.71</td>
<td>2.12</td>
<td>2.82</td>
<td>8.90</td>
</tr>
</tbody>
</table>
Table 6: Correlation Between the Parameters of Stocks

This table reports the correlation between a subset of the parameters associated with the 260 stocks under consideration. IVAR stands for idiosyncratic variance. SNRP, SJRP, IJRP are systematic (S) or idiosyncratic (I) risk premiums associated with normal (N) or jump (J) risk. Correlations between quantities that are available in both the time-series and cross-section are computed across the whole sample, whereas in the other cases, we use firm-by-firm averages.

<table>
<thead>
<tr>
<th></th>
<th>$\beta_{S,J}$</th>
<th>$\kappa_{S,J}$</th>
<th>$\gamma_{S,J}$</th>
<th>IVAR</th>
<th>SNRP</th>
<th>SJRP</th>
<th>IJRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{S,J}$</td>
<td>0.237</td>
<td>0.023</td>
<td>0.164</td>
<td>0.228</td>
<td>0.218</td>
<td>0.987</td>
<td>0.236</td>
</tr>
<tr>
<td>$\beta_{S,J}$</td>
<td>0.128</td>
<td>0.008</td>
<td>0.139</td>
<td>0.046</td>
<td>0.230</td>
<td>0.981</td>
<td>0.116</td>
</tr>
<tr>
<td>$\kappa_{S,J}$</td>
<td>0.511</td>
<td>0.477</td>
<td>0.277</td>
<td>0.037</td>
<td>0.134</td>
<td>0.435</td>
<td></td>
</tr>
<tr>
<td>$\kappa_{S,J}$</td>
<td>0.498</td>
<td>0.215</td>
<td>0.183</td>
<td>0.008</td>
<td>0.618</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{S,J}$</td>
<td>0.107</td>
<td>0.085</td>
<td>0.073</td>
<td>0.203</td>
<td>0.085</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IVAR</td>
<td></td>
<td>0.203</td>
<td></td>
<td></td>
<td>0.146</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SNRP</td>
<td></td>
<td></td>
<td>0.859</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SJRP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7: Firm Parameters Estimated Using Returns and Option Data for each of the eight largest GICS industries

The index parameters are estimated using daily index returns and weekly cross-sections of out-of-the-money options, from January 1996 to August 2015. Parameters are estimated using multiple simplex search method optimizations (fminsearch in Matlab). Robust standard errors are calculated from the outer product of the gradient at the optimal parameter values. For firms, we report statistics on the joint MLE estimates obtained for the 260 individual stocks in our sample across the eight largest GICS industries covered in our sample. Q1 and Q3 report the 25th and 75th percentiles of the estimates.

<table>
<thead>
<tr>
<th>Industry</th>
<th>Average</th>
<th>S.D.</th>
<th>Min</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumer Discretionary (based on 42 firms)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{S_z}$</td>
<td>0.966</td>
<td>0.216</td>
<td>0.595</td>
<td>0.813</td>
<td>0.951</td>
<td>1.086</td>
<td>1.402</td>
</tr>
<tr>
<td>$\beta_{S_y}$</td>
<td>1.071</td>
<td>0.311</td>
<td>0.485</td>
<td>0.817</td>
<td>1.043</td>
<td>1.293</td>
<td>1.937</td>
</tr>
<tr>
<td>$\kappa_{S_z}$</td>
<td>0.958</td>
<td>0.572</td>
<td>0.194</td>
<td>0.599</td>
<td>0.855</td>
<td>1.305</td>
<td>2.631</td>
</tr>
<tr>
<td>$\kappa_{S_y}$</td>
<td>0.404</td>
<td>0.381</td>
<td>0.091</td>
<td>0.106</td>
<td>0.337</td>
<td>0.584</td>
<td>2.201</td>
</tr>
<tr>
<td><strong>Consumer Staples (based on 16 firms)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{S_z}$</td>
<td>0.610</td>
<td>0.184</td>
<td>0.320</td>
<td>0.509</td>
<td>0.604</td>
<td>0.764</td>
<td>0.943</td>
</tr>
<tr>
<td>$\beta_{S_y}$</td>
<td>0.827</td>
<td>0.212</td>
<td>0.594</td>
<td>0.681</td>
<td>0.770</td>
<td>0.893</td>
<td>1.364</td>
</tr>
<tr>
<td>$\kappa_{S_z}$</td>
<td>0.650</td>
<td>0.842</td>
<td>0.151</td>
<td>0.320</td>
<td>0.416</td>
<td>0.655</td>
<td>3.720</td>
</tr>
<tr>
<td>$\kappa_{S_y}$</td>
<td>0.167</td>
<td>0.123</td>
<td>0.091</td>
<td>0.091</td>
<td>0.104</td>
<td>0.198</td>
<td>0.540</td>
</tr>
<tr>
<td><strong>Energy (based on 32 firms)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{S_z}$</td>
<td>0.760</td>
<td>0.241</td>
<td>0.379</td>
<td>0.616</td>
<td>0.711</td>
<td>0.867</td>
<td>1.483</td>
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<tr>
<td>$\beta_{S_y}$</td>
<td>1.357</td>
<td>0.461</td>
<td>0.444</td>
<td>1.050</td>
<td>1.311</td>
<td>1.621</td>
<td>2.503</td>
</tr>
<tr>
<td>$\kappa_{S_z}$</td>
<td>1.352</td>
<td>0.677</td>
<td>0.247</td>
<td>0.919</td>
<td>1.304</td>
<td>1.702</td>
<td>3.367</td>
</tr>
<tr>
<td>$\kappa_{S_y}$</td>
<td>0.342</td>
<td>0.303</td>
<td>0.091</td>
<td>0.108</td>
<td>0.215</td>
<td>0.554</td>
<td>1.238</td>
</tr>
<tr>
<td><strong>Financials (based on 36 firms)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{S_z}$</td>
<td>1.025</td>
<td>0.249</td>
<td>0.514</td>
<td>0.831</td>
<td>1.011</td>
<td>1.192</td>
<td>1.752</td>
</tr>
<tr>
<td>$\beta_{S_y}$</td>
<td>1.203</td>
<td>0.567</td>
<td>0.548</td>
<td>0.890</td>
<td>1.140</td>
<td>1.390</td>
<td>3.928</td>
</tr>
<tr>
<td>$\kappa_{S_z}$</td>
<td>0.576</td>
<td>0.303</td>
<td>0.057</td>
<td>0.404</td>
<td>0.520</td>
<td>0.724</td>
<td>1.385</td>
</tr>
<tr>
<td>$\kappa_{S_y}$</td>
<td>0.198</td>
<td>0.157</td>
<td>0.091</td>
<td>0.098</td>
<td>0.123</td>
<td>0.223</td>
<td>0.852</td>
</tr>
<tr>
<td><strong>Health Care (based on 30 firms)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{S_z}$</td>
<td>0.788</td>
<td>0.260</td>
<td>0.355</td>
<td>0.587</td>
<td>0.786</td>
<td>0.928</td>
<td>1.331</td>
</tr>
<tr>
<td>$\beta_{S_y}$</td>
<td>1.021</td>
<td>0.266</td>
<td>0.502</td>
<td>0.831</td>
<td>1.008</td>
<td>1.166</td>
<td>1.838</td>
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<tr>
<td>$\kappa_{S_z}$</td>
<td>0.545</td>
<td>0.498</td>
<td>0.108</td>
<td>0.238</td>
<td>0.417</td>
<td>0.699</td>
<td>2.659</td>
</tr>
<tr>
<td>$\kappa_{S_y}$</td>
<td>0.267</td>
<td>0.267</td>
<td>0.091</td>
<td>0.091</td>
<td>0.172</td>
<td>0.357</td>
<td>1.332</td>
</tr>
<tr>
<td><strong>Industrials (based on 32 firms)</strong></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>$\beta_{S_z}$</td>
<td>0.945</td>
<td>0.187</td>
<td>0.562</td>
<td>0.820</td>
<td>0.948</td>
<td>1.057</td>
<td>1.315</td>
</tr>
<tr>
<td>$\beta_{S_y}$</td>
<td>0.976</td>
<td>0.309</td>
<td>0.206</td>
<td>0.832</td>
<td>0.969</td>
<td>1.161</td>
<td>1.580</td>
</tr>
<tr>
<td>$\kappa_{S_z}$</td>
<td>0.684</td>
<td>0.522</td>
<td>0.120</td>
<td>0.343</td>
<td>0.589</td>
<td>0.881</td>
<td>2.949</td>
</tr>
<tr>
<td>$\kappa_{S_y}$</td>
<td>0.241</td>
<td>0.247</td>
<td>0.091</td>
<td>0.091</td>
<td>0.138</td>
<td>0.294</td>
<td>1.135</td>
</tr>
<tr>
<td><strong>Information Technology (based on 50 firms)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$\beta_{S_z}$</td>
<td>1.172</td>
<td>0.218</td>
<td>0.705</td>
<td>1.061</td>
<td>1.151</td>
<td>1.326</td>
<td>1.605</td>
</tr>
<tr>
<td>$\beta_{S_y}$</td>
<td>1.150</td>
<td>0.408</td>
<td>0.277</td>
<td>0.845</td>
<td>1.081</td>
<td>1.381</td>
<td>2.420</td>
</tr>
<tr>
<td>$\kappa_{S_z}$</td>
<td>0.452</td>
<td>0.310</td>
<td>0.091</td>
<td>0.231</td>
<td>0.394</td>
<td>0.655</td>
<td>1.505</td>
</tr>
<tr>
<td>$\kappa_{S_y}$</td>
<td>0.879</td>
<td>0.204</td>
<td>0.332</td>
<td>0.815</td>
<td>0.923</td>
<td>1.050</td>
<td>1.108</td>
</tr>
<tr>
<td><strong>Materials (based on 17 firms)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{S_z}$</td>
<td>1.190</td>
<td>0.416</td>
<td>0.532</td>
<td>0.920</td>
<td>1.097</td>
<td>1.552</td>
<td>1.948</td>
</tr>
<tr>
<td>$\beta_{S_y}$</td>
<td>1.248</td>
<td>0.634</td>
<td>0.358</td>
<td>0.807</td>
<td>0.969</td>
<td>1.584</td>
<td>2.573</td>
</tr>
<tr>
<td>$\kappa_{S_z}$</td>
<td>0.595</td>
<td>0.517</td>
<td>0.091</td>
<td>0.270</td>
<td>0.503</td>
<td>0.775</td>
<td>2.137</td>
</tr>
</tbody>
</table>
Table 8: Firm Parameters Estimated Using Returns and Option Data: Neglecting Idiosyncratic Jumps

The index parameters are estimated using daily index returns and weekly cross-sections of out-of-the-money options, from January 1996 to August 2015. Parameters are estimated using multiple simplex search method optimizations (fminsearch in Matlab). Robust standard errors are calculated from the outer product of the gradient at the optimal parameter values. For firms, we report statistics on the joint MLE estimates obtained for the 260 individual stocks in our sample. Q1 and Q3 report the 25th and 75th percentiles of the estimates.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Average</th>
<th>S.D.</th>
<th>Min</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{S,z}$</td>
<td>0.879</td>
<td>0.318</td>
<td>0.163</td>
<td>0.649</td>
<td>0.872</td>
<td>1.087</td>
<td>2.060</td>
</tr>
<tr>
<td>$\beta_{S,y}$</td>
<td>1.456</td>
<td>0.516</td>
<td>0.001</td>
<td>1.164</td>
<td>1.479</td>
<td>1.747</td>
<td>3.958</td>
</tr>
<tr>
<td>$\lambda_{S,z}$</td>
<td>1.789</td>
<td>1.122</td>
<td>-2.318</td>
<td>0.907</td>
<td>1.708</td>
<td>2.488</td>
<td>4.997</td>
</tr>
<tr>
<td>$\kappa_{S,z}$</td>
<td>0.829</td>
<td>0.779</td>
<td>0.045</td>
<td>0.339</td>
<td>0.592</td>
<td>1.122</td>
<td>5.797</td>
</tr>
<tr>
<td>$\alpha_{S,z}$</td>
<td>2.37E-06</td>
<td>2.17E-06</td>
<td>4.55E-09</td>
<td>1.03E-06</td>
<td>1.81E-06</td>
<td>3.20E-06</td>
<td>2.27E-05</td>
</tr>
<tr>
<td>$b_{S,z}$</td>
<td>0.996</td>
<td>0.011</td>
<td>0.910</td>
<td>0.998</td>
<td>0.999</td>
<td>0.999</td>
<td>1.000</td>
</tr>
<tr>
<td>$c_{S,z}$</td>
<td>46.71</td>
<td>92.58</td>
<td>-463.50</td>
<td>-2.84</td>
<td>46.05</td>
<td>81.78</td>
<td>530.90</td>
</tr>
<tr>
<td>Avg, volatility (%)</td>
<td>36.44</td>
<td>9.53</td>
<td>18.36</td>
<td>29.16</td>
<td>34.85</td>
<td>43.32</td>
<td>63.69</td>
</tr>
<tr>
<td>Avg, skewness</td>
<td>-3.29</td>
<td>2.33</td>
<td>-15.97</td>
<td>-4.57</td>
<td>-3.15</td>
<td>-1.58</td>
<td>0.00</td>
</tr>
<tr>
<td>Avg, excess kurtosis</td>
<td>169.97</td>
<td>156.17</td>
<td>0.00</td>
<td>57.60</td>
<td>145.02</td>
<td>234.67</td>
<td>1265.67</td>
</tr>
<tr>
<td>Skewness of normal innovations, $\varepsilon_{S,z}$</td>
<td>-0.37</td>
<td>1.13</td>
<td>-8.14</td>
<td>-0.56</td>
<td>-0.08</td>
<td>0.11</td>
<td>2.08</td>
</tr>
<tr>
<td>Ex, kurtosis of normal innovations, $\varepsilon_{S,t}$</td>
<td>16.46</td>
<td>28.55</td>
<td>0.65</td>
<td>4.99</td>
<td>9.35</td>
<td>16.23</td>
<td>292.95</td>
</tr>
<tr>
<td>RIVRMSE</td>
<td>15.33</td>
<td>6.41</td>
<td>9.94</td>
<td>12.20</td>
<td>13.73</td>
<td>16.50</td>
<td>63.90</td>
</tr>
</tbody>
</table>
Table 9: Excess Returns of Portfolios Based on Idiosyncratic Risk Premiums

Each day, we compute the conditional model-implied risk premium associated with each stock’s idiosyncratic normal risk, first five columns, or idiosyncratic jump risk, last five columns. Stocks are then sorted into quintile portfolios from the lowest (P1) to the highest (P5) level of the risk premium under consideration. Stocks in the portfolios are weighted according to market capitalization. A long-short portfolio is created from a taking long position in P5 and a short position in P1. The daily returns of the long-short portfolio are then regressed on (subsets of) the following seven variables: the Fama-French market (MKT), small minus big (SMB), high minus low (HML), robust minus weak (RMW), and conservative minus aggressive (CMA) factors, the momentum (MOM) factor, and returns on the CBOE volatility index (ΔVIX). Column labels refer to the Fama-French 3-factor (FF3) and 5-factor (FF5) models, the Carhart (MOM) model, and the post-formation regression of Ang, Hodrick, Xing, Zhang (AHXZ, 2006). The regression constant is reported in annualized percentage points ($\Delta t = 1/252$).

**: 5% significance  *: 10% significance.

<table>
<thead>
<tr>
<th>Idiosyncratic Normal Risk Premium</th>
<th>Idiosyncratic Jump Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FF3</td>
</tr>
<tr>
<td>Cst × $100$</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>(0.38)</td>
</tr>
<tr>
<td>MKT</td>
<td>0.12**</td>
</tr>
<tr>
<td></td>
<td>(6.95)</td>
</tr>
<tr>
<td>SMB</td>
<td>0.10**</td>
</tr>
<tr>
<td></td>
<td>(3.68)</td>
</tr>
<tr>
<td>HML</td>
<td>-0.55**</td>
</tr>
<tr>
<td></td>
<td>(-17.23)</td>
</tr>
<tr>
<td>RMW</td>
<td>-0.23**</td>
</tr>
<tr>
<td></td>
<td>(-5.51)</td>
</tr>
<tr>
<td>CMA</td>
<td>-0.47**</td>
</tr>
<tr>
<td></td>
<td>(-7.95)</td>
</tr>
<tr>
<td>MOM</td>
<td>-0.09**</td>
</tr>
<tr>
<td></td>
<td>(-3.58)</td>
</tr>
<tr>
<td>DVIX</td>
<td>0.02*</td>
</tr>
<tr>
<td></td>
<td>(1.73)</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.22</td>
</tr>
</tbody>
</table>
Table 10: Description of Quintile Portfolios
This table describes the quintile portfolios obtained in Table 9. Each day, the following variables are recorded for the firms in each quintile portfolio: market beta, log of market capitalization, book-to-market ratio, operating profitability (OP), investment, trailing 12-month return, and the volatility beta. This table reports the time-series average of these variables for each of the quintile portfolios obtained using the total idiosyncratic risk premium (first five columns) or its component in excess of the common component (last five columns). Standard deviations are reported within square brackets.

<table>
<thead>
<tr>
<th></th>
<th>Idiosyncratic Jump Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P1</td>
</tr>
<tr>
<td>Market beta</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>[0.12]</td>
</tr>
<tr>
<td>log(ME)</td>
<td>25.30</td>
</tr>
<tr>
<td></td>
<td>[0.32]</td>
</tr>
<tr>
<td>BE/ME</td>
<td>1.57</td>
</tr>
<tr>
<td></td>
<td>[0.54]</td>
</tr>
<tr>
<td>OP (%)</td>
<td>5.96</td>
</tr>
<tr>
<td></td>
<td>[1.39]</td>
</tr>
<tr>
<td>Investment (%)</td>
<td>16.27</td>
</tr>
<tr>
<td></td>
<td>[22.14]</td>
</tr>
<tr>
<td>Return [-12,-2] (%)</td>
<td>13.67</td>
</tr>
<tr>
<td></td>
<td>[18.40]</td>
</tr>
<tr>
<td>Volatility beta (%)</td>
<td>-0.28</td>
</tr>
<tr>
<td></td>
<td>[3.38]</td>
</tr>
</tbody>
</table>
On each day $t$, we first sort stocks into quintiles (Q1 to Q5) based on their market beta over the past year. Then, within each market-beta quintile, we sort stocks into terciles based on $R_{P_{S,y,t}}$. We then take a long position in a cap-weighted portfolio of the top-tercile stocks and a short position in a cap-weighted portfolio of the bottom-tercile stocks. This leaves us with five long-short portfolios, each of which is composed of stocks with homogeneous market betas. The first row of this table reports the alphas of regression (4.28) for each of these long-short portfolios. The procedure is repeated for six other variables: log of market capitalization, book-to-market ratio, operating profitability (OP), investment, trailing 12-month return, and the volatility beta.

<table>
<thead>
<tr>
<th>Idiosyncratic Jump Risk Premium</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market beta</td>
<td>7.14*</td>
<td>8.75**</td>
<td>11.63**</td>
<td>9.98**</td>
<td>11.41**</td>
</tr>
<tr>
<td></td>
<td>(1.91)</td>
<td>(2.38)</td>
<td>(3.05)</td>
<td>(2.38)</td>
<td>(2.19)</td>
</tr>
<tr>
<td>log(ME)</td>
<td>11.49**</td>
<td>9.08**</td>
<td>10.51**</td>
<td>18.45**</td>
<td>10.62**</td>
</tr>
<tr>
<td></td>
<td>(2.43)</td>
<td>(2.38)</td>
<td>(3.20)</td>
<td>(5.81)</td>
<td>(3.96)</td>
</tr>
<tr>
<td>BE/ME</td>
<td>15.13**</td>
<td>12.26**</td>
<td>11.90**</td>
<td>15.12**</td>
<td>12.54**</td>
</tr>
<tr>
<td></td>
<td>(3.41)</td>
<td>(2.62)</td>
<td>(2.86)</td>
<td>(3.80)</td>
<td>(3.13)</td>
</tr>
<tr>
<td>OP (%)</td>
<td>17.02**</td>
<td>10.48**</td>
<td>14.18**</td>
<td>6.99</td>
<td>16.34**</td>
</tr>
<tr>
<td></td>
<td>(3.05)</td>
<td>(2.21)</td>
<td>(2.95)</td>
<td>(1.52)</td>
<td>(3.41)</td>
</tr>
<tr>
<td>Investment (%)</td>
<td>11.94**</td>
<td>8.66*</td>
<td>18.11**</td>
<td>20.14**</td>
<td>12.60**</td>
</tr>
<tr>
<td></td>
<td>(2.54)</td>
<td>(2.31)</td>
<td>(4.74)</td>
<td>(4.40)</td>
<td>(2.49)</td>
</tr>
<tr>
<td></td>
<td>(2.07)</td>
<td>(1.37)</td>
<td>(3.54)</td>
<td>(3.42)</td>
<td>(3.03)</td>
</tr>
<tr>
<td>Volatility beta (%)</td>
<td>5.37</td>
<td>10.51**</td>
<td>18.33**</td>
<td>9.63**</td>
<td>19.35**</td>
</tr>
<tr>
<td></td>
<td>(1.05)</td>
<td>(2.51)</td>
<td>(4.80)</td>
<td>(2.57)</td>
<td>(3.89)</td>
</tr>
</tbody>
</table>
APPENDIX

A Innovations’ Cumulant Generating Functions

The convexity correction introduced in equation (2.1), \( \xi_{M,t}^\phi = \xi_{M,t}^\phi (1) + \xi_{y,t}^\phi (1) \), is based on the cumulant generating function of \( z_M \) and \( y_M \). The same holds for \( \xi_{S,t+1}^\phi = \xi_{z_M}^\phi (\beta_{S,t}) + \xi_{y_M}^\phi (\beta_{S,t}) + \xi_{z_S}^\phi (1) + \xi_{y_S}^\phi (1) \), introduced in equation (2.2).

For any \( z_{u,t} \in \{ z_{M,t}, z_{S,t} : S \in \mathbb{S} \} \), the conditional cumulant generating function of \( z_{u,t} \) satisfies

\[
\xi_{z,u,t}^\phi (\phi) = \log E_{\mathcal{F}_{t-1}}^\phi [\exp (\phi z_{u,t})] = \frac{\phi^2}{2} h_{u,z,t}. 
\]

The conditional cumulant generating function of \( y_{u,t} \in \{ y_{M,t}, y_{S,t} : S \in \mathbb{S} \} \) is

\[
\xi_{y,u,t}^\phi (\phi) = \log E_{\mathcal{F}_{t-1}}^\phi [\exp (\phi y_{u,t})] = \Pi_u (\phi) h_{u,y,t}
\]

where

\[
\Pi_u (\phi) = \left( \sqrt{\alpha_u^2 - \delta_u^2} - \sqrt{\alpha_u^2 - (\delta_u + \phi)^2} \right). \tag{A.1}
\]

B Innovations’ Risk Neutral Cumulant Generating Functions

Lemma 2 For any \( e_{u,t} \in \{ e_{M,t}, e_{S,t} : S \in \mathbb{S} \} \), the conditional cumulant generating function of \( e_{u,t} \) under \( Q \) is

\[
\xi_{e,u,t}^Q (\phi) = \log E_{\mathcal{F}_{t-1}}^Q [\exp (\phi e_{u,t})] = \frac{1}{2} \phi^2 - \Lambda_u \sqrt{h_{u,z,t}} \phi
\]

which corresponds to the cumulant generating function of a Gaussian random variable of expectation \( -\Lambda_u \sqrt{h_{u,z,t}} \) and variance 1. To obtain a risk neutral sequence of standard normal innovations, we must set

\[
e_{u,t}^* = e_{u,t} + \Lambda_u \sqrt{h_{u,z,t}}. \tag{B.2}
\]

The proof is in the Online Appendix OA.B. Note that, given that \( h_{u,z,t} \) is \( \mathcal{F}_{t-1}^\mathbb{S} \)–measurable, the conditional cumulant generating function of \( z_{u,t} \) under \( Q \) can easily be shown (by replicating the above) to be that of a normal variable of expectation \( -\Lambda_u h_{u,z,t} \) and variance \( h_{u,z,t} \). In other words, consistent with Christoffersen, Elkmamhi, Feunou, and Jacobs (2010), the risk-neutral \( \mathcal{F}_{t-1}^\mathbb{S} \)–conditional variance of \( z_{u,t}, h_{u,z,t}^2 \), is equal to its physical counterpart, \( h_{u,z,t} \).

Lemma 3 For any \( y_{u,t} \in \{ y_{M,t}, y_{S,t} : S \in \mathbb{S} \} \), the conditional cumulant generating function of \( y_{u,t} \) under \( Q \) is

\[
\xi_{y,u,t}^Q (\phi) = \log E_{\mathcal{F}_{t-1}}^Q [\exp (\phi y_{u,t})] = \Pi_u^y (\phi) h_{u,y,t}^* \tag{B.3}
\]

where

\[
\Pi_u^y (\phi) = \sqrt{\alpha_u^2 - (\delta_u - \Gamma_u)^2} - \sqrt{\alpha_u^2 - (\delta_u - \Gamma_u + \phi)^2. \tag{B.4}
\]

Details are provided in the Online Appendix OA.B. The risk neutral jump component is still a NIG random variable with no location parameter, the tail heaviness parameter \( \alpha_u^* = \alpha_u \) is not affected by the change of measure, the asymmetry parameter becomes \( \delta_u^* = \delta_u - \Gamma_u \) and the scale variable is \( h_{u,y,t}^* = h_{u,y,t} \).
C Risk Premiums

Lemma 4 The mappings between $\lambda_M$ and $\gamma_M$ and their pricing kernel counterparts $\Lambda_M$ and $\Gamma_M$ are

$$
\Lambda_M = \Lambda_M \quad \text{and} \quad \Gamma_M = \Pi_n (1) - \Pi_n^* (1).
$$

For the stock parameters $\lambda_S$ and $\gamma_S$, the relation is

$$
\lambda_S = \Lambda_S, \quad \gamma_M (\beta_{S,y}) = \Pi_M (\beta_{S,y}) - \Pi_M^* (\beta_{S,y}), \quad \gamma_S = \Pi_S (1) - \Pi_S^* (1).
$$

Proof of Lemma 4. Since the proof for the market component is similar, the focus is put on the stock specific parameters. More details are available in the Online Appendix.

Since the discounted stock price should behave as a $Q$–martingale,

$$
1 = \mathbb{E}^Q_{t-1} \left[ \frac{\exp(-r_{f,t}) S_t}{S_{t-1}} \right] = \mathbb{E}^p_{t-1} \left[ \frac{\exp(-\Lambda M z_{M,t} - \gamma_M y_{M,t} - \sum_{S \in S} \Lambda_S z_{S,t} - \sum_{S \in S} \Gamma_S y_{S,t})}{\mathbb{E}^p_{t-1} \left[ \exp(-\Lambda M z_{M,t} - \gamma_M y_{M,t} - \sum_{S \in S} \Lambda_S z_{S,t} - \sum_{S \in S} \Gamma_S y_{S,t}) \right]} \exp(r_{S,t} - r_{f,t}) \right].
$$

Replacing the excess return using (2.2) and the cumulant generating functions, we get

$$
1 = \exp \left[ \frac{\mu^p_{S,t} - r_{f,t} - \xi^p_{z_{M,t}} (\beta_{S,z}) - \xi^p_{y_{M,t}} (\beta_{S,y}) - \xi^p_{z_{S,t}} (1) - \xi^p_{y_{S,t}} (1)}{\xi^p_{z_{M,t}} (\beta_{S,z} - \Lambda_M) + \xi^p_{y_{M,t}} (\beta_{S,y} - \Gamma_M) + \xi^p_{z_{S,t}} (1 - \Lambda_S) + \xi^p_{y_{S,t}} (1 - \Gamma_S)} \right].
$$

Because,

$$
-\xi^p_{z_{M,t}} (\beta_{S,z}) + \xi^p_{z_{M,t}} (\beta_{S,z} - \Lambda_M) - \xi^p_{z_{S,t}} (1) = -\Lambda M h_{S,z,t} h_{M,z,t},
$$

$$
-\xi^p_{z_{S,t}} (1) + \xi^p_{z_{S,t}} (1 - \Lambda_S) = -\Lambda S h_{S,z,t},
$$

$$
-\xi^p_{y_{M,t}} (\beta_{S,y}) + \xi^p_{y_{M,t}} (\beta_{S,y} - \Gamma_M) = -h_{M,y,t} \gamma_{M,S} (\beta_{S,y}),
$$

$$
-\xi^p_{y_{S,t}} (1) + \xi^p_{y_{S,t}} (1 - \Gamma_S) = -h_{S,y,t} \gamma_{S}.
$$

we conclude that

$$
1 = \exp \left[ \mu^p_{S,t} - r_{f,t} - \Lambda M h_{S,z,t} h_{M,z,t} - h_{M,y,t} \gamma_{M,S} (\beta_{S,y}) - \Lambda S h_{S,z,t} - h_{S,y,t} \gamma_{S} \right].
$$

Therefore,

$$
\mu^p_{S,t} = r_{f,t} + \Lambda M h_{S,z,t} h_{M,z,t} + h_{M,y,t} \gamma_{M,S} (\beta_{S,y}) + \Lambda S h_{S,z,t} + h_{S,y,t} \gamma_{S}.
$$

D Risk Neutral Conditional Variances and Jump Intensities

Lemma 5 Let

$$
\eta^*_t = \left[ 1 \quad h^*_{M,z,t} \quad h^*_{M,y,t} \quad h^*_{S,z,t} \quad h^*_{S,y,t} \quad (e^*_M)^2 \quad \sqrt{h^*_{M,z,t} e^*_M} \quad (e^*_S)^2 \quad \sqrt{h^*_{S,z,t} e^*_S} \right]^t.
$$
Then, for any $u \in \{M, S\}$ and $v \in \{z, y\}$,
\begin{equation}
\hat{h}_{u,v,t+1}^{*} = \pi_{u,v} \eta_{t}^{*}
\end{equation}
where $\pi_{u,v}$ is a $1 \times 9$ vector of constants satisfying
\begin{align*}
\pi_{M,z,1} &= w_{M,z} \\
\pi_{M,z,2} &= b_{M,z} + a_{M,z} (c_{M,z} + \Lambda_{M})^{2} \\
\pi_{M,z,6} &= a_{M,z} \\
\pi_{S,z,1} &= \kappa_{z,2} \pi_{M,z,1} - a_{S,z} \\
\pi_{S,z,2} &= \kappa_{z,2} (\pi_{M,z,2} - b_{S,z}) \\
\pi_{S,z,4} &= b_{S,z} + a_{S,z} (2c_{S,z} + \Lambda_{S}) \Lambda_{S} \\
\pi_{S,z,i} &= 0 \text{ for } i \in \{3, 5\}
\end{align*}
For $u \in \{M, S\}$,
\begin{align*}
\pi_{M,y,1} &= w_{M,y} \\
\pi_{M,y,2} &= a_{M,y} (c_{M,y} + \Lambda_{M})^{2} \\
\pi_{M,y,3} &= b_{M,y} \\
\pi_{S,y,1} &= \kappa_{y,3} \pi_{M,y,1} - a_{S,y} \\
\pi_{S,y,2} &= \kappa_{y,3} \pi_{M,y,2} \\
\pi_{S,y,3} &= \kappa_{y,3} (\pi_{M,y,3} - b_{S,y}) \\
\pi_{S,y,4} &= a_{S,y} (2c_{S,y} + \Lambda_{S}) \Lambda_{S} \\
\pi_{S,y,5} &= b_{S,y}
\end{align*}

**Proof of Lemma 5.** The risk neutral market conditional variance $h_{M,z,t+1}^{*}$ and jump intensity variable $\hat{h}_{M,y,t+1}^{*}$ are obtained by replacing (B.2) in (2.3) and (2.5).

In the case of the stocks, for any $v \in \{z, y\}$,
\begin{align*}
\left( \epsilon_{S,t}^{2} - 1 - 2c_{S,v} \sqrt{h_{S,z,t}^{*} \epsilon_{S,t}} \right) \\
= \left( \epsilon_{S,t}^{2} - \Lambda_{S} \sqrt{h_{S,z,t}^{*}} \right)^{2} - 1 - 2c_{S,v} \sqrt{h_{S,z,t}^{*}} \left( \epsilon_{S,t}^{*} - \Lambda_{S} \sqrt{h_{S,z,t}^{*}} \right) \\
= (2c_{S,v} + \Lambda_{S}) \Lambda_{S} h_{S,z,t}^{*} + \left( \epsilon_{S,t}^{*} \right)^{2} - 1 - 2(c_{S,v} + \Lambda_{S}) \sqrt{h_{S,z,t}^{*}} \epsilon_{S,t}^{*}.
\end{align*}

where the first equality arises from (B.2). Replacing back in the conditional variance (2.4) and the jump intensity process (2.6) leads to their risk neutral versions.

**E Moment Generating Function of Risk-Neutral Excess Returns**

**Lemma 6** For $u \in \{M, S\}$, the conditional moment generating function of the excess returns satisfies
\begin{equation}
\varphi_{u,T-t}^{(Q)}(\phi) = \exp \left( \mathcal{A}_{u,T-t}(\phi) + \mathcal{B}_{u,T-t}(\phi) \hat{h}_{M,z,t+1}^{*} + \mathcal{C}_{u,T-t}(\phi) \hat{h}_{M,y,t+1}^{*} + \mathcal{D}_{u,T-t}(\phi) \hat{h}_{S,z,t+1}^{*} + \mathcal{E}_{u,T-t}(\phi) \hat{h}_{S,y,t+1}^{*} \right)
\end{equation}
where the coefficients are found using a backward recursion over time. Indeed, $\varphi_{u,0}^{(Q)}(\phi) = 1$ implies that
\begin{align*}
\mathcal{A}_{u,0}(\phi) &= \mathcal{B}_{u,0}(\phi) = \mathcal{C}_{u,0}(x) = \mathcal{D}_{u,0}(\phi) = \mathcal{E}_{u,0}(\phi) = 0.
\end{align*}
For $i \in \{0, 1, ..., 9\}$, let

$$
\zeta_{u,T^{-1},i} (\phi) = \mathcal{B}_{u,T^{-1}} (\phi) \pi_{M,z,i} + \mathcal{C}_{u,T^{-1}} (\phi) \pi_{M,y,i} + \mathcal{D}_{u,T^{-1}} (\phi) \pi_{S,z,i} + \mathcal{E}_{u,T^{-1}} (\phi) \pi_{S,y,i},
$$

where the $\pi_i$ are as provided in Appendix D. If $\zeta_{s,6} (\phi) < \frac{1}{2}$ and $\zeta_{s,8} (\phi) < \frac{1}{2}$ for any $s \in \{t + 1, ..., T\}$, then

$$
\mathcal{A}_{u,T^{-1}} (\phi) = \mathcal{A}_{u,T^{-1}} (\phi) + \zeta_{u,T^{-1},1} (\phi) - \frac{1}{2} \log (1 - 2\zeta_{u,T^{-1},6} (\phi)) - \frac{1}{2} \log (1 - 2\zeta_{u,T^{-1},8} (\phi)),
$$

$$
\mathcal{B}_{u,T^{-1}} (\phi) = \zeta_{u,T^{-1},2} (\phi) - \frac{1}{2} \beta_{u,z}^2 \phi + \frac{1}{2} \left( \zeta_{u,T^{-1},7} (\phi) + \beta_{u,z} \phi \right)^2,
$$

$$
\mathcal{C}_{u,T^{-1}} (\phi) = \zeta_{u,T^{-1},3} (\phi) - \Pi_M^* \left( \beta_{u,y} \right) \phi + \Pi_M^* \left( \beta_{u,y} \phi \right),
$$

$$
\mathcal{D}_{u,T^{-1}} (\phi) = \zeta_{u,T^{-1},4} (\phi) - \frac{1}{2} \beta_{u,y}^2 \phi + \frac{1}{2} \left( \zeta_{u,T^{-1},8} (\phi) + \beta_{u,y} \phi \right)^2,
$$

$$
\mathcal{E}_{u,T^{-1}} (\phi) = \zeta_{u,T^{-1},5} (\phi) - \Pi_S^* \left( \beta_{u,y} \right) \phi + \Pi_S^* \left( \beta_{u,y} \phi \right).
$$

where for the market case, $\beta_{M,z} = \beta_{M,y} = 1$ and $\beta_{S,z} = \beta_{S,y} = 0$ while for the stock, $\beta_{S,z} = \beta_{S,y} = 1$.

The proof is strongly inspired from the existing literature and is provided in the Online Appendix OA.E.

## F Particle Filter

In the following, whenever the subscript $M$ and $S$ have been dropped, the approach is applicable to both market and stock data.

The filter is based on pure jump particle paths $y_{1,T}^{(i)} = \{y_{1}^{(i)}, y_{2}^{(i)}, ..., y_{T}^{(i)}\}, i \in \{1, ..., N\}$ and the sequential importance resampling (SIR) of Gordon, Salmond, and Smith (1993) is implemented.\footnote{Throughout the paper, $N = 25,000$ particles are used.} A single step of the SIR is now described.

Assume that $N$ jump paths $y_{1,t-1}^{(i)}$, $i \in \{1, 2, ..., N\}$ are available up to time $t - 1$. As a by-product, the conditional variance $\hat{h}_{y,i}^{(t)}$ and the jump scale variable $\hat{h}_{y,i}^{(t)}$ are recovered.

1. For $i \in \{1, 2, ..., N\}$, the time $t$ jump $y_{t}^{(i)}$ is simulated from the proposal distribution\footnote{More precisely,}

$$
f \left( \cdot \mid y_{1:t-1}^{(i)}, r_{1:t-1} \right) = f_{NIG} \left( \cdot ; \alpha, \delta, \hat{h}_{y,i}^{(t)} \right),
$$

$$
f_{NIG} (x, \alpha, \delta, h) = \frac{ahK_1 (\alpha \sqrt{h^2 + x^2})}{\pi \sqrt{h^2 + x^2}} \exp \left( h \sqrt{\alpha^2 - \delta^2} + \delta x \right)
$$

$$
K_1 (x) = \int_0^\infty \exp (-x \cosh (t)) \cosh (t) \, dt.
$$

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2. For $i \in \{1, 2, ..., N\}$, update the importance weights (up to a normalizing constant) to reflect how likely the simulated particles are with respect to the time $t$ information $r_t$:

$$
\tilde{\omega}_t^{(i)} = f \left( r_t \left| r_{1:t-1}, y_{1:t}^{(i)} \right. \right).
$$

More precisely, from equations (2.1) and (2.10), the market returns satisfy

$$
r_{M,t} = r_{f,t} + \left( \lambda_M - \frac{1}{2} \right) h_{M,z,t} + (\gamma_M - \Pi_M (1)) h_{M,y,t} + z_{M,t} + y_{M,t}.
$$

Therefore, conditionally on a simulated path $y_{M,1:t}^{(i)}$ and on the past returns $r_{M,1:t-1}$, the time $t$ market return $r_{M,t}$ is normally distributed with expectation

$$
m_{M,t}^{(i)} = r_{f,t} + \left( \lambda_M - \frac{1}{2} \right) h_{M,z,t} + (\gamma_M - \Pi_M (1)) h_{M,y,t} + y_{M,t}^{(i)}
$$

and variance $h_{M,z,t}^{(i)}$:

$$
f \left( r_{M,t} \left| r_{M,1:t-1}, y_{M,1:t}^{(i)} \right. \right) = \frac{1}{\sqrt{2\pi h_{M,z,t}^{(i)}}} \exp \left\{ - \frac{1}{2} \frac{\left( r_{M,t} - m_{M,t}^{(i)} \right)^2}{h_{M,z,t}^{(i)}} \right\}.
$$

3. For $i \in \{1, 2, ..., N\}$, compute the normalized weights

$$
\omega_t^{(i)} = \frac{\tilde{\omega}_t^{(i)}}{\sum_{k=1}^{N} \tilde{\omega}_t^{(k)}}.
$$

$^{41}$Similarly, the stock returns

$$
r_{S,t+1} \equiv r_{f,t} + \left( \beta_{S,z} \lambda_M - \frac{1}{2} \beta_{S,z}^2 \right) \tilde{h}_{M,z,t} + \left[ y_{M,S} \left( \beta_{S,y} \right) - \Pi_M \left( \beta_{S,y} \right) \right] \tilde{h}_{M,y,t} + \beta_{S,z} \tilde{z}_{M,t+1} + \beta_{S,y} \tilde{y}_{M,t+1}
$$

$$
+ \left( \lambda_S - \frac{1}{2} \right) h_{S,z,t} + \left[ y_{S} - \Pi_S (1) \right] h_{S,y,t} + z_{S,t+1} + y_{S,t+1}
$$

where $\tilde{h}_{M,z,t}, \tilde{h}_{M,y,t}, \tilde{z}_{M,t+1}, \tilde{y}_{M,t+1}$ are the filtered value obtained from the estimation of the market model. Therefore,

$$
f \left( r_{S,t} \left| r_{S,1:t-1}, y_{S,1:t}, \tilde{h}_{M,z,t}, \tilde{h}_{M,y,t}, \tilde{z}_{M,t+1}, \tilde{y}_{M,t+1} \right. \right) = \phi \left( r_{S,t}; m_{S,t}^{(0)}, h_{S,z,t}^{(0)} \right)
$$

with

$$
m_{S,t}^{(0)} = r_{f,t} + \left( \beta_{S,z} \lambda_M - \frac{1}{2} \beta_{S,z}^2 \right) \tilde{h}_{M,z,t} + \left[ y_{M,S} \left( \beta_{S,y} \right) - \Pi_M \left( \beta_{S,y} \right) \right] \tilde{h}_{M,y,t} + \beta_{S,z} \tilde{z}_{M,t+1} + \beta_{S,y} \tilde{y}_{M,t+1}
$$

$$
+ \left( \lambda_S - \frac{1}{2} \right) h_{S,z,t}^{(0)} + \left[ y_{S} - \Pi_S (1) \right] h_{S,y,t}^{(0)} + z_{S,t+1}^{(0)} + y_{S,t+1}^{(0)}
$$

and variance $h_{S,z,t}^{(0)}$.  

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4. For \( i \in \{1, 2, \ldots, N\} \), update the conditional variance and the jump scale variable. For the market, based on (2.3) and (2.5),

\[
\begin{align*}
\hat{h}_{M,z,t+1}^{(i)} &= w_{M,z} + b_{M,z} \hat{h}_{M,z,t}^{(i)} + \frac{a_{M,z}}{\hat{h}_{M,z,t}^{(i)}} \left( \zeta_{M,t} - \hat{c}_{M,z} \hat{h}_{M,z,t}^{(i)} \right)^2 \\
\hat{h}_{M,y,t+1}^{(i)} &= w_{M,y} + b_{M,y} \hat{h}_{M,y,t}^{(i)} + \frac{a_{M,y}}{\hat{h}_{M,y,t}^{(i)}} \left( \zeta_{M,t} - \hat{c}_{M,y} \hat{h}_{M,y,t}^{(i)} \right)^2
\end{align*}
\]

where \( \zeta_{M,t} = r_{M,t} - m_{M,t}^{(i)} \).

5. From normalized importance weights, compute the filtered variables

\[
\begin{align*}
\tilde{z}_{M,t} &= \sum_{i=1}^{N} \hat{z}_{M,t}^{(i)} w_{t}^{(i)}, \\
\tilde{\gamma}_{M,t} &= \sum_{i=1}^{N} \hat{\gamma}_{M,t}^{(i)} w_{t}^{(i)}, \\
\tilde{\eta}_{M,t} &= \sum_{i=1}^{N} \hat{\eta}_{M,t}^{(i)} w_{t}^{(i)}, \\
\tilde{h}_{M,z,t+1} &= \sum_{i=1}^{N} \hat{h}_{M,z,t+1}^{(i)} w_{t}^{(i)}, \\
\tilde{h}_{M,y,t+1} &= \sum_{i=1}^{N} \hat{h}_{M,y,t+1}^{(i)} w_{t}^{(i)}.
\end{align*}
\]

6. Resample the particles using the continuous sampling of Malik and Pitt (2011).\(^{43}\)

(a) Draw \( N \) particles from the current particle set from a smoothed empirical cdf as proposed in Malik and Pitt (2011) and let \( \hat{h}_{M,z,t+1}^{(j)} \) and \( \hat{h}_{M,y,t+1}^{(j)} \) denote the resulting conditional variances and the jump intensity variables once the resampling is accomplished.\(^{44}\)

(b) Replace the current conditional variance and jump intensity with their resampled values:

\[
\hat{h}_{M,z,t+1}^{(j)} \leftarrow \hat{h}_{M,z,t+1}^{(j)}, \quad \text{and} \quad \hat{h}_{M,y,t+1}^{(j)} \leftarrow \hat{h}_{M,y,t+1}^{(j)}.
\]

The log-likelihood is obtained as a by-product of the particle filter. Indeed,

\[
L_{M,\text{returns}}(\Theta_M) = \sum_{t=1}^{T} \log \left( \sum_{i=1}^{N} \tilde{\omega}_{t}^{(i)} \right).
\]

\(^{42}\)For the stock,

\[
\begin{align*}
\hat{h}_{S,z,t+1}^{(i)} &= \kappa_{S,z} \hat{h}_{M,z,t+1} + b_{S,z} \left( \hat{h}_{S,z,t}^{(i)} - \kappa_{S,z} \hat{h}_{M,z,t} \right) + \alpha_{S,z} \left( \hat{h}_{S,z,t}^{(i)} \right)^{-1} \left( \hat{z}_{S,t}^{(i)} \right)^2 - 1 - 2\epsilon_{S,z} \hat{z}_{S,t}^{(i)} \\
\hat{h}_{S,y,t+1}^{(i)} &= \kappa_{S,y} \hat{h}_{M,y,t+1} + b_{S,y} \left( \hat{h}_{S,y,t}^{(i)} - \kappa_{S,y} \hat{h}_{M,y,t} \right) + \alpha_{S,y} \left( \hat{h}_{S,y,t}^{(i)} \right)^{-1} \left( \hat{z}_{S,t}^{(i)} \right)^2 - 1 - 2\epsilon_{S,y} \hat{z}_{S,t}^{(i)}
\end{align*}
\]

where \( \hat{z}_{S,t}^{(i)} = r_{S,t} - m_{S,t}^{(i)} \).

\(^{43}\)As argued in Creal (2012), basic resampling methods are ill-suited for maximum likelihood estimation.

\(^{44}\)Note that when the number of resampled particles is small, we use importance sampling to increase it. To this end, the jump intensity variable is artificially increased and a weight correction is applied accordingly.
G Time-Varying Prices of Idiosyncratic Risk

So far in Appendix A to F, we considered the case in which the market prices of idiosyncratic risk are constant. However, as we will demonstrate shortly, constant prices of risk cannot rule out arbitrage opportunities in a multivariate affine model such as ours.

Accounting for the variation in these prices of risk in Appendix A to F would be relatively straightforward, but notationally cumbersome as it makes the parameters of the risk-neutral model time-varying. In the implementation, we do however account for this variation contemporaneously. Besides, when pricing the options at time $t$, we work under the assumption that agents use the current value of the idiosyncratic prices of risk but ignore their potential future variation over the life of the option. This amounts to assuming that the risk associated with the future variation of the prices of risk is strongly dominated by variance, and variance of variance risk. In essence, the approach mimics the standard approach used (since Bakshi, Cao, and Chen (1997)) to deal with time-varying interest rates when pricing derivatives that are written on underlying assets that are only mildly sensitive to interest rates: at time $t$, the current $r(t, T)$ rate is used, but its potential variation is neglected.

G.1 Proof of Proposition 1

Assume that the market prices of idiosyncratic risk are constant. Since

$$R_{M,t+1} = \sum_{S \in S} \omega_S R_{S,t+1}$$

where $\omega_S$ represents the weight of stock $S$ in the market index $M$, then

$$\sum_{S \in S} \omega_S \exp(r_{S,t+1}) = \exp(r_{M,t+1}) .$$

By taking the conditional expectation on both side, we get a weaker condition:

$$\sum_{S \in S} \omega_S \exp(\mu_{S,t+1}^S) = \sum_{S \in S} \omega_S E^P_t[\exp(r_{S,t+1})] = E^P_t[\exp(r_{M,t+1})] = \exp(\mu_{M,t+1}^P)$$

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or, equivalently,

\[ 0 = \sum_{S \in \mathcal{S}} \omega_S \exp \left( \mu_{S,t+1}^P - \mu_{M,t+1}^P \right) - 1 \]

\[ = \sum_{S \in \mathcal{S}} \omega_S \exp \left( r_{f,t+1} + \beta_{S,z} \lambda_M h_{M,z,t+1} + \gamma_{M,S} h_{M,y,t+1} + \lambda_S h_{S,z,t+1} + \gamma_S h_{S,y,t+1} \right) - 1 \]

\[ = \sum_{S \in \mathcal{S}} \omega_S \exp \left( (\beta_{S,z} - 1) \lambda_M h_{M,z,t+1} + (\gamma_{M,S} - \gamma_M) h_{M,y,t+1} + \lambda_S h_{S,z,t+1} + \gamma_S h_{S,y,t+1} \right) - 1 \]

\[ = \lambda_M h_{M,z,t+1} \left( \sum_{S \in \mathcal{S}} \omega_S \beta_{S,z} - 1 \right) + h_{M,y,t+1} \left( \sum_{S \in \mathcal{S}} \omega_S \gamma_{M,S} - \gamma_M \right) + \sum_{S \in \mathcal{S}} \omega_S \lambda_S h_{S,z,t+1} + \sum_{S \in \mathcal{S}} \omega_S \gamma_S h_{S,y,t+1}. \]

Assuming that \( \sum_{S \in \mathcal{S}} \omega_S \beta_{S,z} = 1 \) and \( \sum_{S \in \mathcal{S}} \omega_S \gamma_{M,S} (\beta_{S,z}) = \gamma_M \), the absence of arbitrage thus imposes that for all \( t \),

\[ \sum_{S \in \mathcal{S}} \omega_S \lambda_S h_{S,z,t} = 0 \quad \text{and} \quad \sum_{S \in \mathcal{S}} \omega_S \gamma_S h_{S,y,t} = 0. \tag{G.7} \]

Note that if all \( \lambda_S \) (or \( \gamma_S \)) have the same sign (as we find it to be the case empirically when assuming constant prices of risk), then Equations (G.7) cannot hold since variances and intensities are positive. If (G.7) holds true, then for two different time \( t_1 \) and \( t_2 \), \( \sum_{S \in \mathcal{S}} \omega_S \lambda_S h_{S,z,t_1} = \sum_{S \in \mathcal{S}} \omega_S \lambda_S h_{S,z,t_2} \). That is equivalent to

\[ \omega_S h_{S,z,t_1} - \omega_S h_{S,z,t_2} = (\lambda_S)^{-1} \sum_{S' \in \mathcal{S} \mid S} \lambda_{S'} (w_{S',z,t_1} h_{S',z,t_1} - w_{S',z,t_2} h_{S',z,t_2}), \]

imposing unrealistically strong restrictions on the joint dynamics of the variances unless we relax the assumption that \( \lambda_S \) and \( \gamma_S \) are constant over time. Hence, in all generality, constant prices of idiosyncratic risk do not rule out arbitrage is a multivariate affine setup.

### G.2 Accounting for Clearing Conditions When One-Stage Estimation is Intractable

The no-arbitrage constraints of Equations (G.7) involve all stocks in the index. While a one-stage procedure could, in theory, straightforwardly accommodate these constraints, it would be absolutely intractable. The two-stage estimation procedure described in Section 3 cannot handle these restrictions since the estimation of each stock’s parameters is dealt with independently. Fortunately, the following algorithm, reminiscent of fixed points algorithms, allows us to impose the no-arbitrage constraints by
relying on the affine dynamics of equations (2.21) and (2.22), which are

$$
\lambda_S h_{S,z,t} = \lambda_S h_{S,z,t} - \sum_{S' \in S} w_{S'} \lambda_{S'} h_{S',z,t} \quad \Leftrightarrow \quad \lambda_S = \lambda_S - \sum_{S' \in S} w_{S'} \lambda_{S'} \frac{h_{S',z,t}}{h_{S,z,t}},
$$

$$
\gamma_S h_{S,y,t} = \gamma_S h_{S,y,t} - \sum_{S' \in S} w_{S'} \gamma_{S'} h_{S',y,t} \quad \Leftrightarrow \quad \gamma_S = \gamma_S - \sum_{S' \in S} w_{S'} \gamma_{S'} \frac{h_{S',y,t}}{h_{S,y,t}}.
$$

Algorithm

**Iteration 0 [Initialization]** Assume that the prices of idiosyncratic risk are constant and apply the second stage of the algorithm described in Section 3 to all stocks in $\mathcal{S}$. Denote by $\Theta^{(0)}_{S}$ the set of parameter estimates from this iteration. In particular, at this stage,

$$
\lambda_{S,t} = \lambda^{(0)}_{S,t}, \quad \text{and} \quad \gamma_{S,t} = \gamma^{(0)}_{S,t}.
$$

**Iteration 1 [From Constants to Time-Varying]**

1. Using the estimates from iteration 0, let

$$
\lambda^{(0)}_{S,t} = \sum_{S' \in S} w_{S'} \lambda^{(0)}_{S'} \frac{h^{(0)}_{S',z,t}}{h^{(0)}_{S,z,t}} \quad \text{and} \quad \gamma^{(0)}_{S,t} = \sum_{S' \in S} w_{S'} \gamma^{(0)}_{S'} \frac{h^{(0)}_{S',y,t}}{h^{(0)}_{S,y,t}},
$$

2. Initialize $\lambda^{(1)}_{S,t}$ and $\gamma^{(1)}_{S,t}$ to

$$
\lambda^{(1)}_{S,t} = \lambda^{(0)}_{S,t} + \frac{1}{T} \sum_{t=1}^{T} \lambda^{(0)}_{S,t} \quad \text{and} \quad \gamma^{(1)}_{S,t} = \gamma^{(0)}_{S,t} + \frac{1}{T} \sum_{t=1}^{T} \gamma^{(0)}_{S,t} \quad (G.9)
$$

such that the initial average values of $\lambda^{(1)}_{S,t}$ and $\gamma^{(1)}_{S,t}$ are respectively $\lambda^{(0)}_{S,t}$ and $\gamma^{(0)}_{S,t}$:

$$
\frac{1}{T} \sum_{t=1}^{T} \lambda^{(1)}_{S,t} = \lambda^{(0)}_{S,t} \quad \text{and} \quad \frac{1}{T} \sum_{t=1}^{T} \gamma^{(1)}_{S,t} = \gamma^{(0)}_{S,t}.
$$

This step is necessary since the parameters $\lambda_S$ and $\gamma_S$ change from being the constant (and thus average) value of the prices of risk to being an intercept in an affine specification for the same prices of risk.

3. Apply the second stage of the algorithm described in Section 3 to all stocks in $\mathcal{S}$, starting from the parameter estimates obtained at iteration 0, $\Theta^{(0)}_{S}$, except for the parameters given by Equa-
tions (G.9). In particular, these two parameters are the only ones being estimated in the time-varying specifications

\[
\lambda_{S,t}^{(1)} = \lambda_{S,t}^{(1)} - \lambda_{S,t}^{(0)} \quad \text{and} \quad \gamma_{S,t}^{(1)} = \gamma_{S,t}^{(1)} - \gamma_{S,t}^{(0)}.
\]

### Iteration \( k > 1 \) [Seeking a Fixed Point]

1. Using the estimates from iteration \( k - 1 \), let

\[
\lambda_{S,t}^{(k-1)} = \sum_{S' \in \mathbb{S}} \lambda_{S',t}^{(k-1)} h_{S',t}^{(k-1)} \quad \text{and} \quad \gamma_{S,t}^{(k-1)} = \sum_{S' \in \mathbb{S}} \gamma_{S',t}^{(k-1)} h_{S',t}^{(k-1)}.
\]

2. Apply the second stage of the algorithm described in Section 3 to all stocks in \( \mathbb{S} \), starting from the parameter estimates obtained at iteration \( k - 1 \). In particular, \( \lambda_{S,t}^{(k)} \) and \( \gamma_{S,t}^{(k)} \) are the only two parameters being estimated in the time-varying specifications

\[
\lambda_{S,t}^{(k)} = \lambda_{S,t}^{(k-1)} - \lambda_{S,t}^{(k-1)} \quad \text{and} \quad \gamma_{S,t}^{(k)} = \gamma_{S,t}^{(k-1)} - \gamma_{S,t}^{(k-1)}.
\]

### Stop Algorithm  
When \( \sum_{t \in \mathbb{S}} \left( \lambda_{S,t}^{(k)} - \lambda_{S,t}^{(k-1)} \right)^2 < \mathcal{E} \) and \( \sum_{t \in \mathbb{S}} \left( \gamma_{S,t}^{(k)} - \gamma_{S,t}^{(k-1)} \right)^2 < \mathcal{E} \) for some tolerance \( \mathcal{E} \).

### H Stock Fundamentals

The market and the volatility betas are obtained by regressing a stock’s excess returns on the S&P 500 excess returns and daily changes on the VIX using the past year of data:

\[
R_{S,t-k} = \alpha + \beta_{mkt,t} R_{MKT,t-k} + \beta_{\Delta VIX,t} \Delta VIX_{t-k} + \epsilon_{t-k}, \quad k = 0, \ldots, 252.
\]

The betas are considered missing if less than 63 data points are available over the past year.

The market equity (ME) is obtained by multiplying the number of outstanding shares by the close price for each stock.

The book equity (BE) is computed as the difference between the total assets of a firm (ATQ in Compustat) and its liabilities. The latter are defined as the sum of the debt in current liabilities (DLCQ) and half of the long-term debt (DLTTQ) as in Bharath and Shumway (2008). Both the debt in current liabilities and the long-term debt are linearly interpolated between quarterly data points to obtain daily estimates. BE is considered missing when negative.
The operating profitability (OP) is defined as the quarterly revenue at time $t$ (REVTQ), minus the cost of goods sold at time $t$ (COGSQ), the interest expense at time $t$ (XINTQ), and selling, general, and administrative expenses at time $t$ (XSGAQ), divided by book equity for the last year (i.e. at $t$ minus 1 year). All the fundamental values used to compute OP were linearly interpolated from quarterly data.

The investment level is obtained from the book value of assets. Specifically, it is computed as the change in total assets over the previous year (from $t$ minus 1 year to $t$), divided by the total assets at the end of the previous year (i.e. at time $t$ minus 1 year). The values of the assets are also linearly interpolated from quarterly data to obtain daily estimates.

Finally, the trailing twelve-month return is obtained by taking the sum of daily excess returns over the last year (i.e. 252 previous business days, when available).
ONLINE APPENDIX

OA.A Continuous-Time Model

In this section, we demonstrate that the market model used in this paper can be seen as a discretization of a standard jump diffusion model. Similar intuition holds for the stock model. Let $U$ be the log-equity value, $V$ be the instantaneous variance, and $\lambda$ be the jump intensity:

$$
\begin{align*}
    dU_t &= \mu(V_t, \lambda_t) dt + \sqrt{V_t} dW_t + dY_t, \\
    dV_t &= a(V_t) dt + b(V_t) dW_t, \\
    d\lambda_t &= c(V_t, \lambda_t) dt + d(V_t) dW_t, \\
    Y_t &= \sum_{n=1}^{N_t} J_n, \quad J_n \sim \text{NIG}(\alpha, \delta, \gamma),
\end{align*}
$$

where $W$ is a Brownian motion, and $\mu(\cdot, \cdot)$, $a(\cdot)$, $b(\cdot)$, $c(\cdot, \cdot)$, as well as $d(\cdot)$ are functions. $\text{NIG}(\alpha, \delta, \gamma)$ represents a normal-inverse Gaussian distribution with a location parameter of 0, a scale parameter of $\gamma$, a tail heaviness parameter of $\alpha$ and an asymmetry parameter of $\delta$ (more details in Section OA.A.1). The stochastic process $N$ is a Cox process with stochastic intensity $\lambda$.

Applying the Milstein discretization to $V$ and $\lambda$, we get the time discretization of the continuous time version:

$$
\begin{align*}
    V_{t+1} \\[0.5ex] \\
    &\approx V_t + a(V_t) \Delta_t + b(V_t) \sqrt{\Delta_t} e^{(t+1)\Delta_t} + \frac{1}{2} b'(V_t) b(V_t) \Delta_t \left(e^{2 \epsilon_{(t+1)\Delta_t}} - 1\right), \\
    \lambda_{t+1} &\approx \lambda_t + c(V_t, \lambda_t) \Delta_t + d(V_t) \sqrt{\Delta_t} e^{(t+1)\Delta_t} + \frac{1}{2} d'(V_t) d(V_t) \Delta_t \left(e^{2 \epsilon_{(t+1)\Delta_t}} - 1\right),
\end{align*}
$$

where $\{\epsilon_{t\Delta_t}\}_{t=1}^{\infty}$ is a sequence of independent standard normal random variables. The Euler-Maruyama discretization of $U$ leads to

$$
U_{t+1} \approx U_t + \mu(V_{t\Delta_t}, \lambda_{t\Delta_t}) \Delta_t + \sqrt{V_{t\Delta_t}} \sqrt{\Delta_t} e^{(t+1)\Delta_t} + \Delta Y_{t+1}. 
$$

where $\Delta Y_{t+1} = Y_{(t+1)\Delta_t} - Y_{t\Delta_t} = \sum_{n=1}^{N_{(t+1)\Delta_t} - N_{t\Delta_t}} J_n$ and $N_{(t+1)\Delta_t} - N_{t\Delta_t} |_{\mathcal{F}_{t\Delta_t}}$ is a Poisson random variable with conditional expectation $\lambda_{t\Delta_t} \Delta_t$.

---

45 These functions depend on the specification of the model.
The SVJ model proposed above is equivalent to the GARCH-like framework proposed in our paper. Starting with $h_{z,t+1}$:

$$h_{z,t+1} = w_z + b_z h_{z,t} + a_z \left( e_t - c_z \sqrt{h_{z,t}} \right)^2$$

$$= w_z + b_z h_{z,t} + a_z \left( e_t^2 - 2e_t c_z \sqrt{h_{z,t}} + c_z^2 h_{z,t} \right)$$

$$= h_{z,t} + \frac{(w_z + a_z) + (b_z + a_z c_z^2 - 1)h_{z,t} - 2a_z c_z \sqrt{h_{z,t}} e_t + a_z (e_t^2 - 1)}{\sqrt{b(\sqrt{V_{x(t-1)\Delta}})\Delta_t}}.$$

A comparison with Equation (A.1) suggests that $h_{z,t} \equiv V_{(t-1)\Delta}, a(V_{(t-1)\Delta}) \Delta_t \equiv (w_z + a_z) + (b_z + a_z c_z^2 - 1) V_{(t-1)\Delta}, b(V_{(t-1)\Delta}) \equiv -2a_z c_z \sqrt{V_{(t-1)\Delta}} / \Delta_t$ and $a_z c_z^2 = 1$. Similarly,

$$h_{y,t+1} = w_y + b_y h_{y,t} + a_y \left( e_t - c_y \sqrt{h_{y,t}} \right)^2$$

$$= w_y + b_y h_{y,t} + a_y \left( e_t^2 - 2e_t c_y \sqrt{h_{y,t}} + c_y^2 h_{y,t} \right)$$

$$= h_{y,t} + (w_y + a_y) + (b_y - 1)h_{y,t} + a_y c_y^2 h_{y,t} - 2a_y c_y \sqrt{h_{y,t}} e_t + a_y (e_t^2 - 1),$$

suggesting that $h_{y,t} \equiv \lambda_{(t-1)\Delta}, c(V_{\lambda(t-1)\Delta}) \Delta_t \equiv (w_y + a_y) + (b_y - 1) \lambda_{\Delta t} + a_y c_y^2 V_{\lambda(t-1)\Delta}) / \Delta_t, d(V_{\lambda(t-1)\Delta}) \equiv -2a_y c_y \sqrt{V_{\lambda(t-1)\Delta}} / \Delta_t$, and $a_y c_y^2 = 1$. Finally,

$$r_{t+1} = \mu_{t+1} - \xi_{t+1} + \epsilon_{t+1} + y_{t+1}$$

$$= \mu(V_{\lambda(t-1)\Delta}) \Delta_t + \sqrt{V_{\lambda(t-1)\Delta}} \sqrt{\lambda_{\Delta t} e_{(t+1)\Delta}} + \Delta Y_{(t+1)\Delta} = U_{(t+1)\Delta} - U_{\lambda(t-1)\Delta}$$

if $\mu(V_{\lambda(t-1)\Delta}) = (\mu_{t+1} - \xi_{t+1}) / \Delta_t$. The jump innovation $\Delta Y_{(t+1)\Delta}$ is close to a NIG($\alpha, \delta, \gamma \lambda(t-1)\Delta$) (see the Online Appendix OA.A.1). Therefore, $\Delta Y_{(t+1)\Delta} \equiv y_{t+1}$ if $\gamma = \Delta_t^{-1}$. In sum, the two models are equivalent, under some parameter restrictions.

**OA.A.1 NIG**

The jump $y_{u,t+1}$ have a NIG distribution with location parameter 0, a scale parameter $h_{u,y,t+1}$, an asymmetry parameter $\delta_u$ and a tail heaviness parameter $\alpha_u$. The first standardized moments are

$$E_1^{\gamma}[y_{u,t+1}] = \frac{\delta_u}{\sqrt{a_u^2 - \delta_u^2}} h_{u,y,t+1}, \quad \text{Var}_1^{\gamma}[y_{u,t+1}] = \frac{a_u^2}{(\sqrt{a_u^2 - \delta_u^2})^3} h_{u,y,t+1}, \quad \text{Skew}_1^{\gamma}[y_{u,t+1}] = \frac{3 \delta_u}{\alpha_u (a_u^2 - \delta_u^2)^{3/2}} \sqrt{h_{u,y,t+1}}$$

**OA-A.2**
and the excess kurtosis is

\[ \text{ExKurt}_t^P \{ y_{u,t+1} \} = 3 \left( 1 + \frac{4\delta_u^2}{\alpha_u^2} \right) \frac{1}{\sqrt{\alpha_u^2 - \delta_u^2}} h_{u,y,t+1}. \]

The moment generating function is

\[ \varphi_{y_{u,t+1}}(\phi) = \exp \left( \sqrt{\alpha_u^2 - \delta_u^2} - \sqrt{\alpha_u^2 - (\delta_u + \phi)^2} \right) h_{u,y,t+1} \]

**Interpretation of the Jump Intensity Parameter**

For comparison, let \( N_{t+1} \) be a Poisson random variable of intensity \( \lambda_{t+1} \), and consider the compound Poisson random variable \( \Sigma_{j=0}^{N_t} J_j \) where the jumps \( J_j \) are independent NIG(0, h', \delta', \alpha') random variables.

The moment generating function of \( \Sigma_{j=0}^{N_t} J_j \) is

\[ \varphi_{\sum_{j=0}^{N_t} J_j}(\phi) = \exp \left( -\lambda_t \right) \sum_{j=1}^{\infty} \exp \left( j \left( \sqrt{(\alpha')^2 - (\delta')^2} - \sqrt{(\alpha')^2 - (\delta' + \phi)^2} \right) h' \right) \left( -\lambda_t \right)^{\frac{j}{j!}} \]

\[ = \exp \left( -\lambda_t \right) \sum_{j=1}^{\infty} \exp \left( \left( \sqrt{(\alpha')^2 - (\delta')^2} - \sqrt{(\alpha')^2 - (\delta' + \phi)^2} \right) h' \right) \frac{1}{j!} \]

\[ = \exp \left( -\lambda_t \right) \exp \left( \left( \sqrt{(\alpha')^2 - (\delta')^2} - \sqrt{(\alpha')^2 - (\delta' + \phi)^2} \right) h' \right) \left( 1 - 1 \right) \]

where the last approximation holds from a first order Taylor expansion, provided that \( \phi \) is close to zero.

Letting \( \alpha' = \alpha_u^2 \), and \( \delta' = \delta_u \), a direct comparison between \( \varphi_{\sum_{j=0}^{N_t} J_j}(\phi) \) and \( \varphi_{y_{u,t+1}}(\phi) \) implies that

\[ h_{u,y,t+1} \equiv \lambda_t h', \]

that is \( h_{u,y,t+1} \) may be interpreted as a scaled version of the jump intensity.
Where

\[ \Pi_u(\phi) = \sqrt{a_u^2 - \delta_u^2} - \sqrt{a_u^2 - (\delta_u + \phi)^2}. \]

Note that

\[ \frac{\partial \Pi_u}{\partial \phi}(\phi) = \frac{(\delta_u + \phi)}{\sqrt{a_u^2 - (\delta_u + \phi)^2}}, \]

\[ \frac{\partial^2 \Pi_u}{\partial \phi^2}(\phi) = \frac{a_u^2}{(a_u^2 - (\delta_u + \phi)^2)^2}, \]

\[ \frac{\partial^3 \Pi_u}{\partial \phi^3}(\phi) = 3 \frac{a_u^2 \delta_u + \phi}{(a_u^2 - (\delta_u + \phi)^2)^2}, \]

\[ \frac{\partial^4 \Pi_u}{\partial \phi^4}(\phi) = 3 \frac{a_u^2 + 4(\delta_u + \phi)^2}{(a_u^2 - (\delta_u + \phi)^2)^2}. \]

The cumulant generating function is therefore

\[ \xi(\phi; a, b, c) = \frac{a^2 \phi^2}{2} h_{M,z,t} + \Pi_M(b \phi) h_{M,y,t} + \frac{c^2 \phi^2}{2} h_{S,z,t} + \Pi_S(c \phi) h_{S,y,t} \]

Note that

\[ \frac{\partial \xi}{\partial \phi}(\phi; a, b, c) = a^2 bh_{M,z,t} + b \frac{\partial \Pi_M}{\partial \phi}(b \phi) h_{M,y,t} + c^2 \phi h_{S,z,t} + c \frac{\partial \Pi_S}{\partial \phi}(c \phi) h_{S,y,t}, \]

\[ \frac{\partial^2 \xi}{\partial \phi^2}(\phi; a, b, c) = a^2 h_{M,z,t} + b^2 \frac{\partial^2 \Pi_M}{\partial \phi^2}(b \phi) h_{M,y,t} + c^2 \frac{\partial^2 \Pi_S}{\partial \phi^2}(c \phi) h_{S,y,t}, \]

\[ \frac{\partial^3 \xi}{\partial \phi^3}(\phi; a, b, c) = b \frac{\partial^3 \Pi_M}{\partial \phi^3}(b \phi) h_{M,y,t} + c^3 \frac{\partial^3 \Pi_S}{\partial \phi^3}(c \phi) h_{S,y,t}, \]

\[ \frac{\partial^4 \xi}{\partial \phi^4}(\phi; a, b, c) = b^4 \frac{\partial^4 \Pi_M}{\partial \phi^4}(b \phi) h_{M,y,t} + c^4 \frac{\partial^4 \Pi_S}{\partial \phi^4}(c \phi) h_{S,y,t}. \]
The first moment of the market and stock returns are
\[
\begin{align*}
E_t^p[r_{M,t+1}] &= \mu_{M,t+1}^p - \xi_{M,t+1}^p + \frac{\partial \hat{\xi}}{\partial \phi}(0; 1, 1, 0), \\
E_t^p[r_{S,t+1}] &= \mu_{S,t+1}^p - \xi_{S,t+1}^p + \frac{\partial \hat{\xi}}{\partial \phi}(0; \beta_{S,z}, \beta_{S,y}, 1).
\end{align*}
\]

Their variances correspond to
\[
\begin{align*}
\text{Var}_t^p[r_{M,t+1}] &= \text{Var}_t^p[z_{M,t+1} + y_{M,t+1}] = \frac{\partial^2 \hat{\xi}}{\partial \phi^2}(0; 1, 1, 0) \\
\text{Var}_t^p[r_{S,t+1}] &= \text{Var}_t^p[\beta_{S,z}z_{M,t+1} + \beta_{S,y}y_{M,t+1} + z_{S,t+1} + y_{S,t+1}] = \frac{\partial^2 \hat{\xi}}{\partial \phi^2}(0; \beta_{S,z}, \beta_{S,y}, 1).
\end{align*}
\]

Similarly, since the third cumulant corresponds to the third centered moment, the third standardized moments are respectively
\[
\begin{align*}
\text{Skew}_t^p[r_{M,t+1}] &= \frac{\partial \hat{\xi}}{\partial \phi}(0; 1, 1, 0) \left( \frac{\partial \hat{\xi}}{\partial \phi}(0; 1, 1, 0) \right)^2 \text{ and } \text{Skew}_t^p[r_{S,t+1}] = \frac{\partial \hat{\xi}}{\partial \phi}(0; \beta_{S,z}, \beta_{S,y}, 1) \left( \frac{\partial \hat{\xi}}{\partial \phi}(0; \beta_{S,z}, \beta_{S,y}, 1) \right)^2.
\end{align*}
\]

Finally, the excess kurtosis are
\[
\begin{align*}
E_t^p\left( \frac{r_{M,t+1} - E_t^p[r_{M,t+1}]}{\sqrt{\text{Var}_t^p[r_{M,t+1}]}} \right)^4 - 3 &= \frac{\partial \hat{\xi}}{\partial \phi}(0; 1, 1, 0) \left( \frac{\partial \hat{\xi}}{\partial \phi}(0; 1, 1, 0) \right)^2, \\
E_t^p\left( \frac{r_{M,t+1} - E_t^p[r_{M,t+1}]}{\sqrt{\text{Var}_t^p[r_{M,t+1}]}} \right)^4 - 3 &= \frac{\partial \hat{\xi}}{\partial \phi}(0; \beta_{S,z}, \beta_{S,y}, 1) \left( \frac{\partial \hat{\xi}}{\partial \phi}(0; \beta_{S,z}, \beta_{S,y}, 1) \right)^2.
\end{align*}
\]

**OA.A.3 Conditional Variance and Jump Intensity Moments**

**Lemma 7**
\[
\begin{align*}
\text{Var}_{t-1}^p[h_{M,z,t+1}] &= 2\alpha_{M,z}^2 (1 + 2c_{M,z}^2 h_{M,z,t}), \\
\text{Var}_{t-1}^p[h_{M,y,t+1}] &= 2\alpha_{M,y}^2 (1 + 2c_{M,y}^2 h_{M,z,t}), \\
\text{Var}_{t-1}^p[h_{S,z,t+1}] &= \kappa_{S,z}^2 \text{Var}_{t-1}^p[h_{M,z,t+1}] + 2\alpha_{S,z}^2 (1 + 2c_{S,z}^2 h_{S,z,t}), \\
\text{Var}_{t-1}^p[h_{S,y,t+1}] &= \kappa_{S,y}^2 \text{Var}_{t-1}^p[h_{M,y,t+1}] + 2\alpha_{S,y}^2 (1 + 2c_{S,y}^2 h_{S,z,t}).
\end{align*}
\]
Proof. Recall that the market conditional variance is
\[
h_{M,z,t+1} = w_{M,z} + b_{M,z}h_{M,z,t} + a_{M,z}\left(\epsilon_{M,t} - c_{M,z}\sqrt{h_{M,z,t}}\right)^2.
\]
Hence, \(\mathbb{E}_{t-1}^P [h_{M,z,t+1}] = w_{M,z} + b_{M,z}h_{M,z,t} + a_{M,z}\left(1 + c_{M,z}^2h_{M,z,t}\right)\) and
\[
\text{Var}_{t-1}^P [h_{M,z,t+1}] = a_{M,z}^2\mathbb{E}_{t-1}^P \left[\left(\epsilon_{M,t} - c_{M,z}\sqrt{h_{M,z,t}}\right)^2 - \left(1 + c_{M,z}^2h_{M,z,t}\right)\right]^2
\]
\[
= a_{M,z}^2\mathbb{E}_{t-1}^P \left[\epsilon_{M,t}^4 - 4c_{M,z}\sqrt{h_{M,z,t}}\epsilon_{M,t}^3 + 2\left(2c_{M,z}^2h_{M,z,t} - 1\right)\epsilon_{M,t}^2 + 4c_{M,z}\sqrt{h_{M,z,t}}\epsilon_{M,t} + 1\right]
\]
\[
= a_{M,z}^2\left(3 + 2\left(2c_{M,z}^2h_{M,z,t} - 1\right) + 1\right)
\]
\[
= 2a_{M,z}^2\left(1 + 2c_{M,z}^2h_{M,z,t}\right).
\]
The market jump scale parameter is
\[
h_{M,y,t+1} = w_{M,y} + b_{M,y}h_{M,y,t} + a_{M,y}\left(\epsilon_{M,t} - c_{M,y}\sqrt{h_{M,z,t}}\right)^2.
\]
Hence \(\mathbb{E}_{t-1}^P [h_{M,y,t+1}] = w_{M,y} + b_{M,y}h_{M,y,t} + a_{M,y}\left(1 + c_{M,y}^2h_{M,z,t}\right)\) and
\[
\text{Var}_{t-1}^P [h_{M,y,t+1}] = a_{M,y}^2\mathbb{E}_{t-1}^P \left[\left(\epsilon_{M,t} - c_{M,y}\sqrt{h_{M,z,t}}\right)^2 - \left(1 + c_{M,y}^2h_{M,z,t}\right)\right]^2
\]
\[
= 2a_{M,y}^2\left(1 + 2c_{M,y}^2h_{M,z,t}\right).
\]
The stock conditional variance satisfies
\[
h_{S,z,t+1} = \kappa_{S,z}h_{M,z,t+1} + b_{S,z} (h_{S,z,t} - \kappa_{S,z}h_{M,z,t}) + a_{S,z}\left(\epsilon_{S,t} - 1 - 2c_{S,z}\sqrt{h_{S,z,t}}\epsilon_{S,t}\right).
\]
Therefore, $E_{t-1}^p [h_{S,z,t+1}] = \kappa_{S,z} E_{t-1}^p [h_{M,z,t+1}] + b_{S,z} (h_{S,z,t} - \kappa_{S,z} h_{M,z,t})$ and

$$\begin{align*}
\text{Var}_{t-1}^p [h_{S,z,t+1}] &= E_{t-1}^p \left[ \left( \kappa_{S,z} (h_{M,z,t+1} - E_{t-1}^p [h_{M,z,t+1}]) + a_{S,z} \left( e_{S,y,t}^2 - 1 - 2c_{S,z} \sqrt{h_{S,z,t} e_{S,y,t}} \right) \right)^2 \right] \\
&= \kappa_{S,z}^2 \text{Var}_{t-1}^p [h_{M,z,t+1}] + 2a_{S,z}^2 (1 + 2c_{S,z}^2 h_{S,z,t}) \\
&
\end{align*}$$

Finally,

$$h_{S,y,t+1} = \kappa_{S,y} h_{M,y,t+1} + b_{S,y} (h_{S,y,t} - \kappa_{S,y} h_{M,y,t}) + a_{S,y} \left( e_{S,y,t}^2 - 1 - 2c_{S,y} z_{S,y,t} \right)$$

implies that $E_{t-1}^p [h_{S,y,t+1}] = \kappa_{S,y} E_{t-1}^p [h_{M,y,t+1}] + b_{S,y} (h_{S,y,t} - \kappa_{S,y} h_{M,y,t})$ and

$$\begin{align*}
\text{Var}_{t-1}^p [h_{S,y,t+1}] &= E_{t-1}^p \left[ \left( \kappa_{S,y} (h_{M,y,t+1} - E_{t-1}^p [h_{M,y,t+1}]) + a_{S,y} \left( e_{S,y,t}^2 - 1 - 2c_{S,y} \sqrt{h_{S,z,t} e_{S,y,t}} \right) \right)^2 \right] \\
&= \kappa_{S,y}^2 \text{Var}_{t-1}^p [h_{M,y,t+1}] + 2a_{S,y}^2 (1 + 2c_{S,y}^2 h_{S,z,t}) .
\end{align*}$$
OA.B  Proofs of Lemmas 2 and 3

**Proof of Lemma 2.** The conditional cumulant generating function of $e_{u,t}$ under $\mathcal{Q}$ is

$$
\xi^Q_{e_{u,t}}(\phi) = \log E_{t-1}^Q \left[ \exp (\phi e_{u,t}) \right] = \log E_{t-1}^P \left[ \frac{\exp (-\Lambda_M \bar{z}_{M,t} - \Sigma (\Lambda_S \bar{z}_{S,t} - \Sigma \bar{y}_S))}{\exp (-\Lambda_M \bar{z}_{M,t} - \Sigma (\Lambda_S \bar{z}_{S,t} - \Sigma \bar{y}_S))} \exp (\phi e_{u,t}) \right] = \log E_{t-1}^P \left[ \exp (-\Lambda_M \bar{z}_{M,t} - \Sigma (\Lambda_S \bar{z}_{S,t} - \Sigma \bar{y}_S)) \exp (\phi e_{u,t}) \right] = \xi^P_{e_{u,t}}(\phi - \Lambda_u \sqrt{h_{u,z,t}}) - \xi^P_{e_{u,t}}(-\Lambda_u \sqrt{h_{u,z,t}}) = \left( \frac{1}{2} (\phi - \Lambda_u \sqrt{h_{u,z,t}})^2 - \frac{1}{2} (-\Lambda_u \sqrt{h_{u,z,t}})^2 \right) = \left( \frac{1}{2} \phi^2 - \Lambda_u \sqrt{h_{u,z,t}} \phi \right). \Box
$$

**Proof of Lemma 3.** For any $y_{u,t} \in \{y_{M,t}, y_{S,t} : S \in \mathcal{S}\}$, the conditional cumulant generating function of $y_{u,t}$ under $\mathcal{Q}$ is

$$
\xi^Q_{y_{u,t}}(\phi) = \log E_{t-1}^Q \left[ \exp (\phi y_{u,t}) \right] = \log E_{t-1}^P \left[ \frac{\exp (-\Lambda_M \bar{z}_{M,t} - \Sigma (\Lambda_S \bar{z}_{S,t} - \Sigma \bar{y}_S))}{\exp (-\Lambda_M \bar{z}_{M,t} - \Sigma (\Lambda_S \bar{z}_{S,t} - \Sigma \bar{y}_S))} \exp (\phi y_{u,t}) \right] = \log E_{t-1}^P \left[ \exp (-\Gamma y_{u,t}) \exp (\phi y_{u,t}) \right] = \xi^P_{y_{u,t}}(\phi - \Gamma) - \xi^P_{y_{u,t}}(-\Gamma) = \Pi_u (\phi - \Gamma) h_{u,y,t} - \Pi_u (-\Gamma) h_{u,y,t} = \left( \sqrt{\alpha_u^2 - (\delta_u - \Gamma)^2} - \sqrt{\alpha_u^2 - (\delta_u + \phi - \Gamma)^2} \right) h_{u,y,t}
$$

which is the cumulant generating function of a NIG of parameter $\mu_u = \mu_u = 0$, $\alpha_u^2 = \alpha_u$, $\delta_u^2 = \delta_u - \Gamma$, and $h_{u,y,t} = h_{u,y,t}$. \Box

OA.C  Proofs of Appendix C’s results

OA.C.1  Market Drift under $\mathcal{P}$

Recall that

$$
\ln \frac{M_t}{M_{t-1}} = r_{M,t} = \mu_{M,t} - \xi^P_{M,t} + \bar{z}_{M,t} + y_{M,t}
$$

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Since the discounted stock price should behave as a $\mathbb{Q}$–martingale,

\[
1 = \mathbb{E}_{t-1}^\mathbb{Q} \left[ \exp \left( -r_{f,t} \right) \frac{M_t}{M_{t-1}} \right] = \mathbb{E}_{t-1}^\mathbb{P} \left[ \exp \left( \mu_{M,t}^\mathbb{P} - \xi_{M,t}^\mathbb{P} \right) \exp \left( -r_{f,t} \right) \right]
\]

\[
= \mathbb{E}_{t-1}^\mathbb{P} \left[ \exp \left( \mu_{M,t}^\mathbb{P} - \xi_{M,t}^\mathbb{P} \right) \exp \left( -r_{f,t} \right) \right] = \exp \left( \mu_{M,t}^\mathbb{P} - r_{f,t} - \xi_{M,t}^\mathbb{P} \right)
\]

Because

\[-\xi_{M,t}^\mathbb{P} (1) + \xi_{M,t}^\mathbb{P} (1 - \Lambda_M) - \xi_{M,t}^\mathbb{P} (-\Lambda_M) = -\frac{1}{2} h_{M,z,t} + \frac{1}{2} h_{M,z,t} (1 - \Lambda_M)^2 - \frac{1}{2} h_{M,z,t} \Lambda_M^2 = -\Lambda_M h_{M,z,t}\]

and

\[-\xi_{M,t}^\mathbb{P} (1) + \xi_{M,t}^\mathbb{P} (1 - \Gamma_M) - \xi_{M,t}^\mathbb{P} (-\Gamma_M) = -h_{M,z,t} \gamma_M\]

where

\[
\gamma_M = \sqrt{\sigma_M^2 - \delta_M^2} = \sqrt{\alpha_M^2 - (\delta_M + 1)^2} + \sqrt{\alpha_M^2 - (\delta_M + 1 - \Gamma_M)^2} = \sqrt{\alpha_M^2 - (\delta_M - \Gamma_M)^2}
\]

\[= \Pi_M (1) - \Pi_M (1), \]

we conclude that

\[
1 = \exp \left( \mu_{M,t}^\mathbb{P} - r_{f,t} - \Lambda_M h_{M,z,t} - h_{M,z,t} \gamma_M \right).
\]

Therefore,

\[
\mu_{M,t}^\mathbb{P} = r_{f,t} + \Lambda_M h_{M,z,t} + \gamma_M h_{M,z,t}.
\]
OA.C.2  Stock Drift under $\mathbb{P}$

Recall that

$$\ln \frac{S_t}{S_{t-1}} = r_{S,t} = \mu^S_{S,t} - \xi^S_{S,t} + \beta_{S,z}Z_{M,t} + \beta_{S,y}Y_{M,t} + z_{S,t} + y_{S,t}$$

Since the discounted stock price should behave as a $\mathbb{Q}$–martingale,

$$1 = \mathbb{E}_{t-1}^Q \left[ \frac{\exp(-r_{f,t}) S_t}{S_{t-1}} \right]$$

$$= \mathbb{E}_{t-1}^P \left[ \exp(\Lambda Mz_{M,t} - \Gamma_M Y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S Z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S Y_{S,t}) \right] \mathbb{E}_{t-1}^P \left[ \exp(-\Lambda Mz_{M,t} - \Gamma_M Y_{M,t} - \sum_{S \in \mathbb{S}} \Lambda_S Z_{S,t} - \sum_{S \in \mathbb{S}} \Gamma_S Y_{S,t}) \right]$$

$$= \mathbb{E}_{t-1}^P \left[ \exp(\mu^S_{S,t} - r_{f,t} - \xi^S_{S,t}) \right]$$

$$= \mathbb{E}_{t-1}^P \left[ \exp(\mu^S_{S,t} - r_{f,t} - \xi^S_{S,t} + \beta_{S,z}Z_{M,t} + \beta_{S,y}Y_{M,t} + z_{S,t} + y_{S,t} - r_{f,t}) \right]$$

$$= \mathbb{E}_{t-1}^P \left[ \exp(\mu^S_{S,t} - r_{f,t} - \xi^S_{M,t} + \beta_{S,z}Z_{M,t} + \beta_{S,y}Y_{M,t} + z_{S,t} + y_{S,t} - r_{f,t}) \right]$$

Because

$$-\xi^S_{z_{M,t}} (\beta_{S,z} - \Lambda_M) + \xi^S_{y_{M,t}} (\beta_{S,y} - \Gamma_M) - \xi^S_{y_{S,t}} (\Lambda_S - \Gamma_S) = \frac{1}{2} h_{M,z,t} \beta_{S,z}^2 + h_{M,z,t} (\beta_{S,z} - \Lambda_M)^2 - \frac{1}{2} h_{M,z,t} \Lambda_M^2$$

$$= -\Lambda_M \beta_{S,z} h_{M,z,t},$$

$$-\xi^S_{z_{S,t}} (1 - \Lambda_S) - \xi^S_{y_{S,t}} (\Lambda_S) = \frac{1}{2} h_{S,z,t}^{1} + h_{S,z,t} (1 - \Lambda_S)^2 - \frac{1}{2} h_{S,z,t} \Lambda_S^2$$

$$= -\Lambda_S h_{S,z,t},$$

$$-\xi^S_{y_{M,t}} (\beta_{S,y} - \Gamma_M) + \xi^S_{y_{M,t}} (\Gamma_M) = (\Pi_M (\beta_{S,y}) + \Pi_M (\beta_{S,y} - \Gamma_M) - \Pi_M (\Gamma_M)) h_{M,y,t}$$

$$= \Pi_M (\beta_{S,y}) - \Pi_M (\Gamma_M) = -\gamma_{MS} (\beta_{S,y}) h_{M,y,t}$$

$$=-\xi^S_{y_{S,t}} (1 - \Gamma_S) - \xi^S_{y_{S,t}} (\Gamma_S) = \Pi_S (1) - \Pi_S (1) = -\gamma_S h_{S,y,t},$$

we conclude that

$$1 = \exp(\mu^P_{S,t} - r_{f,t} - \Lambda_M \beta_{S,z} h_{M,z,t} - h_{M,y,t} \gamma_{MS} (\beta_{S,y}) - \Lambda_S h_{S,z,t} - h_{S,y,t} \gamma_S).$$
where
\[ \gamma_{M,S}(\beta_{S,t}) = \Pi_M(\beta_{S,t}) - \Pi_S^\ast(\beta_{S,t}) \text{ and } \gamma_S = \Pi_S(1) - \Pi_S^\ast(1). \]

Therefore,
\[ \mu_{S,t}^P = r_{f,t} + \Lambda_M \beta_{S,t} h_{M,t} + h_{M,t} \gamma_{M,S}(\beta_{S,t}) + \Lambda_S h_{S,t} + h_{S,t} \gamma_S. \]

**OA.C.3 Co-Skewness**

Note that the pricing kernel may be written as \( \exp\left(r_{f,t+1} \right) \tilde{m}_{t+1} = m_{t+1}^M m_{t+1}^S \) where
\[
m_{t+1}^M = \frac{\exp(-\Lambda_M z_{M,t+1} - \Gamma_M y_{M,t+1})}{\mathbb{E}_F^P[\exp(-\Lambda_M z_{M,t+1} - \Gamma_M y_{M,t+1})]} = \exp\left(-\Lambda_M z_{M,t+1} - \xi_{z,t+1} (-\Lambda_M) - \Gamma_M y_{M,t+1} - \xi_{y,t+1} (-\Gamma_M) \right)
\]
and
\[
m_{t+1}^S = \frac{\mathbb{E}_F^P[\exp(-\Lambda_M z_{M,t+1} - \Gamma_M y_{M,t+1})] \exp(\sum_{S \in S} \Lambda_S z_{S,t+1} - \sum_{S \in S} \Gamma_S y_{S,t+1})}{\mathbb{E}_F^P[\exp(-\sum_{S \in S} \Lambda_S z_{S,t+1} - \sum_{S \in S} \Gamma_S y_{S,t+1})]} = \exp\left(-\sum_{S \in S} \Lambda_S z_{S,t+1} - \sum_{S \in S} \xi_{z,t+1} (-\Lambda_S) - \sum_{S \in S} \Gamma_S y_{S,t+1} - \sum_{S \in S} \xi_{y,t+1} (-\Gamma_S) \right).
\]

Because the discounted price process is a \( \mathbb{Q} \)-martingale, then for \( u \in \{M,S\} \)
\[
u_t = \mathbb{E}_F^\mathbb{Q}[\exp(-r_{f,t+1} u_{t+1})] = \mathbb{E}_F^P[m_{t+1}^M m_{t+1}^S \exp(-r_{f,t+1}) u_{t+1}].
\]

Dividing both sides with \( u_t \) leads to
\[
1 = \mathbb{E}_F^P[m_{t+1}^M m_{t+1}^S \exp(-r_{f,t+1}) u_{t+1} / u_t] = \mathbb{E}_F^P[m_{t+1}^M m_{t+1}^S \exp(r_{u,t+1} - r_{f,t+1})] = \mathbb{E}_F^P[\exp(r_{u,t+1} - r_{f,t+1})] + \text{Cov}_F^P[m_{t+1}^M m_{t+1}^S, \exp(r_{u,t+1} - r_{f,t+1})],
\]

\( \text{OA-11} \)
that is, the expected excess return (simple capitalization) satisfies

\[
E^P_{F_t} \left[ \exp \left( r_{u,t+1} - r_{f,t+1} \right) - 1 \right] = -Cov^P_{F_t} \left[ m^M_{t+1} m^\theta_{t+1}, \exp \left( r_{u,t+1} - r_{f,t+1} \right) \right].
\]

When \( u \) correspond to the market, the last equation becomes\(^{46}\)

\[
E^P_{F_t} \left[ \exp \left( r_{M,t+1} - r_{f,t+1} \right) - 1 \right] = -Cov^P_{F_t} \left[ m^M_{t+1}, \exp \left( r_{M,t+1} - r_{f,t+1} \right) \right],
\]

whereas the stock expected excess return is\(^{47}\)

\[
\text{Cov}^P_{F_t} \left[ m^M_{t+1}, \exp \left( r_{S,t+1} - r_{f,t+1} \right) \right] = \text{Cov}^P_{F_t} \left[ m^M_{t+1}, \exp \left( r_{S,t+1} - r_{f,t+1} \right) \right] + \text{Cov}^P_{F_t} \left[ m^\theta_{t+1}, m^M_{t+1}, \exp \left( r_{S,t+1} - r_{f,t+1} \right) \right]. \tag{C.3}
\]

The first term is what we usually get when idiosyncratic risk is not price. The second covariance term accounts for the price of idiosyncratic risk and vanishes whenever \( \Lambda_S = \Gamma_S = 0 \).

We now relate the first term to the concept of co-skewness. Since \( z_{M,t+1} = r_{M,t+1} - \mu^P_{M,t+1} + \xi^P_{M,t+1} - \)

\[
\begin{align*}
\text{Cov}^P_{F_t} \left[ m^M_{t+1}, m^\theta_{t+1}, \exp \left( r_{M,t+1} - r_{f,t+1} \right) \right] & = E^P_{F_t} \left[ m^M_{t+1}, m^\theta_{t+1}, \exp \left( r_{M,t+1} - r_{f,t+1} \right) \right] - E^P_{F_t} \left[ m^M_{t+1} \right] E^P_{F_t} \left[ m^\theta_{t+1} \right] E^P_{F_t} \left[ \exp \left( r_{M,t+1} - r_{f,t+1} \right) \right] \\
& = E^P_{F_t} \left[ m^M_{t+1}, \exp \left( r_{M,t+1} - r_{f,t+1} \right) \right] E^P_{F_t} \left[ m^\theta_{t+1} \right] - E^P_{F_t} \left[ m^M_{t+1} \right] E^P_{F_t} \left[ m^\theta_{t+1} \right] E^P_{F_t} \left[ \exp \left( r_{M,t+1} - r_{f,t+1} \right) \right] \\
& = \text{Cov}^P_{F_t} \left[ m^M_{t+1}, \exp \left( r_{M,t+1} - r_{f,t+1} \right) \right].
\end{align*}
\]

\[
\begin{align*}
\text{Cov}^P_{F_t} \left[ m^M_{t+1}, m^\theta_{t+1}, \exp \left( r_{S,t+1} - r_{f,t+1} \right) \right] & = E^P_{F_t} \left[ m^M_{t+1}, m^\theta_{t+1}, \exp \left( r_{S,t+1} - r_{f,t+1} \right) \right] - E^P_{F_t} \left[ m^M_{t+1} m^\theta_{t+1} \right] E^P_{F_t} \left[ \exp \left( r_{S,t+1} - r_{f,t+1} \right) \right] \\
& = E^P_{F_t} \left[ m^M_{t+1}, \exp \left( r_{S,t+1} - r_{f,t+1} \right) \right] + \text{Cov}^P_{F_t} \left[ m^\theta_{t+1}, m^M_{t+1}, \exp \left( r_{S,t+1} - r_{f,t+1} \right) \right] - E^P_{F_t} \left[ \exp \left( r_{S,t+1} - r_{f,t+1} \right) \right] \\
& = \text{Cov}^P_{F_t} \left[ m^M_{t+1}, \exp \left( r_{S,t+1} - r_{f,t+1} \right) \right] + \text{Cov}^P_{F_t} \left[ m^\theta_{t+1}, m^M_{t+1}, \exp \left( r_{S,t+1} - r_{f,t+1} \right) \right].
\end{align*}
\]
Replacing (D.6) in the market conditional variance leads to

\[ y_{M,t+1}, \]

\[ m^M_{t+1} = \exp\left(-\Lambda_M \left( r_{M,t+1} - \mu^P_{M,t+1} + \xi^P_{M,t+1} - y_{M,t+1} \right) - \xi_{z,t+1} \right) \]

\[ = \exp\left(-\Lambda_M \left( r_{M,t+1} - r_{f,t+1} \right) - (\Gamma_M - \Lambda_M) y_{M,t+1} \right) \]

\[ = v_{t+1} \exp\left(-\Lambda_M \left( r_{M,t+1} - r_{f,t+1} \right) \right) \exp\left(- (\Gamma_M - \Lambda_M) y_{M,t+1} \right) \]

\[ = v_{t+1} \left( 1 - \Lambda_M \left( r_{M,t+1} - r_{f,t+1} \right) + \frac{1}{2} \Lambda^2_M \left( r_{M,t+1} - r_{f,t+1} \right)^2 \right) \exp\left(- (\Gamma_M - \Lambda_M) y_{M,t+1} \right) \]

where \( v_{t+1} = \exp\left( \Lambda_M \left( \mu^P_{M,t+1} - r_{f,t+1} \right) - \Lambda_M \xi^P_{M,t+1} - \xi_{z,t+1} \right) \) is predictable. If the price of the Gaussian component is the same as the jump one, \( \Gamma_M = \Lambda_M \), then the first term of Equation (C.3) is approximated as follows:

\[ \Cov_{\gamma,t}^P \left[ m^M_{t+1}, \exp\left(r_{S,t+1} - r_{f,t+1} \right) \right] \]

\[ = v_{t+1} \Cov_{\gamma,t}^P \left[ -\Lambda_M \left( r_{M,t+1} - r_{f,t+1} \right) + \frac{1}{2} \Lambda^2_M \left( r_{M,t+1} - r_{f,t+1} \right)^2, r_{S,t+1} - r_{f,t+1} \right] \]

\[ = -\Lambda_M v_{t+1} \Cov_{\gamma,t}^P \left[ r_{M,t+1} - r_{f,t+1}, r_{S,t+1} - r_{f,t+1} \right] + \frac{1}{2} \Lambda^2_M \xi_{z,t+1} \Cov_{\gamma,t}^P \left[ \left( r_{M,t+1} - r_{f,t+1} \right)^2, r_{S,t+1} - r_{f,t+1} \right]. \]

The last covariance term is known as co-skewness.

**OA.D Calculation Associated with Appendix D**

Replacing (D.6) in the market conditional variance leads to

\[ h_{M,z,t+1} = w_{M,z} + b_{M,z} h_{M,z,t} + a_{M,z} \left( e_{M,t} - c_{M,z} \sqrt{h_{M,z,t}} \right)^2 \]

\[ = w_{M,z} + b_{M,z} h_{M,z,t} + a_{M,z} \left( e^*_{M,t} - \Lambda_M \sqrt{h_{M,z,t}} \right)^2 \]

\[ = w_{M,z} + b_{M,z} h_{M,z,t} + a_{M,z} \left( e^*_{M,t} - (c_{M,z} + \Lambda_M) \sqrt{h_{M,z,t}} \right)^2 \]

\[ = w_{M,z} + b_{M,z} h_{M,z,t} + a_{M,z} \left( e^*_{M,t} \right)^2 - 2 (c_{M,z} + \Lambda_M) \sqrt{h_{M,z,t}} e^*_{M,t} + (c_{M,z} + \Lambda_M)^2 h_{M,z,t} \]

\[ = w_{M,z} + \left( b_{M,z} + a_{M,z} (c_{M,z} + \Lambda_M)^2 \right) h_{M,z,t} + a_{M,z} \left( e^*_{M,t} \right)^2 - 2 a_{M,z} (c_{M,z} + \Lambda_M) \sqrt{h_{M,z,t}} e^*_{M,t} \]
A similar argument leads to the stock conditional variance:

\[
\begin{align*}
    h_{S,z,t+1} &= \kappa_S h_{M,z,t+1} + b_{S,z} (h_{S,z,t} - \kappa_S h_{M,z,t}) + a_{S,z} \left( \varepsilon_{S,t}^2 - 1 - 2c_{S,z}z_{z,t} \right) \\
    &= \kappa_S h_{M,z,t+1} + b_{S,z} (h_{S,z,t} - \kappa_S h_{M,z,t}) \\
    &\quad + a_{S,z} \left( (2c_{S,z} + \Lambda_S) \Lambda_S h_{S,z,t}^* + \left( \varepsilon_{S,t}^2 \right)^2 - 1 - 2(c_{S,z} + \Lambda_S) \sqrt{h_{S,z,t}^* e_{S,t}^*} \right) \\
    &= \kappa_S h_{M,z,t+1} + b_{S,z} (h_{S,z,t} - \kappa_S h_{M,z,t}) + a_{S,z} (2c_{S,z} + \Lambda_S) \Lambda_S h_{S,z,t}^* \\
    &\quad + a_{S,z} \left( \varepsilon_{S,t}^2 - 1 - 2c_{S,z} + \Lambda_S \right) \sqrt{h_{S,z,t}^* e_{S,t}^*} \\
    &= \pi_{S,z,1} + \pi_{S,z,2} h_{M,z,t}^* + 0h_{S,z,3}^* + \pi_{S,z,4} h_{S,z,t}^* + 0h_{S,z,t}^* \\
    &\quad + \pi_{S,z,6} \left( \varepsilon_{M,t}^* \right)^2 + \pi_{S,z,7} \sqrt{h_{M,z,t}^* e_{M,t}^*} + \pi_{S,z,8} \left( \varepsilon_{S,t}^* \right)^2 + \pi_{S,z,9} \sqrt{h_{S,z,t}^* e_{S,t}^*}.
\end{align*}
\]

The risk-neutral market jump scale parameter is

\[
\begin{align*}
    h_{M,y,t} &= w_{M,y} + b_{M,y} h_{M,y,t} + a_{M,y} \left( e_{M,t} - c_{M,y} \sqrt{h_{M,z,t}} \right)^2 \\
    &= w_{M,y} + a_{M,y} c_{M,y}^2 h_{M,z,t} + b_{M,y} h_{M,y,t} + a_{M,y} \left( e_{M,t}^2 - 2c_{M,y} \sqrt{h_{M,z,t} e_{M,t}} \right) \\
    &= \left( w_{M,y} + a_{M,y} c_{M,y}^2 h_{M,z,t} + b_{M,y} h_{M,y,t} \right) \\
    &\quad + a_{M,y} \left( \left( e_{M,t}^2 - \Lambda_M \sqrt{h_{M,z,t}} \right)^2 - 2c_{M,y} \sqrt{h_{M,z,t}} e_{M,t}^* \right) \\
    &= w_{M,y} + a_{M,y} \left( c_{M,y} + \Lambda_M \right) h_{M,z,t} + b_{M,y} h_{M,y,t} + a_{M,y} \left( e_{M,t}^2 - 2 \left( c_{M,y} + \Lambda_M \right) \sqrt{h_{M,z,t} e_{M,t}} \right) \\
    &= w_{M,y} + a_{M,y} \left( c_{M,y} + \Lambda_M \right) h_{M,z,t} + b_{M,y} h_{M,y,t} + a_{M,y} \left( e_{M,t}^2 - 2 \left( c_{M,y} + \Lambda_M \right) \sqrt{h_{M,z,t} e_{M,t}} \right) \\
    &= \pi_{M,y,1} + \pi_{M,y,2} h_{M,z,t} + \pi_{M,y,3} h_{M,y,t} + 0h_{S,z,t} + 0h_{S,y,t} \\
    &\quad + \pi_{M,y,6} (\varepsilon_{M,t}^*)^2 + \pi_{M,y,7} \sqrt{h_{M,z,t}^* e_{M,t}^*} + 0(\varepsilon_{S,t})^2 + 0 \sqrt{h_{S,z,t}^* e_{S,t}^*}.
\end{align*}
\]
Similarly, the risk-neutral stock jump scale parameter is

\[ h_{S,t} = \kappa_{S,y} h_{M,y,t+1} + b_{S,y} \left( h_{S,y,t} - \kappa_{S,y} h_{M,y,t} \right) + a_{S,y} \left( e_{S,t}^2 - 1 - 2c_S y z_{S,t} \right) \]

\[ = \kappa_{S,y} h_{M,y,t+1}^* + b_{S,y} \left( h_{S,y,t}^* - \kappa_{S,y} h_{M,y,t}^* \right) \]

\[ + a_{S,y} \left( 2c_{S,y} + \Lambda_S \right) \Lambda_S h_{S,z,t}^* + \left( e_{S,t}^* - 1 - 2c_{S,y} + \Lambda_S \right) \sqrt{h_{S,z,t}^* e_{S,t}^*} \]

\[ = \kappa_{S,y} h_{M,y,t+1}^* - a_{S,y} - b_{S,y} \kappa_{S,y} h_{M,y,t}^* + b_{S,y} h_{S,y,t}^* + a_{S,y} \left( 2c_{S,y} + \Lambda_S \right) \Lambda_S h_{S,z,t}^* \]

\[ + a_{S,y} \left( e_{S,t}^* \right)^2 - 2a_{S,y} \left( c_{S,y} + \Lambda_S \right) \sqrt{h_{S,z,t}^* e_{S,t}^*} \]

\[ = \pi_{S,y,1} + \pi_{S,y,2} \pi_{S,y,3} h_{M,y,t}^* + \pi_{S,y,4} h_{S,y,t}^* + \pi_{S,y,5} h_{S,z,t}^* \]

\[ + \pi_{S,y,6} \left( e_{M,t}^* \right)^2 + \pi_{S,y,7} \sqrt{h_{M,z,t}^* e_{M,t}^*} + \pi_{S,y,8} \left( e_{S,t}^* \right)^2 + \pi_{S,y,9} \sqrt{h_{S,z,t}^* e_{S,t}^*} \]

**OA.E  Proofs of Appendix E’s results**

**Lemma 8** if \( \varepsilon \) represents a standard normal random variable, then

\[ E \left[ \exp \left( a \varepsilon^2 + b \varepsilon \right) \right] = \exp \left( -\frac{1}{2} \ln \left( 1 - 2a \right) + \frac{1}{2} \frac{b^2}{1 - 2a} \right) \]

provided that \( a < \frac{1}{2} \).

**Proof of Lemma 8.**

\[ E \left[ \exp \left( a \varepsilon^2 + b \varepsilon \right) \right] \]

\[ = \int \exp \left( a \varepsilon^2 + b \varepsilon \right) \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} \varepsilon^2 \right) d\varepsilon \]

\[ = \int \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( 1 - 2a \right) \left( \varepsilon^2 - 2 \frac{b}{1 - 2a} \varepsilon \right) \right) d\varepsilon \]

\[ = \exp \left( \frac{b^2}{2 \left( 1 - 2a \right)} \right) \sqrt{\frac{1}{1 - 2a}} \int \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{1 - 2a}} \exp \left( -\frac{1}{2} \left( \frac{\varepsilon - \frac{b}{1 - 2a}}{1 - 2a} \right)^2 \right) d\varepsilon. \]

If \( 1 - 2a > 0 \), then the integral is one since it corresponds to the area under the density function of a \( \mathcal{N} \left( \frac{b}{1 - 2a}, \frac{1}{1 - 2a} \right) \) random variable. Hence,

\[ E \left[ \exp \left( a \varepsilon^2 + b \varepsilon \right) \right] = \exp \left( \ln \left( \frac{1}{1 - 2a} \right) + \frac{b^2}{2 \left( 1 - 2a \right)} \right). \]
**OA.E.1 Moment Generating Function**

For $u \in \{M, S\}$, The risk neutral returns process is

$$
\log \frac{u_{t+1}}{u_t} = r_{f,t+1} - \xi_{u,t+1}^Q + \beta_{u,z}^* z_{M,t+1}^* + \beta_{u,y}^* y_{M,t+1}^* + \beta_{u,z}^* z_{S,t+1}^* + \beta_{u,y}^* y_{S,t+1}^*
$$

$$
\zeta_{u,t+1}^* = \sqrt{h_{u,z,t+1}^* e_{u,t+1}^*}, \ e_{u,t+1}^* \sim \mathcal{N}(0, 1)
$$

$$
y_{u,t+1}^* \sim \text{NIG}\left(0, \alpha_{u}^*, \delta_{u}^2, h_{u,z,t+1}^* \right)
$$

where the convexity correction\(^{48}\) is

$$
\xi_{u,t}^Q = \xi_{M,t}^Q (\beta_{u,z}) + \xi_{S,t}^Q (\beta_{u,y}) + \xi_{z,t}^Q (\beta_{u,z}^*) + \xi_{y,t}^Q (\beta_{u,y}^*)
$$

For the market case, $\beta_{M,z} = \beta_{M,y} = 1$ and $\beta_{M,z}^* = \beta_{M,y}^* = 0$. For the stock, $\beta_{S,z} = \beta_{S,y} = 1$.

**Proof of Lemma 6.** For $u \in \{M, S\}$, let $\bar{r}_{u,t+j}$ denotes the excess return. Its risk neutral dynamics is

$$
\bar{r}_{u,t+j} = r_{u,t+j} - r_{f,t+j}
$$

$$
= \frac{1}{2} \beta_{u,z}^* h_{M,z,t+j}^* - \Pi_{M} (\beta_{u,z}) h_{M,y,t+j}^* \left( \beta_{u,y}^* \right)^2 h_{S,z,t+j}^* - \Pi_{S} (\beta_{u,y}^*) h_{S,y,t+j}^* + \beta_{u,z}^* z_{M,t+j}^* + \beta_{u,y}^* y_{M,t+j}^* + \beta_{u,z}^* z_{S,t+j}^* + \beta_{u,y}^* y_{S,t+j}^*
$$

For the market case, $\beta_{M,z} = \beta_{M,y} = 1$ and $\beta_{M,z}^* = \beta_{M,y}^* = 0$. For the stock, $\beta_{S,z} = \beta_{S,y} = 1$. The proof is based on a backward recursion over time. Indeed, the moment generating function of $\sum_{j=1}^{T-t} \bar{r}_{u,t+j}$ given

\(^{48}\) Appendix B shows that

$$
\xi_{u,t}^Q (\phi) = \frac{\phi^2}{2} h_{u,z}^* \quad \text{and} \quad \xi_{z,t}^Q (\phi) = \Pi_{u} (\phi) h_{u,y}^*
$$

where $\Pi_{u} (\phi) = \exp\left( \frac{1}{2} \delta_{u}^2 \phi^2 + \alpha_{u}^* \phi \right)$ and $\alpha_{u}^* = \alpha_{u} - \delta_{u}^2 \Gamma_{u}$.
Therefore,  

\[ \varphi_{r,t}^Q (\phi) = E_t^Q \left[ \exp \left( \phi \sum_{j=1}^{T-t} \tilde{r}_{u,t+j} \right) \right] = E_t^Q \left[ \exp (\phi \tilde{r}_{u,t+1}) E_{t+1}^Q \left[ \exp \left( \phi \sum_{j=1}^{T-t-1} \tilde{r}_{u,t+j+1} \right) \right] \right. \]

\[ = E_t^Q \left[ \exp \left( \phi \right) \left( -\frac{1}{2} \beta_{u,c}^2 h_{M,z,t+1}^* - \Pi_M^S (\beta_{u,y}) h_{M,y,t+1}^* - \frac{1}{2} \left( \beta_{u,z}^* \right)^2 h_{S,z,t+1}^* + \frac{1}{2} \left( \beta_{u,y}' \right)^2 h_{S,y,t+1}^* \right) \right] \]

\[ \times \exp \left( \mathcal{A}_{u,T-t-1} (\phi) + \mathcal{B}_{u,T-t-1} (\phi) h_{M,y,t+1}^* + \mathcal{C}_{u,T-t-1} (\phi) h_{S,y,t+1}^* + \mathcal{D}_{u,T-t-1} (\phi) h_{S,z,t+1}^* + \mathcal{E}_{u,T-t-1} (\phi) h_{S,y,t+1}^* \right) \]

(from the induction hypothesis).

Therefore,  

\[ \varphi_{r,t}^Q (\phi) = E_t^Q \left[ \exp \left( \phi \left( -\frac{1}{2} \beta_{u,c}^2 h_{M,z,t+1}^* - \Pi_M^S (\beta_{u,y}) h_{M,y,t+1}^* - \frac{1}{2} \left( \beta_{u,z}^* \right)^2 h_{S,z,t+1}^* + \frac{1}{2} \left( \beta_{u,y}' \right)^2 h_{S,y,t+1}^* \right) \right) \right. \]

\[ \times \exp \left( \mathcal{A}_{u,T-t-1} (\phi) + \mathcal{B}_{u,T-t-1} (\phi) h_{M,y,t+1}^* + \mathcal{C}_{u,T-t-1} (\phi) h_{S,y,t+1}^* + \mathcal{D}_{u,T-t-1} (\phi) h_{S,z,t+1}^* + \mathcal{E}_{u,T-t-1} (\phi) h_{S,y,t+1}^* \right) \]

\[ \left. \mathcal{A}_{u,T-t-1} (\phi) + \mathcal{B}_{u,T-t-1} (\phi) h_{M,y,t+1}^* + \mathcal{C}_{u,T-t-1} (\phi) h_{S,y,t+1}^* + \mathcal{D}_{u,T-t-1} (\phi) h_{S,z,t+1}^* + \mathcal{E}_{u,T-t-1} (\phi) h_{S,y,t+1}^* \right) \].

But the moment generating function of the risk-neutral jump component (B.3) gives  

\[ E_{F_t}^Q \left[ \exp \left( \beta_{u,y} h_{y,t+1}^* \right) \right] = \]
\[ \exp \left( \Pi_\nu^* (\beta \phi) h^\nu_{u,y,j+1} \right) \]. Therefore,

\[
E^{\nu}_{T_i^j} \left[ \exp \left( \beta_{u,y} \phi y^*_{M,j+1} \right) \right] = \exp \left( \Pi_M^* (\beta_{u,y} \phi) h^*_M, y, j+1 \right)
\]

and

\[
E^{\nu}_{T_i^j} \left[ \exp \left( \beta_{u,y} \phi y^*_{S,j+1} \right) \right] = \exp \left( \Pi_S^* (\beta_{u,y} \phi) h^*_S, y, j+1 \right)
\]

Moreover, Lemma 8 implies that

\[
\begin{align*}
E^{\nu}_{T_i^j} \left[ \exp \left( \zeta_{u,T-t-1,6} (\phi) \left( e^*_{M,j+1} \right)^2 + \left( \zeta_{u,T-t-1,7} (\phi) + \beta_{u,z} \phi \right) \sqrt{h^*_{M,z,t+1} e^*_{M,t+1}} \right) \right] &= \exp \left( -\frac{1}{2} \ln \left( 1 - 2 \zeta_{u,T-t-1,6} (\phi) \right) + \frac{1}{2} \frac{\left( \zeta_{u,T-t-1,7} (\phi) + \beta_{u,z} \phi \right)^2}{\left( 1 - 2 \zeta_{u,T-t-1,6} (\phi) \right)} h^*_{M,z,t+1} \right) \\
\end{align*}
\]

if \( \zeta_{u,T-t-1,6} (\phi) < \frac{1}{2} \) and

\[
\begin{align*}
E^{\nu}_{T_i^j} \left[ \exp \left( \zeta_{u,T-t-1,8} (\phi) \left( e^*_{S,j+1} \right)^2 + \left( \zeta_{u,T-t-1,9} (\phi) + \beta_{u,z} \phi \right) \sqrt{h^*_{S,z,t+1} e^*_{S,t+1}} \right) \right] &= \exp \left( -\frac{1}{2} \ln \left( 1 - 2 \zeta_{u,T-t-1,8} (\phi) \right) + \frac{1}{2} \frac{\left( \zeta_{u,T-t-1,9} (\phi) + \beta_{u,z} \phi \right)^2}{\left( 1 - 2 \zeta_{u,T-t-1,8} (\phi) \right)} h^*_{S,z,t+1} \right) \\
\end{align*}
\]

provided that \( \zeta_{u,T-t-1,8} (\phi) < \frac{1}{2} \). Therefore,

\[
\varphi^{\nu}_{T_i^j} (\phi) = \exp \left\{ \begin{array}{l}
\mathcal{A}_{u,T-t-1,1} (\phi) + \zeta_{u,T-t-1,1} (\phi) \\
+ \left( \zeta_{u,T-t-1,2} (\phi) - \frac{1}{2} \beta_{u,z}^2 \phi \right) h^*_{M,z,t+1} + \left( \zeta_{u,T-t-1,3} (\phi) - \Pi_M^* (\beta_{u,y} \phi) h^*_M, y, t+1 \right) \\
+ \left( \zeta_{u,T-t-1,4} (\phi) - \frac{1}{2} \beta_{u,z}^2 \phi \right) h^*_{S,z,t+1} + \left( \zeta_{u,T-t-1,5} (\phi) - \Pi_S^* (\beta_{u,y} \phi) h^*_S, y, t+1 \right)
\end{array} \right\} 
\]

\[
\exp \left( -\frac{1}{2} \ln \left( 1 - 2 \zeta_{u,T-t-1,6} (\phi) \right) + \frac{1}{2} \frac{\left( \zeta_{u,T-t-1,7} (\phi) + \beta_{u,z} \phi \right)^2}{\left( 1 - 2 \zeta_{u,T-t-1,6} (\phi) \right)} h^*_{M,z,t+1} \right) \exp \left( \Pi_M^* (\beta_{u,y} \phi) h^*_M, y, t+1 \right) \\
\exp \left( -\frac{1}{2} \ln \left( 1 - 2 \zeta_{u,T-t-1,8} (\phi) \right) + \frac{1}{2} \frac{\left( \zeta_{u,T-t-1,9} (\phi) + \beta_{u,z} \phi \right)^2}{\left( 1 - 2 \zeta_{u,T-t-1,8} (\phi) \right)} h^*_{S,z,t+1} \right) \exp \left( \Pi_S^* (\beta_{u,y} \phi) h^*_S, y, t+1 \right) .
\]

A comparison with

\[
\varphi^{\nu}_{T_i^j} (\phi) = \exp \left\{ \begin{array}{l}
\mathcal{A}_{u,T-t} (\phi) + \mathcal{B}_{u,T-t} (\phi) h^*_{M,z,t+1} + \mathcal{C}_{u,T-t} (\phi) h^*_M, y, t+1 \\
+ \mathcal{D}_{u,T-t} (\phi) h^*_{S,z,t+1} + \mathcal{E}_{u,T-t} (\phi) h^*_S, y, t+1
\end{array} \right\} 
\]

leads to the recursive formulation of the coefficients.
OA.E.2 European Call Option Price

Given the moment generating function (MGF) \( \varphi^Q_{r,t,T}(\phi) \) of the risk-neutral excess returns, we build on the work of Heston and Nandi (2000) and obtain a closed-form solution for the price of European index and stock options. More precisely, for \( u_t \in \{ M_t, S_t \} \), the price of an European call option is

\[
C_t(u_t, K, T) = e^{-r_{f,t,T}(T-t)} E_t^Q \left[ \max(u_T - K, 0) \right]
= e^{-r_{f,t,T}(T-t)} \left( E_t^Q [u_T I[u_T > K]] - KE_t^Q [I[u_T > K]] \right),
\]

where \( r_{f,t,T} = \frac{1}{T-t} \sum_{j=1}^{T-1} r_{f,t+j} \), in which \( r_{f,t+j} \) is the deterministic risk-free rate at time \( t + j \) and \( I[A] \) is the indicator function that worth 1 if the event \( A \) is realized and 0 otherwise. Since

\[
u_T = u_t \exp \left( \sum_{j=1}^{T-t} r_{u,t+j} \right) = u_t \exp \left( r_{f,t,T} (T-t) \right) \exp \left( \sum_{j=1}^{T-t} \tilde{r}_{u,t+j} \right),
\]

then

\[
P_{2,t,T} = E_t^Q [I[u_T > K]] = Q \left[ u_t e^{r_{f,t,T}(T-t)} \exp \left( \sum_{j=1}^{T-t} \tilde{r}_{u,t+j} \right) > K \right]
= Q \left[ \sum_{j=1}^{T-t} \tilde{r}_{u,t+j} > \ln \tilde{K}_{t,T} \right]
= 1 - F_{\tilde{r},t,T}^Q (\log(\tilde{K}_{t,T}))
\]

where \( \tilde{K}_{t,T} = \frac{K e^{-r_{f,t,T}(T-t)}}{u_t} \), and \( F_{\tilde{r},t,T}^Q \) is the cumulative distribution function associated with the MGF \( \varphi_{\tilde{r},t,T}^Q (\phi) \). Using results from Feller (1971) and Kendall and Stuart (1977),

\[
P_{2,t,T} = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left[ \frac{1}{\phi i} e^{-i \phi \log \tilde{K}_{t,T}} \varphi_{\tilde{r},t,T}^Q (\phi) \right] d\phi.
\]

Moreover,

\[
E_t^Q [u_T I[u_T > K]] = u_t e^{r_{f,t,T}(T-t)} E_t^Q \left[ \exp \left( \sum_{j=1}^{T-t} \tilde{r}_{u,t+j} \right) I[u_T > K] \right] = u_t e^{r_{f,t,T}(T-t)} P_{1,t,T}
\]
where

\[ P_{1,t,T} = E_Q^t \left[ \exp \left( \sum_{j=1}^{T-t} \tilde{r}_{u,j} + j \right) I[u_T > K] \right] = \int_{-\infty}^{\infty} \exp (\phi) \hat{f}_{f_{t,T}}(\phi) \, d\phi = \int_{-\infty}^{\infty} \tilde{p}(x) \, dx \]

where \( \hat{f}_{f_{t,T}} \) is the density function of the excess returns and the last equality stands by letting \( \tilde{p}(x) = \exp (\phi) \hat{f}_{f_{t,T}}(\phi) \). Note that \( \tilde{p} \) is a well-defined distribution since \( \exp (\phi) \) is always positive and

\[ \int_{-\infty}^{\infty} \tilde{p}(x) \, dx = \int_{-\infty}^{\infty} \exp (\phi) p(x) \, dx = \varphi_{\tilde{p}_{t,T}}(1) = 1, \]

given that \( \hat{f}_{f_{t,T}}(1) \) is the gross expected excess return under \( Q \), that is

\[ \int_{-\infty}^{\infty} \tilde{p}(x) \, dx = E_Q^t \left[ \exp \left( \sum_{j=1}^{T-t} \tilde{r}_{u,j} + j \right) |u_T| u_t = 1 \right] = 1. \]

Hence, following Heston and Nandi (2000), we note that MGF corresponding to \( \tilde{p} \) is simply

\[ \varphi_{\tilde{p}_{f_{t,T}}} (\phi) = \int_{-\infty}^{\infty} \exp (\phi x) \tilde{p}(x) \, dx = \int_{-\infty}^{\infty} \exp (\phi x) \exp (\phi x) \varphi_{\tilde{p}_{f_{t,T}}}(x) \, dx = \varphi_{\tilde{p}_{f_{t,T}}}(\phi + 1) \]

and thus

\[ P_{1,t,T} = \frac{1}{2} + \frac{1}{\pi} \int_{0}^{\infty} \text{Re} \left[ \frac{1}{\phi i} e^{-i\phi \log \tilde{K}} \varphi_{\tilde{p}_{f_{t,T}}} (\phi i) \right] \, d\phi. \]

Finally, we have that

\[ C_t(u_t, K, T) = u_T P_{1,t,T} - K e^{-r_{f,t,T}(T-t)} P_{2,t,T}. \]

**OA.F Results Without Clearing Conditions**

Table OA.1 reports summary statistics on the parameters associated with the 260 stocks under consideration, as obtained in iteration 0 of the algorithm described in section Appendix G.2. While there is substantial cross-sectional variation, the average value of the parameters of the variance and intensity processes are comparable to the parameters obtained for the market model.

Figure OA.1 decomposes, for each of the 260 stocks in our sample, the stock’s equity risk premium
in terms of the premiums associated with the four different risk factors in the model: (i) systematic normal, \( \beta_{SZ} \lambda_M h_{MZ,t} \), (ii) systematic jump, \( \gamma_{MS}(\beta_{SZ}) h_{MZ,t} \), (iii) idiosyncratic normal, \( \lambda_{SZ} h_{SZ,t} \), and (iv) idiosyncratic jump, \( \gamma_{SZ} h_{SZ,t} \). The most striking result illustrated in Figure OA.1 is that the normal component of idiosyncratic risk, which is easily diversifiable, is not priced once other sources of risk are accounted for. This is consistent with the average value of \( \lambda_{SZ} \) being very small at \( 5.079 \times 10^{-5} \).

Hence, it is already clear at this stage of the estimation procedure that the only impact of idiosyncratic risk on the cross-sectional variation in the equity risk premium is through idiosyncratic jump risk. Another striking feature of results in Figure OA.1 is that the premium associated with the idiosyncratic jump factor, while it varies significantly from firm to firm, is always positive. This is a consequence of the constant price of risk \( \gamma_{SZ} \), estimated to be positive for all the 260 firms under investigation.

Figure OA.2 highlights why this is problematic. At any given point in time, the no-arbitrage constraints of equation (2.17) are violated. That is, buying a long portfolio of the stocks in our sample and dynamically hedging it with a short position on the index always has positive expected return, represented by the yellow line in the both panels of Figure OA.2. Although our sample does not cover the full set of constituents of the index at any point in time, this virtually cannot be the cause of this apparent arbitrage. It would have to be that, although \( \gamma_{SZ} \) is positive for all stocks in our sample, it is for some reason negative for a substantial proportion of the stocks we do not cover. Even then, as highlighted in the proof of Proposition 1 (Appendix G), the model could not be arbitrage free at all points in time.

Figure OA.3 compares the cross-sectional distribution of equity risk premiums (ERP) applying (left panel) and without applying (right panel) the clearing conditions. In each panel, the solid vertical line reports the average ERP on the market; the dashed vertical line, the average ERP of a long position in a cap-weighted portfolio of comprising of the 260 in our sample. Theoretically, the lines should be superposed in the absence of arbitrage. It is almost the case when applying the clearing conditions; the 0.41% difference comes from the fact that our sample covers only 260 out of the 500 in the index. As made clear by the right panel of Figure OA.3, without applying the clearing conditions, parameter estimates would substantial deviate from no-arbitrage constraints.

\[ \text{OA-21} \]
This figure is based on the parameter estimates obtained in iteration 0 of the algorithm described in section Appendix G.2. This figure presents, for each of the 260 stocks, the decomposition of its equity risk premium in terms of the premiums associated with the four different risk factors in the model: (i) systematic normal, (ii) systematic jump, (iii) idiosyncratic normal, and (iv) idiosyncratic jump. Firms are grouped by industry, based on the Global Industry Classification Standard (GICS). Results for telecommunication services and utilities are not reported since they concern only five firms.
Figure OA.2: Decomposition of the Equity Risk Premium on Stocks Without Clearing Conditions: Time-Series of Cross-Sectional Averages

This figure is based on the parameter estimates obtained in iteration 0 of the algorithm described in section Appendix G.2. The top panel of this figure presents, at each point in time, the cap-weighted average of the systematic and idiosyncratic risk premiums. The bottom panel describes the distribution of idiosyncratic jump risk premiums (IJRP) across time. The average IJRP is cap-weighted.
Figure OA.3: Comparison of the Equity Risk Premium by Firm: With and Without Clearing Conditions
The left panel of this figure reports the cross-sectional distribution of equity risk premiums (ERP) as reported in the paper (i.e. it reproduces the left panel of Figure 8). The right panel reports the corresponding cross-sectional distribution when the ERPs are based on the parameter estimates obtained in iteration 0 of the algorithm described in section Appendix G.2. The solid vertical line reports the average ERP on the market; the dashed vertical line, the average ERP of a long position in a cap-weighted portfolio of comprising of the 260 in our sample.
This figure reports the parameter estimates obtained in iteration 0 of the algorithm described in section Appendix G.2. Parameters for each stock are estimated using daily index returns and available weekly cross-sections of out-of-the-money options, from January 1996 to August 2015. This table reports statistics on the cross-section of joint MLE estimates obtained for the 260 individual stocks in our sample. Q1 and Q3 report the 25th and 75th percentiles of the estimates.

<table>
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<tr>
<th>Parameter</th>
<th>Average</th>
<th>S.D.</th>
<th>Min</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Max</th>
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<tr>
<td>$\beta_{S,z}$</td>
<td>0.908</td>
<td>0.296</td>
<td>0.206</td>
<td>0.676</td>
<td>0.925</td>
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<td>$\beta_{S,y}$</td>
<td>1.082</td>
<td>0.417</td>
<td>0.202</td>
<td>0.805</td>
<td>1.029</td>
<td>1.282</td>
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<td>0.000</td>
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<td>0.010</td>
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<td>$\gamma_{S}$</td>
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<td>0.300</td>
<td>0.083</td>
<td>0.791</td>
<td>0.976</td>
<td>1.086</td>
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<tr>
<td>$\kappa_{S,z}$</td>
<td>0.959</td>
<td>0.704</td>
<td>0.083</td>
<td>0.516</td>
<td>0.778</td>
<td>1.200</td>
<td>4.665</td>
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<td>$a_{S,z}$</td>
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<td>Avg. excess kurtosis</td>
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<td>52.63</td>
<td>114.23</td>
<td>229.01</td>
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<td>0.06</td>
<td>0.12</td>
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