Credit Default Swaps and Debt Overhang

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Abstract

We analyze the impact of credit default swaps (CDS) trading on firm investment and financing in a dynamic contingent claims model. CDS trading opens the empty creditor channel: it increases debt capacity and allows the firm to capture a larger tax shield by deterring strategic default, but it also accelerates endogenous default and worsens debt overhang. We measure the agency cost of CDS as the gain in firm value under the equity’s commitment to a value-maximizing investment policy. For firms with grim growth prospects, the agency cost can exceed 1% of firm value and is even more substantial for firms with high business risk or asset tangibility. Moreover, we argue that debt overhang decreases with creditors’ bargaining powers, renegotiation frictions, and the debt’s commitment to the socially optimal credit insurance. Overall, our dynamic analysis suggests that empirical tests on the real impact of CDS trading should distinguish debt issuance (refinance) times, for which the ex-ante positive effect applies, from the true dynamics that contain only the ex-post negative effect on investment.

Keywords: Credit Default Swaps, Debt Overhang, Investment, Empty Creditor, Credit Risk
JEL Classification: G31, G33, G34

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1 Introduction

Credit default swaps (CDS) have become one of the most important financial innovations over the past two decades. As the credit derivative allows creditors to transfer credit risk out of their balance sheets and speculators to gamble on the prospects of the reference entities, the CDS market has grown tremendously since the advent of the instrument.\(^1\) Despite the literature has done works on the pricing of CDS contracts, and there are public debates about the desirability of the innovation, the implications of CDS trading on the real economic activities of firms have not been fully explored, particularly in dynamic environments. In this paper, we study the real impacts of credit derivative trading on current and future investment decisions as well as capital structure choices of CDS-referenced firms.

We introduce a competitive CDS market and dynamic investment opportunities in an otherwise standard Leland-type model. The key message of our paper is that there are both positive and negative sides of CDS trading. On the one hand, the availability of CDS contracts expands a firm’s debt capacity. The \textit{ex-ante} positive effect allows any financially-constrained firms to acquire more assets and better exploit tax shields at debt issuance times. On the other hand, on top of the adverse effect of CDS trading on default risk, we argue that it also worsens debt overhang: the credit protection indirectly increases the transfer of future cash flows from shareholders to creditors, thereby undermining the former’s incentives to invest. The \textit{ex-post} negative effect forces the firm to forgo some positive net present value (NPV) projects once the credit-protected debt is in place.

We identify CDS trading as a new source of debt overhang cost and measure the loss in value, which we call the \textit{agency cost of CDS}, using the relative difference in the values between the equity-maximizing and the value-maximizing CDS firms. Using the baseline calibration, we argue that the magnitude of the agency cost of CDS, as compared to the debt overhang cost estimated in the existing literature, is substantial. In fact, our comparative statics analysis shows that CDS-induced debt overhang destroys 1% to 3% value for firms with grim growth prospects and high business risk or high liquidation value. We conclude that in evaluating the impact of CDS trading, it is necessary to take into account the trade-off between increased debt capacity and increased debt overhang cost.

\(^1\)The CDS market peaked at USD 58,244 billion in a total notional amount in 2007. After the financial crisis, the market has been shrinking. As of the first half of 2016, the market stood at USD 11,777 billion in notional amounts outstanding. The statistics are available at the Bank for International Settlements. (http://www.bis.org/statistics/derstats.htm, Table D10 OTC credit default swaps)
Our analysis builds on the well-known debt overhang problem analyzed by Myers (1977), and the empty creditor problem pointed out by Hu and Black (2008a,b) and first formalized in Bolton and Oehmke (2011). The debt overhang problem refers to situations in which the equity holders have reduced incentives to undertake valuable investment because the debt holders share the return of equity-financed investment in default. The empty creditor problem arises when a firm’s creditor, who has obtained insurance against default, has no incentives to continue the firm efficiently and forces the debtor into inefficient liquidation. The CDS market provides the creditors with such an insurance instrument. In fact, credit-protected debt holders have strengthened bargaining positions in out-of-court restructurings and their incentives to obtain the credit insurance stem from the possibility of the equity to strategically default on its debt obligations.\(^2\)

To study the interaction between debt overhang and empty creditor, we employ Leland (1994)’s model of capital structure. On top of the standard trade-off between tax shields and bankruptcy costs, we endow the firm with dynamic investment opportunities. At any point in time, the equity holders can make an all-or-nothing decision regarding the growth of assets-in-place and have the option to liquidate the firm. Additionally, the debt is renegotiable: The equity holders have limited commitment to fulfilling the debt obligations and hence can default strategically and renegotiate the coupon with the creditors. As strategic debt service specifies a linear sharing rule, under our parametric restriction, the levered firm is free of debt overhang problem and always invests at the maximum level without the CDS market.

When the debt holders have access to a competitive CDS market that allows them to hedge against the firm’s credit risk after the debt is in place, the CDS contracts purchased by the creditors increase their bargaining positions in private workouts. The reason is that once they reject the equity’s proposal and exercise their liquidation right, they receive the CDS payment from the protection sellers. The increased outside option weakens the threat of liquidation imposed by the equity holders and allows the CDS-protected empty creditors to demand higher interest payments in renegotiation. CDS trading thus reduces the equity’s incentives to renegotiate and default strategically. More importantly, as the credit insurance transfers wealth from the firm to the debt holders in private workouts, the equity holders have to absorb more significant losses as the firm’s fundamental deteriorates and hence accelerates the ex-post optimal default time. The endogenous

\(^2\)There is evidence that CDS trading affects corporate restructuring outcomes. Danis (2015) document that firms with traded CDS have lower bondholders’ participation rate in restructurings. Bedendo, Cathcart, and El-Jahel (2016) document increases in recovery prices in distress exchanges with empty creditors.
default mechanism in Leland (1994) allows us to capture this logic conveniently.³

The CDS market then affects the firm’s dynamic investment decisions through the empty creditor channel: Debt overhang arises from the increased likelihood of endogenous default with the inception of CDS trading. As the CDS accelerates the equity’s default time, the credit derivative endogenously shifts the distribution of investment benefits towards the debt holders and exacerbates the ex-post conflict of interests between the firm’s owners and the lenders. Consequently, the equity stops investing inefficiently early as the firm’s fundamental deteriorates and moves closer to the default boundary, and the reduced asset growth rate post-introduction of the CDS market is the negative effect of credit insurance.

Despite the negative effect of CDS trading on the real side of the firm, the credit derivative commits the equity holders to renegotiate the debt contract less frequently. This ex-ante commitment benefit expands the firm’s debt capacity and increases the debt value. Consistent with Bolton and Oehmke (2009), the increased borrowing ability allows financially-constrained firms to finance a broader set of positive NPV projects initially. Additionally, as the debt holders are more willing to inject capital ex-ante, the firm can better exploit the tax shield and increases its value in general.

The dynamic nature of our model allows us to quantify the real effects of CDS trading. The optimal capital structure trades off the standard benefits from tax shields and costs of financial distress, including the amplified agency costs arise from debt overhang and accelerated default through the empty creditor channel. With our baseline parameter constellations, the introduction of the CDS market raises the optimal market leverage from 37.04% to 58.15% and reduces the credit spread from 384 basis points to 117 basis points (bps).⁴ The investment threshold and default threshold increase from 0 to 3.83 and 2.77 respectively, resulting in a substantial non-investment region.

Consistent with the existing empirical literature, the increase in the optimal leverage and bankruptcy likelihood matches evidence provided by Hirtle (2009), Saretto and Tookes (2013), and Subrahmanyan, Tang, and Wang (2014). The reduction in the credit spread is consistent with some of the evidence reported in Ashcraft and Santos (2009) and Kim (2016). The real effects of CDS

³We refer to strategic default as debt renegotiation to differentiate it from a formal default (bankruptcy).
⁴The effect of CDS trading on debt market value is two-fold. First, Hackbarth, Hennessy, and Leland (2007) show that firms borrow up to the maximum coupon that triggers an immediate renegotiation. Any contractual coupon greater than the maximum is not credible and does not increase pledgeable income. In our model, this constraint is relaxed by creditors’ CDS protection, resulting in an increase in the optimal coupon. Second, by deterring debt regeneration, it increases the market value of debt for a given coupon. These two sources and their interaction increase debt value significantly.
on dynamic investment are in line with Colonnello, Efing, and Zucchi (2016), Guest, Karapatsas, Petmezas, and Travlos (2017), and Batta and Yu (2017). Notably, Batta and Yu (2017) document that post-CDS introduction, the sampled firms have an average decline in asset growth by 2.1% and an overall reduction in net investment. However, when focusing on the CDS-introduction years, the evidence shows that both net investment and debt issuance increase in response to CDS trading. Our analysis thus provides CDS-driven debt overhang as one potential explanation for their observed dynamic investment pattern.

Importantly, we quantify the debt overhang cost with two measures. First, we quantify the agency cost of CDS as the relative difference in the equity-maximizing and value-maximizing firm values. We find that when the equity holders can commit to a firm value-maximizing investment policy after the debt is in place, the firm value can increase by 0.376% under the base case. When we analyze firms that are subject to strong potential debt overhang, that is, firms with poor investment opportunities, the agency cost of CDS can increase to 1% to 3% depending on the firm’s characteristics including cash-flow volatility and bankruptcy cost.

Second, we measure the reduction in asset value due to CDS trading. Under the baseline parameters, the reduction is $-2.008\%$ when we scale the reduction with the value of non-CDS firms and is $-2.18\%$ when we take the unlevered asset of the non-CDS firms as the scale. Both quantities show that the asset reduction is non-trivial and possibly is massive in dollar terms for large corporations.

The baseline model allows us to provide a few more results. First, debt overhang decreases with the bargaining power of the debt holders. Intuitively, as the key benefit of CDS protection to the debt is the strength it provides in negotiation, so the increase in the debt’s bargaining power substitutes out the use of CDS contract. In the extreme when the debt holders can make a take-it-or-leave-it offer to the equity, the debt effectively becomes the residual claimant; and to minimize the liquidation costs, the debt holders forgo the hedging opportunities. Thus our analysis implies a minimal overhang in this case.

Second, debt overhang decreases with renegotiation frictions. The reason is that when the renegotiation cost is high, the incentive for the equity to restructure its debt is low. This reduces the creditors’ use of CDS protection and thus the likelihood of bankruptcy, and hence the debt overhang cost.

Lastly, the debt holders excessively hedge against credit risk. When the creditors can freely choose
their CDS positions, they trade off the commitment benefits of reducing strategic default (debt renegotiation) and the costs of CDS premium from increasing endogenous default. Nevertheless, the privately optimal level of credit protection is inefficient: The creditors over-insure against default as they do not fully internalize the equity’s loss of the option value to renegotiate out-of-court and the bankruptcy costs. Compared to the social optimum, the excessive credit protection chosen by the empty creditors exaggerates debt overhang.

Our paper contributes to a growing literature that examines the impact of CDS trading from a corporate finance perspective. First, we provide a structural model with CDS being non-redundant securities. On the theoretical side, we embed the empty creditor problem in Bolton and Oehmke (2011) into the dynamic contingent claims model with renegotiable debt (Mella-Barral and Perraudin (1997), Fan and Sundareasan (2000), Hackbarth, Hennesy, and Leland (2007), Davydenko and Strebulaev, 2007) and investment options (Hennessey (2004), Diamond and He (2014)). We are among the first to study CDS as a non-redundant security in the structural credit risk model. Kim (2016) introduces the CDS contract the dynamic model with debt-for-equity swaps. However, he only uses the model to motivate the test hypotheses for his empirical investigation of CDS trading and credit risk without studying investment opportunities.

Second, we identify debt overhang as a new source of inefficiency through the empty creditor channel. The most closely related paper to us is Bolton and Oehmke (2011) who analyze the ex-ante commitment benefits of reducing strategic default and the ex-post cost of inefficient debt renegotiation from CDS trading. However, the model features a one-shot investment. We expand their analysis to firms facing future investment opportunities and study the interaction of the empty creditor problem and the debt tax shield. In a contemporaneous and related work, Danis and Gamba (2016) extend Bolton and Oehmke (2011) into a dynamic setting. Using simulation-based evidence, they find that investment, leverage, and firm value increase with the inception of CDS trading. The implication on investment is in sharp contrast with our predictions. The reason is that their firms only issue one-period debt contract, and simultaneously make the investment and financing decisions after they repay all the existing debts. Thus the debt-overhang effect is absent in their model.

In addition, Oehmke and Zawadowski (2015) analyze the effect of CDS introduction on bond prices and CDS-Bond basis in an equilibrium model based on market liquidity and heterogeneous

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5For a detailed overview, Augustin, Subrahmanyam, Tang, and Wang (2016) provide an excellent summary.
beliefs. Parlour and Winton (2013) analyze the trade-off between loan sales and CDS regarding credit risk transfers and monitoring incentives in a banking model. Campello and Matta (2016) argue that CDS over-insurance is pro-cyclical and CDS trading leads to a more significant increase in debt capacity during economic booms. Fostel and Geanakoplos (2016) show that uncovered CDS positions may lead to under-investment. In contrast, covered CDS positions that distort the debtor-creditor relationship drives our debt-overhang result. Overall, our model sheds light on these important theoretical works that study CDS markets.

Third, we contribute to the empirical literature by providing several new testable hypotheses regarding the CDS firms’ dynamic paths and calling attention to the distinction between the CDS firms’ debt issuance times and their true dynamics. Our dynamic model suggests that CDS firms capture the ex-ante positive effect of debt capacity, therefore, debt-financed investment, at any debt issuance or refinancing times. Once the debt is in place, recapitalization costs prevent a CDS firm tapping the debt market often, and in those times, the ex-post negative effect of investment applies. The distinction between debt issuance/ refinancing times and the true dynamics is reminiscent of Strebulaev (2007); and along with this line, our framework provides a consistent explanation of estimations reported by Colonnello, Efing, and Zucchi (2016), Batta and Yu (2017), and Guest, Karampatsas, Petmezas, and Travlos (2017). As for financial implications, our work captures several empirical findings that relate CDS and corporate finance, such as Ashcraft and Santos (2009), Hirtle (2009), Saretto and Tookes (2013), Subrahmanyam, Tang, and Wang (2014), and Kim (2016).

2 The Model

Technology. Consider a firm with assets that generate pre-tax cash flows at rate $\delta_t$. The cash-flow process $\{\delta_t : t \geq 0\}$ evolve as a geometric Brownian motion under the risk-neutral measure

$$\frac{d\delta_t}{\delta_t} = (\mu + i_t) dt + \sigma dZ_t.$$

The baseline growth rate is $\mu$ and the volatility $\sigma$ is a positive constant, and $\{Z_t : t \geq 0\}$ is a standard Brownian motion.$^6$ Endogenous investment $i_t \in \{0, i\}$ affects the asset growth rate. The

$^6$All stochastic processes are defined on a complete probability space $(\Omega, \mathcal{F}, P)$ with the filtration $\mathcal{F} = \{\mathcal{F}_t : t \geq 0\}$ satisfying the usual conditions. Diamond and He (2014) use this cash-flow specification to examine the implication
investment cost is given by \( \phi_i \delta_t \) because the asset growth scales with \( \delta_t \). Following Myers (1977), the equity, or the firm’s manager who acts in the best interest of the shareholders, controls the investment decisions. Investment costs are equity-financed.

As in other Leland-type models, we let \( \tau \in [0, 1] \) be the corporate tax rate and \( \alpha \in [0, 1] \) be a proportional bankruptcy cost.\(^7\) If an unlevered firm always invests, its asset value is

\[
\mathbb{E}_t \left[ \int_t^\infty e^{-r(s-t)} \left((1 - \tau) \delta_s - \phi_i \delta_s\right) ds \right] = \frac{(1 - \tau)(1 - \tilde{\phi}_i)}{r - (\mu + i)} \delta_t. \tag{1}
\]

As opposed, if the firm never invests, its asset value is given by \( \frac{1 - \tau}{r - \mu} \delta_t \). In (1), we define \( \tilde{\phi} \equiv \frac{\phi}{1 - \tau} \) and assume \( r > \mu + i \) for convergence. Also, for notational convenience, we define \( U_i \equiv \frac{1 - \tilde{\phi}_i}{r - (\mu + i)} \), \( U_0 \equiv \frac{1}{r - \mu} \), and assume \( \Pi \equiv U_i - U_0 > 0 \). Hence, \( \Pi > 0 \), or equivalently \( U_0 > \tilde{\phi} \), captures a positive marginal value of investment; and (1) provides the unlevered firm value. In the quantitative analysis, we use the parameter \( \Pi/U_0 \) to measure the profitability of investment opportunities and it provides a proxy for growth options.

The liquidation value of the asset is \( L \delta_t \), where \( L \equiv (1 - \alpha) \frac{1 - \tau}{r - \mu} \). That is, once the firm liquidates, it loses its investment opportunities or its existing manager who has superior skills in investment. We maintain the following parametric restriction throughout the paper.

**Assumption 1.** \( U_i - \tilde{\phi} > L \).

While \( \Pi \) captures the investment value from the firm’s perspective, \( U_i - L - \tilde{\phi} \) measure the net investment value accruing to the shareholders of a levered firm given the interest payment per unit of cash flows is \( L \). To see this, consider an increase in the investment from 0 to a level such that \( \int_t^{t+dt} i \delta_t dt = 1 \) over a small time interval \( (t, t + dt) \). The additional unit of fundamental generates a present value of \( U_i \) and it costs \( \tilde{\phi} \). Moreover, suppose the extra interest is \( L \), which will happen in a benchmark without the credit default swaps (CDS) market (see Section 3.3.1), then Assumption 1 ensures that the equity holders have sufficient incentives to investment under the given debt service.

**Financing.** The firm has access to a frictionless equity market and a debt market. At time 0, the firm borrows from outside investors by issuing a perpetual debt contract that promises a contractual of debt maturity on overhang.

\(^7\)As in Myers (1977), corporate taxes are not crucial for our key results on debt overhang: Proposition 1 and 5 hold with \( \tau = 0 \). But the tax shield provides incentives for the firm to use debt financing and thus taxes are important for optimal leverages and credit risks in the quantitative analysis.
coupon at rate $c_B$. When the equity holders declare bankruptcy, the debt holders have absolute priority in liquidation and continue to run the firm as an ongoing entity or sell the firm outright. Hence, the debt holders obtain the liquidation value $Lδ$.\footnote{Following other Leland-type models, we assume the new firm’s owner does not relever.}

However, the equity holders can initiate a private workout to renegotiate the interest payment at any point in time because of their limited commitment to fulfilling the debt obligations. We follow the approach by Mella-Barral and Parraudin (1997) and Hackbarth et al. (2007) in modeling renegotiation (strategic default). Let $s(δ)$ be the debt service flow function. In renegotiation, the equity holders make a take-it-or-leave-it offer $s(δ)$ to the debt holders.\footnote{Fan and Sundarasean (2000) and Sundarasean and Wang (2007) model renegotiation as a Nash bargaining game. In contrast, the equity holders make a take-it-or-leave-it offer and they have all the bargaining power in our model. In Section 5.1, we consider take-it-of-leaving-it offers made by debt holders.} The incentive for the debt to accept the offer stems from the threat of liquidation. The threat is credible because the debt bears part of the bankruptcy cost in a negotiation breakdown. Since the debt holders obtain their reservation value $R(δ)$ upon exercising the liquidation right, they accept the proposal $s(δ)$ if it delivers at least the reservation value.\footnote{We follow the previous studies and restrict attention to debt service functions that are piecewise right continuous in $δ$ and $s(δ) < c_B \Rightarrow b(δ) ≥ R(δ)$, where $b(δ)$ is the debt’s payoffs defined in Section 3.2.1.}

**The CDS market.** The novel element our model is that the creditors may enter into a single-name CDS contract that references the firm’s debt as protection buyers to (partially) transfer their losses in default to CDS sellers. Specifically, we model the CDS market as a competitive market with risk-neutral protection buyers and sellers. Right after the debt issuance and before the firm operates, the debt holders can decide their CDS position $θ \in [0, \infty)$, which represents a lump-sum payment made by the protection seller to the buyer in a credit event, and in exchange, the protection buyer pays a CDS premium at rate $p$ to the seller. Here, only bankruptcy qualifies as a credit event, and a successful renegotiation does not trigger the credit protection. We make this assumption because the International Swaps and Derivatives Association (ISDA) no longer recognizes debt restructuring as a credit event since Spring 2009. For tractability, we assume that CDS contracts are perpetual to match the debt’s maturity.

In a competitive market, CDS contracts are fairly priced. Given a position $θ$, the seller sets a CDS premium $p$ to break even:

$$
\mathbb{E} \left[ \int_0^{τ(θ)} e^{-rt}(-p)dt + e^{-rτ(θ)}θ \right] = 0.
$$

(2)
Here, $\tau_d(\theta)$ is the default time chosen by the equity given the debt’s credit protection. Therefore, the novelty of the pricing equation (2) is that it incorporates the interaction between the equity’s ex-post decisions and CDS trading and highlights the role of endogenous decisions as an essential mechanism in determining the CDS premium with renegotiable debt. The competitive market pricing mechanism is in sharp contrast with reduced-form credit risk models, which typically price CDS contracts based on exogenously specified default intensity and liquidity process.

### 3 Model Analysis

In this section, we first fix the capital structure and derive the closed-form valuation formulas for CDS contracts and corporate securities. In the process, we characterize the equity’s optimal decisions after the debt is in place, the key debt-overhang result, as well as the debt’s optimal hedging strategy given the anticipation of the equity’s ex-post optimal policy. Then we analyze two benchmarks: non-CDS firms and CDS firms with a commitment to an efficient investment policy. The benchmarks allow us to characterize the debt overhang cost induced by CDS trading. We end the section by defining the optimal leverage.

We start by setting up a few notations. Since three endogenous thresholds characterizing the optimal ex-post policies, we denote $\delta_i$ as an investment threshold, $\delta_n$ as a renegotiation threshold, and $\delta_d$ as a default threshold. Moreover, suppose the equity’s investment policy takes the form:

$$i(\delta) = \begin{cases} i, & \text{if } \delta \geq \delta_i; \\ 0, & \text{if } \delta < \delta_i. \end{cases}$$

Throughout the paper, we assume $\delta_n > \delta_i$.\footnote{We make this assumption because for all reasonable parameter values we work with, we find that $\delta_n > \delta_i$.} In what follows, $z_1 < 0$ is the negative root, and $a_1 > 1$ is the positive root of the fundamental quadratic $Q_1(x) = \frac{1}{2}\sigma^2 x^2 + (\mu + i - \frac{1}{2}\sigma^2) x - r = 0$; and $z_0 < 0$ is the negative root, and $a_0 > 1$ is the positive root of the fundamental quadratic $Q_0(x) = \frac{1}{2}\sigma^2 x^2 + (\mu - \frac{1}{2}\sigma^2) x - r = 0$. Note that as $i > 0$, $0 > z_0 > z_1$ and $a_0 > a_1 > 1$.\footnote{This is a typical assumption in financial modeling.}
3.1 Valuation of Credit Default Swaps

The value of a CDS contract contains its protection leg and its premium leg. For the protection leg, the value is \( C(\delta) = \mathbb{E}^\delta [e^{-rt_d(\theta)}\delta] \) because the CDS seller pays zero prior to bankruptcy, and since the default event triggers the CDS payment, \( C(\delta_d) = \theta \). The value has a closed-form solution:

\[
C(\delta) = \begin{cases} 
\theta P^i_d(\delta), & \text{if } \delta \geq \delta_i; \\
\theta P^0_d(\delta), & \text{if } \delta_d < \delta < \delta_i.
\end{cases}
\]

Here, \( P^i_d(\delta) \) is the present value of a contingent claim that pays one dollar at default for \( \delta \) in the investment region. Similarly, \( P^0_d(\delta) \) is for \( \delta \) in the non-investment region. Intuitively, the quantities capture the probability of default at state \( \delta \); and the Appendix provides their closed-form expressions.

The value of the premium leg is \( P(\delta) = \mathbb{E} \left[ \int_0^{\tau_d(\theta)} e^{-rt_d(\delta)} \, dt \right] \). It is the expected discounted value of the CDS premium paid to the protection seller. Hence, \( P(\delta) \) is a contingent claim that pays \( p \) before default and zero at liquidation. Given the equity’s decisions, the CDS price is:

\[
P(\delta) = \begin{cases} 
p \left( 1 - P^i_d(\delta) \right), & \text{if } \delta \geq \delta_i; \\
p \left( 1 - P^0_d(\delta) \right), & \text{if } \delta_d < \delta < \delta_i.
\end{cases}
\]

In sum, a CDS contract has an expected value of \( C(\delta) - P(\delta) \) to the protection buyer and \( P(\delta) - C(\delta) \) to the protection seller. Let \( T(\theta, c_B) \) be the collection of thresholds \( \{\delta_i, \delta_n, \delta_d\} \) as functions of the CDS position \( \theta \) and the coupon \( c_B \), and \( \theta(c_B) \) be the debt’s optimal CDS position in response to \( c_B \), then the pricing condition (2) can be written as

\[
C(\delta_0; T(\theta(c_B), c_B)) = P(\delta_0; T(\theta(c_B), c_B)).
\] (3)

Compared to (2), condition (3) explicitly states that the endogenous CDS premium capitalizes the market expectation regarding the impact of the optimal CDS position \( \theta(c_B) \) on the equity’s ex-post investment and default decisions. Moreover, since the creditors choose the position after the debt issuance, the pricing equation (3) threats the coupon \( c_B \) as a parameter.
3.2 Valuation of Corporate Securities

3.2.1 Debt Value and the Optimal CDS Position

Given a CDS position \( \theta \), the debt has a reservation value of \( R(\delta) = \min \left( \frac{c_B}{r}, \theta + L\delta \right) \). The idea is that the creditors receive either the principal \( \frac{c_B}{r} \) when the liquidation value is sufficiently high or the liquidation value together with the CDS payment when the firm is unable to cover the principal. Thus, \( R(\delta) \) is the value of the debt holders in a negotiation breakdown, and it constitutes the minimal amount that the debt holders have to receive to accept the equity’s take-it-or-leave-it offer. It also makes clear that the availability of the CDS contracts improves the debt holders’ outside option and thus their bargaining position in renegotiation.

Let \( b(\delta) \) be a contingent claim that represents the present value of future cash flows accruing to debt holders, including the payment made by the CDS protection seller upon default. The standard argument implies that \( b(\delta) \) must satisfy the ordinary differential equation (ODE):

\[
rb(\delta) = s(\delta) + (\mu + i(\delta))\delta b'(\delta) + \frac{1}{2} \sigma^2 \delta^2 b''(\delta).
\]

The left-hand side is the required return on the claim. The right-hand side is the debt service plus the expected capital gain from holding this claim, given the debt’s anticipation of the equity’s ex-post optimal investment policy. Because the equity holders have full bargaining power, the debt payoff is pushed down to its reservation value during renegotiation: \( b(\delta) = R(\delta) = \theta + L\delta \).

Substituting this into (4), we have the optimal debt service function:

\[
s(\delta) = \begin{cases} 
  c_B, & \text{if } \delta > \delta_n; \\
  r\theta + (r - (\mu + i))L\delta, & \text{if } \delta_i \leq \delta \leq \delta_n; \\
  r\theta + (r - \mu)L\delta, & \text{if } \delta_d < \delta < \delta_i. 
\end{cases}
\]

Thus, the interest payment equals the flow of the reservation value in a private workout. It follows that the contingent claim has a closed-form solution:

\[
b(\delta) = \begin{cases} 
  \frac{c_B}{r} + \left( \theta + L\delta_n - \frac{c_B}{r} \right) \left( \frac{\delta}{\delta_n} \right)^{21}, & \text{if } \delta > \delta_n; \\
  \theta + L\delta, & \text{if } \delta_d < \delta \leq \delta_n. 
\end{cases}
\]
Here, the first line is the present value of the default-free coupon and the expected change in value from renegotiation, and the second line represents the reservation value.

We define $B(\delta) \equiv b(\delta) - C(\delta)$ as the value of a debt contract. The idea is that the debt value, by definition, accounts only for the interest payment and the liquidation value paid out from the firm’s assets. The claim $b(\delta)$ captures the interest payment before bankruptcy. However, it includes the CDS coverage from the protection seller at liquidation as well. Therefore, we subtract the expected injection from the third party to obtain the market value of debt.

Debt holders hold a portfolio of a debt contract, with value $b(\delta) - C(\delta)$, and a CDS contract, with value $C(\delta) - P(\delta)$. The portfolio value is the net payoff $b(\delta) - P(\delta)$ of the debt holders:

$$b(\delta) - P(\delta) = \begin{cases} \frac{c_B}{r} + \left( \theta + L\delta_n - \frac{c_B}{r} \right) \left( \frac{\delta}{\delta_n} \right)^{z_1} - \frac{P_i}{r} \left( 1 - P^j_d(\delta) \right), & \text{if } \delta > \delta_n; \\ \theta + L\delta - \frac{P_i}{r} \left( 1 - P^j_d(\delta) \right), & \text{if } \delta_i \leq \delta \leq \delta_n; \\ \theta + L\delta - \frac{P_i}{r} \left( 1 - P_0^j_d(\delta) \right), & \text{if } \delta_d < \delta < \delta_i. \end{cases}$$

At time 0, the debt holders choose $\theta$ to maximize their portfolio value

$$\theta(c_B) = \arg \max_{\theta \in (0, \infty)} \left\{ b(\delta_0; \theta, T(\theta, c_B)) - P(\delta_0; \theta, T(\theta, c_B)) \right\},$$

given the contractual coupon $c_B$, the rational anticipation of the equity’s ex-post decision $T(\theta, c_B)$, and the competitive CDS pricing (3). Note that at time 0, the net payoff of the debt holders equals to the market value of debt because of the pricing condition (3). Moreover, CDS is a non-redundant security because of the strengthened bargaining position it provides to counter strategic default. To see this, the debt value, $B(\delta) = L\delta + \theta \left( 1 - P^j_d(\delta) \right)$ on $(\delta_d, \delta_n)$, is strictly increasing in the CDS position. As opposed, the debt value in Leland (1994) is independent of the CDS trading: the debt’s portfolio consists merely of two non-interacting securities.\(^{13}\)

\(^{12}\)More precisely, for $\delta > \delta_d$, the present value of the interest payments $\theta + L\delta$ is paid out of the firm’s cash flows. At $\delta_d$, $\theta$ is paid out from the CDS contract and $L\delta_d$ comes from the firm’s liquidated assets. The transfer at default captures the cash settlement procedure in an actual credit event.

\(^{13}\)Formally, to obtain Leland (1994), we can set $\delta_n = \delta_d$ in (6) to remove strategic default. Then $B(\delta) = b(\delta) - C(\delta)$ becomes $B(\delta) = \frac{c_B}{r} + \left( L\delta_d - \frac{c_B}{r} \right) \left( \frac{\delta}{\delta_d} \right)^{z_1}$ and the market debt is independent of the CDS.
### 3.2.2 Equity Value and the Endogenous Decisions

The equity value $E(\delta)$ satisfies the Hamilton-Jacobi-Bellman equation

$$rE(\delta) = \max_{i_t \in \{0, i\}, \delta_d, \delta_n} \left\{ (1 - \tau)(\delta - s(\delta)) - \phi i_t \delta + (\mu + i_t) \delta E'(\delta) + \frac{1}{2} \sigma^2 \delta^2 E''(\delta) \right\}.$$  \hspace{1cm} (7)

The left-hand side is the required return on equity, which equals the sum of expected dividend net of the investment cost and the expected increment in the equity value on the right-hand side. In (7), the debt service is given by (5). The maximization with respect to $i_t$ yields $i(\delta) = i$ if $E'(\delta) \geq \phi$, and $i(\delta) = 0$ otherwise. Since $E'(\delta)$ is the increase in equity value when the firm invests at $\delta$, a unique threshold $\delta_t$ satisfying $E'(\delta_t) = \phi$ is ex-post optimal. Given $\{\delta_d, \delta_t, \delta_n\}$, the equity value takes the following form. For $\delta_d < \delta < \delta_t$, the firm restructures its debt continuously and does not invest. We have

$$E(\delta) = (1 - \tau) \left( U_0 \delta - (\theta + L\delta) \right) + (1 - \tau) \left( \delta + \frac{1}{2} \Gamma_0(\delta, \delta_d) \right),$$  \hspace{1cm} (8)

where $\Gamma_0(\delta, \delta_d)$ is in the Appendix. On the right-hand side of (8), the first term is the present value of the default-free dividend for the non-investing firm and the second term is the value of the default option. The difference with Leland (1994) is that upon default, the equity saves the credit protection-dependent debt service flow $\theta + L\delta_d$ instead of the regular coupon $c_B$. The last term is the value of investment option, which captures both the benefit from investment driven by the increase in the asset growth rate and the loss of the investment option when the firm bankrupts.

For $\delta_t \leq \delta \leq \delta_n$, the firm invests and restructures its debt continuously, and the equity value is

$$E(\delta) = (1 - \tau) \left( U_i \delta - (\theta + L\delta) \right) + (1 - \tau) \left( \theta + L\delta_d - U_0\delta_d \right) P^i_d(\delta) + (1 - \tau) \Pi \delta_t \Gamma_i(\delta_t, \delta_d),$$  \hspace{1cm} (9)

where $\Gamma_i(\delta_t, \delta_d)$ is in the Appendix. In (9), the first term is the present value of the default-free dividend for the investing firm and the second term captures the value of the default option. Note that as the fundamental $\delta_t$ deteriorates and crosses $\delta_t$, the equity holders stop investing, implying a reduction in the growth rate of cash flows and an increased likelihood to lose the investment opportunities. Thus, the last term represents the value of the option to reduce investment.
Outside the renegotiation region $\delta > \delta_n$, the firm invests, and the equity value is

$$E(\delta) = (1 - \tau) \left( U_i \delta - \frac{c_B}{r} \right) + (1 - \tau) \left( \theta + L \delta_d - U_0 \delta_d \right) P^i_d(\delta) + (1 - \tau) \Pi_i \Gamma_i(\delta_i, \delta_d) \left( \frac{\delta}{\delta_i} \right)^{z_1} + (1 - \tau) \left( \frac{c_B}{r} - (\theta + L \delta_n) \right) \left( \frac{\delta}{\delta_n} \right)^{z_1}. \quad (10)$$

Similarly, the first and second terms are the present value of the default-free dividend and the value of the default option respectively, and the third term is the value of the option to reduce investment. Additionally, the last term represents the value of the renegotiation option stemming from the limited commitment of the equity holders to pay the contractual debt service. The value depends on the optimally chosen strategic default time and the bargaining position of the CDS-protected debt holders.

The ex-post decisions of the equity holders are as follows. The smooth-pasting of (9) and (10) at $\delta_n$ delivers the endogenous renegotiation threshold $\delta_n$ in closed-form:

$$\delta_n = \frac{z_1}{z_1 - 1} \left( \frac{c_B}{r} - \theta \right) \frac{1}{L}. \quad (11)$$

Thus, the introduction of a CDS market reduces the incentive for the equity holders to default strategically since an increase in the creditors’ position in the CDS contract improves their bargaining power. In other words, CDS tradings commit the equity holders to pay the regular debt service more often. Importantly, our result generalizes the insight of the empty creditor problem in Bolton and Oehmke (2011) to a dynamic contingent claims model with current and future investment opportunities.\(^\text{14}\)

The endogenous investment threshold $\delta_i$ solves the optimality condition $E'(\delta_i) = 0$, and the endogenous default threshold $\delta_d$ satisfies the smooth-pasting condition $E'(\delta_d) = 0$. We cannot derive the two endogenous thresholds $\delta_i$ and $\delta_d$ in closed-form because they solve two nonlinear equations simultaneously, but we can establish the monotonicity of the thresholds in debt’s CDS position.

\(^{14}\)Under the renegotiation policy (11), the debt service function (5) must exhibit discontinuities at $\delta_n$ and $\delta_i \in (\delta_d, \delta_n)$. To see this, note that

$$\lim_{\delta \downarrow \delta_n} s(\delta) - \lim_{\delta \uparrow \delta_n} s(\delta) = (\epsilon_B - \tau \theta) \left( 1 - \frac{z_1}{z_1 - 1} \right) > 0, \text{ and } \lim_{\delta \downarrow \delta_i} s(\delta) - \lim_{\delta \uparrow \delta_i} s(\delta) = -i \cdot L \delta_i < 0.$$

The discontinuities imply that the dividend paid to equity jumps upward at $\delta_n$. This reflects the concessions made by the debt holders when the equity initiates the private workout. When the equity stops investing at $\delta_i$, the dividend jumps downward because the equity holders need to compensate the debt holders for the slowdown in the growth rate of the fundamental grows at a lower rate. Hence, the debt service jumps upward.
**Proposition 1.** The equity’s ex-post optimal choice of the investment threshold $\delta_i$ and the default threshold $\delta_d$ are strictly increasing in the debt’s CDS protection $\theta$ and have a fixed ratio $\delta_i/\delta_d$. The latter fact implies the non-investment region $(\delta_d, \delta_i)$ expands in $\theta$.

The proposition states our main qualitative result that firms with traded CDSs face worsened debt overhang problem through the empty creditor channel. With the opportunity to purchase the credit derivative, the empty creditor increases the likelihood of endogenous default. In fact, we show in the proof that that $\theta > 0$ implies $\delta_d > 0$. The increased chance of default affects the distribution of overhang; the debt holders can capture the benefit of equity-financed investment more frequently with the presence of the CDS market, and more so when the firm’s distance-to-default is short. The overhang becomes more severe than the non-CDS firms when the debt holders can take a stronger bargaining position by off-loading credit risks. Therefore, as $\theta$ increases, $E'(\delta)$ decreases and $D'(\delta)$ increases, which in turn imply a larger non-investment region. Consequently, the equity holders reduce investment earlier and the empty creditor problem exacerbates the under-investment problem.

In our model, the firm takes $i_t = 0$ only in the renegotiation region $(\delta_d, \delta_i)$. Intuitively, the equity may reduce investment when the firm’s asset deteriorates. However, as investment creates persistent values, the equity holders would prefer to negotiate down the debt service first and keep the investment alive as the firm becomes financially distressed. Technically, our setup with binary investment levels drives this feature.\(^{15}\)

The recent literature on the real effects of CDS trading has not discussed the possibility of debt overhang. Our model formally embeds the empty creditor problem formalized by Bolton and Oehmke (2011) in Leland (1994). In Bolton and Oehmke (2011), there is a single project to be financed by debt. The strategic benefit of the CDS contract in reducing the limited commitment frictions increases the ex-ante debt value of the firm. The real impact is positive: the availability of CDS contracts allows the firm to finance a broader set of projects. However, the paper does not discuss the connection between the CDS contracts and future investment opportunities. Danis and Gamba (2016) study the empty creditor problem in a dynamic structural model. The firm

\(^{15}\)The formulation allows us to characterize the renegotiation threshold in closed-form in order to study the impact of the CDS market. In an earlier version of the paper, we showed that the renegotiation threshold takes the same form as (11) and debt overhang is increased by CDS trading in a one-shot real option investment framework, for example, Hackbart and Mauer (2012). Our results should hold in a continuous investment setup, for example, Hennessy (2004). In that case, we expect the firm will under-invest even outside the renegotiation region.
finances with one-period debt which matures before the next investment opportunity arrives. As in Hennessy and Whited (2005), the firm makes financing and investment decisions simultaneously, and the CDS trading does not change the distribution of overhang.\footnote{Both Danis and Gamba (2016) and we assume the maturity of CDS matches the debt maturity. In practice, CDS contracts typically have a five-year maturity. The average debt maturity in Saretto and Tookes (2013)'s sample of S&P 500 firms is 8.68 years. The “maturity mismatch” between CDS contracts and debt suggests that illiquid and long-term debt investors may face “rollover risk” that depend on the liquidity of the CDS markets. These factors may affect the strategic benefits of CDS the debt holders anticipate and the debt overhang effect.} In contrast, our model features under-investment driven by CDS trading through the empty creditor channel.

3.3 Benchmarks

3.3.1 Benchmark: Non-CDS Firms

Suppose there are no CDS contracts that reference the firm’s debt. We can take $\theta = 0$. In this case, the equity holders never default $\delta_d = 0$. The reason is the following. From equation (5), the debt service at $\delta$ becomes either
$$s(\delta) = (r - \mu)L\delta = (1 - \alpha)(1 - \tau)\delta$$
for non-investing firms or
$$s(\delta) = (1 - \alpha)(r - (\mu + i))\frac{1 - \tau}{1 - \mu}L\delta$$
for investing firms. Regardless of the investment decision, the debt service specifies a linear sharing rule of the cash flows between the debt and equity in the absence of credit protection. Therefore, the equity holders absorb no losses and thus never default.\footnote{When $\theta = 0$, then the left-hand side of (18), the equation that pins down the endogenous default threshold, scales with $\delta_d$ and hence $\delta_d = 0$ is the unique solution.} From (11), $\delta_n = \frac{\delta_i}{\frac{c_B}{\gamma} - \frac{\delta_i}{\gamma} - \delta}L$ is the renegotiation threshold.

Proposition 2. Under Assumption 1, there is no debt overhang without the CDS market.

During a private workout, the present value of strategic debt service is $L$ per unit of $\delta$. Assumption 1 guarantees that the present value of dividends with investment net of the investment cost is positive. In fact, it ensures that the benefit of investment shared by the debt holders is sufficiently small. Therefore, it is optimal for the equity holders to invest at all times without the CDS market.\footnote{For the non-CDS firms, under-investment may occur without default because investment increases the growth rate of the cash flows and hence the chance that the debt holders is paid the higher regular coupon $c_B$. Technically, for any investment threshold, $E'(\delta_i)$ is a constant. Assumption 1 guarantees that $E'(\delta_i)$ is sufficiently large when $\delta_i \to 0$, and this is shown analytical in the appendix. See Sundaresan and Wang (2007) and Pawlina (2010) for related discussions on under-investment with renegotiable debt.}

The proposition implies that we can measure the overhang by comparing the asset value of non-CDS firms and that of CDS firms. We emphasize that Assumption 1 is not crucial for our main
results: The assumption is made to sharpen the impact of investment with CDS trading. If the hypothesis fails to hold, there are some under-investment with renegotiable debts, and the empty creditor problem causes increased debt overhang.

3.3.2 Benchmark: Value-Maximizing Investment

Another benchmark that allows us to examine the negative impact of CDS trading is the first-best benchmark. Here, we assume the equity holders choose an investment policy to maximize the firm value after the debt is in place. The comparison of the first-best firm value to the firm value with equity-maximizing investment policy (second-best firm, Section 3.2.2) allows us to derive the agency cost of CDS that captures the investment inefficiency with CDS trading. Note that Proposition 2 implies the agency cost contains only the inefficiency induced by the availability of CDSs.

Suppose the equity holders commit to a first-best investment policy that maximizes the firm value $V(\delta) = E(\delta) + B(\delta)$ after the debt is in place. The firm value must satisfy the ODE

$$ rV(\delta) = \max_{i_t \in \{0, i\}} \left\{ (1 - \tau)\delta - \phi i_t \delta + \tau s(\delta) + (\mu + i_t)\delta V'(\delta) + \frac{1}{2}\sigma^2\delta^2 V''(\delta) \right\}. $$

Similar to (7), the required return on the firm is the sum of the after-tax cash flows net of investment costs, the tax saving, and the expected capital gain. The key difference with (7) is that the maximization problem involves the investment level only, and the equity holders still choose the renegotiation and default threshold to maximize the market value of equity ex-post. In other words, the equity commits to an efficient investment policy.

The maximization with respect to $i_t$ yields $i^{FB}(\delta) = i$ if $V'(\delta) \geq \phi$ and $i^{FB}(\delta) = 0$ if $V'(\delta) < \phi$. This condition, together with the smooth-pasting at default $E'(\delta_d) = 0$ and the smooth-pasting of (9) and (10) at renegotiation, determine the policies $\{\delta_d, \delta_i, \delta_n\}$ when the equity holders commit to the first-best investment rule. As the first-best investment decision internalizes the debt value, there is less under-investment under the equity’s commitment.\textsuperscript{19} In fact, we have the following result.

\textsuperscript{19}To see this, fix a $\delta_d$. The optimality condition for an interior solution can be written as $V'(\delta^{FB}) = E'(\delta^{FB}) + b'(\delta^{FB}) - C'(\delta^{FB}) = \phi$. Note that $b'(\delta) - C'(\delta) > 0$ for all $\delta > \delta_d$: On the one hand, the debt faces less credit risks when the firm has a stronger fundamental and so a higher debt payoff. On the other hand, the expected payout of the credit derivative decreases as the distance-to-default increases. It follows that $E'(\delta_i) = \phi > E'(\delta^{FB})$ and since the equity value is convex, we have $\delta_i > \delta^{FB}$. 

17
Proposition 3. Under Assumption 1, first-best firms invest all the time until default: \( \delta_i = \delta_d \).

With the equity’s commitment to the value-maximizing investment until default, we can derive the associated equity value from (8), (9), and (10) by sending \( \delta_i \to \delta_d \). Outside the renegotiation region \( \delta > \delta_n \), the equity value is

\[
E(\delta) = (1 - \tau) \left[ \left( \frac{U_i \delta - cB}{r} \right) + \left( \frac{cB}{r} - (\theta + L\delta_n) \right) \left( \frac{\delta}{\delta_n} \right)^{z_1} + (\theta + L\delta_d - U_i \delta_d) \left( \frac{\delta}{\delta_d} \right)^{z_1} \right];
\]  

(12)

and in the renegotiation region and before default \( \delta_d < \delta \leq \delta_n \),

\[
E(\delta) = (1 - \tau) \left[ (U_i \delta - (\theta + L\delta)) + (\theta + L\delta_d - U_i \delta_d) \left( \frac{\delta}{\delta_d} \right)^{z_1} \right].
\]  

(13)

The usual smooth-pasting conditions of the equity values (12) and (13) at \( \delta_n \) and \( E'(\delta_d) = 0 \) characterize the endogenous renegotiation and default threshold, \( \delta_n = \frac{z_1}{z_1 - 1} \frac{cB}{r} - \theta \frac{1}{L} \) and \( \delta_d = \frac{z_1}{z_1 - 1} \frac{cB}{r} - \theta \). Note that default occurs on the equilibrium path because the default decision maximizes the equity’s claims. Moreover, the bankruptcy cost affects the debt service in out-of-court restructurings and hence the default decision. In contrast, the equity does not internalize the debt value and cannot renegotiate the interest payments in Leland (1994), and thus the default threshold is independent of the bankruptcy cost.

Remark. For notations, we denote \( V_{CDS} \) as an equity-maximizing (second-best) CDS firm value, \( V_{FB} \) as a value-maximizing (first-best) CDS firm value, and \( V_0 \) as the value of a non-CDS firm. We may drop the subscripts whenever the context poses no ambiguity.

3.4 Optimal Capital Structure

The optimal financial leverage balances the benefit of debt tax shield and the cost of bankruptcy. At time \( t = 0 \), the equity holders choose a coupon to maximize the ex-ante equity value (firm value)

\[
\max_{cB} V(\delta_0; \theta(c_B), T(\theta(c_B), c_B)),
\]

given their ex-post optimal investment and financial policies, and the debt’s optimal hedging strategy. Denote \( c_B^* \) as the optimal coupon and \( \theta^* = \theta(c_B^*) \) as the optimal CDS position given the optimal coupon. For any coupon, CDS position, and the associated endogenous thresholds, we
define the firm’s market leverage ratio and credit spread are defined

\[
ML(\delta_0) = \frac{B(\delta_0; \theta, T(\theta, c_B))}{E(\delta_0; \theta, T(\theta, c_B)) + B(\delta_0; \theta, T(\theta, c_B))}, \quad \text{and} \quad CS(\delta_0) = \frac{c_B}{B(\delta_0; \theta, T(\theta, c_B))} - r
\]

respectively. The ratio of the CDS premium to the notional amount \(p/\theta(c_B)\) defines the CDS spread. These objects are defined analogously for the first-best CDS firm in which the equity holders optimize the capital structure given their commitment to the first-best investment policy once the debt is in place. Following the discussion in the first-best benchmark, the CDS-induced agency cost of debt as \(AC(\delta_0) = V_{FB}(\delta_0)/V_{CDS}(\delta_0) - 1\). Lastly, as in Leland (1994), our model features scale invariance.

**Proposition 4 (Scale Invariance).** Under the optimal choice, the thresholds \(T = \{\delta_d, \delta_i, \delta_n\}\), the CDS position \(\theta^*\), the coupon \(c_B^*\), and the market values \(E(\delta_0)\) and \(B(\delta_0)\) are functions of homogeneous of degree one in the initial cash flows \(\delta_0\). It follows that \(ML(\delta_0), CS(\delta_0), \text{and } AC(\delta_0)\) are homogeneous of degree zero in \(\delta_0\).

Although CDS trading increases the debt’s payoff linearly in the renegotiation states, the additional interest \(r\theta\) scales with the default threshold. The implication is that these thresholds are still independent of the firm scale. Therefore, the optimal leverage ratio \(ML(\delta_0)\), credit spread \(CS(\delta_0)\), and the agency cost of CDS \(AC(\delta_0)\) are independent of the initial cash flows \(\delta_0\).

4 **Quantitative Analysis**

In this section, we calibrate the model to parameters that match previous studies. We set the risk-free interest rate to \(r = 5\%\), the baseline risk-neutral drift to \(\mu = 1\%\), the volatility of the cash-flow shock to \(\sigma = 25\%\). For the investment option, we set \(i = 2.5\%\) so that the growth rate of unlevered firms is \(\mu + i = 3.5\%\), which is comparable to the simulated growth rate of 3.31% in He (2011)’s analysis of dynamic agency and debt overhang. The investment cost parameter is chosen to be \(\phi = 12.5\), which implies the firm reinvests \(\phi i = 31.25\%\) of cash flows in the investment region, and an investment value of \(\Pi/U_0 = 62.5\%\). We calibrate the effective tax rate (including personal taxes) to \(\tau = 20\%\) and the bankruptcy cost to \(\alpha = 35\%\). The choice here satisfies Assumption 1.
Optimal Investment and Default Policies

The Market Value of Debt and The CDS Price

Figure 1: The endogenous investment and default policy and the debt value. Left Panel: The solid line is $\delta_i$ and the dashed line is $\delta_d$. The dotted line shows the default threshold $\delta_d$ for the first-best CDS firms. Right Panel: The solid lines are the market values of debt at time 0 and the dashed lines are the expected payments to the CDS seller, with the thick lines for the second-best firms and the thin lines for the first-best firms. The parameters are $r = 5\%$, $\mu = 1\%$, $\sigma = 25\%$, $i = 2.5\%$, $\tau = 20\%$, $\alpha = 35\%$. We choose $\phi = 12.5$, which gives $\Pi / U_0 = 62.5\%$, and set the coupon at the optimum: $c^*_B = 12.7$. Under the optimal CDS $\theta^* = 131.06$, the renegotiation threshold is $\delta_n = 5.39$.

4.1 The Impact of CDS Trading and Calibration

Figure 1 graphs the impact of CDS trading on decisions and the debt’s net payoff under a fixed capital structure. The left panel, which depicts the endogenous investment and default policy, verifies Proposition 1 and 2. In particular, it shows that the equity holders of a CDS firm choose higher investment (blue solid) and default (red dashed) thresholds as $\theta$ increases, resulting in an expansion of the under-investment region ($\delta_d, \delta_i$). For non-CDS firms, we have $\delta_i(\theta = 0) = \delta_d(\theta = 0) = 0$ because of the linear sharing of cash flows in renegotiation and the absence of default on the equilibrium path. Holding fixed the capital structure, the default threshold associated with the equity’s commitment to the first-best investment policy (thin dotted) mostly overlaps with the second-best default threshold. This value-maximizing benchmark informs us that the key driver for endogenous default is the CDS trading rather than the inability for the equity holders to switch to an optimal investment level.

The right panel of Figure 1 depicts the time-0 market value of debt (the solid lines) and the CDS price $C(\delta_0)$ (the dashed lines). The CDS premium are increasing in the hedging position because a higher $\theta$ accelerates the default time. In anticipation of an earlier default, the protection sellers
Figure 2: The optimal capital structure and the debt’s optimal CDS position. For CDS firms, the optimal leverage ratio is $ML(\delta_0) = 58.15\%$ and the credit spread is $CS(\delta_0) = 117$ bps; and for non-CDS firms, the optimal leverage ratio is $37.04\%$ and the credit spread is $384$ bps. The thin lines in both panels are for first-best CDS firms. The percentage difference in values between the first-best and the second-best CDS firms under the respective optimal leverage, $AC(\delta_0) = 0.376\%$, captures the debt overhang cost. The parameters are $r = 5\%$, $\mu = 1\%$, $\sigma = 25\%$, $i = 2.5\%$, $\phi = 12.5$, $\tau = 20\%$, $\alpha = 35\%$, and the initial cash flows $\delta_0 = 10$.

charge a higher CDS premium. The debt’s net payoff is concave in $\theta$ because an excessive CDS position induces a price that outweighs the strategic benefit. Moreover, the first-best investment decisions internalize the debt, and the high asset-growth rate results in a higher debt value (thin black solid) and a lower CDS price (thin red dashed).

Figure 2 illustrates the firm’s optimal capital structure and the debt’s optimal CDS position. The left panel of Figure 2 shows that $\theta(c_B)$ is increasing in the contractual coupon $c_B$ in both the equity-maximizing CDS firm and the value-maximizing benchmark. Facing a highly levered firm, it is beneficial for the creditors to strengthen their bargaining position because of a stronger strategic default incentive.

The right panel of Figure 2 show the firm values and the associated ex-ante optimal coupon. The non-CDS firms (black dotted-dash) issues an optimal coupon of $c_B^* = 11.5$ that exhausts the debt capacity: $c_B^* = c_B^{\text{max}} = \arg \max_{c_B} B(\delta_0; \theta = 0, T(\theta = 0, c_B))$. The intuition is that for any coupon larger than $c_B^*$, the equity initiates a private workout immediately once the firm starts its operation. It follows that the creditors are unwilling to lend more than the debt capacity.\(^{21}\)

\(^{20}\)The formal argument for this linear equilibrium is provided in the proof of Proposition 4.

\(^{21}\)Formally, the firm promises a coupon $c_B^*$ such that $\delta_0(c_B^*) = \frac{z_1 c_B^*}{z_1 + 1} = \delta_0$ in the absence of the CDS market.
The CDS firms (blue solid) sets an optimal coupon of \( c^*_B = 12.7 \) and the value-maximizing CDS firms (thin red) promises \( c^*_B = 13.3 \). Compared to a non-CDS firm, a CDS firm has an expanded debt capacity because of the strategic benefit enjoyed by the credit-protected debt holders.\(^{22}\) The debt capacity is more substantial when the equity holders could commit to a value-maximizing investment policy that internalizes the debt value: a higher asset growth rate implies a higher debt value.

Table 1 reports the calibration of the model. In panel A, we report the model-implied financial variables and the decision thresholds under the optimal capital structure. Because of the CDS-induced expansion in debt capacity, the optimal leverage ratio increases from 37.04\% to 58.15\% for the equity-maximizing firm, and to 60.31\% for the value-maximizing firm. Also, the CDS firm faces non-trivial under-investment: \( \delta_i = 3.83 > \delta_d = 2.77 \); and the default time for a value-maximizing is slightly earlier than an equity-maximizing firm: \( \delta^{FB}_d = 2.95 > \delta_d = 2.77 \). As seen in the left panel of Figure 1, the equity holders choose a lower default threshold because of the loss in the commitment to valuable investment. However, the first-best firm has a higher leverage that accelerates the equity’s ex-post optimal default time.

Additionally, the model-implied credit spread is the same as the CDS spread \( p/\theta^* = 116.88 \) basis points. The reason is that the primary driving factor for a non-zero CDS-bond basis is the liquidity of the bond and CDS market, not the investment opportunities of the firm. Moreover, \( \frac{\theta^*}{c_B/\gamma} = 51.60\% \) which matches the mean Net CDS/bonds (face value; direct issuance) of 50.3\% reported in Oehmke and Zawadowski (2016).\(^{23}\) Lastly, the increased in the debt value implies a lower credit spread for CDS firms as well: with the introduction of the CDS market, the spread decreases from 384.62 basis points to 116.88 basis points for the equity-maximizing firm, and to 120.44 basis points for the value-maximizing firm.

Despite the adverse effect of under-investment and accelerated default, the firm value still increases from \( V_0(\delta_0) = 351 \) to \( V_{CDS}(\delta_0) = 354.07 \), which amounts to a 0.875\% increase, in the baseline

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\(^{22}\) As in Leland (1994), \( c^*_B < c^{max}_B \equiv \arg \max_c B(\delta_0; \theta(c_B), T(\theta(c_B), c_B)) \) since a coupon rate higher than the optimal one induces a higher bankruptcy cost.

\(^{23}\) See, for example, Longstaff, Mithal, and Neis (2005) for a reduced-form model that captures CDS spread as a measure of default component of bond spread in illiquid bond markets; and Oehmke and Zawadowski (2015) show that in market equilibrium, negative CDS-bond basis is driven by bond trading costs and disagreement of the market participants about the bond’s default probability.
Table 1: Baseline calibration. The parameters are $r = 5\%$, $\mu = 1\%$, $\sigma = 25\%$, $i = 2.5\%$, $\phi = 12.5\%$, $\tau = 20\%$, $\alpha = 35\%$, and the initial cash flows $\delta_0 = 10$. The investment option increases the value of assets-in-place by $\Pi/U_0 = 62.5\%$. The first two columns report the quantitative results for CDS firms. The second-best (SB) investment policy maximizes the equity value and the first-best (FB) investment policy maximizes the firm value. The last column reports the result for non-CDS firms. Panel A reports the optimal initial coupon $c^*_B$, the debt’s optimal CDS position $\theta^*$, the hedge ratio $\theta^*_c$, the market leverage $ML(\delta_0) = \frac{D(\delta_0)}{V_0(\delta_0)}$, the quasi-market leverage $\text{quasi}-ML(\delta_0) = \frac{c^*_B/r}{c^*_B/r + E(\delta_0)}$, the credit spread $CS(\delta_0) = c^*_B/D(\delta_0) - r$, the endogenous default, investment, and renegotiation thresholds ($\delta_d, \delta_i, \delta_n$), the firm value $V(\delta_0)$, and the unlevered asset value $A(\delta_0)$. Appendix A provides the expression for $A(\delta_0)$. Panel B provides the decomposition of firm value and we report the percentage of firm value contributed by various sources. There, net tax shield (NTS) is the tax shield $TS(\delta_0)$ minus the bankruptcy cost $BC(\delta_0)$. The last three rows report the difference in the quantities relative to non-CDS firms over the value of non-CDS firms, and they satisfy the relation $\Delta V(\delta_0)/V_0(\delta_0) = \Delta \text{Asset}/V_0(\delta_0) + \Delta \text{NTS}/V_0(\delta_0)$. The agency cost of CDS is $AC(\delta_0) = (V_{FB} - V_{CDS})/V_{CDS} \times 100\%$.

<table>
<thead>
<tr>
<th>CDS firms</th>
<th>Non-CDS firms</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Financial variables and endogenous decisions</strong></td>
<td></td>
</tr>
<tr>
<td>Equity max. (SB)</td>
<td>Value max. (FB)</td>
</tr>
<tr>
<td><strong>Coupon $c^*_B$</strong></td>
<td>12.7</td>
</tr>
<tr>
<td><strong>CDS position $\theta^*$</strong></td>
<td>131.06</td>
</tr>
<tr>
<td><strong>Hedge ratio (%)</strong></td>
<td>51.60</td>
</tr>
<tr>
<td><strong>Market leverage (%)</strong></td>
<td>58.15</td>
</tr>
<tr>
<td><strong>Quasi-market leverage (%)</strong></td>
<td>63.15</td>
</tr>
<tr>
<td><strong>Credit spread (bps)</strong></td>
<td>116.88</td>
</tr>
<tr>
<td>$\delta_d/\delta_i/\delta_n$</td>
<td>2.77/3.83/5.39</td>
</tr>
<tr>
<td><strong>Firm value $V(\delta_0)$</strong></td>
<td>354.07</td>
</tr>
<tr>
<td><strong>Asset value $A(\delta_0)$</strong></td>
<td>317.93</td>
</tr>
</tbody>
</table>

**Panel B: Value decomposition**

Percentage of firm value from:

<table>
<thead>
<tr>
<th></th>
<th>CDS firms</th>
<th>Non-CDS firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity value</td>
<td>41.85%</td>
<td>39.69%</td>
</tr>
<tr>
<td>Debt value</td>
<td>58.15%</td>
<td>60.31%</td>
</tr>
<tr>
<td>Unlevered asset</td>
<td>89.79%</td>
<td>89.46%</td>
</tr>
<tr>
<td>Tax shield</td>
<td>11.24%</td>
<td>11.65%</td>
</tr>
<tr>
<td>Bankruptcy cost</td>
<td>1.04%</td>
<td>1.11%</td>
</tr>
<tr>
<td>Net tax shield (NTS)</td>
<td>10.21%</td>
<td>10.54%</td>
</tr>
<tr>
<td>Agency Cost of CDS (%)</td>
<td>0.3760</td>
<td></td>
</tr>
<tr>
<td>Difference with non-CDS Firms:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm value $\Delta V(\delta_0)/V_0(\delta_0)$</td>
<td>0.875%</td>
<td>1.256%</td>
</tr>
<tr>
<td>Unlevered asset $\Delta \text{Asset}/V_0(\delta_0)$</td>
<td>-2.015%</td>
<td>-2.008%</td>
</tr>
<tr>
<td>Net tax shield $\Delta \text{NTS}/V_0(\delta_0)$</td>
<td>2.890%</td>
<td>3.264%</td>
</tr>
</tbody>
</table>
calibration. To better understand the sources of value, panel B of Table 1 provides a value decomposition. There are a few observations. First, the CDS trading induces a significant wealth transfer from the equity holders to the debt holders. Second, the increase in the debt tax shield is the primary driver of the increase in the firm value. In particular, the net tax shield only accounts for 7.41% of the value of a non-CDS firm but 10.21% for the CDS firm. The last three rows of the table (“difference with non-CDS firms”) decompose the percentage increase in firm value into the reduction in asset value and the gain in the net tax shield. The percentage change in firm value attributable to the increase in the net tax shield is 2.89%.

Importantly, we quantify the CDS-induced debt overhang cost with two measures. First, we compute the agency cost of CDS that captures the marginal gain in the firm value when the equity holders of a CDS firm could commit to the value-maximizing investment policy. In the baseline calibration, the agency cost of CDS is $AC(\delta_0) = 0.376\%$. While the magnitude may seem to be small, we emphasize that our agency cost should be interpreted as the debt overhang cost induced by CDS trading. Therefore, our model captures a new dimension of the agency cost of debt.\footnote{Section 4.2 reports that there are substantial variations in the agency cost of CDS with respect to investment opportunities and other parameters.}

The existing literature with long-term debt estimates the debt overhang cost to be 2\% in Mauer and Ott (2000), 4.7\% in Moyen (2007), and about 0.2\% to 1\% in Hackbarth and Mauer (2012). More recently, Hackbarth, Rivera, and Wong (2017) quantify a 1\% of agency cost in a dynamic agency model with short-termism. Taking a simple “adding-up” view, the inception of CDS trading could lead to an 8\% to more than 50\% increase in the debt overhang cost. Hence, the debt overhang problem implied by the empty creditor channel is non-trivial.

Second, we compute the reduction in the asset value. Inspection of table 1 gives $A_{CDS}(\delta_0) - A_0(\delta_0) = -7.07$. In panel B, we report the loss in asset relative to the value of a non-CDS firm is $(A_{CDS}(\delta_0) - A_0(\delta_0))/V(\delta_0) \times 100\% = -2.015\%$. It measures the percentage of the non-CDS firm value destroyed due to the negative impact. For completeness, we calculate the percentage change in asset value $(A_{CDS}(\delta_0) - A_0(\delta_0))/A(\delta_0) \times 100\% = -2.18\%$ as well.\footnote{The comparison of $A_{FB}(\delta_0) = 317.95$ and $A_{CDS}(\delta_0) = 317.93$ seems to suggest the commitment of the first-best investment policy does not improve the asset value. This is misleading because a first-best firm has a higher debt capacity and issues a higher coupon than the corresponding second-best firm. This implies the equity holders will default earlier ($\delta_{FB}^d = 2.91 > \delta_{CDS}^d = 2.77$), and this hurts the asset.} Consequently, the reduction in the asset value is relatively large: the negative impact erodes the gain in the tax shield.

Our observations highlight a two-fold effect of CDS protection on firm value, that is, a trade-off
between increased tax benefits and debt overhang. Therefore, the inception of CDS trading is only beneficial from the firm’s perspective when the tax shield is sufficiently strong, and it always make the shareholders worse off.

In sum, our qualitative results are consistent with the empirical evidence provided by the literature. Saretto and Tookes (2013) estimate that the increase in debt ratios due to CDS tradings is between 0.9% to 5.5%, with the average market leverage being 13% for non-CDS firms and 18% for CDS firms in their sample. Subrahmanyam et al. (2014) observe increases in leverages with the inception of CDS trading and report that firms with CDS actively traded have the probability of bankruptcies increases from 0.14% to 0.47%. Ashcraft and Santos (2009) find that the CDS market lowers credit spreads for firms with high credit quality, and Kim (2016) document that firms with strong strategic default incentives have their credit spreads decrease with the introduction of CDS referencing the firm.\(^\text{26}\)

4.2 Comparative Statics

This section provides two comparative statics results. Specifically, we examine the heterogeneous impact of CDS trading for firms with different business risks, liquidation values, and profitability of investment opportunities. We document several key findings. First, CDS tradings result in higher market leverages, smaller equity values, and larger debt values; and the effects are stronger for firms with more efficient investment technologies. Second, CDS-induced debt overhangs are stronger for high-risk firms, and firms with low growth potentials and low bankruptcy costs. Third, firms subject to strong debt overhang have a reduction of firm value up to 2.5% post-CDS introduction. In the figures below, we vary the investment cost \(\phi \in [2, 16]\) to obtain \(\Pi/U_0\).

Figure 3 shows the effect of CDS trading on the firm, equity, and debt value (the upper panel), the agency cost of CDS, the hedge ratio, and the credit spread (the lower panel) by varying the investment opportunities. The solid lines are for the high-risk firm, \(\sigma_H = 28\%\), and the dash lines

\(^{26}\)As is well-known, Leland (1994) produces the low-leverage puzzle. Under reasonable parameter values, the model-implied optimal leverage is 60%-80%. Under our baseline parameters, Leland’s model generates a 57.14% leverage. As we adopt Leland (1994)’s framework, we do not aim to match the observed leverage ratios and instead we focus on the impact of the inception of CDS trading. Fortunately, the scale-invariance property of our model allows the usual tricks that address the low-leverage puzzle, for example, upward debt restructuring (Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001), Strebulaev (2007)), to be embeddable in our framework. We leave the dynamic capital structure decisions for CDS firms for further research.
The investment value is measured as $\Pi / U$ (where fixed parameters are).

Figure 3: The heterogeneous impact of CDS trading on high- and low-risk firms. The σ are for the low-risk firm, CDS tradings increase the debt capacity and the tax shield more for the high-growth firms, and for growth firms allow their debt holders to transfer more wealth from the shareholders. As a result, because they are less subject to debt overhang and default risk. The less costly strategic positions

Second, firms having more profitable investment opportunities benefit more from the CDS trading, the debt holders choose a lower hedge to economize the cost of credit protection when a firm has a poor investment opportunity and is thus more likely to default, the price effect also implies that the debt holders choose a lower hedge to economize the cost of credit protection.

There are several implications. First, the debt holders of a high-risk firm hedge less. The intuition comes from an equilibrium price effect: an increase in the cash-flow volatility increases the default likelihood, and thus the protection sellers would charge a higher CDS premium. When a firm has a poor investment opportunity and is thus more likely to default, the price effect also implies that the debt holders choose a lower hedge to economize the cost of credit protection.

Second, firms having more profitable investment opportunities benefit more from the CDS trading, because they are less subject to debt overhang and default risk. The less costly strategic positions for growth firms allow their debt holders to transfer more wealth from the shareholders. As a result, CDS tradings increase the debt capacity and the tax shield more for the high-growth firms, and
the shareholders become worse off.

Importantly, the inception of CDS trading does not always increase the firm value. In particular, the value of a high-risk CDS firm with poor investment opportunities can be 1% to 2% lower than the corresponding non-CDS firm. On the one hand, the volatility-implied price effect forces the debt holders to purchase less credit protection, so the positive impact on the debt capacity is smaller for the high-risk firms. On the other hand, the higher default likelihood implies a stronger debt overhang effect once the firm has CDSs traded. We can observe this from the bottom left panel. The agency cost of CDS has a negative relationship with the profitability of investment opportunities, and the pattern is more pronounced for high-risk firms. Specifically, the agency cost of the high-risk firm can be 0.6-0.7% higher (at $\Pi/U_0 = 33.3\%$) than the low risks.

Figure 2 examines the impact of CDS trading on firms with a different degree of bankruptcy cost. Moreover, the cost parameter proxies for the asset intangibility: firms with more tangible assets tend to have a lower bankruptcy cost. The panel for the hedging ratio shows the behavior of the creditors. Note that the threat posed by the shareholders in a private workout is more credible in firms with a high bankruptcy cost. Thus, without the CDS market, the debt’s reservation value decreases with the bankruptcy cost. As a result, for the high bankruptcy cost firms, the CDS contracts provide a more significant strategic benefit to the creditors, and they choose a higher hedge ratio.

The heterogeneous strategic benefit of the CDS contract implies that firms with a low liquidation value benefit more from the inception of CDS trading. The debt capacity and the tax shield increase more, and the wealth transfer from the equity to the debt is also stronger. In addition, low-growth CDS firms with a high liquidation value have a firm value 1% to 2.5% lower than the corresponding non-CDS firms. While the creditors of firms with a low bankruptcy cost hedge less, a high liquidation value still implies that they make less concession in renegotiation and thus the equity chooses a higher default threshold. Therefore, the debt overhang effect implies that the loss in value from the inability of the equity to commit to the first-best policy can exceed 1% for firms with less efficient investment technologies. For $\Pi/U_0 = 33.3\%$, the agency cost of CDS for the low bankruptcy cost firms (3%) is twice the cost for the high bankruptcy cost firm (1.5%).
Figure 4: The heterogeneous impact of CDS trading on firms with high-and low-bankruptcy costs. The fixed parameters are \( r = 5\% \), \( \mu = 1\% \), \( \tau = 20\% \), \( \alpha = 35\% \) and the initial cash flow is \( \delta_0 = 10 \). The investment value is measured as \( \Pi = \frac{U_i - U_0}{U_0} \). The agency cost of CDS is measured as \( \frac{V_{FB}(\delta_0)/V_{CDS}(\delta_0) - 1}{100\%} \) and the hedge ratio is given by \( \frac{\theta^*}{(c_B^*/r)} \). A high (low) bankruptcy cost firm has \( \alpha_H = 40\% \) (\( \alpha_L = 30\% \)) and is plotted in dashed (solid) lines.

5 Extensions and Discussions

5.1 Debt Holders Offers

Consider a situation where the debt holders have all the bargaining power in renegotiation. In this case, debt holders have the ability to make a take-it-or-leave-it offer to the equity holders in the private workouts. The key result in this section is that CDS becomes redundant securities and thus does not affect the firm valuation and real decisions. Intuitively, as the debt holders have all the bargaining power, their outside option, which is affected by their positions in CDS, becomes irrelevant for the bargaining outcomes.

As a first step of the analysis, we assume for the moment that the firm cannot disinvest. As the
debt has the full bargaining power, the equity will be pushed down to its value of outside option, which is zero under limited liability and absolute priority. This implies that for any \( \delta \in [\delta_d, \delta_n] \), the equity value is zero and the debt extracts all the cash flows, leading to the debt service function:

\[
s(\delta) = \begin{cases} 
(1 - \tilde{\phi}i)\delta & \text{for } \delta \in [\delta_d, \delta_n]; \\
c_B & \text{for } \delta > \delta_n.
\end{cases}
\] (14)

As the debt holders are paid all cash flows of the firm during renegotiation, we expect them to behave as residual claimants. As a consequence, it is the debt holders that effectively choose when to liquidate the firm. To understand the debt’s incentives to initiate the formal bankruptcy procedure, first, note that the debt would never declare the firm’s bankruptcy when all they receive is the positive cash flows \((1 - \tilde{\phi}i)\delta\) in the private workout. This implies that the only incentive for the debt to liquidate the firm must come from the collection of CDS coverage \(\theta\).

However, since the CDS contract does not affect the debt’s bargaining position and given that the contract is fairly priced, the ex-ante debt value only depends on the CDS position through its effect on liquidation time. The presence of the bankruptcy cost then forces the debt holders to reduce their CDS trading to minimize the possibility of liquidation. This logic yields the following proposition.

**Proposition 5.** When the debt holders can make a take-it-or-leave-it offer to the equity holders, the optimal debt service function is given by (14). The debt holders do not hedge \((\theta = 0)\), and never liquidate the firm \((\delta_d = 0)\). Consequently, the inception of CDS trading does not worsen debt overhang when the equity holders have no bargaining power.

When the equity holders have the flexibility to choose the investment level, the benefits of investment do not accrue to the debt because there is no default on the equilibrium path. In fact, as the debt holders behave like the residual claimants, they have even lower incentives to hedge against the firm’s credit risk with CDS contracts as they anticipate that the equity will adopt an inefficient investment policy.
5.2 Renegotiation Frictions

This subsection analyzes the impact of renegotiation frictions on the firm’s financial and investment policies with the presence of the CDS market. In particular, we go back to the base case with the equity holders making take-it-or-leave-it offers. We capture renegotiation frictions as an exogenous probability \( q \in [0, 1) \) of negotiation failure. When renegotiation fails, the firm liquidates, in which case the debt holders claim \( \theta + L\delta \) and the equity holders get nothing. In other words, the equity can capture the value \( E(\delta) \) with probability \( 1 - q \) in a private workout, and \( q \) reflects the magnitude of renegotiation costs, which are borne by the equity holders. The modeling advantage of a proportional renegotiation cost is that as the equity value scales with \( q \) in the renegotiation region, the magnitude of \( q \) has no direct consequence on the default decision. It allows us to isolate the direct effect of \( q \) on CDS price and focus on the strategic benefit of hedging.

For the valuation, renegotiation frictions have no direct impact on the debt portfolio, and the market value of debt is still given by (6). The equity value satisfies

\[
re(\delta) = \max_{i_t \in \{0, i\}, \delta_d, \delta_n} \left\{ (1 - \tau)(\delta - s(\delta)) - \phi i_t \delta + (\mu + i_t) \delta E'(\delta) + \frac{1}{2} \sigma^2 \delta^2 E''(\delta) \right\}. \tag{15}
\]

Denote \( E^q(\delta) \) as the solution to equation (15). Proportionality implies that for \( \delta \in [\delta_d, \delta_n] \), \( E^q(\delta) = (1 - q) \cdot E(\delta) \) with \( E(\delta) \) given by (9) or (8). It follows that the default threshold \( \delta_d \) is independent of \( q \) because of the smooth-pasting condition \( E^q(\delta_d) = (1 - q) \cdot E(\delta_d) = 0 \), which holds given \( E'(\delta_d) = 0 \). Moreover, the investment threshold \( \delta_i \) is also independent of \( q \) because \( E^q(\delta) = (1 - q)\phi \) if and only if \( E'(\delta_i) = \phi \), for \( \delta_i \in (\delta_d, \delta_n) \).

The renegotiation threshold is

\[
\delta_n = \frac{z_1}{z_1 - 1} \left( \frac{cB}{r} - (1 - q)\theta \right) \frac{1}{L + q(U_i - L)}. \tag{16}
\]

The renegotiation policy (16) reduces to (11) when \( q = 0 \). Suppose there is no CDS market, \( \theta = 0 \). Because the shareholders risk failure in private workouts, they have reduced incentives to default strategically when the renegotiation cost increases.\(^{27}\)

\(^{27}\)Alternatively, we can follow Mella-Barral and Perraudin (1997) in assuming a fixed cost \( k \) per unit of time during renegotiation. This formulation implies that the dividend is \( (1 - \tau)(\delta - (r - \mu - i)L\delta - k) \) for \( \delta < \delta_n \) and the equity has increased incentives to default because they absorb extra losses \( k \). However, as the renegotiation cost affects the default decision directly, its effects on CDS hedging is less clear.

\(^{28}\)This result holds given a positive CDS position \( \theta \) that is not “too large”. Formally, we can show \( \frac{\partial \delta_n}{\partial q} < 0 \) for
More importantly, the presence of renegotiation costs weakens the commitment effect of the CDS trading in reducing strategic defaults. To see this, note that \( \frac{\partial \delta^n}{\partial q} = -\frac{z_1}{z_1-1} \frac{1-q}{L+q(U_i-L)} \) and \( \frac{\partial}{\partial q} \left| \frac{\partial \delta^n}{\partial q} \right| < 0 \). Intuitively, the equity pays the additional debt service induced by the debt’s credit protection, \( \theta \), with probability \( 1-q \). So, given a fixed CDS position, a higher renegotiation cost implies that the equity pays less to the debt in the private workout because the expected interest payment is \( (1-q)\theta \). This effect weakens the strategic benefit of the creditor’s CDS position.

Panel A of Table 2 summarizes the effect of the renegotiation cost on non-CDS firms under the optimal capital structure. As argued above, the renegotiation threshold is decreasing in the renegotiation cost. We also observe a non-monotonicity of the optimal leverage and the debt value in the renegotiation cost. As \( q \) increase, there are two effects. On the one hand, the equity holders finance the firm with less debt ex-ante because they anticipate a high leverage may trigger renegotiation more frequently. On the other hand, the increase in the negotiation friction deters strategic defaults and makes the debt value more sensitive to the contractual coupon. As the ex-ante optimal leverage internalizes the debt value, it becomes higher than the \( q = 0 \) case when the strategic default the value-maximizing firms, in which case the default threshold is \( \delta_d = \frac{z_1}{1+z_1} \theta \frac{1}{U_i-L} \). Differentiating (16) with respect to \( q \), we have \( \frac{\partial \delta^n}{\partial q} = \frac{1+z_1}{z_1-1} \frac{\theta^*}{(L+q(U_i-L))r} \). Hence \( \frac{\partial}{\partial q} \left| \frac{\partial \delta^n}{\partial q} \right| \leq 0 \) if and only if \( \frac{\partial}{\partial q} \left( \frac{1-q}{L+q(U_i-L)} \right) \geq \theta \), which holds if \( \delta_d \leq \delta_n \). Hence, \( \theta \) being not “too large” means the creditor does not choose a \( \theta \) such that \( \delta_d > \delta_n \). In fact, the creditor at most hedges up to a \( \tilde{\theta} \) such that \( \delta_d(\tilde{\theta}) = \delta_n(\tilde{\theta}) \). Any \( \theta > \tilde{\theta} \) is not optimal because the renegotiation does not occur on the equilibrium path and hedges no longer provide strategic benefits.
incentive is low. That is when the renegotiation cost is sufficiently high.

Panel B of Table 2 is for CDS firms. Consider an increase in the renegotiation cost. An important observation is that the optimal hedge ratio will decrease due to the reduced strategic benefit of CDS trading. As the creditors become less tough in bargaining, the equity holders delay formal bankruptcy. While for low renegotiation costs the equity holders deleverage to avoid the risk of negotiation failure, the reduced bankruptcy cost for high renegotiation costs is sufficiently large to incentivize the equity holders to lever up.

Nevertheless, the renegotiation frictions destroy values. In both panels, we report the loss in value due to costly on-path renegotiation in the “loss” column. The losses could be significant for both types of firms but smaller for CDS firms. The reason is that the increase in the renegotiation cost not only reduces the default risk but also mitigates the debt overhang implied by the empty creditor channel. The last two columns of panel B quantify the under-investment problem. In particular, the agency cost of CDS decreases with the renegotiation friction in general, and the percentage reduction in asset value relative to the asset of non-CDS firms decreases monotonically.29

5.3 Socially Optimal Credit Protection

This section analyzes the level of credit protection that maximizes the firm value. Formally, for each coupon rate, we define the socially optimal CDS position as

$$\theta^s(c_B) = \arg\max_{\theta \in [0, \infty)} E(\delta_0; T(\theta, c_B), c_B) + B(\delta_0; T(\theta, c_B), c_B)),$$

given the equity’s ex-post decisions. Note that we can always decompose the firm value as

$$E(\delta) + B(\delta) = E(\delta) + \underbrace{(b(\delta) - P(\delta))}_{\text{debt’s expected payoff}} + \underbrace{(P(\delta) - C(\delta))}_{\text{seller’s payoff}}.$$ 

The expression shows that the socially optimal CDS position maximizes the sum of the payoffs of the equity, the debt, and the CDS seller. At $\delta_0$, competitive pricing implies that the CDS seller breaks even, and the firm’s trade-off of the socially optimal CDS position is as follows. On the one hand, an increase in $\theta$ improves the ex-ante firm value, since the credit protection strengthens the

---

29From $q = 0\%$ to $q = 10\%$, the agency cost of CDS increases because of the scaling in the firm value. Indeed, the difference $V_{FB}(\delta_0) - V_{CDS}(\delta_0)$ decreases monotonically with the renegotiation cost.
The socially optimal and private optimal CDS position. The parameters are $r = 5\%$, $\mu = 1\%$, $\sigma = 25\%$, $i = 2.5\%$, $\phi = 12.5\%$, $\tau = 20\%$, $\alpha = 35\%$, and the initial cash flows $\delta_0 = 10$. We plot the values under the optimal coupon, $c^*_B = 12.7$, of the baseline CDS firm.

creditor’s bargaining position. The benefits are reflected in the increased in $b(\delta)$. On the other hand, a higher $\theta$ reduces the equity value and increases the CDS price due to an increased default chance and a more severe under-investment. Moreover, the equity value reduces because of the shareholders can take less advantage of the strategic default option.

Compared to the socially optimal CDS position, the creditor does not internalize the reduction in the equity value when choosing the privately optimal credit protection. Consequently, the creditor over-hedges against credit risk and the excessively high default likelihood yields inefficiency.

Proposition 6. The debt holders always over-hedge against credit risk $\theta^*(c_B) > \theta^s(c_B)$.

Figure 5 depicts the socially- and privately-optimal credit protection, for both the equity-maximizing and value-maximizing firms. The figure shows that even without the debt-overhang effect, the creditor still over-insures. This is because the creditors have no incentive to internalize the equity value holds regardless of the investment option. Also, the difference in the slopes of the increasing part of the firm value and debt value reveals, again, the wealth transfer effect of CDS contracts. The steeper debt value reflects that the debt holders benefit disproportionately more than the firm when the credit protection reduces the limited commitment frictions in the firm.
5.4 Predicting Investment

Our model provides a new perspective on dynamic investment: compared to the dynamic paths of non-CDS firms, CDS firms have a higher debt-financed investment at debt issuance times and have lower equity-or internal-financed investment once the debt is in place. Specifically, suppose at time 0, the cost of setting up the assets is $K$ and the entrepreneur finances the initial investment cost with debt. One can conceive situations in which $B_0(\delta_0) < K$ but $B_{CDS}(\delta_0) \geq K$. Hence the positive effect of CDS trading on the debt capacity allows the firm to break the initial financial constraint. However, over the paths where the CDS firms are not actively issuing debt, debt overhang implies a lower investment rate.

Several empirical studies have investigated the real impacts of credit derivatives recently. All of these works motivate their test hypotheses from the empty creditor problem. However, none of them explicitly links the empirical findings to the debt overhang problem. Colonello, Efing, and Zucchi (2016) document that for CDS firms with strong shareholders cut investment (measured by capital expenditures over lagged property, plant, and equipment (PPE)) by 0.003. This observation is consistent with the debt overhang after the initiation of CDS trading.

On the other hand, Guest, Karapatsas, Petmezas, and Travlos (2017) investigate how CDS affects corporate acquisitions and find CDS firms have a 4.99% higher acquisition propensity than non-CDS firms. The result is consistent with the hypothesis of increased debt capacity. Batta and Yu (2017) investigate firms’ investment in a more detailed way. Post-CDS introduction, they find that (i) asset growth declines by about 2.1% as well as a significant decrease in net investment and cash paid for acquisitions; (ii) net debt issuance declines by an average of 1.1%. Interestingly, they also estimate a positive effect of CDS trading on investment, cash paid for acquisitions and debt issuance at the CDS introduction years. The last finding is consistent the hypothesis that debt-financed acquisitions can be executed more efficiently with expanded debt capacity due to the initiation of CDS trading.

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30 Their paper provides a static model without corporate taxes that generates under-investment with CDS trading and their mechanism is different from ours. Their firm uses both equity and debt in financing an investment project: That is, $E(\delta_0) + D(\delta_0) \geq K$ needs to be satisfied. They prove that the firm value will decrease with traded CDS, and hence under-investment occurs as the financing condition could be violated. We argue that the financing condition is likely to be satisfied with the usual corporate tax rate and the optimal choice of capital structure.

31 Our model assumes a one-shot capital structure choice and thus does not capture the declines in debt issuance. We believe that the extended model that incorporates dynamic capital structure decision is able to explain the result. See footnote 26. Also, recall that in Table 1, CDS trading transfers values from the equity holders to the debt holders. Facing future debt issuance opportunities, the equity holders have reduced incentives to issue new debts as they expect dilution caused by CDS trading.

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34
Our model provides a consistent explanation of the empirical findings: firms with CDS traded have increased investment at any debt issuance time or refinancing point, and the debt overhang effect implies reduced investment and asset growth. In spirit, this logic is in line with the distinction between the true dynamics and refinancing points as discussed in Strebelueav (2007). Consider the regression model used in some of the above empirical studies

\[ I_{it} = \beta_0 + \beta_1 \cdot CDStrading_{it} + \gamma X_{it} + \epsilon_{it}, \]

where \( I_{it} \) measures investment, \( CDStrading_{it} = 1 \) if the firm has CDS traded at time \( t \) and 0 otherwise, and \( X_{it} \) is the control of the firm’s characteristics. On the one hand, the ex-ante positive effect implies that \( \beta_1 > 0 \) when we perform the regression analysis using data at the debt issuance times. On the other hand, a panel regression using all the available data is likely to incorporate a significant amount of ex-post negative effect because of infrequent debt issuance, and thus \( \beta_1 < 0 \).

Our extensions in Section 5.1 and 5.2 suggest further implications. Consider including an interaction term \( CDStrading_{it} \times Z_{it} \), where \( Z_{it} \) can be a variable that controls for the CEO or institutional shareholdings that measure the equity’s bargaining power; or the number of creditors and the number of bond issues that measure the renegotiation frictions (Davydenko and Strebulaev, 2007). Our analysis of bargaining powers and renegotiation frictions implies that the coefficient of the interaction term is \( \beta_2 < 0 \).

Overall, we add to the empirical design that the timing of the initiation of CDS contracts and debt issuance, CDS trading periods, financing methods for investment and capital expenditure as essential controls in identifying the real effects of CDS.

6 Conclusion

We analyze both the financial and real implications of the introduction of a CDS market in a contingent claims model with dynamic investment opportunities. We show that with renegotiable debts, the strategic benefit of CDS tradings expands the debt capacity and increases the optimal leverage. Moreover, the credit spread, which measures the cost of corporate debt, decreases as a response to the inception of CDS trading when the equity’s ex-post default decision is taken into account. This positive effect allows ex-ante financially constrained firms to undertake a larger set
of positive NPV projects, consistent with the prediction of the static model provided by Bolton and Oehmke (2001). We also find that as long as the corporate tax rate is not too lower, firms with actively traded CDS have higher firm value.

Our dynamic analysis uncovers a novel and a negative real impact of CDS tradings stemming from the empty creditor channel. The empty creditor problem implies that the equity holders face tougher debt holders in the future when the firm’s fundamental is weak; and the incentive for the equity to declare bankruptcy increase with the notional amount of the CDS held by the debt holders. This drives debt overhang and reduces the equity’s incentive to invest in the future. We further show that the debt overhang implied by the empty creditor channel becomes more severe if the equity holders have higher bargaining power or face higher renegotiation costs and the debt holders hedge excessively.

In sum, we provide a tractable model with endogenous CDS positions and the optimal capital structure in the spirit of Leland (1994). CDSs are non-redundant securities because the contracts reduce limited commitment frictions. The model sheds light on bond spread, credit risk, and real investment. The framework is useful for further studies that explore the dynamic interaction of the CDS market and the financial and real policies of firms. Our model can be structurally estimated and the exercise is likely to provide further insights.
Appendix A

Appendix for Section 3.1 (General contingent claims and the valuation of CDS contracts).
Consider a contingent claim $C(\delta)$ that pays $\rho$ continuously and $p_D$ at default. The value $C(\delta)$ solves the ODE

$$rC(\delta) = \begin{cases} (\mu + i)\delta C'(\delta) + \frac{1}{2}\sigma^2 \delta^2 C''(\delta) & \text{if } \delta \geq \delta_i; \\ \mu \delta C'(\delta) + \frac{1}{2}\sigma^2 \delta^2 C''(\delta) & \text{if } \delta_d < \delta < \delta_i. \end{cases}$$

The general solution is given by

$$C(\delta) = \begin{cases} \frac{\hat{\rho} + K_1 \delta z_i}{r} & \text{if } \delta \geq \delta_i; \\ \frac{\hat{\rho} + K_2 \delta z_o + K_3 \delta z_0}{r} & \text{if } \delta_d < \delta < \delta_i. \end{cases} \quad (17)$$

We pin down the coefficients $K_1$, $K_2$, and $K_3$ using the boundary conditions: First, value-matching and smooth-pasting at $\delta_i$ gives $K_1 = K_2 \delta z_o - z_1 + K_3 \delta z_o - z_1$, and $K_2 = \frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 K_3$. Observe that the payment $\hat{\rho}$ and $p_D$ only affect these two equations through their effect on $K_3$. And second, value-matching $C(\delta_d) = p_D$ at default gives $K_3 = \frac{p_D - \hat{\rho}/r}{\left(\frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 + \delta z_o\right)}$. Hence, given $\delta_i$ and $\delta_d$, $K_3$ is linear in $\hat{\rho}$ and $p_D$.

Then for the expected value of credit protection $C(\delta)$ can be obtained by setting $p_D = \theta$ and $\hat{\rho} = 0$. Therefore, $C_3 = \frac{\theta}{\left(\frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 + \delta z_0\right)}$, $C_2 = \frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 C_3$, and $C_1 = C_2 \delta z_o - z_1 + C_3 \delta z_o - z_1$. Then for $\delta_d < \delta < \delta_i$, $C(\delta) = \theta \cdot \left(\frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 + \delta z_o\right) / \left(\frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 + \delta z_o + \delta z_0\right)$; and for $\delta \geq \delta_i$, $C(\delta) = \theta \cdot \left(\frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 + \delta z_o + \delta z_0\right) / \left(\frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 + \delta z_0\right)$. Similarly, for the premium leg, we have $P_3 = \frac{-p/r}{\left(\frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 + \delta z_0\right)}$, $P_2 = \frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 P_3$, and $P_1 = P_2 \delta z_o - z_1 + P_3 \delta z_o - z_1$ by setting $\hat{\rho} = p_D = \theta$.

Hence, for $\delta_d < \delta < \delta_i$, $P(\delta) = \rho \cdot \left(1 - \left(\frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 + \delta z_o\right) / \left(\frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 + \delta z_0 + \delta z_o\right)\right)$; and for $\delta \geq \delta_i$, $P(\delta) = \rho \cdot \left(1 - \left(\frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 + \delta z_o + \delta z_0\right) / \left(\frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 + \delta z_0\right)\right)$. The expressions suggest that we can define

$$P^i_d(\delta) = \frac{a - z_1}{z_1 - \zeta_0} \delta z_o - a_0 \delta z_0 + \delta z_0 + \delta z_0 \quad \text{as } \delta_0 \to \infty.$$
for $\delta_d < \delta_0 < \delta_1$. ■

Appendix for Section 3.2.1 and 3.2.2 (The valuation of debt and equity). For the ODE for the debt payoff (4), it can be written more explicitly as

$$rb(\delta) = \begin{cases} 
  c_B + (\mu + i)\delta b'(\delta) + \frac{1}{2} \sigma^2 \delta^2 b''(\delta) & \text{if } \delta > \delta_n; \\
  s(\delta) + (\mu + i)\delta b'(\delta) + \frac{1}{2} \sigma^2 \delta^2 b''(\delta) & \text{if } \delta_i \leq \delta \leq \delta_n; \\
  s(\delta) + \mu \delta b'(\delta) + \frac{1}{2} \sigma^2 \delta^2 b''(\delta) & \text{if } \delta_d < \delta < \delta_i.
\end{cases}$$

As $b(\delta) = R(\delta) = \theta + L\delta$ for $\delta \in (\delta_d, \delta_n]$, the general solution is

$$b(\delta) = \begin{cases} 
  \frac{c_B}{\tau} + B_1 \delta^{z_1} & \text{if } \delta > \delta_n; \\
  \theta + L\delta & \text{if } \delta_d < \delta \leq \delta_n.
\end{cases}$$

Note that $b(\delta)$ automatically satisfies value-matching and smooth-pasting at $\delta_i$; and value-matching at $\delta_n$ yields $B_1 = (\theta + L\delta_n - \frac{c_B}{\tau}) \delta_n^{-z_1}$. This gives the debt payoff (6).

To derive the solution for the equity value (7), we use $V(\delta) = E(\delta) + B(\delta)$. According to the Trade-off theory, the firm value can be written as

$$V(\delta) = \underbrace{A(\delta)}_{\text{unlevered asset}} + \tau \underbrace{(B(\delta) - L(\delta))}_{\text{tax shield}} - \underbrace{BC(\delta)}_{\text{bankruptcy cost}},$$

where $L(\delta)$ is the liquidation value of the firm. So $B(\delta) - L(\delta)$ captures the present value of interest payments before default. The claim $L(\delta)$ and $BC(\delta)$ have general solutions (17) with $\bar{p} = 0$ and values at default $L(\delta_d) = L_0$ and $BC(\delta_d) = \alpha(1 - \tau)U_0\delta_d$. Thus, for $\delta \geq \delta_i$, $L(\delta) = L_0 \cdot P_d^0(\delta)$ and $BC(\delta) = \alpha(1 - \tau)U_0\delta_d \cdot P_d^0(\delta)$; and for $\delta_d < \delta < \delta_i$, $L(\delta) = L_0 \cdot P_d^0(\delta)$ and $BC(\delta) = \alpha(1 - \tau)U_0\delta_d \cdot P_d^0(\delta)$. Define a claim $A_d(\delta)$ that pays $(1 - \tau)U_0\delta_d$ at default and zero before default, then $L(\delta) = A_d(\delta) - BC(\delta)$. The equity value can be expressed as

$$E(\delta) = \underbrace{A(\delta) - (1 - \tau)b(\delta)}_{\text{dividend+invest. option}} + (1 - \tau) \left( C(\delta) + L(\delta) - \frac{A_d(\delta)}{1 - \tau} \right).$$

Now for the unlevered asset value, it has a general solution

$$A(\delta) = \begin{cases} 
  \frac{(1 - \tau)(1 - \delta_d)}{\tau - \mu + 1}\delta + A_1 \delta^{z_1} & \text{if } \delta \geq \delta_i; \\
  \frac{1 - \tau}{\tau - \mu}\delta + A_2 \delta^{z_0} + A_3 \delta^{z_0} & \text{if } \delta_d < \delta < \delta_i.
\end{cases}$$

To determine the coefficients $A_1$, $A_2$, and $A_3$, we impose three boundary conditions. First, value-matching
at default: \(A(\delta_d) = U_d \delta_d\). This yields \(\frac{1 - \tau}{r - \mu} \delta_d + A_2 \delta_d^{a_0} + A_3 \delta_d^{a_0} = \frac{1 - \tau}{r - \mu} \delta_d\). Second, value-matching and smooth-pasting at investment: \((1 - \tau)(1 - \phi_i) \delta_i + A_1 \delta_i^{z_1} = (1 - \tau) \frac{\delta_i}{r - \mu} + A_2 \delta_i^{a_0} + A_3 \delta_i^{z_0}\) and \((1 - \tau)(1 - \phi_i) \delta_i + z_1 A_1 \delta_i^{z_1} = \frac{1 - \tau}{r - \mu} \delta_i + a_0 A_2 \delta_i^{a_0} + z_0 A_3 \delta_i^{z_0}\). With some algebra, we can show that

\[
A_2 = \frac{(1 - z_1)(1 - \tau)(U_i - U_0)\delta_i}{(a_0 - z_1)\delta_i^{a_0} - (z_0 - z_1) \left(\frac{\delta_i}{\delta_d}\right)^{a_0} \delta_d^{a_0}},
\]

and using the value-matching at default, we have

\[
A(\delta) = \frac{1 - \tau}{r - \mu} \delta + A_2 \left(\delta^{a_0} - \left(\frac{\delta}{\delta_d}\right)^{a_0} \delta_d^{a_0}\right) = \frac{1 - \tau}{r - \mu} \delta + (1 - \tau)(U_i - U_0)\delta_i \cdot \Gamma_0(\delta, \delta_d),
\]

for \(\delta_d < \delta < \delta_i\). Here, \(\Gamma_0(\delta, \delta_d) = \frac{(1 - z_1)\delta^{a_0} - (1 - z_1) \left(\frac{\delta}{\delta_d}\right)^a \delta_d^a}{(a_0 - z_1)\delta^{a_0} - (z_0 - z_1) \left(\frac{\delta}{\delta_d}\right)^a \delta_d^a}\). Similarly, for \(\delta \geq \delta_i\),

\[
A(\delta) = \frac{(1 - \tau)(1 - \phi_i)}{r - (\mu + i)} \delta + A_2 \left(\delta^{a_0} - \left(\frac{\delta}{\delta_d}\right)^{a_0} \delta_d^{a_0}\right) - (1 - \tau)(U_i - U_0)\delta_i \left(\frac{\delta}{\delta_i}\right)^{z_1},
\]

where \(\Gamma_i(\delta_i, \delta_d) = \frac{(1 - a_0)\delta^{a_0} - (1 - z_0) \left(\frac{\delta}{\delta_d}\right)^a \delta_d^a}{(a_0 - z_1)\delta^{a_0} - (z_0 - z_1) \left(\frac{\delta}{\delta_d}\right)^a \delta_d^a}\). Substituting the all the solved components, we obtain the equity value (8), (9), and (10).

**Endogenous thresholds and the proof of Proposition 1.** For the default threshold, the smooth-pasting condition \(E'(\delta_d) = 0\) is equivalent to \(E'(\delta_d) \delta_d = 0\). Therefore, given \(\delta_i, \delta_d\) solves

\[
(1 - \tau) (U_i - L) \delta_d + (1 - \tau) \left(\theta + L \delta_d - \frac{\delta_d}{r - \mu}\right) \frac{\partial P_d^d(\delta_d)}{\partial \delta} \delta_d + (1 - \tau) \Pi \delta_i \Gamma_0(\delta, \delta_d) = 0. \tag{18}
\]

In (18), \(\frac{\partial P_d^d(\delta_d)}{\partial \delta} \delta_d = \frac{a_0}{a_0 - z_1} \left(\frac{\delta_d}{\delta_d}\right)^{a_0 - a_0 + \delta_d^{a_0}} + \frac{a_0^{a_0}}{a_0 - z_1} \delta_d^{a_0} + \delta_d^{a_0 + \delta_d^{a_0}}\) and \(\Gamma_0(\delta, \delta_d) = \frac{(a_0 - z_1)\delta_d^{a_0} - (z_0 - z_1) \left(\frac{\delta_d}{\delta_d}\right)^a \delta_d^a}{(a_0 - z_1)\delta^{a_0} - (z_0 - z_1) \left(\frac{\delta_d}{\delta_d}\right)^a \delta_d^a}\). For the investment threshold, the optimality condition \(E'(\delta_i) = \phi\) is equivalent to \(E'(\delta_i) \delta_i = \phi \delta_i\). Therefore, given \(\delta_d, \delta_i\) solves

\[
(1 - \tau) (U_i - L) \delta_i + (1 - \tau) \left(\theta + L \delta_d - \frac{\delta_d}{r - \mu}\right) \frac{\partial P_d^i(\delta_i)}{\partial \delta} \delta_i + z_1(1 - \tau) \Pi \delta_i \cdot \Gamma_i(\delta_i, \delta_d) = \phi \delta_i. \tag{19}
\]

In (19), \(\frac{\partial P_d^i(\delta_i)}{\partial \delta} \delta_i = \frac{z_1}{a_0 - z_1} \left(\frac{\delta_d}{\delta_d}\right)^{z_1 + \delta_d^{z_1}}\). Note that in deriving \(E'(\delta_i)\), we use (8). We obtain the same condition if we use (9). This is because the equity value satisfies the smooth-pasting condition at the investment threshold. The solution to the system of nonlinear equations (18) and (19) is the optimal default and investment thresholds.

To obtain more information, we write the system (18) and (19) as follows. Denote \(y = \frac{\delta_i}{\delta_d}\). Note that both \(\Gamma_0(\delta_d, \delta_d)\) and \(\Gamma_i(\delta_i, \delta_d)\) depends only on the ratio \(\delta_i/\delta_d\), hence we denote them as \(\Gamma_0(y)\) and \(\Gamma_i(y)\)
respectively. Moreover,

\[
\frac{\partial P_i^g(\delta_i)}{\partial \delta} \delta_i = \frac{z_1 \left(\frac{\theta - \delta_i}{\theta} + 1\right) \left(\frac{\theta}{\theta} \right)_{\theta > 0} + 1}{a_0 \left(\frac{\theta - \delta_i}{\theta} + 1\right) \left(\frac{\theta}{\theta} \right)_{\theta > 0} + z_0 \left(\frac{\theta}{\theta} \right)_{\theta > 0}} = G_1(y); \quad \text{and} \quad \frac{\partial P_d^g(\delta_d)}{\partial \delta} \delta_d = \frac{a_0 \left(\frac{\theta - \delta_i}{\theta} + 1\right) \left(\frac{\theta}{\theta} \right)_{\theta > 0} + z_0 \left(\frac{\theta}{\theta} \right)_{\theta > 0}} = G_2(y).
\]

Then using (18) to eliminate \(\theta + L\delta_d - \frac{\delta_d}{r - \mu}\) in (19), we obtain

\[
(U_i - L) - (\Pi \cdot \Gamma_0(y) + (U_0 - L)) G(y) + z_1 \Pi \cdot \Gamma_i(y) = \phi,
\]

and from equation (18), we have

\[
\frac{\theta}{\delta_d} = \frac{(G_2(y) - 1) (U_0 - L) - \Pi \cdot y \cdot \Gamma_0(y)}{G_2(y)},
\]

respectively. The system implies that given a fixed set of parameters, the nonlinear equation (20) first determines \(y = \delta_i/\delta_d\) regardless of \(\theta\). Then the default boundary \(\delta_d\) can be solved from equation (21), given the solution \(y\) and the CDS position \(\theta\). Observe that \(\Gamma_0(y) > 0\) and \(G_2(y) < 0\). Then \(\theta/\delta_d > 0\) and it follows that \(\delta_d\) is increasing in \(\theta\); and through the fixed ratio \(y\), \(\delta_i\) is also increasing in \(\theta\).

**Proof of Proposition 2.** Within the renegotiation region \(0 < \delta \leq \delta_n\), the equity value is given by

\[
E(\delta) = \begin{cases} 
(1 - \tau) \left(\frac{1 - \delta}{r - (\mu + i)} \delta - L\delta \right) - \frac{a_0 - 1}{a_0 - z_1} \Pi \delta_i \left(\frac{\delta}{\delta_i}\right)_{\theta > 0} & \text{if } \delta_i \leq \delta \leq \delta_n; \\
(1 - \tau) \left(\frac{\delta}{r - \mu} - L\delta \right) + \frac{1 - z_1}{a_0 - z_1} \Pi \delta_i \left(\frac{\delta}{\delta_i}\right)_{\theta > 0} & \text{if } 0 < \delta < \delta_i,
\end{cases}
\]

where \(\Pi = (1 - \tau) \left(\frac{1 - \delta_i}{r - (\mu + i)} - \frac{1}{r - \mu}\right)\) is the increment of the present value of cash flows per unit of \(\delta\) from investment. The second terms on the right-hand side of the expression capture the value of investment and disinvestment option respectively. The equity value can be obtained by letting \(\theta = 0\) and \(\delta_d = 0\) in (8), (9), and (10). Note that for \(\delta \geq \delta_i\),

\[
E'(\delta) = (1 - \tau) (U_i - L) + \frac{a_0 - 1}{a_0 - z_1} \Pi \left(\frac{\delta}{\delta_i}\right)_{\theta > 0}^{1 - z_1}
\]

and hence for \(\delta_i \to 0\), the second term on the right-hand side converges to 0 for any \(\delta\) because \(z_1\) is the negative root and \(\Pi > 0\). Therefore, if \(E'(\delta; \delta_i = 0) = (1 - \tau) (U_i - L) > \phi\), then it is optimal for the equity holders to invest at all times and \(\delta_i = 0\).

In fact, the proof of Proposition 1 implies the same result. From there, let \(\theta \to 0\). Equation (21) implies that \(\delta_d \to 0\) as well for any \(y > 0\). Then the left-hand side of (19) converges to

\[
((1 - \tau)(U_i - L) - \phi) \delta_i + z_1 (1 - \tau) \Pi \cdot \frac{1 - a_0}{a_0 - z_1} \cdot \delta_i = 0.
\]
The coefficient in front of $\delta_i$ in the first term is strictly positive by assumption 1; and the coefficient attached to $\delta_i$ in the second term is also strictly positive. The only solution for $\delta_i$ is 0. ■

**Proof of Proposition 3.** First, we make a few observations when $\delta_i \to \delta_d$: (i) $P_d^1(\delta) \to \left(\frac{\phi}{\phi z}\right)^{z_1}$ and $P_d^0(\delta) \to \left[\frac{z_0 - z_1}{z_1} + a_0 + \left(\frac{\phi}{\phi z}\right)^{z_0}\right] / \left[\frac{z_0 - z_1}{z_1} + 1\right]$. As the length of $(\delta_d, \delta_i)$ converges to 0, any $\delta$ in this interval is effectively $\delta_d$. So $P_d^0(\delta) \to 1$. (ii) $\Gamma_1(\delta_i, \delta_d) \to -1, \Gamma_0(\delta_d, \delta_d) \to 1 - z_1$, $\frac{\partial P_d^0(\delta)}{\partial \delta} \delta_d \to z_1$, $\frac{\partial P_d^1(\delta)}{\partial \delta} \delta_i \to z_1$, and similar to the first observation, $\Gamma_0(\delta, \delta_d) \to \left[(1 - z_1) \left(\frac{\phi}{\phi z}\right)^{a_0} - (1 - z_1) \left(\frac{\phi}{\phi z}\right)^{z_0}\right] / [a_0 - z_0] = 0$ because $\delta = \delta_d$ on $(\delta_d, \delta_i)$. Using these observations, it is easy to check that the equity value (10) on $(\delta_n, \infty)$ converges to (12); and the equity value (9) on $(\delta_i, \delta_n)$ converges to (13). Also, the equity value (8) becomes 0. Moreover, given $\delta_i = \delta_d$, the smooth-pasting condition for default (18) becomes

$$(U_0 - L)\delta_d - (\theta + L\delta_d - U_0\delta_d)z_1 + \Omega \delta_d (1 - z_1) = 0 \Rightarrow \delta_d = \frac{z_1}{z_1 - 1} \frac{1}{U_i - L} \theta.$$ 

Second, we show that under Assumption 1, $V'(\delta^{FB}; \delta^{FB} = \delta^{FB}_d) > \phi$ and hence the investment threshold is a corner solution at $\delta^{FB}_d$. To simplify notation, we suppress the FB superscript. By definition, $V'(\delta) = E'(\delta) + B'(\delta)$. Since $E'(\delta^{FB}_d) = 0$,

$$V'(\delta_d) \cdot \delta_i = B'\delta_d) \cdot \delta_i = L \delta_i - \theta \frac{\partial P_d^1(\delta_i)}{\partial \delta} \delta_i$$

$$\Rightarrow V'(\delta_d; \delta_i = \delta_d) \cdot \delta_d = L \delta_d - \theta z_1.$$

because $\frac{\partial P_d^1(\delta_i)}{\partial \delta} \delta_i \to z_1$ as $\delta_i \to \delta_d$. Eliminate $\theta$ using $\delta_d = \frac{z_1}{z_1 - 1} \frac{1}{U_i - L} \theta$, we have $V'(\delta_d; \delta_i = \delta_d) \cdot \delta_d = [L + (1 - z_1)(U_i - L)]\delta_d > \phi \delta_d$ if $L + (1 - z_1)(U_i - L) > \phi$. But the strict inequality is always satisfied under Assumption 1 because of $z_1 < 0$ and $\tau < 1$, so $L + (1 - z_1)(U_i - L) > (1 - \tau)(U_i - L) > \phi$. Therefore, $\delta_i = \delta_d$ in the first-best. ■

**Proof of Proposition 4.** We prove the scale invariance in three steps.

**Step 1.** We first show that the value functions are homogeneous of degree one in $(\delta, c_B, \theta, T)$ where we recall that $T = (\delta_n, \delta_i, \delta_d)$ is the collection of thresholds. That is, for any constant $\beta > 0$,

$$b(\beta \delta; \beta c_B, \beta \theta, \beta T) = \beta b(\delta; c_B, \theta, T)$$

$$C(\beta \delta; \beta c_B, \beta \theta, \beta T) = \beta C(\delta; c_B, \theta, T)$$

$$E(\beta \delta; \beta c_B, \beta \theta, \beta T) = \beta E(\delta; c_B, \theta, T)$$

and, therefore, firm value function $V(\delta; c_B, \theta, T)$ is homogeneous of degree one in $(\delta, c_B, \theta, T)$.

To show this, first observe that $b(\delta; c_B, \theta, T)$ (in Equation (6)) is homogeneous of degree one in $(\delta, c_B, \theta, T)$. 41
Moreover, \( P^\delta_d(\delta) \) and \( P^\delta_b(\delta) \) are homogeneous of degree zero in \( (\delta, c_B, \theta, T) \). Hence,

\[
C(\beta \delta; \beta c_B, \beta \theta, \beta T) = \theta P^\delta_d(\beta \delta; \beta c_B, \beta \theta, \beta T) \quad \text{and} \quad C(\beta \delta; \beta c_B, \beta \theta, \beta T) = \theta P^\delta_b(\beta \delta; \beta c_B, \beta \theta, \beta T)
\]

are homogeneous of degree one in \( (\delta, c_B, \theta, T) \). Similarly, \( \Gamma_0(\delta) \) and \( \Gamma_i(\delta) \) are homogeneous of degree zero in \( (\delta, c_B, \theta, T) \). Then from equations (8), (9), and (10), \( E(\delta; c_B, \theta, T) \) is homogeneous of degree one in all of the three regions.

**Step 2.** We show that the optimal coupon and CDS protection are linear in the initial cash flow \( \delta_0 \). That is, \( c_B = \gamma \delta_0 \) and \( \theta = \eta \delta_0 \), where \( \gamma \) and \( \eta \) are constants independent upon \( \delta_0 \) and are the respective optimal coupon and CDS protection when \( \delta_0 = 1 \). This also implies that \( \theta(c_B) = \frac{\eta}{\gamma} c_B \) is linear in \( c_B \).

To show this, simply notice that given \( c_B = \gamma \delta_0 \) and \( T = \rho \delta_0 \), the optimal CDS Protection is linear in \( \delta_0 \):

\[
\theta = \arg\max_\delta b(\delta_0; c_B, \theta, T) - C(\delta_0; c_B, \theta, T) = \arg\max_\delta \delta_0 b(1; c_B/\delta_0, \bar{\theta}/\delta_0, T/\delta_0) - \delta_0 C(1; c_B/\delta_0, \bar{\theta}/\delta_0, T/\delta_0)
\]

\[
= \delta_0 \arg\max_\gamma b(1; \gamma, \bar{\eta}, \rho) - C(1; \gamma, \bar{\eta}, \rho) = \delta_0 \eta.
\]

In addition, given \( \theta = \eta \delta_0 \) and \( T = \rho \delta_0 \), the optimal coupon is linear in \( \delta_0 \):

\[
c_B = \arg\max_\delta b(\delta_0; c_B, \theta, T) - C(\delta_0; c_B, \theta, T) + E(\delta_0; c_B, \theta, T) = \arg\max_\delta \delta_0 b(1; c_B/\delta_0, \theta/\delta_0, T/\delta_0) - \delta_0 C(1; c_B/\delta_0, \theta/\delta_0, T/\delta_0) + \delta_0 E(1; c_B/\delta_0, \theta/\delta_0, T/\delta_0)
\]

\[
= \delta_0 \arg\max_\gamma b(1; \gamma, \eta, \rho) - C(1; \gamma, \eta, \rho) + E(1; \gamma, \eta, \rho) = \delta_0 \gamma.
\]

**Step 3.** We claim that the collection of thresholds is linear in \( \delta_0 \). That is,

\[
T = (\delta_n, \delta_i, \delta_d) = (\nu \delta_0, \iota \delta_0, \lambda \delta_0) = (\nu, \iota, \lambda) \delta_0 = \rho \delta_0
\]

where \( \nu, \iota, \) and \( \lambda \) are constants independent upon \( \delta_0 \) and are the respective ex post renegotiation, (dis)investment, and default threshold when \( \delta_0 = 1 \). This also implies that these thresholds are linear in \( \theta \) and \( c_B \). To prove the claim, we substitute \( \iota \delta_0 \) for \( \delta_i \) and \( \lambda \delta_0 \) for \( \delta_d \) in the expressions of \( \frac{\partial P^\delta_d(\delta)}{\partial \delta} \delta_d, \frac{\partial P^\delta_b(\delta)}{\partial \delta} \delta_i, \Gamma'(\delta_d, \delta_d), \) and \( \Gamma_i(\delta_i, \delta_d) \). This results in the independence of these function on \( \delta_0 \). Then together with \( \theta = \lambda \delta_0 \), \( \iota = \iota \delta_0 \), and \( \delta_i = \lambda \delta_0 \), both sides of equations (18) and those of (19) are linear in \( \delta_0 \). Now dividing both sides of the equations by \( \delta_0 \), we obtain a non-linear system of equations of \( (\iota, \lambda) \) that is independent of \( \delta_0 \). This verifies
the existence of a pair of \((\delta_i = \nu \delta_0, \delta_d = \lambda \delta_0)\) solving the original non-linear system of equations (18) and (19). Finally, given \(c_B = \gamma \delta_0\) and \(\theta = \lambda \delta_0\) from step 2, it is evident that \(\delta_n\) as given by equation (11) is linear in \(\delta_0\). That is, \(\delta_n = \nu \delta_0\).

Combing all three steps, we can see that all values are homogenous of degree one in \(\delta_0\) under the optimal choices of \(c_B, \theta\), and the thresholds \(T\).  

**Proof of Proposition 5.** To simplify the equations, we assume that \(\tau = 0\) so that \(\phi = \phi_i\). When there is no disinvestment, the value of the CDS contract to the debt holders is \(CDS(\delta) = \theta \left( \frac{\delta}{\delta_0} \right)^{z_1} - \frac{p}{r} \left( 1 - \left( \frac{\delta}{\delta_0} \right)^{z_1} \right)\).

Let \(\tilde{B}(\delta)\) be the value of the debt contract without CDS protection. For \(\delta \in [\delta_d, \delta_n]\), the debt’s portfolio value is

\[
D(\delta) = \frac{1 - \phi}{r - (\mu + i)} \delta + \left( L \delta_d - \frac{1 - \phi}{r - (\mu + i)} \delta_d \right) \left( \frac{\delta}{\delta_d} \right)^{z_1} + CDS(\delta).
\]

Since the default time is chosen by the debt holders effectively, the smooth-pasting condition, \(D'(\delta_d) = L\), characterizes the default threshold: \(\delta_d = \frac{z_1}{z_1 - 1} (\theta + \frac{p}{r}) / \left( \frac{1 - \phi}{r - (\mu + r)} - L \right)\). For \(\delta > \delta_n\), the debt’s portfolio value is given by

\[
D(\delta) = \frac{c_B}{r} + \left( \tilde{b}(\delta_n) - \frac{c_B}{r} \right) \left( \delta \right) \left( \frac{\delta}{\delta_n} \right)^{z_1} + CDS(\delta).
\]

Moreover, the equity value is zero on \([\delta_n, \delta_n+1]\) and \(E(\delta) = \frac{1 - \phi_i}{r - (\mu + i)} \delta - \frac{c_B}{r} + \left( \frac{c_B}{r} - \frac{1 - \phi_i}{r - (\mu + r)} \delta_n \right) \left( \frac{\delta}{\delta_n} \right)^{z_1}\) for \(\delta > \delta_n\). Smooth-pasting of the equity value, \(\lim_{\delta \uparrow \delta_n} E'(\delta) = \lim_{\delta \downarrow \delta_n} E'(\delta)\), implies that \(\delta_n = \frac{z_1}{z_1 - 1} \frac{c_B}{r} - \frac{1 - \phi_i}{r - (\mu + i)} \delta_n\). We now argue that in maximizing the ex-ante debt holder’s portfolio value \(D(\delta_0)\), it is optimal for the debt holders to choose \(\theta = 0\). First, the independence of \(\delta_n\) with respect to \(\theta\) and the fact that \(CDS(\delta_0) = 0\) implies that \(D(\delta_0)\) is independent of \(\theta\) for \(\delta_0 > \delta_n\). Hence the debt holders have weak incentives to choose \(\theta = 0\). Second, for \(\delta_0 \in [\delta_d, \delta_n]\), \(D(\delta_0) = \tilde{B}(\delta_0)\) and it is decreasing in \(\delta_d\) because under our parametric restrictions, \(\frac{1 - \phi_i}{r - (\mu + i)} > L\). By the competitive pricing condition \((p\) is increasing in \(\theta\)), the default threshold \(\delta_d\) is increasing in \(\theta\). So, a higher \(\theta\) reduces the debt value. Therefore, it is optimal for the debt holders not to hedge. It follows that \(\delta_d = 0\) and the fact that \(\theta = 0\), together with lemma 2, implies there is no debt overhang.  

**Appendix for Section 5.2.** With the specification of the renegotiation friction, the choices of the equity holders remain to be the thresholds \((\delta_d, \delta_n, \delta_i)\). Given the thresholds, the debt valuation remains the same as in Section 3.2.1 because the debt does not bear the renegotiation cost. Given the baseline equity value \(E(\delta)\), the general solution of the equity value with renegotiation frictions \(E^g(\delta)\) is given by

\[
E^g(\delta) = \begin{cases} 
(1 - \tau) \left( \frac{1 - \phi}{r - (\mu + r)} \delta - \frac{c_B}{r} \right) + A^g \delta^{z_1} & \text{if } \delta > \delta_n; \\
(1 - q) \cdot E(\delta) & \text{if } \delta_d \leq \delta \leq \delta_n.
\end{cases}
\]

The coefficient \(A^g\) is pinned down by the value-matching condition at \(\delta_n\): \(E^g(\delta_n) = (1 - q) \cdot E(\delta)\). With
some algebra, we can show that for $\delta > \delta_n$,

$$E^q(\delta) = (1 - \tau) \left( U_i \delta - \frac{c_B}{r} \right) + (1 - \tau) \left( \frac{c_B}{r} - (1 - q) \theta - (L + q (U_i - L)) \delta_n \right) \left( \frac{\delta}{\delta_n} \right)^{z_1} + (1 - q) O(\delta_n) \left( \frac{\delta}{\delta_n} \right)^{z_1},$$

where $O(\delta) \equiv (1 - \tau) \left( \theta + L \delta_d - \frac{\delta_d}{r_n} \right) P_d^i(\delta) + (1 - \tau) \Pi \delta_i \cdot \Gamma_i(\delta_i, \delta_d) \left( \frac{\delta}{\delta_n} \right)^{z_1}$ is the option value in (9). Notice that the smooth-pasting condition, $E^q'(\delta_d) = 0$, is equivalent to $E^p'(\delta_d) = 0$; and the maximization of (15) with respect to $\delta_i$ requires $E^q'(\delta_i) = (1 - q) \phi$, which is equivalent to $E^p'(\delta_i) = \phi$. Therefore, equations (21) and (20) still determine both $\delta_i$ and $\delta_d$ simultaneously. The smooth-pasting condition at $\delta_n$ characterizes the optimal renegotiation boundary and gives (16).

Appendix B: Sensitivity Analysis

Table 3 decomposes explicitly the direct effect of variation in the investment rate $i$ on the financial and policy variables by holding the choice of coupon and the CDS position constant; and the associated indirect effects through the endogenous responses of the coupon and the CDS position.

For CDS firms, the increase in the value of the growth option encourages the equity to delay the exercise of the default option and increase investment, leading to a decrease in the credit spread. This can be seen in the row of “direct effect” in Table 3. The reduced costs of financial distress and the increase in asset growth rate then encourage the firm to issue a higher coupon to exploit the tax shields. This is shown in the row of “indirect effect through the optimal coupon” in Table 3. The row of “indirect effect through the optimal hedging” shows that the debt holders strategically increase their CDS positions in response to the equity’s stronger strategic default incentives. Summing up these effects, the firm increases its investment and the increase in the CDS position delays the renegotiation time. With more investment, both the equity and debt value increase; with the equity increases more relative to the debt as the convex claim is more sensitive to changes in fundamental. This suggests that the debt holders can transfer wealth generated from more profitable investment opportunities by positioning themselves strategically. This result is in sharp contrast with non-CDS firms for which the debt values are independent of the investment parameters.

Table 4 reports model outcomes for variation in the baseline growth rate of cash flows, volatility of cash flows, investment rates, investment costs, corporate tax rate, and bankruptcy costs for both CDS firms (Panel A) and non-CDS firms (Panel B). Although we have not reported the
Table 3: Decomposition of the effect of the variation in the exogenous investment rate. Here, we compute the effect of a 5% deviation from \( i = 1.5\% \). When the parameter varies, both the coupon and the CDS position respond to it. We isolate the effect of parameter variation as follows. For direct effect, we hold \( c_B \) and \( \theta \) at the pre-variation levels. For the indirect effect through the optimal coupon, we hold \( c_B \) at the post-variation level and let \( \theta \) be fixed. The channel captures the response of the firm’s capital structure to changes in \( i \). For the indirect effect through the optimal hedging, we hold \( \theta \) at the post-variation level and \( c_B \) fixed. This gives us the debt’s response on hedging. The bolded numbers in percentage are the percentage variation in the respective endogenous quantities. For example, -5.56% of the leverage ratio in the row of direct effect is computed as \( \frac{ML(\delta_0;i=1.58\%)-ML(\delta_0;i=1.43\%)}{ML(\delta_0;i=1.58\%)} \); so \( \frac{57.97\%-61.29\%}{59.68\%} = -5.56\% \). Note that as coupon increases, the CDS position increases as well due to the strategic effect (Recall the left panel of Figure 2); and the increased bankruptcy cost will in turn affect the optimal coupon. The table does not isolate such interactions. The other parameters are set at the baseline values.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( c_B )</th>
<th>( \theta )</th>
<th>Leverage</th>
<th>Spread</th>
<th>( E(\delta_0) )</th>
<th>( D(\delta_0) )</th>
<th>( \delta_n )</th>
<th>( \delta_i )</th>
<th>( \delta_d )</th>
</tr>
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<tr>
<td><strong>Baseline</strong></td>
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</tr>
<tr>
<td>1.5%</td>
<td>17.5</td>
<td>183.84</td>
<td>59.68%</td>
<td>120 bps</td>
<td>190.87</td>
<td>282.49</td>
<td>5.47</td>
<td>3.39</td>
<td>2.88</td>
</tr>
<tr>
<td><strong>Fixed ( c_B ) and ( \theta ) (Direct effect)</strong></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1.43%</td>
<td>17.5</td>
<td>183.84</td>
<td>61.29%</td>
<td>127 bps</td>
<td>176.28</td>
<td>279.11</td>
<td>5.44</td>
<td>3.62</td>
<td>3.03</td>
</tr>
<tr>
<td>1.58%</td>
<td>17.5</td>
<td>183.84</td>
<td>57.97%</td>
<td>112 bps</td>
<td>207.16</td>
<td>285.76</td>
<td>5.49</td>
<td>3.18</td>
<td>2.72</td>
</tr>
<tr>
<td>-5.56%</td>
<td>-12.23%</td>
<td>16.18%</td>
<td>2.36%</td>
<td>0.81%</td>
<td>-13.14%</td>
<td>-10.68%</td>
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<td><strong>Optimal ( c_B ) and fixed ( \theta ) (Indirect effect through the optimal coupon)</strong></td>
<td></td>
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</tr>
<tr>
<td>1.43%</td>
<td>17.2</td>
<td>183.84</td>
<td>60.64%</td>
<td>124 bps</td>
<td>178.97</td>
<td>275.74</td>
<td>5.25</td>
<td>3.62</td>
<td>3.03</td>
</tr>
<tr>
<td>1.58%</td>
<td>17.9</td>
<td>183.84</td>
<td>58.74%</td>
<td>117 bps</td>
<td>203.72</td>
<td>290.07</td>
<td>5.75</td>
<td>3.18</td>
<td>2.72</td>
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<tr>
<td>2.38%</td>
<td>6.64%</td>
<td>-3.21%</td>
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<td>8.43%</td>
<td>0.00%</td>
<td>0.00%</td>
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<td><strong>Fixed ( c_B ) and optimal ( \theta ) (Indirect effect through the optimal hedging)</strong></td>
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<tr>
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<td>175.65</td>
<td>61.06%</td>
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<td>279.16</td>
<td>5.71</td>
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<td>17.5</td>
<td>193.49</td>
<td>58.21%</td>
<td>112 bps</td>
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<td>0.77%</td>
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<tr>
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<td>3.46</td>
<td>2.89</td>
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<td>193.49</td>
<td>59.03%</td>
<td>116 bps</td>
<td>201.55</td>
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<td>3.34</td>
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<td>10.99%</td>
<td>5.10%</td>
<td>-1.50%</td>
<td>-3.47%</td>
<td>-1.01%</td>
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Table 4: The effect of parameter variation on investment, financing, and hedging decisions. The baseline parameters are \( r = 5\% \), \( \mu = 2\% \), \( \sigma = 25\% \), \( i = 1.5\% \), \( \phi = 10 \), \( \tau = 20\% \), \( \alpha = 35\% \), and the initial cash flow is \( \delta_0 = 10 \). Panel A reports the comparative statics results for firms with traded CDS. Panel B is for non-CDS firms. As these firms issue coupon such that \( \delta_0 = \delta_n(c_B) \), \( \delta_n = 10 \).

### Panel A: CDS Firms

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( c_B^* )</th>
<th>( \theta^* )</th>
<th>Leverage</th>
<th>Spread</th>
<th>( V(\delta_0) )</th>
<th>( E(\delta_0) )</th>
<th>( B(\delta_0) )</th>
<th>( \delta_n )</th>
<th>( \delta_i )</th>
<th>( \delta_d )</th>
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<td></td>
</tr>
<tr>
<td>( \mu = 1.5% )</td>
<td>14.0</td>
<td>130.76</td>
<td>61.75%</td>
<td>137 bps</td>
<td>355.96</td>
<td>136.16</td>
<td>219.80</td>
<td>5.57</td>
<td>3.76</td>
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<tr>
<td>( \mu = 2.5% )</td>
<td>24.1</td>
<td>292.86</td>
<td>56.62%</td>
<td>103 bps</td>
<td>706.50</td>
<td>306.51</td>
<td>399.99</td>
<td>5.32</td>
<td>3.12</td>
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<td>481.91</td>
<td>166.65</td>
<td>315.26</td>
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<td>( \sigma = 30% )</td>
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<td>4.80</td>
<td>2.89</td>
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<tr>
<td>( i = 1.0% )</td>
<td>15.9</td>
<td>138.99</td>
<td>64.17%</td>
<td>143 bps</td>
<td>385.28</td>
<td>138.06</td>
<td>247.23</td>
<td>5.73</td>
<td>3.87</td>
<td>2.96</td>
</tr>
<tr>
<td>( i = 2.0% )</td>
<td>21.4</td>
<td>273.49</td>
<td>54.95%</td>
<td>99 bps</td>
<td>650.07</td>
<td>292.87</td>
<td>357.20</td>
<td>5.22</td>
<td>3.07</td>
<td>2.79</td>
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<td>( \phi = 8 )</td>
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<td>58.95%</td>
<td>118 bps</td>
<td>494.45</td>
<td>202.99</td>
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<td>5.41</td>
<td>3.21</td>
<td>2.86</td>
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<td>( \phi = 12 )</td>
<td>17.0</td>
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<td>60.46%</td>
<td>122 bps</td>
<td>452.20</td>
<td>178.79</td>
<td>273.42</td>
<td>5.53</td>
<td>3.64</td>
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<tr>
<td>( \tau = 15% )</td>
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<td>158.42</td>
<td>53.09%</td>
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<td>493.42</td>
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<td>261.97</td>
<td>4.75</td>
<td>2.92</td>
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<td>205.23</td>
<td>64.86%</td>
<td>142 bps</td>
<td>453.79</td>
<td>159.44</td>
<td>294.35</td>
<td>6.06</td>
<td>3.81</td>
<td>3.19</td>
</tr>
<tr>
<td>( \alpha = 30% )</td>
<td>18.2</td>
<td>180.88</td>
<td>61.40%</td>
<td>124 bps</td>
<td>474.88</td>
<td>183.28</td>
<td>291.60</td>
<td>5.59</td>
<td>3.49</td>
<td>2.94</td>
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<tr>
<td>( \alpha = 40% )</td>
<td>16.9</td>
<td>187.42</td>
<td>58.15%</td>
<td>116 bps</td>
<td>471.86</td>
<td>197.48</td>
<td>274.38</td>
<td>5.37</td>
<td>3.32</td>
<td>2.82</td>
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</tbody>
</table>

### Panel B: Non-CDS Firms

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( c_B^* )</th>
<th>( \theta^* )</th>
<th>Leverage</th>
<th>Spread</th>
<th>( V(\delta_0) )</th>
<th>( E(\delta_0) )</th>
<th>( B(\delta_0) )</th>
<th>( \delta_n )</th>
<th>( \delta_i )</th>
<th>( \delta_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm characteristics</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( \mu = 1.5% )</td>
<td>13.4</td>
<td>-</td>
<td>41.88%</td>
<td>402 bps</td>
<td>354.71</td>
<td>206.14</td>
<td>148.57</td>
<td>10.00</td>
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<tr>
<td>( \mu = 2.5% )</td>
<td>17.8</td>
<td>-</td>
<td>30.08%</td>
<td>356 bps</td>
<td>691.60</td>
<td>483.60</td>
<td>208.00</td>
<td>10.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \sigma = 20% )</td>
<td>13.0</td>
<td>-</td>
<td>37.04%</td>
<td>250 bps</td>
<td>468.00</td>
<td>294.67</td>
<td>173.33</td>
<td>10.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \sigma = 30% )</td>
<td>17.9</td>
<td>-</td>
<td>37.04%</td>
<td>533 bps</td>
<td>468.00</td>
<td>294.67</td>
<td>173.33</td>
<td>10.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( i = 1.0% )</td>
<td>15.7</td>
<td>-</td>
<td>45.06%</td>
<td>406 bps</td>
<td>384.67</td>
<td>211.33</td>
<td>173.33</td>
<td>10.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( i = 2.0% )</td>
<td>14.9</td>
<td>-</td>
<td>27.31%</td>
<td>360 bps</td>
<td>634.67</td>
<td>461.33</td>
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<td>0</td>
</tr>
<tr>
<td>( \phi = 8 )</td>
<td>15.3</td>
<td>-</td>
<td>35.52%</td>
<td>383 bps</td>
<td>488.00</td>
<td>314.67</td>
<td>173.33</td>
<td>10.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \phi = 12 )</td>
<td>15.3</td>
<td>-</td>
<td>38.69%</td>
<td>383 bps</td>
<td>448.00</td>
<td>274.67</td>
<td>173.33</td>
<td>10.00</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Tax shield and bankruptcy cost</td>
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<tr>
<td>( \tau = 15% )</td>
<td>16.2</td>
<td>-</td>
<td>37.26%</td>
<td>380 bps</td>
<td>494.29</td>
<td>310.13</td>
<td>184.17</td>
<td>10.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \tau = 25% )</td>
<td>14.3</td>
<td>-</td>
<td>36.88%</td>
<td>380 bps</td>
<td>440.63</td>
<td>278.13</td>
<td>162.50</td>
<td>10.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \alpha = 30% )</td>
<td>16.4</td>
<td>-</td>
<td>39.66%</td>
<td>379 bps</td>
<td>470.67</td>
<td>284.00</td>
<td>186.67</td>
<td>10.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \alpha = 40% )</td>
<td>14.1</td>
<td>-</td>
<td>34.38%</td>
<td>381 bps</td>
<td>465.33</td>
<td>305.33</td>
<td>160.00</td>
<td>10.00</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
decomposition of the effect of the variation in the other parameters, the comparative statics results reported in Table 4 can be understood similarly. An increase in the volatility of cash flows has no impact on the optimal leverage and the values of corporate securities of non-CDS firms except that it doubles the credit spread. For CDS firms, the equity holders delay the exercise of the default and renegotiation option, which encourage more investment. The potential increase in the default probability due to the higher asset volatility reduces the firm’s coupon. Then the debt holders optimally hold a small CDS position in response to the reduced strategic default, resulting in a significant transfer of values from the debt to the equity which lowers the optimal leverage. Nevertheless, the increased volatility still exposes the firm to credit risk, and the credit spread increases sharply.

An increase in the baseline growth rate implies that the firm has a higher profitability. The standard logic of the trade-off theory then implies that the firm issues a higher coupon. Moreover, for CDS firms, the equity’s incentive to default strategically increases and the debt responses to this by purchasing more credit protection. Although the default threshold increases slightly, the chance to default decreases as reflected on the credit spread. The direct effect of the baseline growth rate on the equity’s default decision drives the result. It follows that the equity has improved incentives to invest. In contrast, an increase in the corporate tax rate increases the contractual coupon and the debt’s CDS position analogously, it does not directly reduce default and hence causes a more severe under-investment problem.

Finally, an increase in the bankruptcy has two effects. On the one hand, it reduces the optimal coupon based on the trade-off theory and thus reduces the CDS position. On the other hand, it also reduces the outside option of the debt holders, and this induces them to increase their CDS position. The combined effect causes a slight increase in hedging. As the equity internalizes part of the bankruptcy costs, they delay default, and the debt overhang channel implies an increase in the firm’s investment.

Table 4 contains an additional observation. The debt values of CDS firms are more sensitive to parameter variation than non-CDS firms given the creditors’ ability to position themselves with the hedging opportunities. This implies a significant transfer of values from the equity to the debt as the change in the economic environment is beneficial to the firm as a whole. It follows that the optimal leverage ratio is less sensitive to exogenous changes in parameters in general.32

32Except for the changes in volatility or the corporate tax rate. This is because non-CDS firms never default on
References


the equilibrium path and always exhaust the debt capacity.


[34] Saretto, Alessio, and Heather Tookes, 2013, Corporate Leverage, Debt Maturity, and Credit Supply: The Role of Credit Default Swaps, Review of Financial Studies 26, 1190-1247.

