Market Equilibrium of Loan Securitization with Information Asymmetry and Interaction of Banks

Rui-hui Xu and Rose Neng Lai

Abstract:

This article models market equilibrium of loan securitization under different scenarios, where equilibrium is co-determined by banks and security investors. Banks face information asymmetry in granting loans, and investors rationally choose acceptable security quality. The key insight is an information indicator for each bank, which is a combination of the bank’s information accuracy level and level of optimism. By theoretically and numerically analysis, this article sheds light to some controversial issues. First, securitization enhances information asymmetry and banks’ optimism, and further lead to conflict of interests between banks and investors. Second, banks which have information advantage do not always outperform banks those at a disadvantage. Moreover, security price could be less informative on security quality, and less sensitive to information accuracy if the loan market is mostly occupied by information disadvantaged banks. The models provide insights into bank's aggressive securitization strategy that led up to financial instability.

Keywords: Securitization, Information Asymmetry, Liquidity Premium

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1. Introduction

The financial sector has experienced substantial changes over the past three decades of deregulation, re-regulation, liberalization and globalization. Securitization together with its costs and benefits has received increasing attention, especially after the 2008 financial crisis (Altunbasa et al. (2009), Loutskina (2011)). In this paper, we theoretically and numerically investigate securitization market equilibrium under information asymmetry, as well as the interaction among banks.

Securitization is believed to benefit both originators and firms whose loans have been securitized. Originators' benefits of securitization have been well documented, including: higher liquidity in banks’ balance sheets, lower reserve and capital requirements, reduced funding costs and information costs, tool for managing credit risk and duration mismatch (see, for example, Pennacchi (1988), Hess and Smith (1988), Fabozzi and Dunlevy (2001), Altunbasa et al. (2009), Loutskina (2011)). Fabozzi and Dunlevy (2001) state that originating banks could set up a Special Purpose Vehicle (SPV) for securitization that is designed to avoid bankruptcy. Moreover, securitization enhances credit supply and alleviates a firm's financial constraints (Drucker and Puri (2009)). For investors, Caballero and Krishnamurthy (2009) states that the originate-to-distribute model of securitization helps to satisfy a growing demand for safe assets. However, impact of securitization on financial stability is still controversial. Although Duffie (2007) states that securitization increases risk diversification and has improved financial stability,

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2 Schwarcz (1994) advocates that securitization is alchemy, rather than a "zero-sum game".
3 Gorton and Souleles (2005) provides an overview of the institutional features of SPVs and securitization, which paper highlights the SPVs(special purpose vehicles) advantage of bankruptcy remoteness. Refer also to Ayotte and Gaon (2010).
it is also accused for its contribution in exaggerating market enthusiasm and financial instability, as well as the price paid by the economy (Keys et al. (2010), Valverde et al. (2012)).

Literature documents mix evidence for the relationship between securitization and information asymmetry. On one hand, it is argued that the presence of information asymmetry encourages securitization (Ambrose et al. (2005)), and securitization in return ameliorate the effects of asymmetric information (DeMarzo (2005)), since banks have to disclose more information about the related loans or assets by issuing securities than keeping assets on the balance sheet. On the other hand, securitization may lead to greater information opacity, since information cannot be credibly transmitted to the market and banks might lack incentives to screen borrowers at origination or to keep monitoring them (Gorton and Pennacchi (1995), Foley et al. (1999), Schwarcz (2004), Cheng et al. (2008)). On the contrast, we build up static models to analyze the interrelationship between information asymmetry and securitization in the following two steps. Firstly, the impact from information asymmetry to securitization is analyzed using comparative static equilibrium analysis of security price and volume, and we find that it is really depends on the level of information indicator. Secondly, the impact from securitization to information asymmetry is considered using banks’ tendency in adjusting their information indicator levels, instead of banks’ screening and monitoring incentive as analyzed in literature. And we find that banks may rationally choose imperfect information, that is, securitization enhances information inaccuracy.

This article is distinct from literature from several angles. First, instead of considering banks’ retention decisions as well as its signaling effect, we simplify the analysis by endogenizing bank's retention decision. Although some literature (such as, DeMarzo and Duffie (1999) and
DeMarzo (2005), Albertazzi et al. (2011)) show how retaining riskier junior tranche can serve as a signal of banks’ private information, Jeon and Nishihara (2014) argue that the risk retention requirement with a fixed ratio (by Dodd-Frank Act) might induce losses of social. In contrast, we set our equilibrium to maximize both banks’ and investors' profits (hence a bank could buy its own security as an investor), thus bank's retention decision is properly endogenized.

Second, instead of considering moral hazard and adverse selection problems in security market, we focus on rational equilibrium in security market and banks’ incentive in retrieving information about loan applicants. Banks are believed to have an informational advantage over a loan investor, thus have less incentive to monitor and service loans after sold⁴ (Pennacchi (1988), Carletti (2004)), and adverse selection occurs (Passmore and Sparks (1996)). By setting a simple and transparent decision rule for banks, we avoid analyzing of information asymmetry induced moral hazard problem between banks and investors, which is not our main concern. Moreover, adverse selection problem is also reconciled in our model, since we assume that investors are able to choose acceptable security quality, and banks take account of this into their securitization decisions.

Although some literature assume that banks can increase expected asset quality by exerting effort (Innes (1990), Fender and Mitchell (2010), Chemla and Hennessy (2014)), we do not parameterize banks’ screening and monitoring efforts. Instead, we assume that the distribution of loans’ repayment probability is common knowledge, and banks’ monitoring and screening incentives are considered through the analysis of information indicator. In our model, we firstly

⁴ Some papers also investigate bank managers’ behavior. For example, in Rajan (1994), bank managers with short-term concerns select the bank's credit policies. Acharya and Naqvi (2012) argue that volume-based compensation under abundant liquidity lead to excessive credit volume and asset price bubbles.
set initial information asymmetry levels for banks to obtain the general equilibrium in securitization market, and then consider banks' monitoring and screening incentive by analyzing their profit increment against information indicator (which is also the maximum information cost that banks are willing to pay).

In this paper, informational asymmetries both across banks and between banks and borrowers are considered. In primary loan market, banks observe the probability distribution of loan repayments, and for a specific loan, they make an inaccurate estimation of the true probability based on its own information level, which could be results of accumulated past relationship with good borrowers. Then base on this estimated risk level of loans, banks choose to reject, accept and hold, or securitize the loan. In secondary loan market, banks compete to sell securities. For interaction of banks with different information levels, we focus on the impact of liquidity gap and information gap on market equilibrium. In addition, information asymmetry between banks and investors is also discussed, and found not affecting our results under our model setting.

Benefit of securitization is also parameterized in our models. By converting illiquid loans into liquid capital, securitization provides banks with an additional source of funding and increases their lending ability (Loutskina (2011)). Moreover, Agostino and Mazzuca (2011) states that the need for funding and capital arbitrage motivation could be main factors of securitization decision.

To this end, we adopt a parameter to capture these benefit of securitization, called liquidity premium, which is the additional benefits for banks when they securitize loans rather than hold loans on balance sheet. This is consistent with Heuson et al. (2001) and Passmore et al. (2001),

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5 This is different from DeMarzo and Duffie (1999), which assumes information asymmetry level vary from loan to loan.
which assume the payoff to a securitized bank equals to the MBS effective coupon yield plus the liquidity premium. In addition, we assume liquidity premium in our model differs across banks and scenarios, affected by bank's balance sheet strength, potential investment opportunities and market cycle.

In our paper, securitization volume is co-determined by banks and investors. Banks set upper bound (of no-default probability) for securitization\(^6\), and investors are rational and have minimum requirement for the quality of loans in the security pool. Furthermore, we assume banks and investors agree on a swap between varying return loans and fixed return securities. Investors tend to maximize their profit in security investment, and banks also maximize their overall profit which includes profit from securitization, liquidity premium provided by securitization, and profit from bank's holding loans. Finally, securitization market equilibrium is attained.

Our base model elaborates the effect of loan market condition, information asymmetry, and bank's liquidity premium on securitization market equilibrium. We analyze how securitization price, quality and volume, and investors' profit vary with these factors. We also check a bank's incentive of improving loan market information accuracy by investigating its profit increment against information asymmetry level. Our analysis shed light to bank's strategies in securitization market, and one of our key findings is that although investors' welfare deteriorates with increasing information asymmetry, a bank may earn optimal profit with optimistic and inaccurate information.

\(^6\) It is more profitable for banks to hold safer loans in portfolios than to securitize as analyzed in our model setting.
An extension of the model is then developed to consider the interaction of banks with different characteristics (information, liquidity and market share), and see how these differences affect market equilibrium. Our analysis shows that those banks that have information advantage do not always outperform banks that are disadvantaged. Banks' screening incentive is largely influenced by bank's characteristics such as the relative information advantage. Specifically, information advantaged banks may prefer either large information gap or no gap, while information disadvantaged banks may prefer intermediate level of information gap. Moreover, when the loan market share of information disadvantaged banks increase, the quality gap of securities offered by different types of banks increase, but the spread of banks' security price decreases.

Note that we build static models for general equilibrium in securitization market, while some literature in loan market emphasizes learning effects among banks, which leads to herding or overlending, because bank managers try to avoid jeopardizing their position if following wrong choices (, Rajan (1994), Rajan (2005)). Ogura (2006) find that the lending competition and learning effect can drive bankers to adopt aggressive strategy. Acharya and Yorulmazer (2008) and Nakagawa and Uchida (2011) build up theoretical and empirical models respectively to show the existence of herding among banks. Different from these papers that focus on dynamic evolution in lending market, we simultaneously consider loan and securitization markets, and show how banks with different characteristics choose strategies to reduce their relative disadvantage.

The remainder of the paper is organized as follows. Section 2 presents the base model and its equilibrium analysis. Section 3 specifically discuss information asymmetry between investors
and banks, and present numerical analysis for the base model. Section 4 extends the base model into the market that consists of banks with different endowments, and elaborates the interaction of banks. Section 5 discuss remaining issues, and Section 6 concludes.

2 Modeling Securitization Market Equilibrium

2.1 Market Structure and Bank’s Role

Assume there are $N$ households in the market, each household having a capital requirement of $\$1$ for its loan. A representative bank grants loans to households. Let $q \in [0,1]$ be the probability of a household not defaulting on a loan, and assume $q$ distributes according to a continuous, differentiable, and single-peaked density function $f(q)$. And denote the corresponding cumulative distribution function as $F(q)$. The bank will receive a payoff $SR_\sigma (R_\sigma > 1)$ if the borrower does not default, and get collateral $c (c < 1)$ if default happen. When default actually happens, it costs the borrower $r_c (r_c > c)$, which should be higher than the expected return from default ($r_c > qR$), so that households cannot gain from default. Assume that each household's $q$ is potentially private information, while $R$ is common knowledge.

The bank knows the distribution of loan quality, and makes its estimation of no-default probability $q$ as:

$$\beta = q\sigma \quad (0 \leq \beta \leq 1, \text{ and } \sigma > 0),$$

where information indicator $\sigma$ could be greater than 1 (overestimating $q$), equal to 1 (means no information asymmetry), or less than 1 (underestimating $q$). The extent of information asymmetry is measured by $|\sigma - 1|$, and the relative position of $\sigma$ and 1 indicates bank’s
optimistic level. Suppose that a representative bank has an initial level of $\sigma$ due to its specialization and accumulated past relationships with good borrowers. Information indicator $\sigma$ plays its role in the model like this: the bank makes decision according to the estimated risk level $1 - \beta$, but the outcome follows true probability $q$. Since our focus is on securitization market, we choose bank's average loan rate $R_\sigma$ to simplify loan market analysis.\footnote{Otherwise, if we set loan rate as risk adjusted, $R_\sigma = R_0 + k/\beta$, where $k$ is risk premia per unit. Then, in following analysis, the upper and lower bounds of securitization will both be shifted down by $k/R_\sigma - c$, that is, the upper and lower bounds are respectively $\beta' = \frac{R_\sigma + \sigma - c}{R_\sigma - c} - \frac{k}{R_\sigma - c}$ $\bar{\beta} = \frac{\sigma(R_\sigma - c)}{R_\sigma - c} - \frac{k}{R_\sigma - c}$. Meanwhile, the implicit solution for rate $r_s$ in equation (4) still holds. Hence, we claim that, our simplification in loan rate $R_\sigma$ will not significantly affect our results.}

Under the assumption of risk neutrality, a borrower applies a loan if and only if the benefit is not less than cost, that is,

$$qR \geq qR_\sigma + (1 - q) r_c$$  \hspace{1cm} (1)$$

The bank will accept the application only when it incrementally adds to banks' expected profit:

$$\beta(R_\sigma - r) + (1 - \beta)(c - r) \geq 0$$  \hspace{1cm} (2)$$

where bank's cost of capital is $r$, which is larger than collateral rate $c$, indicating that bank cannot profit from borrowers' default. While literature (for example, Fender and Mitchell (2010), Innes (1990), Pennacchi (1988), Chemla and Hennessy (2014)) assume that banks can exert costly effort to influence the probability of success of the underlying projects, we instead endogenize
the screening incentive into the equilibrium analysis of bank's profit, and we also provide explanations for bank's strategies in securitization market.

While the bank chooses to securitize part of its loans to enhance liquidity, investors rationally buy securitization products only when they are sold at a reasonable price and the quality of related loan pool is above a certain threshold. Specifically, while the bank sets criteria for accepting loan applications $\beta_{\text{min}}$ and for securitization (upper bound $\beta'$), investors choose acceptable security quality (no-default probability) $\tilde{\beta} \in [0,1]$. Moreover, banks and investors agree on a swap between varying return loans $(qR_\sigma + (1-q)c)$ and fixed return securities $r_\delta$ ($r_\delta > c$), which means investor should pay $r_\delta$ so as to be entitled to incomes from the loan pool. Meanwhile, bank's total return from securitizing a loan is $r_\delta + \delta$, where $\delta > 0$ is the value of liquidity to the bank from holding a security as opposed to loan (i.e. liquidity premium). Suppose bank's return from securitizing a loan is greater than its cost of capital, that is, $r_\delta + \delta > r_\delta$. With this constraint, we mean banks have incentive to securitize loans if they are acceptable to investors, compared to rejecting a loan application. We propose a lemma for bank's decision.

**Lemma 1: (Probability Partitioning of Bank's Decision)**

Given no-default probability $q \in [0,1]$, the corresponding regions for bank's decision are:

i) reject a loan application if $\beta \in (0, \beta_{\text{min}})$, where $\beta_{\text{min}} = (r - c)/(R_\sigma - c)$;

ii) accept and hold the loan if $\beta \in (\beta_{\text{min}}, \tilde{\beta}) \cup (\beta', 1)$, where $\beta' = (r_\delta + \delta - c)/(R_\sigma - c)$, and $\tilde{\beta}$ is the quality threshold for securitization; and

iii) accept and securitize the loan if $\beta \in (\tilde{\beta}, \beta')$. 
The intuition of the above lemma is that, a bank will reject a loan application if it is not profitable \((\beta R_\sigma + (1 - \beta)c < \tau)\), accept and hold the loan when it is either profitable to hold but not qualify for securitization \(\beta \in [\beta_{min}, \bar{\beta})\), or more profitable to hold than to securitize \((\beta R_\sigma + (1 - \beta)c > \tau + \delta)\), securitize the loan only when it qualifies for securitization and more profitable than holding \((\beta R_\sigma + (1 - \beta)c < \tau + \delta)\). Figure 1 shows this probability partitioning.

By adopting this partition in Lemma 1, we formulate bank's decision under information asymmetry in loan market. Our results will not be altered even when there is information asymmetry between banks and security investors (to be discussed in Section 3.1), because under our model setting, investors can choose acceptable security quality, and securitization contract is designed for investors' interest. Therefore, we assume that banks share their information with investors and see whether banks still have the incentive to improve information.

2.2 Security Price and Quality Threshold: Market Equilibrium of Base Model

Based on the model setting proposed in last subsection, there are \(N\) applicants, each applying for $1 loan. Given loans' repayment probability distribution, a representative bank makes decision according to the estimated probability (Lemma 1). General securitization market equilibrium will be attained by maximizing both bank and security investors' profits. Note that the bank's overall profit includes profit from securitization, liquidity premium provided by securitization, and profit

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\[8\] If banks hold large amounts of super-senior securities (Acharya and Schnabl (2009)), then under the setting of our model, banks earn the liquidity premium, and also maximize their profit from investing in securities (like investors).

\[9\] Heuson et.al. (2001) consider probability partitioning under perfect information, and discuss whether liquidity benefit of loan securitization passes to borrowers.
from bank's holding loans. This problem could be seen as the bank choosing competitive rate $r_s$ and securitization cutoff $\beta'$ on the security to maximize profit, while knowing investors' optimal response in setting the securitization threshold $\bar{\beta}$.

We study how the securitization equilibrium is affected by market condition, bank's characteristics and information asymmetry level, and whether bank have incentive to improve information. To this end, we consider securitization contract items, which are quality threshold $\bar{\beta}$ and security price $r_s$ from investors' angle, and investigate information incentive from bank's point of view. Under initial information level $\sigma$, the securitization market equilibrium is achieved with loan quality threshold $\bar{\beta}$ and rate $r_s$ which maximize investors' expected profits:

$$\max_{r_s, \bar{\beta}} \pi_l = N \int_{\bar{\beta}/\sigma}^{\beta'/\sigma} [qR_{\sigma} + (1 - q)c - r_s]f(q)dq$$

where $\beta' = \frac{r_s + \delta - c}{R_{\sigma} - c}$, $\beta_{\text{min}} = \frac{r - c}{R_{\sigma} - c}$. With all the proof and conditions for a maximum given in Appendix A1, the solutions for $\beta$ and $r_s$ are implicitly given below:

$$\bar{\beta} = \frac{\sigma (r_s - c)}{R_{\sigma} - c}$$  \hspace{1cm} (3)

$$\frac{(r_s - c)(1 - \sigma) + \delta}{\sigma^2 (R_{\sigma} - c)} f\left(\frac{\beta'}{\sigma}\right) = F\left(\frac{\beta'}{\sigma}\right) - F\left(\frac{\bar{\beta}}{\sigma}\right)$$  \hspace{1cm} (4)

Equation (3) gives investors' buying threshold for maximizing profit, and equation (4) is an explicit solution for security price $r_s$. For the bank and investors to have mutual agreement on
securitization, it must satisfy that $\beta < \beta'$, i.e. $\delta + (1 - \sigma)(r_s - c) > 0$. But banks may choose to securitize some loans that are not acceptable for "hold" if $\beta < \beta_{\text{min}}$. This case happens because $r_s$ is fixed for the loan pool, and $r_s + \delta > r$. For loans with estimated no-default probability $\beta \in [\bar{\beta}, \beta_{\text{min}}]$, although the marginal profit for acceptance is negative ($\beta R_\sigma + (1 - \beta)c - r < 0$), securitization always yields positive marginal profit ($r_s + \delta - r > 0$). As shown in Figure 2, we plot this effect as the bold supply function, where $\bar{\beta} = \frac{r - \delta - c}{R_\sigma - c}$ is the critical value for bank to choose securitizing a loan over rejection. This shift (expansion) in credit supply could happen when bank has high liquidity premium (capital shortfall, facing better investment opportunity, etc.) or during an economic boom.

2.3 Comparative Static Analysis of Base Model

Now we proceed to comparative static analysis, and leave all the proof in the Appendix A1. We evaluated the effects of loan rate $R_\sigma$, liquidity premium $\delta$, and information indicator $\sigma$ on securitization market equilibrium (security price $r_s$ and quality threshold $\bar{\beta}$, securitization volume $V$, as well as profits of investors and the bank). The three parameters correspond to macroeconomic conditions, bank specific characteristics (balance strength, liquidity need, etc.), and information asymmetry and bank optimism.

Among the results, it is surprisingly to see that, with the present of securitization, perfect information could not be optimal for bank's profit under certain economic environment (probability distribution, liquidity premium, loan rate level, collateral, etc.). And this is later demonstrated by numerical analysis.
For the effect of loan rate $R_\sigma$, both security price and standard will rise as a response to increase in loan rate $\left( \frac{\partial r_s}{\partial R_\sigma} \cdot \frac{\partial \bar{p}}{\partial R_\sigma} > 0 \right)$, if the following condition holds:\(^{10}\)

$$
\frac{(r_s-c)(1-\sigma)+\delta}{\sigma^2(R_\sigma-c)} f' + \frac{1-\sigma}{\sigma} f < 0 \tag{5}
$$

Note that information accuracy $\sigma$, liquidity premium $\delta$ and distribution of no-default probability all affect this reaction of investor. Condition (5) is more likely to be satisfied when $\sigma$ is larger, which means information asymmetry enhances the impact of loan market on securitization market. For convenience, denote $H_1 = \frac{(r_s-c)(1-\sigma)+\delta}{\sigma^2(R_\sigma-c)} f' + \frac{1-\sigma}{\sigma} f$.

Volume of securitization at equilibrium is $V = N \int_{\bar{\beta}/R_\sigma}^{\beta' / R_\sigma} f(q) dq$ (proportion of securitization times total number of firms $N$), and is determined by the lower bound $\bar{\beta}$ chosen by investors and the upper bound $\beta'$ set by bank. As shown under condition (5), when loan rate $R_\sigma$ rises, the lower bound $\bar{\beta}$ will rise, which tend to reduce the volume securitized. The upper bound $\beta'$ is also directly connected with liquidity premium $\frac{\partial \beta'}{\partial R_\sigma} = \frac{1}{\sigma} \frac{\partial \bar{p}}{\partial R_\sigma} - \frac{\delta}{(R_\sigma-c)^2}$. The combined outcome is that any increment in $R_\sigma$ will reduce the securitization volume if imposing an additional condition $(f - \sigma f^0) > 0$ (proof shown in appendix).

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\(^{10}\) Note that while this is a sufficient condition for $\frac{\partial r_s}{\partial R_\sigma}$, it is a necessary and sufficient condition for $\frac{\partial \bar{p}}{\partial R_\sigma}$. 

The effects of liquidity premium $\delta$ on equilibrium security price and threshold have the same direction, as shown in appendix. Specifically, the effect on $\tilde{\beta}$ is a multiplier $\frac{\sigma}{R_\sigma - c}$ on $r_5$, and both have the same sign as $H_1$. Moreover, the liquidity premium effect on securitization volume and investors' profit are positive if $H_1 > 0$, which means liquidity premium will push up both the securitization volume and profit of investors, but the extent of effect varies with the level of information asymmetry and macroeconomic conditions. The intuition is that an increment in $\delta$ will induce the bank to raise the upper bound of securitization $\beta'$, and issue more securities so as to earn more profit. If investors know well about the liquidity premium value $\delta$, they will raise the acceptable securitization standard $\tilde{\beta}$. Hence the average quality of securitized loan pool increase, and investors are willing to pay higher security price $r_5$.

In terms of whether the bank or investors have incentive to improve information, straightforward calculation yields:\textsuperscript{11}

$$\frac{\partial \pi_I}{\partial \sigma} = -N \frac{r_5 + \delta - c}{\sigma^2 (R_\sigma - c)} \frac{(r_5 - c)(1 - \sigma) + \delta}{\sigma} f \left( \frac{\beta'}{\sigma} \right) < 0$$

It indicates that investors have incentive to retrieve more information when information is inaccurate and optimistic ($\sigma > 1$), and they prefer conservative estimation to perfect information ($\sigma \leq 1$). Effects of information indicator ($\sigma$) on other equilibrium variables show that when

\textsuperscript{11} Comparative static analysis is taken on information indicator $\sigma$. We argue that it is possible to separate the effect of optimistic and the pure effect of information asymmetry extent $|\sigma - 1|$, but this is not done due to the complexity of expressions. Yet, since the extent of information asymmetry must have symmetric impact on equilibrium, since

$$\frac{\partial y}{\partial |\sigma - 1|} = \begin{cases} \frac{\partial y}{\partial (\sigma - 1)} & \text{if } \sigma > 1 \\ -\frac{\partial y}{\partial (\sigma - 1)} & \text{if } \sigma < 1 \end{cases}.$$
$H_1 > 0$, investors prefer higher securitization threshold $\bar{\beta}$ and lower securitization price $r_s$ if they believe that the information shared by bank is inaccurate, and the bank will also raise its retaining level $\beta'$. Moreover, although an increment of $\sigma$ tends to increase both the lower bound $\bar{\beta}$ and upper bound $\beta'$ of securitization, equilibrium securitization volume $V$ and average quality $Q$ still decrease. Table 1 summarizes all the equilibrium analysis results. Note that $\sigma = 1$ is a special case of condition $H_1 < 0 < (f - \sigma f^0)$.

Table 1: Summary of Comparative Static Analysis for Market Equilibrium (Base Model)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Mortgage rate $R_\sigma$</th>
<th>Information indicator $\sigma$</th>
<th>Liquidity premium $\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(f - \sigma f^0) &lt; 0$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$H_1 &lt; 0 &lt; (f - \sigma f^0)$</td>
<td>$?$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>$H_1 &gt; 0$</td>
<td>$?$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\sigma = 1$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

Note: "?" means undetermined, $H_1 = (r_s-c)(1-\sigma)+\delta f' + 1-\sigma f$, $f = f \left( \frac{\beta'}{\sigma} \right)$ is a notation for the probability distribution function (pdf) evaluated at $\left( \frac{\beta'}{\sigma} \right)$, $f^0$ as the pdf evaluated at $\left( \frac{\bar{\beta}}{\sigma} \right)$, and $f' = f' \left( \frac{\beta'}{\sigma} \right)$ is the first derivative of the pdf evaluated at $\frac{\beta'}{\sigma}$.

2.4 Bank's Incentive to Improve Information

In order to check the bank's incentive in improving loan information, we investigate its profit increment against information indicator. The bank's total profit consists of three parts: profit from bank's holding loans, profit from securitization and the related liquidity premium.
\[
\pi_{\text{bank}} = N \left\{ \int_{\tilde{\beta}/\sigma}^{\beta'/\sigma} \left[ r_s + \delta - r \right] f(q) dq \right. + \left. \left( \int_{\tilde{\beta}/\sigma}^{\beta'/\sigma} + \int^{1}_{\tilde{\beta}/\sigma} \right) \left[ qR_s + (1 - q)c - r \right] f(q) dq \right\}
\]

where \( \beta' = \frac{r_s + \delta - c}{R_s - c}, \beta_{\min} = \frac{r - c}{R_s - c}, \tilde{\beta} = \frac{\sigma(r_s - c)}{R_s - c} \). As shown in Appendix A1, the bank's profit increment against information indicator is:

\[
\frac{\partial \pi_{\text{bank}}}{\partial \sigma} = N \left[ \frac{\delta}{\sigma(R_s - c)} (f - \sigma f^0) \frac{\partial r_s}{\partial \sigma} - (1 - \frac{1}{\sigma})(r - c) f \left( \frac{\beta_{\min}}{\sigma} \right) \frac{\beta_{\min}}{\sigma^2} - \left( 1 - \frac{1}{\sigma} \right) (r_s + \delta - c) \frac{\beta'}{\sigma^2} f \right]
\]

When \( \sigma > 1 \) and \( H_1 > 0 \), the bank's equilibrium profit will decrease with increasing information asymmetry \( \left( \frac{\partial \pi_{\text{bank}}}{\partial \sigma} < 0 \right) \), and the bank has incentive to improve information. The maximum cost that the bank is willing to pay is limited by the absolute value of marginal increment in profit with respect to information level, that is \( |\partial \pi_{\text{bank}}/\partial \sigma| \).

The bank may not have incentive to improve information in other cases. Specifically, we consider bank's tendency in information retrieving when it initially has perfect information \((\sigma = 1)\) in loan market. When \( \sigma = 1 \), we have \( H_1 = \frac{1}{(R_s - c)} f' < 0 \), and \( G_{r_s} < 0 \) according to the second order condition for equilibrium. Then

\[
\left. \frac{\partial \pi_{\text{bank}}}{\partial \sigma} \right|_{\sigma = 1} = N \frac{\delta^2 (f - f^0)}{(R_s - c)^2 G_{r_s}} \left( r_s + \delta - c \right) \left( f' + f \right)
\]
is positive if 
\[(f - f^0) \left( \frac{\tau + \delta - c}{\sigma - c} \frac{f'}{f} \right) < 0.\]
When this condition holds, bank's profit will increase with more asymmetric information, and the extent of this effect is enhanced by liquidity premium. It means that perfect information may not be optimal for a bank which participates in securitization market, especially when the bank has stronger liquidity needs. This situation is demonstrated with simulation analysis that is presented in section 3.2.

By carrying out equilibrium analysis on profit, we endogenize the bank's incentive to improve information. This is distinct from literature that emphasizes moral hazard problem and assumes bank's screening could affect the success probability of projects. For example, Fender and Mitchell (2010) extend the principal agent problem of Innes (1990) to the case of asset securitization, and derive optimal screening effort under various retention. Hartman-Glaser et al. (2012) focus on the optimal contract for loan backed securities between an originator and investors, under its dynamic model. Instead, we find that bank's incentive is constrained by bank's liquidity premium and macroeconomic situations.

3 Extension and Discussion of Base Model

This section discusses information asymmetry between banks and security investors, and present numerical analysis for base model.

3.1 Information Asymmetry between Bank and Investors
Suppose a bank and investors hold different information accuracy, say $\sigma_b$ and $\sigma_s$. According to the bank’s loan quality estimation, its rejection threshold is $eta_{b,\text{min}} = \frac{r-c}{R_{eb}-c}$, and acceptance and securitization region is $\bar{\beta}_s < \beta \leq \beta_b' = \frac{r_s + \delta - c}{R_{eb}-c}$, where securitization threshold $\bar{\beta}_s$ is set by or for investors. Now the profit maximization problem becomes:

$$\max \pi_l = N \int_{\bar{\beta}_s}^{\beta_b'} \frac{\beta_b'}{\sigma_b} \left[ qR_{\sigma_b} + (1 - q)c - r_x \right] f(q) dq$$

where $\beta_b' = \frac{r_s + \delta - c}{R_{eb}-c}$, $\beta_{b,\text{min}} = \frac{r-c}{R_{eb}-c}$, and average loan rate $R_{\sigma_b}$ is public knowledge. The optimal securitization threshold and rates are such that:

$$\bar{\beta}_s = \frac{\sigma_b (r_s - c)}{R_{eb}-c}$$

$$\frac{(r_s - c)(1 - \sigma_b)}{\sigma_b^2 (R_{eb}-c)} \int \frac{\beta_b'}{\sigma_b} f(\frac{\beta_b'}{\sigma_b}) = F(\frac{\beta_b'}{\sigma_b}) - F(\frac{\bar{\beta}_s}{\sigma_s})$$

for which the proof and corresponding second-order conditions are given in Appendix A2.

To check the impact of information asymmetry between the bank and investors, we carry out the equilibrium analysis with respect to the bank’s and investors’ information indicator $\sigma_b$ and $\sigma_s$, as well as their information gap $|\sigma_s - \sigma_b|$, and organize the results into the following proposition, with all the proofs in Appendix A2. The result is intuitive.\textsuperscript{12} It implies that when investors enter

\textsuperscript{12} Since the retention threshold $\beta_b'$ is set by the bank and not affected by investors' information accuracy, while investors' attempt of raising securitization acceptance threshold is offset by their information disadvantage.
the securitization market, they are mainly choosing banks and their information about loan quality.

**Theorem 2: (Irrelevance of Investors’ Information)** When there is information asymmetry between a bank and investors, the optimal contract for maximizing investors' profit is designed without affected by investors' information. Specifically,

1. security price \( r_S \), volume \( V \), quality \( Q \) and investors' profit \( \pi_I \) are neither affected by investors' own information accuracy \( \sigma_s \), nor by information gap between the bank and investors \( \sigma_s - \sigma_B \);

2. investors tend to increase securitization threshold \( \bar{\beta}_s \), but this effect is offset by the information disadvantage \( \sigma_s \);

3. investors' profit \( \pi_I \) is negatively related to bank's information inaccuracy \( \sigma_B \).

In the following subsections, the extension of the model takes the original assumption that the bank and investors have the same information about loan quality. This simple assumption does not affect our results even if there is information asymmetry.
3.2 Numerical Analysis of Base Model

This subsection numerically demonstrates the equilibrium of base model under different scenarios, and analyzes the corresponding implications. Results are presented for varying information asymmetry level, liquidity premium level, and loan rates.

Assume there are $N = 10000$ borrowers in the economy\(^{13}\), and each applies for a $1 loan as set in base model. For the distribution of repayment probability $f(q)$, \(^{14}\) we adopt the Beta density function, which is a continuous probability distribution defined compactly on the interval $[0,1]$\(^{15}\), and is parameterized by two positive shape parameters $a$ and $b$:

$$f(q) = \frac{1}{B(a,b)} q^{a-1}(1-q)^{b-1}$$

where the beta function $B(a,b)$ is a normalization constant to ensure that the total probability integrates to 1. We set the shape parameters $a=10$, $b=2$ for the numerical demonstrations.

It is well documented that cost of borrowing (lending rate and other charges) and the amount of collateral required varies with business cycle (Aivazian et al. (2013)). Our analysis is then mainly focused on two scenarios as proxy for different macroeconomic environments. For

\(^{13}\) Note that $N=10000$ is only an arbitrary value chosen for convenience, and the number does not change our analysis.

\(^{14}\) Kiefer (2011) uses a Bayesian approach to make inference about default rates, using prior distributions assessed from industry experts.

\(^{15}\) Since the no-default probability $q$ takes values in $[0,1]$, we could not adopt normal distribution. And the commonly used uniform distribution is also not practical, and will make the differentiation of probability distribution equal to zero ($f' = 0$).
scenario 1 (boom market), we assumed that the gross loan rate $R_\sigma$ is 1.11, deposit rate $r$ is 1.05, collateral rate $c$ is 0.8; for scenario 2, the parameters are correspondingly set as $R_\sigma = 1.09, r = 1.03, c = 0.9$. And liquidity premium $\delta$ is set as 0.015 or 0.020. When varying liquidity premium or loan rate, information indicator $\sigma$ is set to be either greater than 1 or less than 1 for robustness consideration.

Figure 3 shows the equilibrium effects of information indicator $\sigma$ in the base model, with $\sigma$ varying from 0.95 to 1.05 (the x-axis). As displayed on the top part of Panel A, when information indicator increases, security price $r_s$ first increase then start to decrease after a certain level of information indicator $\sigma$. Meanwhile, as banks become more optimistic and facing more inaccurate information, average quality of securitization pool becomes worse. It implies that security price is not necessarily an indicator of quality, and this is consistent with the sharp decline in the quality of securitized loans prior to the recent crisis.\(^{16}\) On the other hand, in scenario 1, the sharp decline in securitization threshold $\bar{\beta}$ indicates that investors tend to lower their loan quality acceptance level along with the decline in security price. Figure 3 also shows the consistent trend of securitization standard $\bar{\beta}$ and bank's retention level $\beta'$, and the narrowing of their gap lead to lower securitization volume.

The bottom layer of Panel A provides an answer under the given scenarios: despite that investors' welfare deteriorates all the way along increasing information indicator, banks may earn optimal profit with not-so-perfect information, and have screening incentive only when the information deviates away from this optimal asymmetry level. This situation is more severe when bank's

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\(^{16}\) Demyanyk and Van Hemert (2011) find the loan quality deteriorated in the years leading up to the financial crisis, by using subprime loan loans that issued between 2001 and 2007.
liquidity premium is higher (for example, \( \delta = 0.020 \) as shown in Panel B). In other words, banks may gain at the cost of investors, especially for banks with greater liquidity needs since securitization loosens bank's capital rationing. Hence, we argue that there is an overlooked mechanism that the existence of securitization to some extent leads to rational information asymmetry choice of banks. This complements the argument that information asymmetry encourages securitization (DeMarzo and Duffie (1990), Ambrose et al. (2005)).

Notice that the results around perfect information \( \sigma = 1 \) is interestingly not symmetric. Under given scenarios, there is no reason for banks to make conservative estimation of loans repayment probability \( (\sigma < 1, \text{inaccurate information and pessimistic estimation}) \), although it benefit investors with higher security quality and lower price. Moreover, Panel C of Figure 3 traces \( H_1 \) and \( (f - \sigma f^0) \), which serve as critical values in Table 1 for theoretical analysis.

In line with theoretical analysis, Figure 4 shows the equilibrium effects of liquidity premium \( \delta \), whose increase simultaneously push up securitization quality, volume and profits of bank and investors. It also shows that the bank may raise security price \( r_s \) when liquidity premium \( \delta \) is small, but lower the price when liquidity premium is large enough to compensate for price drop. Moreover, the common trend of security price \( r_s \) and quality threshold \( \bar{\beta} \) shed light to investors’ reaction on bank's pricing strategy: when bank requires lower security price, investor tend to lower their acceptable threshold of securitized loans. These results contribute to an explanation for bank's aggressive strategy in securitization market. If the market is in large liquidity needs, and banks could gain additional liquidity from securitization, then its security marketing strategy would be "lower threshold, lower price". And this strategy could be beneficial for both bank and
investors. This is consistent with the argument of Adrian and Shin (2010) that banks have incentive to increase the leverage (more securitization), just as U.S. banks behaved before the sub-prime loan debacle in 2007.

Figure 5 shows the equilibrium effects of loan rate $R_f$. While increasing in loan rate lead to higher security price, and hence higher bank profit, it lowers securitization volume, average quality and thus investors' profit. The inverse correlation between securitization volume and loan rate is consistent with Heuson et al. (2001), which focus on loan securitization under perfect information. Securitization market fluctuations during business cycle could be decomposed into effects of the above three parameters, as well as general market conditions such as loan quality distribution.

4 Interaction of Banks

This section focuses on the interaction between banks with different characteristics — information asymmetry level, liquidity premium and loan market power — and the impacts that these characteristic differences make for the securitization market, such as the security price spread offered by the two banks, difference in profit per loan, and market share in securitization market. We shall see that different banks may deviate in information incentives; investors' profit worsens with more distinct securitization originators, and this can be enhanced by higher market share of information disadvantaged banks.
4.1 Extension Model

Banks may differ in capitalization, reputation, specialization, and relationship with good borrowers, etc. Assume there are two types of banks in the market, having different liquidity premium (caused by balance sheet strength, credit rationing, etc.), different level of information asymmetry (caused by relation banking, specialization, economics of scale, etc.), and possibly different loan market shares. Table 2 lists the difference.

Table 2: Banks with Different Characteristics

<table>
<thead>
<tr>
<th></th>
<th>Mortgage Rate</th>
<th>Information Indicator</th>
<th>Liquidity</th>
<th>Security Price</th>
<th>Mortgage Market Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank 1</td>
<td>$R_1$</td>
<td>$\sigma_1$</td>
<td>$\delta_1$</td>
<td>$r_{5,1}$</td>
<td>$N - m$</td>
</tr>
<tr>
<td>Bank 2</td>
<td>$R_2$</td>
<td>$\sigma_2 = \sigma_1 + \varepsilon$</td>
<td>$\delta_2 = \delta_1 + \Delta$</td>
<td>$r_{5,2}$</td>
<td>$m$</td>
</tr>
</tbody>
</table>

Note: $\varepsilon$ is the information gap between banks, $\Delta$ is the difference in liquidity premium between banks, and we assume that $\varepsilon$ and $\Delta$ are independent.

Suppose investors rationally choose the acceptable intrinsic risk level in a consistent manner (i.e. do not discriminate against specific banks), that is, $\frac{\bar{\sigma}_1}{\sigma_1} = \frac{\bar{\sigma}_2}{\sigma_2} \equiv \bar{q}_s$. Then the securitization market equilibrium is determined by three variables: security prices $r_{5,1}$ and $r_{5,2}$, and securitization threshold $\bar{q}_s$. That is,

$$\max_{r_{5,1}, r_{5,2}, \bar{q}_s} \pi_I = (N - m) \int_{\bar{q}_s}^{\bar{q}_1} \left[ qR_1 + (1 - q)c - r_{5,1} \right] f(q) \, dq$$

$$+ m \int_{\bar{q}_s}^{\bar{q}_2} \left[ qR_2 + (1 - q)c - r_{5,2} \right] f(q) \, dq$$
where the upper bounds $\beta_1' = \frac{r_{s,1}+\delta_1-c}{R_1-c}$, $\beta_2' = \frac{r_{s,2}+\delta_2-c}{R_2-c}$ are determined by banks according to their own characteristics. The minimum quality for banks to accept loan application are respectively $\beta_{1,\text{min}} = \frac{r-c}{R_1-c}$ and $\beta_{2,\text{min}} = \frac{r-c}{R_2-c}$. The derivation of optimal solution for this problem is given in Appendix A3, which also provides the necessary second order conditions.

**Proposition 3:** With information asymmetry, two types of banks competing in securitization market yields the following equilibrium for securitization threshold $\bar{q}_s$ and rates $r_{s,1}, r_{s,2}$:

$$\bar{q}_s = \frac{(N-m)(r_{s,1}-c) + m(r_{s,2}-c)}{(N-m)(R_1-c) + m(R_2-c)}$$  \hspace{1cm} (10)

$$\frac{(r_{s,1}-c)(1-\sigma_1)+\delta_1}{\sigma_1^2(R_1-c)} f \left( \frac{\beta_1'}{\sigma_1} \right) = F \left( \frac{\beta_1'}{\sigma_1} \right) - F \left( \bar{q}_s \right)$$  \hspace{1cm} (11)

$$\frac{(r_{s,2}-c)(1-\sigma_2)+\delta_2}{\sigma_2^2(R_2-c)} f \left( \frac{\beta_2'}{\sigma_2} \right) = F \left( \frac{\beta_2'}{\sigma_2} \right) - F \left( \bar{q}_s \right)$$  \hspace{1cm} (12)

The impacts of each bank’s characteristics on its own equilibrium variables are analogous to the base model, in which only one type of banks is considered. Therefore, in the remaining parts of this section, our focus is on the equilibrium effects of liquidity premium difference $\Delta$ and information gap $\varepsilon$, as well as loan market shares $(N-m)$ and $m$. For notation simplicity, we denote $f_i'$ as the first derivative of the probability distribution function $f(q)$ evaluated at $\frac{\beta_i'}{\sigma_i}$ for $i=1,2$, $f_i$ as the probability distribution function evaluated at $\frac{\beta_i'}{\sigma_i}$, $f^0$ as the probability distribution function evaluated at $\bar{q}_s$. 
4.2 Impacts of Banks' Characteristic Differences

In order to obtain the impact of information difference on equilibrium security prices and threshold, we solve for the partial derivatives and obtain the following results, with all the proof left in Appendix A3. Denote $\Phi_2 = \left[ \frac{(r_{s,2} - c)(1 - \sigma_2) + \delta_2}{\sigma_2^2(R_2 - c)} f_2^2 + 2\left( \frac{1}{\sigma_2} - 1 \right) f_2 \right]$, and

$$K_1 = \frac{(N-m)}{(N-m)(R_1-c) + m(R_2-c)} f^0, K_2 = \frac{m}{(N-m)(R_1-c) + m(R_2-c)} f^0,$$

we have the impact on securitization contract as:

$$\frac{\partial r_{s,2}}{\partial \varepsilon} = \frac{\tilde{G}_1}{\sigma_2^2 \tilde{G}_1 \tilde{G}_2 - K_1 K_2} \left[ \beta_2' \Phi_2 + \frac{\delta_2}{R_2 - c} f_2 \right]$$

$$\frac{\partial r_{s,1}}{\partial \varepsilon} = -\frac{K_2}{\tilde{G}_1} \frac{\partial r_{s,2}}{\partial \varepsilon} = -\frac{K_2}{\sigma_2^2 \tilde{G}_1 \tilde{G}_2 - K_1 K_2} \left[ \beta_2' \Phi_2 + \frac{\delta_2}{R_2 - c} f_2 \right]$$

and

$$\frac{\partial \tilde{q}_s}{\partial \varepsilon} = \frac{\tilde{G}_1 - K_1}{\sigma_2^2 \tilde{G}_1 \tilde{G}_2 - K_1 K_2} \frac{K_2}{f^0} \left[ \beta_2' \Phi_2 + \frac{\delta_2}{R_2 - c} f_2 \right]$$

where $\tilde{G}_1 \tilde{G}_2 < 0$ are notations for second order conditions defined in Appendix A3. A necessary condition for positive sign for all these three is

$$\left[ \frac{(r_{s,2} - c)(1 - \sigma_2) + \delta_2}{\sigma_2^2(R_2 - c)} f_2^2 + \left( \frac{1}{\sigma_2} - 2 \right) f_2 \right] > 0.$$

While interest rate spread has been largely investigated for loan market (Hertzel and Officer (2012), Ivashina (2009), Lima, Minton and Weisbach, (2014)), we analyze the impact of bank differences on security price spread. In our model, the security price spread $r_{s,2} - r_{s,1}$ is marginally affected by banks' information accuracy gap as:
\[ \frac{\partial (r_{s,2} - r_{s,1})}{\partial e} = \frac{\tilde{G}_1 + K_2}{\sigma_2^2 (\tilde{G}_1 \tilde{G}_2 - K_1 K_2)} \left[ \beta_2 \sigma_2 + \frac{\delta_2}{R_2 - c} f_2 \right] \]

We shall illustrate the results by numerical examples in the next subsection. For simplicity, the cross effect of banks’ characteristics is not listed. For example, Bank 1 makes decision not just based on its own characteristics, but also considers the general equilibrium of the whole market, including the reaction of rival Bank 2. This interaction will be demonstrated in the numerical analysis.

For the impact of information gap on securitization volume, quality and profits, we leave them all in the appendix, and only list the original expression below. Further discussion will be given in numerical analysis. Volume of securitization and the average quality of the securitization pool at equilibrium are correspondingly:

\[ V = (N - m) \int_{q_S} \frac{\beta_1'}{\sigma_1} f(q) dq + m \int_{q_S} \frac{\beta_2'}{\sigma_2} f(q) dq \]

\[ Q = (N - m) \int_{q_S} q f(q) dq + m \int_{q_S} q f(q) dq \]

and the profits of both banks are respectively:

\[ \pi_1 = (N - m) \left\{ \int_{q_S} \frac{\beta_1'}{\sigma_1} [r_{s,1} + \delta_1 - r] f(q) dq + \left( \int_{q_S} \frac{\beta_1}{\sigma_1} + \frac{\beta_1'}{\sigma_1} \right) [q R_1 + (1 - q)c - r] f(q) dq \right\} \]
\[
\pi_2 = m \left\{ \frac{\sigma_2}{\sigma_{z_2}} \int_{q_s}^{q_s'} \left[ r_{s,2} + \delta_2 - r \right] f(q) dq + \left( \frac{\sigma_{s,\min}}{\sigma_2} + \frac{1}{\sigma_2} \right) \left[ q R_2 + (1 - q) c - r \right] f(q) dq \right\}
\]

The market equilibrium based on banks' initial information levels may not enable banks to achieve their global maximal profit. Hence banks have incentive to deviate from their initial information levels. We may get an inspiration of banks' different information incentives from the partial differentiations of banks' profits with respect to information, which cannot be equal to zero by using a same set of information indicators. Banks even have opposite information incentives under certain scenarios, as demonstrated in next subsection.

For the equilibrium impact on investors' profit,

\[
\frac{\partial \pi_I}{\partial \epsilon} = -m \frac{r_{s,2} + \delta_2 - c (r_{s,2} - c) (1 - \sigma_2)}{\sigma_2^2 (R_2 - c)} f_2 < 0
\]

indicating that investors' profit will worsen with more distinct securitization originators. Moreover, this effect could be enhanced by higher \( m \), that is, by increasing market share of information disadvantaged banks.

The impact of banks' liquidity premium difference are similarly obtained,

\[
\frac{\partial r_{s,2}}{\partial \Delta} = - \frac{\hat{G}_1}{\hat{G}_1 \hat{G}_2 - K_1 K_2} \psi_2
\]

\[
\frac{\partial r_{s,1}}{\partial \Delta} = - \frac{K_2}{\hat{G}_1} \frac{\partial r_{s,2}}{\partial \Delta} = \frac{K_2}{\hat{G}_1 \hat{G}_2 - K_1 K_2} \psi_2
\]
and
\[
\frac{\partial \bar{q}_s}{\partial \Delta} = -\frac{K_2}{f \hat{G}_1 \hat{G}_2 - K_1 K_2} \frac{\psi_2}{\sigma_1 (R_1 - c)} \frac{1}{\sigma_1 (R_1 - c)} \left[ \frac{(r_{s,1} - c)(1 - \sigma_1) + \delta_2}{\sigma_1^2 (R_1 - c)} f'_1 + \left( \frac{1}{\sigma_1} - 2 \right) f_1 \right]
\]

These effects are quite consistent with each other, and it shows that changes in liquidity gap $\delta$ will push the security prices $r_{s,1}, r_{s,2}$ into the same direction. And when $\left[ \frac{(r_{s,1} - c)(1 - \sigma_1) + \delta_2}{\sigma_1^2 (R_1 - c)} f'_1 + \left( \frac{1}{\sigma_1} - 2 \right) f_1 \right] > 0$, any increment in liquidity gap $\delta$ will lead to higher security price as well as threshold. However, the effect on security price spread is unclear:

\[
\frac{\partial (r_{s,2} - r_{s,1})}{\partial \Delta} = -\frac{\hat{G}_1 + K_2}{\hat{G}_1 \hat{G}_2 - K_1 K_2} \frac{\psi_2}{\sigma_1^2 (R_1 - c)} \left[ \frac{(r_{s,1} - c)(1 - \sigma_1) + \delta_2}{\sigma_1^2 (R_1 - c)} f'_1 + \left( \frac{1}{\sigma_1} - 2 \right) f_1 \right]
\]

which is also affected by the market shares of information advantaged and disadvantaged banks.

Due to the complexity of the expression of equilibrium impact of liquidity premium on securitization volume, quality and profits, they are all left in Appendix A3, and analyzed in numerical demonstration.

### 4.3 Numerical Analysis of Extension Model

In this subsection, by controlling other parameters, we analyze the impacts of information asymmetry difference, liquidity premium gap, and loan market power. Our focus will be on the effects that are not explained by base model, for example, the two banks' security price spread, difference in information incentives, etc.
Considering the two scenarios as adopted in base model, we set $R_1 = R_2$ for simplicity.\(^\text{17}\) The equilibrium effects of information gap are summarized in Figure 6, where we vary the information indicator of Bank 1 $\sigma_1$, set $\sigma_2 = 1 + \epsilon$, and control liquidity premium gap ($\Delta = 0$). Panel A to Panel D are respectively for four cases $\sigma_1 = 1, \sigma_1 = 0.97, \sigma_1 = 0.95,$ and $\sigma_1 = 1.02$.

Since we fix $\sigma_1$, the increasing in information gap will lead to a rise in the average information asymmetry level in the market, and hence downward movement of investors' profit and average security quality as shown in base model analysis. And there is still coexistence of decline in average quality of securitized loans and increasing price $r_s$.

For the interaction of banks, we find that although Bank 1 has preset characteristics, its securitization strategy and information incentive are both affected by the information level of Bank 2. First, security prices $r_{s,1}$ and $r_{s,2}$ have similar trend when varying with information gap $\epsilon$. Although Bank 1 has fixed information asymmetry level, its security quality still decreases as a response to increasing information gap $\epsilon$ (or alternatively, average market information level).

While the securitization strategy of information advantaged bank (Bank 1 when $\epsilon > 0$, Bank 2 when $\epsilon < 0$) is relatively higher quality securities with lower price $r_{s,2}$, banks at information disadvantage require higher security price $r_{s,1}$ but providing lower quality securities. The implication is that, when investors pay higher securitization price to buy "safe" securities, it may turn out to be a low quality pool.

\(^{17}\) Otherwise, the difference in $R_1, R_2$ could be interacted with loan market concentration indicator $m$. 
Second, Bank 1 and Bank 2 may have opposite tendency in information incentives. While Bank 2 could outperform Bank 1 in a certain region of information gap, Bank 1 may prefer to either keep a large information advantage against its rival, or have no information difference. One possible level of information advantage $\varepsilon$ that Bank 1 prefers is the level that maximizes the security prices spread $(r_{s,2} - r_{s,1})$. Moreover, although banks have equal loan market share ($m = 0.5*N$), Bank 1’s security market share decreases when the information gap increases. By keeping a mediate level of information gap, Bank 1 may not lose too much security market share.

To see the impact of loan market share (competition or concentration), we check above information gap impacts in a market which is mainly occupied by Bank 2. We set unequal market shares for the two types of banks ($m = 0.8*N$), and show a variation of above results of Panel A in Figure 7. Since all other parameters are kept constant except information gap and loan market share, the relative trends of equilibrium variables are analogue for cases when information gap $\varepsilon > 0$ and $\varepsilon < 0$, we just focus on the case when $\varepsilon > 0$ and Bank 2 is information disadvantaged bank. Now investors raise acceptable security quality, then the securitization volume decrease, and investors' welfare $\pi_t$ is worse. They are now facing smaller spread in security price $(r_{s,2} - r_{s,1})$ but larger average security quality difference $(Q_1 - Q_2)$, which may be more misleading for investors since prices are less sensitive to information accuracy. This could be the case during economic boom or mania in securitization market when market average information level is lower, and information advantage becomes less important. On the other side, Bank 2’s security

\[\text{footnotesize 18 The impacts of competition on banks’ risk taking and credit standard has been investigated by Jiménez et al. (2013) and Ruckes (2004).}\]
\[\text{footnotesize 19 This is contradictory to Horvath et. al (2013), which state that enhanced competition reduces liquidity creation. In our case, lower competition from information advantage banks will reduces liquidity creation.}\]
market share now declines slower as a response to the rise in information gap, and its profit per loan over Bank 1 becomes smaller, indicating less efficient securitization market equilibrium.

Figure 8 shows the impact of liquidity premium difference $\Delta$, when we set $\delta_1 = 0.015, \delta_2 = \delta_1 + \Delta$ and control all other parameters. The benefit of liquidity premium advantage is demonstrated by profits of banks ($\pi_1 > \pi_2$ when $\Delta < 0$, and $\pi_1 < \pi_2$ when $\Delta > 0$), indicating banks that have liquidity advantage outperform banks at disadvantage. Moreover, it shows that liquidity advantaged banks (Bank 2 when $\Delta > 0$) ask for lower security price and securitize less loans. While the profits of both banks increase along with liquidity premium, the profit of investors varies dramatically, and decrease when liquidity difference become larger. This variation is due to the rebalance of related factors along with increasing liquidity premium gap. Specifically, the positive effect of bank's retaining level $\beta'$'s, negative effect of investors acceptable security quality threshold $\tilde{q}_2$ and security price $r_2$. Liquidity premium has consistent impact on all these three factors (referring to results in theoretical and numerical analysis of base model). However, their impact on investors' profit is diverse: bank's retaining level $\beta'$ has positive impact on investors' profit, but quality threshold and security price have negative impact. These contradictory impacts contribute to the convex and concave shape of investors' profit, by their relative importance under different liquidity premium gaps. For example, when liquidity premium is not large, impacts of security price and quality threshold overweight that of bank's retaining choice.

The combining effects of both information gap and liquidity premium gap under the two scenarios are listed in Appendix Figure A1. The "flight to quality" effect of deposit (Acharya and
Yorulmazer (2008)) may grant Bank 1 with stronger balance sheet than Bank 2, and hence obtain lower liquidity premium. Under Scenario 1, Bank 2 have a higher liquidity premium than Bank 1 ($\delta_1=0.015, \delta_2=0.020$), banks offer competitive security prices ($r_{s1}, r_{s2}$), and these rates are quite stable, and hardly affected by the increasing information gap ($\varepsilon$ ranges from 0 to 0.05). On the other hand, the average quality of securitized loan pool decrease, especially for information disadvantaged banks (Bank 2), but Bank 2 could still outperform Bank 1 within a certain range of information indicator range. This could be the case during economic expansion, when it is difficult make implication of the risk level of securities by prices. Moreover, when the economy is not good (Scenario 2), the interest rates could be intertwined, and Bank 2 again has incentive to maintain a certain level of information asymmetry against Bank 1.

5 Discussion

5.1 Impact of Securitization on Mortgage Rate

In our models, we base our securitization market equilibrium on loan market settings such as banks' average loan rate and collateral rate. In this subsection, we discuss the impact of securitization on loan rate, which is similar to Heuson et al. (2001).

Figure 9 shows the equilibrium of loan market with different demand level and availability constraint. When credit demand is relatively low (Inverse Demand, $D$), securitization lowers the loan rate, since the risk level of marginal borrower is relatively low, and banks are willing to securitize it (earn more to securitize than hold). On the contrary, if the credit demand is relatively
high (Inverse Demand, $D'$), bank will reject high risk level loans, and charge a relatively high loan rate, since they cannot earn liquidity premium from these loans which are not qualified for securitization.

Government actions in securitization market could also influence loan market. US government carried out the Federal Reserve's Mortgage-Backed Security (MBS) Purchase Program ($500 billion) in 2008, with the goal to "reduce the cost and increase the availability of credit for the purchase of houses". Hancock and Passmore (2011) empirically claim that this program has significant downward pressure on loan rates. Consistent with their claim, we could treat this program as an increment in supply function (shift down), which lead to lower loan rate, as demonstrated in Figure 9.

5.2 Tranching

Different from literature that focus on the tranching and retention decision of banks, as well as the moral hazard problem between banks and investors, we endogenize bank's proportional retention decision by setting our equilibrium to maximize both bank's and investors' profits (hence the bank could buy its own security as an investor), and we also avoid to consider the signaling effect and moral hazard problem by assuming bank share the same information as investors.

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20 DeMarzo (2005) shows that if assets are not only pooled but also tranche, banks can signal the quality of the sold loan portfolio by retaining interest in the equity tranche. Gorton and Pennacchi (1995) focus on the subsequent adverse selection problem in loan sales, and they show that proportional retention or implicit guarantees can reduce agency problems.
In this subsection, we shortly discuss the tranching equilibrium rather than pooling equilibrium. Further research on this part may provide answers for whether tranching will affect investors' and bank's profits or not, how information asymmetry affect the optimal threshold for each tranche, and also the impacts on volume and average quality of securities.

Suppose the bank plans to tranche the securitization into equity tranche and mezzanine tranche, and there are interest rate spreads of loan and securitization between the two tranches, as shown in Table 3.

**Table 3: Security Tranches**

<table>
<thead>
<tr>
<th>Security</th>
<th>Security price</th>
<th>Tranche thresholds</th>
<th>Mortgage rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity tranche</td>
<td>$r_{s,eq} = r_s + \omega$</td>
<td>$\bar{\beta}_{eq}$</td>
<td>$R_{eq} = R_\sigma + \mu$</td>
</tr>
<tr>
<td>Mezzanine tranche</td>
<td>$r_{s,mez} = r_s$</td>
<td>$\bar{\beta}_{mez}$</td>
<td>$R_{mez} = R_\sigma$</td>
</tr>
</tbody>
</table>

Note that $\bar{\beta}_{eq} < \bar{\beta}_{mez}$, and assume $\omega(\bar{\beta}_{eq}, \bar{\beta}_{mez}) = r_{s,eq} - r_{s,mez} = \omega(\bar{\beta}_{mez} - \bar{\beta}_{eq})$, i.e. function on the threshold difference ($\bar{\beta}_{mez} - \bar{\beta}_{eq}$). Then

$$
\frac{\partial \omega(\bar{\beta}_{eq}, \bar{\beta}_{mez})}{\partial \bar{\beta}_{eq}} = -\omega' < 0, \quad \frac{\partial \omega(\bar{\beta}_{eq}, \bar{\beta}_{mez})}{\partial \bar{\beta}_{mez}} = \omega' > 0.
$$

Suppose loan spread $\mu$ is exogenously determined. Then the securitization contract contains three parameters: $\bar{\beta}_{eq}, \bar{\beta}_{mez}$ and $r_s$. In this scenario, securitization region is $\bar{\beta}_{eq} < \beta \leq \beta' = \frac{r_s + \delta - c}{R_\sigma - c}$, and $\beta_{min} = \frac{r - c}{R_\sigma + \mu - c}$. The problem of maximizing investors' profit is:
\[
\max_{r_s, \bar{\beta}_{eq}, \bar{\beta}_{mez}} \pi_I = N \left\{ \int_{\bar{\beta}_{eq}}^{\bar{\beta}_{mez}} \frac{\bar{\beta}_{mez}}{\sigma} \left[ q R_{eq} + (1 - q)c - r_{eq} \right] f(q) dq + \int_{\bar{\beta}_{mez}}^{\bar{\beta}_{eq}} \left[ q R_{mez} + (1 - q)c - r_{s, mez} \right] f(q) dq \right\}
\]

or equivalently,

\[
\max_{r_s, \bar{\beta}_{eq}, \bar{\beta}_{mez}} \pi_I = N \left\{ \int_{\bar{\beta}_{eq}}^{\bar{\beta}_{mez}} \frac{\beta'}{\sigma} f(q) dq + \int_{\bar{\beta}_{mez}}^{\bar{\beta}_{eq}} \left[ \mu q - \omega(\bar{\beta}_{eq}, \bar{\beta}_{mez}) \right] f(q) dq \right\}
\]

(13)

Then the implicit solutions of the contract parameters \( \bar{\beta}_{eq}, \bar{\beta}_{mez} \) and \( r_s \) are: 21

\[
\int_{\bar{\beta}_{eq}}^{\bar{\beta}_{mez}} f(q) dq = \frac{(r_s - c)(1 - \sigma) + \delta}{\sigma} \frac{1}{f(\bar{\beta}_{mez} - (R_{\sigma} - c))}
\]

\[
\omega' \int_{\bar{\beta}_{eq}}^{\bar{\beta}_{mez}} f(q) dq = \left[ \mu \frac{\bar{\beta}_{mez}}{\sigma} - \omega(\bar{\beta}_{eq}, \bar{\beta}_{mez}) \right] f\left(\frac{\bar{\beta}_{mez}}{\sigma}\right) \frac{1}{\sigma}
\]

\[
\omega' \int_{\bar{\beta}_{eq}}^{\bar{\beta}_{mez}} f(q) dq = \left[ \frac{\bar{\beta}_{eq}}{\sigma} \right] \left( R_{\sigma} - c \right) + c - r_s + \frac{\mu(\bar{\beta}_{eq})}{\sigma} - \omega(\bar{\beta}_{eq}, \bar{\beta}_{mez}) \right] f\left(\frac{\bar{\beta}_{eq}}{\sigma}\right) \frac{1}{\sigma}
\]

Note that, when \( \omega = \mu = 0 \), it degenerate to base model. Due to the complexity of this tranching equilibrium and the length of current paper, we leave the discussion and numerical analysis to further research.

21 Proofs are not reported for concise, and are available upon request.
5.3 Policy Consideration

We consider bank's capital cost as its weighted average cost of capital (WACC) by taking account of both regulatory restrictions on capital-to-asset ratio and bank's required return on equity. Since required return of equity could be greater than the cost of deposit, the bank prefers to fund with deposit if possible, implying that banks will hold the minimum amount of equity capital required by regulators, and

\[ r_{WACC} = (1 - k_0)r + k_0r_e \]

where \( k_0 \) is the regulatory minimum capital-to-asset ratio, \( r_e \) is the minimum expected return to bank equity. On the other hand, if loan risk is transferred to investors through securitization (Passmore et al. (2002)), the minimum capital ratio could be lowered to \( k_1 \) (\( k_1 < k_0 \)).

Now the investors' profit becomes:

\[
\max \pi_{i,WACC} = N \int_{\beta}^{\beta'} \left[ qR_o + (1 - q)c - r_s \right] f(q) dq
\]

where \( \beta' = \frac{r_s + \delta - c + (k_0 - k_1)(r_e - r)}{R_o - c} \). The corresponding optimal solutions are:

\[
\bar{\beta} = \frac{\sigma(r_s - c)}{R_o - c} 
\]

\[
\frac{(r_s - c)(1-\sigma) + (k_0 - k_1)(r_e - r) + \delta}{\sigma^2(R_o - c)} f \left( \frac{\beta'}{\sigma} \right) = F \left( \frac{\beta'}{\sigma} \right) - F \left( \frac{\bar{\beta}}{\sigma} \right)
\]

This is analogue to the base model, except that liquidity premium \( \delta \) is now replaced by \( (k_0 - k_1)(r_e - r) + \delta \), and the opportunity cost of capital is substituted by \( r + k_0(r_e - r) \). All
equilibrium effects of loan rate $R_\sigma$, information indicator $\sigma$, and liquidity premium $\delta$ are parallel to base model summarized in Table 1, with $H_2 = H_{1,W,ACC} = \frac{(r_e-c)(1-\sigma)+(k_0-k_1)(r_e-r)+\delta}{\sigma^2(R_\sigma-c)} f' + \frac{1-\sigma}{\sigma} f^2$.\(^{22}\)

For policy effect analysis, our focus is on the impact of regulation parameters $k_0, k_1$ and the wedge between bank's equity and debt (deposit) cost $(r_e - r)$. It turns out that both $(k_0 - k_1)$ and $(r_e - r)$ enhance all the equilibrium effects, no matter what direction the effects are. Moreover, the negatively impacts of information indicator $\sigma$ on investor's profit is enhanced by the level and square of regulation parameters $k_0, k_1$, and bank's cost wedge $(r_e - r)$. That is, in the presence of information asymmetry, regulatory policy could deteriorate the welfare of securitization buyer. This is consistent with Athanasoglou et al. (2014), which identifies three main channels through which monetary policy affects the banking sector, and state that market imperfections and Basel-type regulations, accounting standards and leverage exacerbate it.

6 Conclusion

The current paper presents a framework wherein information asymmetry in loan market interacts with bank characteristics in determining securitization market equilibrium. We investigate how information asymmetry, liquidity premium and loan market conditions affect securitization market equilibrium in determining investors' decision on security investment, banks' decision on

\(^{22}\) Proofs are not reported for concise, and are available upon request.
loan retention and securitization, as well as profits of both parties. Our numerical analysis serves as supporting and complementary demonstrations for our theoretical analysis.

We consider first market equilibrium with one type of banks, and have some interesting findings. Firstly, although investors' profit can be enhanced with lower information asymmetry level or banks’ conservative estimation, banks may rationally choose in-accurate information to maximize their overall profit from both retained loans and sold securities under certain scenarios. And this situation will be more severe for those banks who can obtain higher liquidity premium from securitization. Secondly, relation between security price and quality differs with the level of information asymmetry. When information asymmetry level increases within certain range, there could be coexistence of decrease in average quality of securitization pool and increase in security price. When information asymmetry is even larger, then market is full of low quality and low price securities. Finally, liquidity premium provides an explanation for bank's aggressive strategy. If the market is in large liquidity needs, and banks could obtain additional liquidity from securitization, then its security marketing strategy would be "lower threshold, lower price", and this strategy could be beneficial for both the bank and investors. This finding is consistent with argument that securitization enhance credit supply and is beneficial for market participants.

Based on these findings, we not only provide explanation for bank's aggressive strategy in securitization market, but also highlight the causal relationship from securitization to information asymmetry. As a supplementary to literature that information asymmetry encourages securitization (DeMarzo and Duffie (1990), Ambrose et al. (2005)), we argue that the existence of securitization also leads to rational information asymmetry choice of banks.
Then we consider market equilibrium with two types of banks which have different characteristics, and analyze the interaction among banks. First, banks may have opposite reverse information incentives. While information advantaged banks want to either keep a large advantage or to be in line with information disadvantaged banks, information disadvantaged banks may prefer to keep a mediate level of information gap. Moreover, investors' profit worsens with more distinct securitization originators, and this effect could be enhanced by higher market share of information disadvantaged banks. Second, different banks' security prices could be misleading in terms of security quality. With increasing information gap, investors may pay higher price but end up with buying low quality securities under certain scenarios. Third, when loan market becomes mostly occupied by information disadvantaged banks, security prices are less informative. Specifically, investors face smaller spread in security prices but larger average security quality difference.

Some other implications of current paper are consistent with existing literatures. For example, regulation policy enhances the impact of information asymmetry on securitization market equilibrium, and thus weakens bank's liquidity creation and deteriorates investors' welfare (Athanasoglou et al. (2014), Blommestein et al. (2011)). Moreover, we show that the information structure of loan market affects the securitization market, in determining securitization standard and prices, and therefore could provide implications for market efficiency and financial stability.

Note that several issues are not discussed in current paper for reasons. Firstly, the liquidity and solvency of banks is not involved, since our focus is on the static equilibrium analysis of
securitization market. And for the same reason, we do not consider other tools for bank to transfer their credit risk, such as credit default swaps (Parlour and Winton (2013). Second issue not discussed is the interbank market. Although Acharya and Richardson (2009) shows by statistic data that most securities are retained or traded in interbank market, we treat this interbank part of securitization trading in the same way as investors buying decision, since investors' profit are maximized in equilibrium. Finally, the tranching equilibriums is left for our further research.

References


Appendix A: Proofs of Theoretical Models

A1: Proofs of Base Model

Proof: Second-order Conditions for Base Model

In base model, first order conditions are:

\[ \bar{\beta} = \frac{\sigma(r_\sigma - c)}{R_\sigma - c} \quad (A1.1) \]

\[ \frac{(r_\sigma - c)(1 - \sigma) + \delta}{\sigma^2(R_\sigma - c)} f \left( \frac{\bar{\beta}}{\sigma} \right) = F \left( \frac{\bar{\beta}}{\sigma} \right) - F \left( \frac{\bar{\beta}}{\sigma} \right) \quad (A1.2) \]

For a maximum to exist, the Hessian matrix must be negative definite. Total differentiation first order conditions leads to:

\[ \frac{\partial^2 \pi_t}{\partial \bar{\beta}^2} = - \frac{N}{\sigma^2} \frac{\bar{\beta}}{R_\sigma} + \left( 1 - \frac{\bar{\beta}}{\sigma} \right) c - r_z f' \left( \frac{\bar{\beta}}{\sigma} \right) - \frac{N}{\sigma^2(R_\sigma - c)} f \left( \frac{\bar{\beta}}{\sigma} \right) \]

\[ = 0 - \frac{N}{\sigma^2} (R_\sigma - c) f \left( \frac{\bar{\beta}}{\sigma} \right) < 0 \text{ always hold.} \]

\[ \frac{\partial^2 \pi_{SPV}}{\partial r_z^2} = \frac{N}{\sigma^2(R_\sigma - c)^2} \left( \frac{r_\sigma - c}{\sigma} \right) \left( 1 - \frac{\bar{\beta}}{\sigma} \right) + \frac{N}{\sigma(R_\sigma - c)} \left( 1 - 2 \right) f' \left( \frac{\bar{\beta}}{\sigma} \right) \]

Then \( \frac{\partial^2 \pi_{SPV}}{\partial r_z^2} < 0 \) requires:

\[ \frac{(r_\sigma - c)(1 - \sigma) + \delta}{\sigma^2(R_\sigma - c)} f'' + \left( \frac{1}{\sigma} - 2 \right) f < 0 \quad (A1.3) \]

Alternatively, substitute equation (A1.1) into (A1.2) eliminates \( \bar{\beta} \) as an explicit choice variable, we could state second-order conditions to the single choice variable \( r_z \). Let

\[ G = \frac{(r_\sigma - c)(1 - \sigma) + \delta}{\sigma^2(R_\sigma - c)} f' \left( \frac{\bar{\beta}}{\sigma} \right) - F \left( \frac{\bar{\beta}}{\sigma} \right) + F \left( \frac{\bar{\beta}}{\sigma} \right) \text{ with } \bar{\beta} = \frac{\sigma(r_\sigma - c)}{R_\sigma - c} \]

It follows that the partial differentiation of \( G \) with respect to \( r_z \) is:

\[ G_{r_z} = \frac{(r_\sigma - c)(1 - \sigma) + \delta}{\sigma^2(R_\sigma - c)^2} f' \left( \frac{\bar{\beta}}{\sigma} \right) + \left( 1 - \frac{\bar{\beta}}{\sigma} \right) f \left( \frac{\bar{\beta}}{\sigma} \right) - \frac{1}{\sigma(R_\sigma - c)} f' \left( \frac{\bar{\beta}}{\sigma} \right) + \frac{1}{(R_\sigma - c)} f \left( \frac{\bar{\beta}}{\sigma} \right) \]

Denote \( f' \) as the first derivative of the pdf evaluated at \( \frac{\bar{\beta}}{\sigma} \), \( f \) as the pdf evaluated at \( \frac{\bar{\beta}}{\sigma} \), \( f^0 \) as the pdf evaluated at \( \frac{\bar{\beta}}{\sigma} \), then we derive another second-order condition:

\[ (R_\sigma - c) G_{r_z} = \frac{1}{\sigma} \left[ \frac{(r_\sigma - c)(1 - \sigma) + \delta}{\sigma^2(R_\sigma - c)^2} f' + \left( \frac{1}{\sigma} - 2 \right) f + f^0 \right] < 0 \quad (A1.4) \]

Specifically, when \( \sigma = 1 \) it becomes:
\[(R_\sigma - c)G_{r_s} = \left(\frac{\delta}{R_\sigma-c}f' - f \right) + f^0 < 0. \]

**Proof:** Equilibrium Analysis of Loan Rate \( R_\sigma \)

Take partial differentiation of first order condition (A1.1) and (A1.2) with respect to loan rate \( R_\sigma \), then straightforward calculation yields:

\[
\frac{\partial r_s}{\partial R_\sigma} = \frac{r_s + \delta - c}{R_\sigma - c} + \frac{1}{G_{r_s}} \frac{\delta}{\sigma^2(R_\sigma - c)}(f - \sigma f^0)
\]

\[
= \frac{r_s - c}{R_\sigma - c} + \frac{1}{G_{r_s}} \frac{\delta}{\sigma^2(R_\sigma - c)^2} \left[ \sigma(R_\sigma - c)G_{r_s} + (f - \sigma f^0) \right]
\]

\[
= \frac{r_s - c}{R_\sigma - c} + \frac{1}{G_{r_s}} \frac{\delta}{\sigma^2(R_\sigma - c)^2} \left[ \frac{(r_s - c)(1-\sigma) + \delta}{\sigma^2(R_\sigma - c)} f' + \frac{1-\sigma}{\sigma} f \right]
\]

where \( G_{r_s} < 0 \) as in second order condition (A1.4). And it is positive if \( \left[ \frac{(r_s - c)(1-\sigma) + \delta}{\sigma^2(R_\sigma - c)} f' + \frac{1-\sigma}{\sigma} f \right] < 0 \), otherwise undeterminate. For notation simplicity, we denote \( H_1 = \left[ \frac{(r_s - c)(1-\sigma) + \delta}{\sigma^2(R_\sigma - c)} f' + \frac{1-\sigma}{\sigma} f \right] \).

Note that when \( \sigma = 1 \), then \((R_\sigma - c)G_{r_s} = (\frac{\delta}{R_\sigma-c}f' - f) + f^0 < 0 \) and \( f' < 0 \), and hence \( \frac{\partial r_s}{\partial R_\sigma} = \frac{r_s - c}{R_\sigma - c} + \frac{1}{(R_\sigma - c)^2 f'} + f^0(\frac{\delta^2}{\sigma^2(R_\sigma - c)^2} f' > 0 \). The intuition is that, when perfect information, banks require higher security price as a response to any increment in loan rate.

Moreover,

\[
\frac{\partial \bar{\sigma}}{\partial R_\sigma} = \frac{\sigma}{R_\sigma - c} \frac{\partial r_s}{\partial R_\sigma} - \frac{\sigma(R_\sigma - c)}{(R_\sigma - c)^2} G_{r_s}
\]

\[
= \frac{\sigma}{(R_\sigma - c)^2 G_{r_s}} \left[ \sigma(R_\sigma - c)G_{r_s} + (f - \sigma f^0) \right]
\]

\[
= \frac{\delta}{(R_\sigma - c)^2 G_{r_s}} \left[ \frac{(r_s - c)(1-\sigma) + \delta}{\sigma^2(R_\sigma - c)} f' + \frac{1-\sigma}{\sigma} f \right]
\]

which has opposite sign as \( H_1 \). Hence, when \( H_1 < 0 \), both securitization standard and security price raise, in response to the increases in loan rate.
Volume of securitization at equilibrium is: \( V = N \int_q \frac{\beta'}{\sigma} f(q) dq \). For the upper bound \( \beta' \), \( \frac{\partial \beta'}{\partial R_\sigma} = \frac{1}{\sigma} \frac{\partial \beta}{\partial R_\sigma} - \frac{\delta}{(R_\sigma - c)^2} \)

\[
\frac{\partial V}{\partial R_\sigma} = N f \left( \frac{\beta'}{\sigma} \right) \frac{1}{\sigma} \frac{\partial \beta'}{\partial R_\sigma} - N f \left( \frac{\tilde{\beta}}{\sigma} \right) \frac{1}{\sigma} \frac{\partial \tilde{\beta}}{\partial R_\sigma} = \frac{N}{\sigma^2} (f - \sigma f^0) \frac{\partial \tilde{\beta}}{\partial R_\sigma} - \frac{N \delta}{\sigma (R_\sigma - c)^2} f
\]

From second order condition (A1.4), we have

\[
\frac{(r_s - c) (1 - \sigma) + \delta}{\sigma^2 (R_s - c)} f' + \frac{1 - \sigma}{\sigma} f < f - \sigma f^0
\]  \hspace{1cm} (A1.5)

Hence, condition \( H_1 < 0 < (f - \sigma f^0) \) will lead to: \( \frac{\partial V}{\partial R_\sigma} < 0 \).  \( \blacksquare \)

Average quality \( Q = N \int_q \frac{\beta'}{\sigma} q f(q) dq \)

\[
\frac{\partial Q}{\partial R_\sigma} = N \frac{\beta'}{\sigma} f \left( \frac{\beta'}{\sigma} \right) \frac{1}{\sigma} \frac{\partial \beta'}{\partial R_\sigma} - N f \left( \frac{\tilde{\beta}}{\sigma} \right) \frac{1}{\sigma} \frac{\partial \tilde{\beta}}{\partial R_\sigma}
\]

\[
= \frac{N}{\sigma^2} \left( f - \tilde{\beta} f^0 \right) \frac{\partial \tilde{\beta}}{\partial R_\sigma} - \frac{N \delta \beta'}{\sigma^2 (R_s - c)^2} f
\]

When \( H_1 > 0 \), then \( f - \sigma f^0 > 0, \frac{\partial \beta'}{\partial R_\sigma} > 0, \frac{\partial \tilde{\beta}}{\partial R_\sigma} < 0, \) and hence \( \frac{\partial Q}{\partial R_\sigma} < 0 \).

**Proof: Equilibrium Analysis of Liquidity Premium \( \delta \)**

For the effect of liquidity premium on equilibrium, since \( \frac{\partial \beta'}{\partial \delta} = \frac{1}{R_\sigma - c} \left( \frac{\partial r_s}{\partial \delta} + 1 \right), \frac{\partial \beta_{\min}}{\partial \delta} = 0, \frac{\partial \beta}{\partial \delta} = \frac{\sigma}{R_\sigma - c} \frac{\partial r_s}{\partial \delta} \), then partially differentiate the first order conditions with respect to \( \delta \) yields:

\[
\frac{(r_s - c) (1 - \sigma) + \delta}{\sigma^2 (R_s - c)} f \left( \frac{\beta'}{\sigma} \right) \frac{1}{\sigma (R_s - c)} \left( \frac{\partial r_s}{\partial \delta} + 1 \right) + \frac{(1 - \sigma)}{\sigma^2 (R_s - c)} f \left( \frac{\tilde{\beta}}{\sigma} \right) \frac{1}{\sigma (R_s - c)} \left( \frac{\partial r_s}{\partial \delta} + 1 \right)
\]

\[
= f \left( \frac{\beta'}{\sigma} \right) \frac{1}{\sigma (R_s - c)} \left( \frac{\partial r_s}{\partial \delta} + 1 \right) - f \left( \frac{\tilde{\beta}}{\sigma} \right) \frac{1}{\sigma (R_s - c) - c} \left( \frac{\partial r_s}{\partial \delta} + 1 \right)
\]

which yields:

\[
\frac{\partial r_s}{\partial \delta} = - \frac{1}{\sigma (R_s - c) G_{r_s}} \left( \frac{(r_s - c) (1 - \sigma) + \delta}{\sigma^2 (R_s - c)} f' + \frac{1 - \sigma}{\sigma} f \right)
\]

It has the same sign as \( H_1 \). Besides, \( \frac{\partial \beta}{\partial \delta} = \frac{\sigma}{R_\sigma - c} \frac{\partial r_s}{\partial \delta} \) indicates that the effects of liquidity premium on equilibrium security price and threshold are consistent.

\[
\frac{\partial V}{\partial \delta} = N f \left( \frac{\beta'}{\sigma} \right) \frac{1}{\sigma \delta} \frac{\partial \beta'}{\partial \delta} - N f \left( \frac{\tilde{\beta}}{\sigma} \right) \frac{1}{\sigma \delta} \frac{\partial \tilde{\beta}}{\partial \delta}
\]
which is positive if $H_1 > 0$. And

$$
\frac{\partial Q}{\partial \delta} = N[\frac{\beta'}{a} R_a + (1 - \frac{\beta'}{a} c - r_s) f(\frac{\beta'}{a}) ] \frac{1}{a} - N[\frac{\beta}{a} R_a + (1 - \frac{\beta}{a} c - r_s) f(\frac{\beta}{a}) ] \frac{1}{a} \frac{\partial \beta}{\partial \delta} + N[\frac{\beta'}{a} R_a + (1 - \frac{\beta'}{a} c - r_s) f(\frac{\beta'}{a}) ] \frac{1}{a} \frac{\partial r_s}{\partial \delta}
$$

When $H_1 > 0$, then $f - \sigma f^0 > 0$, $\frac{\beta'}{a} f - \beta f^0 > 0$, $\frac{\beta}{a} f - \beta f^0 > 0$, and hence $\frac{\partial Q}{\partial \delta} > 0$.

Finally, the impact of liquidity premium on investors profit is always positive.

$$
\frac{\partial \pi_1}{\partial \sigma} = N[\frac{\beta'}{a} R_a + (1 - \frac{\beta'}{a} c - r_s) f(\frac{\beta'}{a}) ] \frac{1}{a} - N[\frac{\beta}{a} R_a + (1 - \frac{\beta}{a} c - r_s) f(\frac{\beta}{a}) ] \frac{1}{a} \frac{\partial \beta}{\partial \sigma} + N[\frac{\beta'}{a} R_a + (1 - \frac{\beta'}{a} c - r_s) f(\frac{\beta'}{a}) ] \frac{1}{a} \frac{\partial r_s}{\partial \sigma} f(q) dq
$$

$$
= N[\frac{\beta'}{a} R_a + (1 - \frac{\beta'}{a} c - r_s) f(\frac{\beta'}{a}) ] \frac{1}{a} - N[\frac{\beta}{a} R_a + (1 - \frac{\beta}{a} c - r_s) f(\frac{\beta}{a}) ] \frac{1}{a} \frac{\partial \beta}{\partial \sigma} + N[\frac{\beta'}{a} R_a + (1 - \frac{\beta'}{a} c - r_s) f(\frac{\beta'}{a}) ] \frac{1}{a} \frac{\partial r_s}{\partial \sigma} f(q) dq
$$

Equilibrium Analysis of Information Indicator $\sigma$

By straight forward calculation and using equilibrium conditions, we obtain:

$$
\frac{\partial \pi_1}{\partial \sigma} = N[\frac{\beta'}{a} R_a + (1 - \frac{\beta'}{a} c - r_s) f(\frac{\beta'}{a}) ] \frac{1}{a} - N[\frac{\beta}{a} R_a + (1 - \frac{\beta}{a} c - r_s) f(\frac{\beta}{a}) ] \frac{1}{a} \frac{\partial \beta}{\partial \sigma} + N[\frac{\beta'}{a} R_a + (1 - \frac{\beta'}{a} c - r_s) f(\frac{\beta'}{a}) ] \frac{1}{a} \frac{\partial r_s}{\partial \sigma}
$$

$$
= 0 - N \frac{1}{a} \frac{\partial \beta}{\partial \sigma} \left[ \frac{\beta'}{a} R_a + (1 - \frac{\beta'}{a} c - r_s) f(\frac{\beta'}{a}) \frac{r_s - c}{a} \frac{\sigma^2}{(R_a - c)} f(q) \right]
$$

$$
= -N \frac{r_s + \sigma - c}{a} \left[ \frac{\beta'}{a} R_a + (1 - \frac{\beta'}{a} c - r_s) f(\frac{\beta'}{a}) \frac{r_s - c}{a} \frac{\sigma^2}{(R_a - c)} f(q) \right] < 0
$$

In order to find $\frac{\partial r_s}{\partial \sigma}$, we firstly have

$$
\frac{\partial \beta}{\partial \sigma} = \frac{r_s - c}{R_a - c} + \frac{\sigma}{R_a - c} \frac{\partial r_s}{\partial \sigma}, \text{ and } \frac{\partial \beta'}{\partial \sigma} = \frac{1}{\sigma} \frac{\partial r_s}{\partial \sigma}
$$

which gives $\frac{\partial \beta'}{\partial \sigma} = \frac{1}{\sigma} \frac{\partial r_s}{\partial \sigma} - \frac{1}{\sigma} \frac{r_s + \sigma - c}{R_a - c}$, and $\frac{\partial \beta}{\partial \sigma} = \frac{1}{\sigma} \frac{\partial r_s}{\partial \sigma}$.

Recall the first order condition of $r_s$ (implicit solution) in (A1.2), differentiate both sides with respect to $\sigma$ gives:
Bank's Perspective

when and Furthermore If Use the notation which leads to

\[
\left(\frac{r_s-c}{\sigma^2(R_s-c)}\right)^{\frac{\delta}{\sigma}} f'\left(\frac{\beta}{\sigma}\right) \left[\frac{1}{\sigma(R_s-c)\partial \sigma} - \frac{1}{\sigma^2} \frac{r_s+c}{R_s-c}\right] + \frac{\left(1-\sigma\right) \frac{\partial r_s}{\partial \sigma} - \left(r_s-c\right)\sigma^2 - 2\left(\frac{r_s-c}{1-\sigma}\right) + \delta\right] \sigma^4(R_s-c) f'\left(\frac{\beta}{\sigma}\right)
\]

\[
= f\left(\frac{\beta}{\sigma}\right) \left[\frac{1}{\sigma(R_s-c)\partial \sigma} - \frac{1}{\sigma^2} \frac{r_s+c}{R_s-c}\right] - f\left(\frac{\beta}{\sigma}\right) \frac{1}{\sigma(R_s-c)\partial \sigma}
\]

which leads to

\[
\left(\frac{r_s-c}{\sigma^2(R_s-c)}\right)^{\frac{\delta}{\sigma}} f'\left(\frac{\beta}{\sigma}\right) + \frac{1}{\sigma^2} (\frac{\beta}{\sigma} - 2) f + \sigma f^0 \right] \frac{\partial r_s}{\partial \sigma}
\]

\[
= \frac{r_s+c}{\sigma^2} \left\{ \left(\frac{r_s-c}{1-\sigma}\right) + \delta \right\} \sigma^4(R_s-c) f'\left(\frac{\beta}{\sigma}\right) + \frac{1}{\sigma^2} f
\]

Use the notation \( H_1 \equiv \frac{r_s-c}{\sigma^2(R_s-c)}\sigma f'\left(\frac{\beta}{\sigma}\right) + \frac{1}{\sigma^2} \frac{\beta}{\sigma} - 1) f \), then

\[
\sigma(R_s-c) \frac{\partial r_s}{\partial \sigma} = \frac{r_s+c}{\sigma^2} \left\{ \left(\frac{r_s-c}{1-\sigma}\right) + \delta \right\} \sigma^4(R_s-c) f'\left(\frac{\beta}{\sigma}\right) + \frac{1}{\sigma^2} f \]

\[
\frac{\partial r_s}{\partial \sigma} = \frac{r_s+c}{\sigma^2} \frac{\sigma(H_1 \equiv \frac{r_s-c}{\sigma^2(R_s-c)}\sigma f'\left(\frac{\beta}{\sigma}\right) + \frac{1}{\sigma^2} \frac{\beta}{\sigma} - 1) f}{\sigma^2 + \frac{\sigma^2}{\sigma^2} \frac{r_s-c}{1-\sigma} f} + \frac{(r_s-c)(1-\sigma) + \delta}{\sigma^2 f}
\]

If \( H_1 > 0 \), then \( \frac{\partial r_s}{\partial \sigma} < 0 \). Moreover, under the same condition that \( \sigma^2 \frac{\partial r_s}{\partial \sigma} > 0 \), we have \( \frac{\partial \varphi}{\partial \sigma} > 0 \), \( \frac{\partial \varphi'}{\partial \sigma} > 0 \), where

\[
\frac{\partial \varphi}{\partial \sigma} = \frac{r_s-c}{\sigma^2} \frac{\partial r_s}{\partial \sigma} \quad \text{and} \quad \frac{\partial \varphi'}{\partial \sigma} = \frac{1}{\sigma} \frac{\partial r_s}{\partial \sigma}
\]

Furthermore, when \( H_1 > 0 \),

\[
\frac{\partial V}{\partial \sigma} = f'\left(\frac{\beta}{\sigma}\right) \left[\frac{1}{\sigma} \frac{\partial r_s}{\partial \sigma} - f'\left(\frac{\beta}{\sigma}\right) \frac{r_s-c}{\sigma(R_s-c)\partial \sigma} + \frac{\sigma}{\sigma^2} \frac{\partial r_s}{\partial \sigma} \right] = \frac{1}{\sigma} \frac{\partial r_s}{\partial \sigma} \frac{r_s-c}{\sigma(R_s-c)\partial \sigma} - \frac{r_s-c}{\sigma(R_s-c)\partial \sigma} f^0 < 0
\]

and for securitization quality: \( Q = \int_{\theta}^{\beta} f(q) dq \),

\[
\frac{\partial Q}{\partial \sigma} = f'\left(\frac{\beta}{\sigma}\right) \left[\frac{1}{\sigma} \frac{\partial r_s}{\partial \sigma} - f'\left(\frac{\beta}{\sigma}\right) \frac{r_s-c}{\sigma(R_s-c)\partial \sigma} + \frac{\sigma}{\sigma^2} \frac{\partial r_s}{\partial \sigma} \right]
\]

\[
= \left[ f'\left(\frac{\beta}{\sigma}\right) - \sigma \frac{\partial f}{\partial \sigma} \left(\frac{\beta}{\sigma}\right) \right] \frac{1}{\sigma} \frac{\partial r_s}{\partial \sigma} - \frac{\partial \varphi}{\partial \sigma} \frac{r_s-c}{\sigma(R_s-c)\partial \sigma} - \frac{r_s-c}{\sigma(R_s-c)\partial \sigma} f^0 < 0
\]

when \( H_1 > 0 \) (then \( f - \sigma f^0 > 0 \), \( \beta f'\left(\frac{\beta}{\sigma}\right) - \sigma \beta f'\left(\frac{\beta}{\sigma}\right) > 0 \), and \( \frac{\partial r_s}{\partial \sigma} < 0 \))

Bank's Perspective
Bank's overall profit with securitization is:

\[
\pi_{\text{bank}} = N \left( \int_{\beta_{\text{min}}/\alpha}^{\beta_{\text{max}}/\alpha} [q_r + 2 \delta - r] f(q) dq + \int_{\beta_{\text{min}}/\alpha}^{1} [q R_\sigma + (1 - q) c - r] f(q) dq \right)
\]

\[
= N \left( \int_{\beta_{\text{min}}/\alpha}^{\beta_{\text{max}}/\alpha} f(q) dq + \int_{\beta_{\text{min}}/\alpha}^{1} [q R_\sigma + (1 - q) c - r] f(q) dq \right) - \pi_1
\]

where \( \beta_{\text{min}} = \frac{\tau_c + \delta - c}{R_\sigma - c}, \beta_{\text{max}} = \frac{\tau_c - c}{R_\sigma - c}, \tilde{\beta} = \frac{\alpha (\tau_c - c)}{R_\sigma - c} \). In order to check bank's incentive in improving information,

\[
\frac{\partial \pi_{\text{bank}}}{\partial \sigma} = N \left( \delta \left( f \left( \frac{\alpha}{\sigma} \right) \frac{\partial f}{\partial \sigma} - f \left( \frac{\tilde{\beta}}{\sigma} \right) \frac{\partial \tilde{\beta}}{\partial \sigma} \right) + \left[ \frac{\beta_{\text{min}}}{\sigma} R_\sigma + (1 - \frac{\beta_{\text{min}}}{\sigma} c - r \right] f \left( \frac{\beta_{\text{min}}}{\sigma} \right) \frac{\beta_{\text{min}}}{\sigma^2} - (1 - \frac{1}{\sigma}) (r_\sigma + \delta - c) \frac{\beta_{\text{min}}}{\sigma^2} \right) \right) - \frac{\partial \pi_1}{\partial \sigma}
\]

When \( H_1 > 0 \), then \( \frac{\partial \pi_1}{\partial \sigma} < 0 \) and \( (f - \sigma f') > 0 \), which lead to \( \frac{\partial \pi_{\text{bank}}}{\partial \sigma} < 0 \). In this case, the bank also has incentive to improve information. In contrast, when \( \sigma = 1 \), then \( H_1 |_{\sigma=1} = -\frac{\delta}{R_\sigma - c} f' < 0 \), and \( \frac{\partial \pi_1}{\partial \sigma} |_{\sigma=1} = -\frac{\delta}{(R_\sigma - c) G_{r_\sigma}} (r_\sigma + \delta - c) f' + f \), then

\[
\left. \frac{\partial \pi_{\text{bank}}}{\partial \sigma} \right|_{\sigma=1} = \frac{N \delta}{R_\sigma - c} (f - \sigma f') \left. \frac{\partial \pi_1}{\sigma} \right|_{\sigma=1} = \frac{N \delta^2 (f - \sigma f')}{(R_\sigma - c)^2 G_{r_\sigma}} (r_\sigma + \delta - c) f' + f \]

where \( (R_\sigma - c) G_{r_\sigma} < 0 \) as required by second order condition. Since when \( \sigma = 1, (f - f') > 0 \), and the bank will have incentive to departure from perfect information if \( \left( \frac{r_\sigma + \delta - c}{R_\sigma - c} f' + f \right) < 0 \). This case is demonstrated in numerical analysis.

**A2: Asymmetry between Bank and Investor**

Suppose bank and investor hold different information accuracy are respectively \( \sigma_b \) and \( \sigma_s \), and specifically, bank has relative better information about loan quality \( \sigma_b < \sigma_s \). Now the profit maximization problem becomes:

\[
\max_{r_s, \beta_s} \pi_1 = N \left( \int_{\beta_s/\alpha}^{\beta_{s,\text{max}}/\alpha} [q R_{s,\sigma} + (1 - q) c - r_s] f(q) dq \right)
\]

where \( \beta_{s,\text{max}} = \frac{r_s + \delta - c}{R_{s,\sigma} - c}, \beta_{s,\text{min}} = \frac{r_s - c}{R_{s,\sigma} - c} \). Assume positive solutions for \( \beta_s \) and \( r_s \), first order conditions yield:
\[
\frac{\partial \pi_{1}}{\partial \beta_{\varepsilon}} = -\frac{N}{\sigma_{\varepsilon}} \frac{\beta_{\varepsilon}}{\sigma_{\varepsilon}} R_{\sigma_{b}} + (1 - \frac{\beta_{b}}{\sigma_{b}}) c - r_{s} f \left( \frac{\beta_{b}}{\sigma_{b}} \right) = 0
\]

and

\[
\frac{\partial \pi_{1}}{\partial r_{s}} = \frac{N}{\sigma_{b} (R_{\sigma_{b}} - c)} \left[ \frac{\beta_{b}}{\sigma_{b}} R_{\sigma_{b}} + (1 - \frac{\beta_{b}}{\sigma_{b}}) c - r_{s} f \left( \frac{\beta_{b}}{\sigma_{b}} \right) - N \int_{\frac{\beta_{b}}{\sigma_{b}}} \frac{\beta_{b}'}{\sigma_{b}} \ f(q) \ dq \right]
\]

which gives:

\[
\beta_{\varepsilon} = \frac{\sigma_{b} (r_{s} - c)}{R_{\sigma_{b}} - c}
\]

(A2.1)

\[
\frac{(r_{s} - c)(1 - \sigma_{b}) + \delta}{\sigma_{b} (R_{\sigma_{b}} - c)} f \left( \frac{\beta_{b}}{\sigma_{b}} \right) = F \left( \frac{\beta_{b}}{\sigma_{b}} \right) - F \left( \frac{\beta_{b}'}{\sigma_{b}} \right)
\]

(A2.2)

(i) Second-order Conditions

Similar as base model, we obtain second-order conditions shown as below:

\[
\frac{1}{\sigma_{b} (R_{\sigma_{b}} - c)} \left[ \frac{(r_{s} - c) (1 - \sigma_{b}) + \delta}{\sigma_{b} (R_{\sigma_{b}} - c)} f' + \left( \frac{1}{\sigma_{b}} - 2 \right) f \right] < 0
\]

(A2.3)

or

\[
(R_{\sigma_{b}} - c) G_{r_{s}} = \frac{1}{\sigma_{b}} \left[ \frac{(r_{s} - c) (1 - \sigma_{b}) + \delta}{\sigma_{b} (R_{\sigma_{b}} - c)} f' + \left( \frac{1}{\sigma_{b}} - 2 \right) f \right] + f^{0} < 0
\]

(A2.4)

where \( f' \) denotes the first derivative of the probability distribution function evaluated at \( \left( \frac{\beta_{b}}{\sigma_{b}} \right) \), \( f \) as the pdf evaluated at \( \left( \frac{\beta_{b}}{\sigma_{b}} \right) \), \( f^{0} \) as the pdf evaluated at \( \left( \frac{\beta_{b}}{\sigma_{b}} \right) \).

(ii) Equilibrium analysis

Differentiate both sides of first order conditions with respect to bank’s information asymmetry indicator \( \sigma_{b} \) gives:

\[
\left[ \frac{(r_{s} - c) (1 - \sigma_{b}) + \delta}{\sigma_{b}^{2} (R_{\sigma_{b}} - c)} f' + \left( \frac{1}{\sigma_{b}} - 2 \right) f + \sigma_{b} f^{0} \right] \frac{\partial \sigma_{b}}{\partial r_{s}} = \frac{r_{s} + \delta - c}{\sigma_{b}} \sigma_{b} \left[ \frac{(r_{s} - c) (1 - \sigma_{b}) + \delta}{\sigma_{b}^{2} (R_{\sigma_{b}} - c)} f' + \left( \frac{1}{\sigma_{b}} - 1 \right) f \right] + \frac{(r_{s} - c) (1 - \sigma_{b}) + \delta}{\sigma_{b}^{2} (R_{\sigma_{b}} - c)} \frac{f}{G_{r_{s}}}
\]

Denote

\[
G_{1} = \frac{(r_{s} - c) (1 - \sigma_{b}) + \delta}{\sigma_{b}^{2} (R_{\sigma_{b}} - c)} f' + \left( \frac{1}{\sigma_{b}} - 1 \right) f,
\]

\[
\frac{\partial r_{s}}{\partial \sigma_{b}} = \frac{r_{s} + \delta - c}{\sigma_{b}} \frac{G_{1}}{\sigma_{b} (R_{\sigma_{b}} - c) G_{r_{s}}} + \frac{(r_{s} - c) (1 - \sigma_{b}) + \delta}{\sigma_{b}^{2} (R_{\sigma_{b}} - c) G_{r_{s}}} \frac{f}{G_{r_{s}}}
\]
Due to second order condition (A2.4), we have 

\[ \frac{\partial^2 V}{\partial \sigma_b^2} = \frac{1}{\sigma_b} \frac{\partial^2 G_1}{\partial \sigma_b^2} \text{ and } \frac{\partial^2 V}{\partial \sigma_a \frac{\partial Z}{\partial \sigma_b}} = \frac{\sigma_a - \sigma_b}{\sigma_b - c} \frac{\partial^2 G_1}{\partial \sigma_a \partial \sigma_b} \text{ have same sign as } \frac{\partial^2 G_1}{\partial \sigma_b^2}. \]

Note that these results are exactly the same as base case, in which investors and banks share the same information.

For investors’ information asymmetry indicator, we take partial derivative of the first order conditions with respect to \( \sigma_z \):

\[
\frac{(r_z - c)(1 - \sigma_b) + \delta}{\sigma_b^2 (R_{\sigma_b} - c)} f' \left( \frac{\beta_b}{\sigma_b} \right) \frac{1}{\sigma_b} \frac{\partial r_z}{\partial \sigma_z} + \frac{(1 - \sigma_b)}{\sigma_b^2 (R_{\sigma_b} - c)} f' \left( \frac{\beta_b}{\sigma_b} \right) \frac{\partial r_s}{\partial \sigma_z} = \frac{1}{\sigma_b^2 (R_{\sigma_b} - c)} f' \left( \frac{\beta_b}{\sigma_b} \right) \frac{\partial r_z}{\partial \sigma_z} - \frac{1}{\sigma_b} \frac{\partial r_z}{\partial \sigma_z} = 0
\]

which leads to

\[
\left( \frac{(r_z - c)(1 - \sigma_b) + \delta}{\sigma_b^2 (R_{\sigma_b} - c)} f' \left( \frac{1}{\sigma_b} \right) f + \sigma_b f' \right) \frac{\partial r_z}{\partial \sigma_z} = 0
\]

that is, \( \frac{\partial r_z}{\partial \sigma_z} = 0 \), indicating that \( r_z \) is not affected by investors' own information accuracy. Moreover,

\[
\frac{\partial \beta'_b}{\partial \sigma_z} = \frac{1}{\sigma_b} \frac{\partial r_z}{\partial \sigma_z} = 0 \quad \frac{\partial \beta'_b}{\partial \sigma_z} = 0 \quad \frac{\partial \beta'_b}{\partial \sigma_z} = \frac{\sigma_z}{\sigma_b} \frac{\partial r_z}{\partial \sigma_z} + \frac{r_z - c}{R_{\sigma_b} - c} \frac{\partial r_z}{\partial \sigma_z} = 0
\]

which are quite intuitive: \( \beta'_b \) is set by bank and not affect by investors' information accuracy; \( \frac{\partial \beta'_b}{\partial \sigma_z} > 0 \) means that investors tend to raise acceptable threshold with less accurate information (\( \sigma_z \), dec., \( \beta_z \), inc.). But in fact, the increment in loan quality is offset be investors’ information disadvantage.

\[
\frac{\partial (\beta'_b \sigma_z)}{\partial \sigma_z} = \frac{\partial r_z - c}{R_{\sigma_b} - c} \frac{\partial \sigma_z}{\partial \sigma_z} = 0
\]

Volume of securitization at equilibrium becomes

\[
V = N \int_{\sigma_b} \beta'_b f(q) dq, \quad \frac{\partial V}{\partial \sigma_b} = N f' \left( \frac{\beta'_b}{\sigma_b} \right) \left[ \frac{1}{\sigma_b} \frac{\partial (r_z - c)}{\partial \sigma_z} + \frac{1}{\sigma_b} \frac{\partial r_z}{\partial \sigma_z} \right] - \frac{1}{\sigma_b} \frac{\partial r_z}{\partial \sigma_z} = 0
\]

Due to second order condition (A2.4), we have \( G_1 < (f - \sigma_b f') \). When \( G_1 > 0 \), then \( \frac{\partial r_z}{\partial \sigma_z} < 0 \), \( (f - \sigma_b f') > 0 \), and hence \( \frac{\partial V}{\partial \sigma_b} < 0 \). However, since \( \frac{\partial r_z}{\partial \sigma_z} = 0 \),
\[ \frac{1}{N} \frac{\partial V}{\partial \sigma_s} = f \left( \frac{\beta'_b}{\sigma_b} \right) \frac{1}{\sigma_b(\sigma_{ab} - c)} \frac{\partial r_s}{\partial \sigma_b} - f \left( \frac{\beta'_s}{\sigma_s} \right) \frac{1}{\sigma_s(\sigma_{ab} - c)} \frac{\partial r_s}{\partial \sigma_s} = 0 \]

indicating that \( V \) is also not affected by investors' information accuracy.

For the effect on investors' profit, we have \( \frac{\partial \pi_i}{\partial \sigma_s} = 0 \) by substituting first order conditions into the expression of \( \frac{\partial \pi_i}{\partial \sigma_s} \)

\[
\frac{\partial \pi_i}{\partial \sigma_s} = N\left[ \frac{\beta'_b}{\sigma_b} R_{ab} + (1 - \frac{\beta'_b}{\sigma_b}) c - r_s \right] f \left( \frac{\beta'_b}{\sigma_b} \right) \frac{\partial \beta'_b}{\partial \sigma_b} - N \frac{\partial r_s}{\partial \sigma_s} \left[ \frac{\beta'_s}{\sigma_s} R_{ab} + (1 - \frac{\beta'_s}{\sigma_s}) c - r_s \right] f \left( \frac{\beta'_s}{\sigma_s} \right) \frac{\partial \beta'_s}{\partial \sigma_s} 
\]

\[
= N\left[ \frac{\beta'_b}{\sigma_b} R_{ab} + (1 - \frac{\beta'_b}{\sigma_b}) c - r_s \right] f \left( \frac{\beta'_b}{\sigma_b} \right) \frac{1}{\sigma_b(\sigma_{ab} - c)} \frac{\partial r_s}{\partial \sigma_b} 
\]

\[= 0 \]

On the contrary, bank's information level \( \sigma_b \) still have impact on investors profit.

\[
\frac{1}{N} \frac{\partial \pi_i}{\partial \sigma_b} = \frac{\beta'_b}{\sigma_b} R_{ab} + (1 - \frac{\beta'_b}{\sigma_b}) c - r_s \frac{\beta'_b}{\sigma_b} \frac{1}{\sigma_b(\sigma_{ab} - c)} \frac{\partial r_s}{\partial \sigma_b} 
\]

\[
= -N\left[ \frac{\beta'_b}{\sigma_b} R_{ab} + (1 - \frac{\beta'_b}{\sigma_b}) c - r_s \right] f \left( \frac{\beta'_b}{\sigma_b} \right) \frac{1}{\sigma_b(\sigma_{ab} - c)} \frac{\partial r_s}{\partial \sigma_b} 
\]

\[
= \frac{r_s + c - \sigma_b}{\sigma_b(\sigma_{ab} - c)} \frac{\partial r_s}{\partial \sigma_b} \left[ f \left( \frac{\beta'_b}{\sigma_b} \right) \right] < 0 
\]

Investors want bank to improve its information, but do not have incentive to improve their own information. In short, security investors rely too much on bank.

If we look into impacts of difference in information indicator, the results are analogous. Set \( \sigma_s - \sigma_b = \varepsilon \), that is, \( \sigma_s = \sigma_b + \varepsilon \), straight forward calculation yields:

\[
\frac{\partial \beta'_b}{\partial \varepsilon} = \frac{1}{\sigma_{ab} - c} \frac{\partial r_s}{\partial \varepsilon} \frac{\partial \beta'_s}{\partial \varepsilon} = \frac{\sigma_s}{\sigma_{ab} - c} \frac{\partial r_s}{\partial \varepsilon} + \frac{r_s - c}{\sigma_{ab} - c}
\]

and

\[
\left[ \frac{(r_s - c)(1 - \sigma_b) + \delta}{\sigma_b^2(\sigma_{ab} - c)} f' \left( \frac{1}{\sigma_b} \right) + \left( \frac{1}{\sigma_b} - 2 \right) f + \sigma_b f' \right] \frac{\partial r_s}{\partial \varepsilon} = 0
\]

that is, \( \frac{\partial r_s}{\partial \varepsilon} = 0 \). Moreover,

\[
\frac{\partial \pi_i}{\partial \varepsilon} = -N\left[ \frac{\beta'_s}{\sigma_s} R_{ab} + (1 - \frac{\beta'_s}{\sigma_s}) c - r_s \right] f \left( \frac{\beta'_s}{\sigma_s} \right) \frac{1}{\sigma_s(\sigma_{ab} - c)} \frac{\partial r_s}{\partial \varepsilon}
\]
A3: Banks with Different Characteristics

The securitization problem in this case is:

$$\max_{\pi_1, \pi_2, q_3} \pi_j = (N - m) \int \frac{\beta'_1}{q_3} [q R_1 + (1 - q)c - r_{x,1}] f(q) dq + m \int \frac{\beta'_2}{q_3} [q R_2 + (1 - q)c - r_{x,2}] f(q) dq$$

where the upper bounds $\beta'_1 = \frac{r_{x,1} + q_3 - c}{R_1 - c}$, $\beta'_2 = \frac{r_{x,2} + q_3 - c}{R_2 - c}$, and the minimum quality for banks to accept loan application are respectively $\beta_{1,\text{min}} = \frac{r - c}{R_1 - c}$ and $\beta_{2,\text{min}} = \frac{r - c}{R_2 - c}$.

First order conditions yields:

$$\frac{\partial \pi_j}{\partial q_3} = -(N - m)[\tilde{q}_3 R_1 + (1 - \tilde{q}_3)c - r_{x,1}] f(\tilde{q}_3) - m[\tilde{q}_3 R_2 + (1 - \tilde{q}_3)c - r_{x,2}] f(\tilde{q}_3) = 0$$

and

$$\frac{\partial \pi_1}{\partial r_{x,1}} = \frac{N - m}{\sigma_1(R_1 - c)} \left( \frac{\beta'_1}{\sigma_1} R_1 + (1 - \frac{\beta'_1}{\sigma_1})c - r_{x,1} \right) f(\frac{\beta'_1}{\sigma_1}) - (N - m) \int \frac{\beta'_1}{q_3} f(q) dq$$

$$\frac{\partial \pi_2}{\partial r_{x,2}} = \frac{m}{\sigma_2(R_2 - c)} \left( \frac{\beta'_2}{\sigma_2} R_2 + (1 - \frac{\beta'_2}{\sigma_2})c - r_{x,2} \right) f(\frac{\beta'_2}{\sigma_2}) - m \int \frac{\beta'_2}{q_3} f(q) dq = 0$$

which in turns yields:

$$\frac{(N - m)(r_{x,1} - c) + m(r_{x,2} - c)}{(N - m)(R_1 - c) + m(R_2 - c)}$$

and

$$\frac{(r_{x,1} - c)(1 - q_3) + q_3}{q_3 (R_1 - c)} f \left( \frac{\beta'_1}{\sigma_1} \right) = F \left( \frac{\beta'_1}{\sigma_1} \right) - F(\tilde{q}_3)$$
\[
\frac{\partial r_{s,2} - c}{\partial \sigma_2} \frac{\partial \sigma_{s,2}}{\partial \sigma_2} f \left( \frac{\beta_i'}{\sigma_i} \right) = F \left( \frac{\beta_i'}{\sigma_i} \right) - F (\tilde{q}_s)
\]  

(A3.3)

And the second order conditions are:

\[
\tilde{g}_1 = \frac{1}{\sigma_1(R_1-c)} \left[ \left( r_{s,1} - (1-\sigma_1) + \frac{\sigma_1}{\sigma_2} \sigma_{s,2} \right) f'_1 + \left( \frac{1}{\sigma_1} - 2 \right) f_1 \right] + \frac{N-m}{(N-m)(R_1-c) + m(R_1-c)} f^0 < 0 \tag{A3.4}
\]

\[
\tilde{g}_2 = \frac{1}{\sigma_2(R_2-c)} \left[ \left( r_{s,2} - (1-\sigma_2) + \frac{\sigma_2}{\sigma_2} \sigma_{s,2} \right) f'_2 + \left( \frac{1}{\sigma_2} - 2 \right) f_2 \right] + \frac{m}{(N-m)(R_1-c) + m(R_1-c)} f^0 < 0 \tag{A3.5}
\]

where \( f'_i \) denotes the first derivative of the probability distribution function \( f(q) \) evaluated at \( \frac{\beta_i'}{\sigma_i} \) for \( i=1,2 \), \( f_i \) is the pdf evaluated at \( \frac{\beta_i'}{\sigma_i} \), \( f^0 \) is the pdf evaluated at \( \tilde{q}_s \).

To check how the differences in liquidity premium and information affect the equilibrium, we list the partial derivatives for convenience:

Liquidity premium: \( \frac{\partial \beta_1'}{\partial \Delta} = \frac{1}{\Delta} \frac{\partial r_{s,1}}{\partial \Delta} \), \( \frac{\partial \beta_2'}{\partial \Delta} = \frac{1}{\Delta} \frac{\partial r_{s,2}}{\partial \Delta} \) + 1

Information difference: \( \frac{\partial \beta_1'}{\partial \xi} = \frac{1}{\Delta} \frac{\partial r_{s,1}}{\partial \xi} \), \( \frac{\partial \beta_2'}{\partial \xi} = \frac{1}{\Delta} \frac{\partial r_{s,2}}{\partial \xi} \)

and

\[
\frac{\partial (\beta_1')}{\partial \Delta} = \frac{1}{\Delta} \frac{\partial r_{s,1}}{\Delta} = \frac{1}{\Delta} \frac{\partial r_{s,1}}{\Delta} + 1
\]

\[
\frac{\partial (\beta_2')}{\partial \Delta} = \frac{1}{\Delta} \frac{\partial r_{s,2}}{\Delta} = \frac{1}{\Delta} \frac{\partial r_{s,2}}{\Delta} + 1
\]

\[
\frac{\partial (\beta_1')}{\partial \xi} = \frac{1}{\Delta} \frac{\partial r_{s,1}}{\Delta} - \frac{1}{\sigma_1} \frac{\partial r_{s,1}}{\Delta} \frac{\partial r_{s,1}}{\Delta} = \frac{1}{\Delta} \frac{\partial r_{s,1}}{\Delta} + 1
\]

\[
\frac{\partial (\beta_2')}{\partial \xi} = \frac{1}{\Delta} \frac{\partial r_{s,2}}{\Delta} - \frac{1}{\sigma_2} \frac{\partial r_{s,2}}{\Delta} \frac{\partial r_{s,2}}{\Delta} = \frac{1}{\Delta} \frac{\partial r_{s,2}}{\Delta} + 1
\]

\[
\frac{\partial q_s}{\partial \xi} = \frac{(N-m) \frac{\partial r_{s,1}}{\Delta} + m \frac{\partial r_{s,2}}{\Delta}}{(N-m)(R_1-c) + m(R_2-c)} \frac{\partial q_s}{\partial \Delta} = \frac{(N-m) \frac{\partial r_{s,2}}{\Delta} + m \frac{\partial r_{s,1}}{\Delta}}{(N-m)(R_1-c) + m(R_2-c)}
\]

Difference in Information Asymmetry level

(i) impact on security prices, threshold and volume
Take partial derivatives of first order condition equations (A3.2) and (A3.3) with respect to \( \varepsilon \) gives:

\[
\frac{(r_{s,1} - c)(1 - \sigma_1) + \delta_1}{\sigma_1^2 (R_1 - c)} f' \left( \frac{\beta_1^*}{\sigma_1} \right) \frac{1}{\sigma_1 (R_1 - c)} \frac{\partial r_{s,1}}{\partial \varepsilon} + \frac{(1 - \sigma_1) \frac{\partial r_{s,1}}{\partial \varepsilon}}{\sigma_1^2 (R_1 - c)} \frac{f' \left( \frac{\beta_1^*}{\sigma_1} \right)}{\sigma_1} = \frac{f \left( \frac{\beta_1^*}{\sigma_1} \right)}{\sigma_1 (R_1 - c)} \frac{1}{\sigma_1} \frac{\partial r_{s,1}}{\partial \varepsilon} - \frac{\partial}{\partial \varepsilon} f(\tilde{q}_s)
\]

and

\[
\frac{(r_{s,2} - c)(1 - \sigma_2) + \delta_2}{\sigma_2^2 (R_2 - c)} f' \left( \frac{\beta_2^*}{\sigma_2} \right) \left[ \frac{1}{\sigma_2 (R_2 - c)} \frac{\partial r_{s,2}}{\partial \varepsilon} - \frac{1}{\sigma_2^2} \frac{r_{s,2} + \delta_2 - c}{R_2 - c} \right] \sigma_2^2 (R_2 - c) = \frac{f \left( \frac{\beta_2^*}{\sigma_2} \right)}{\sigma_2 (R_2 - c)} \frac{1}{\sigma_2} \frac{\partial r_{s,2}}{\partial \varepsilon} - \frac{\partial}{\partial \varepsilon} f(\tilde{q}_s)
\]

Adopt the notations in second order conditions (A3.4) and (A3.5), denote \( \Phi_2 = \frac{(r_{s,2} - c)(1 - \sigma_2) + \delta_2}{\sigma_2^2 (R_2 - c)} \frac{f''}{\sigma_2^2} + 2 \left( \frac{1}{\sigma_2} - 1 \right) f_2 \),

\[
\frac{\partial}{\partial \varepsilon} + \frac{m f^0}{(N - m)(R_1 - c) + m (R_2 - c)} \frac{\partial r_{s,2}}{\partial \varepsilon} = 0
\]

Also denote \( K_1 = \frac{(N - m)(R_1 - c) + m (R_2 - c)}{m} \), \( K_2 = \frac{m}{(N - m)(R_1 - c) + m (R_2 - c)} \), we have

\[
\frac{\partial r_{s,2}}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon} = \frac{1}{\sigma_2^2 (G_1 G_2 - K_1 K_2)} \left[ \beta_2^* \Phi_2 + \delta_2 \right]
\]

Then the spread difference \( r_{s,2} - r_{s,1} \) of the two banks with respect to information asymmetry difference is:

\[
\frac{\partial}{\partial \varepsilon} \frac{r_{s,2} - r_{s,1}}{\partial \varepsilon} = \frac{1}{\sigma_2^2 (G_1 G_2 - K_1 K_2)} \left[ \beta_2^* \Phi_2 + \delta_2 \right]
\]

And

\[
\frac{\partial}{\partial \varepsilon} = \frac{(N - m) \frac{\partial}{\partial \varepsilon}}{(N - m)(R_1 - c) + m (R_2 - c)} + \frac{K_1 \frac{\partial r_{s,1}}{\partial \varepsilon}}{f_1^0} + \frac{K_2 \frac{\partial r_{s,2}}{\partial \varepsilon}}{f_2^0} = \frac{\tilde{q}_s}{\sigma_2^2 (G_1 G_2 - K_1 K_2)} \left[ \beta_2^* \Phi_2 + \delta_2 \right]
\]

where \( \frac{\partial}{\partial \varepsilon} = \frac{1}{\sigma_2 (R_2 - c)} \left( \frac{r_{s,1} - c}{\sigma_1 (R_1 - c)} \right) f_1 + \left( \frac{1}{\sigma_1} - 2 \right) f_1 \).
Volume of securitization at equilibrium is: $V = (N - m) \int \frac{\rho_1'}{\sigma_1} f(q) \, dq + m \int \frac{\rho_2'}{\sigma_2} f(q) \, dq$, then partial derivative with respect to information gap $\varepsilon$ yields:

$$
\frac{\partial V}{\partial \varepsilon} = \frac{f_1}{\sigma_1(R_1 - c)}(N - m) \frac{\partial r_{x,1}}{\partial \varepsilon} + \frac{f_2}{\sigma_2(R_2 - c)}m \frac{\partial r_{x,2}}{\partial \varepsilon} - \frac{Nf^0}{\sigma_1((N - m)(R_1 - c) + m(R_2 - c))} \frac{m}{\sigma_1(R_1 - c)} \left( (r_{x,1} - c)(1 - \sigma_1) + \delta_1 \right) f_1' 
+ \left( \frac{1}{\sigma_1} - 2 \right) f_1 \frac{\partial r_{x,2}}{\partial \varepsilon} - mf_2 \frac{1}{\sigma_2^2} \frac{r_{x,2} + \delta_2 - c}{R_2 - c}
$$

(ii) Impact on Profits and Security Quality

For the effect on investors’ profit, we have:

$$
\frac{\partial \pi_1}{\partial \varepsilon} = (N - m) \left( \frac{\beta_1'}{\sigma_1} R_1 + (1 - \frac{\beta_1'}{\sigma_1})c - r_{x,1} \right) f(q) \, dq + \frac{\beta_2'}{\sigma_2} R_2 + (1 - \frac{\beta_2'}{\sigma_2})c - r_{x,2} f(q) \, dq
- (N - m) \left( \frac{\beta_1'}{\sigma_1} R_1 + (1 - \frac{\beta_1'}{\sigma_1})c - r_{x,1} \right) \frac{\partial q}{\partial \varepsilon} + \frac{\beta_2'}{\sigma_2} R_2 + (1 - \frac{\beta_2'}{\sigma_2})c - r_{x,2} \frac{\partial q}{\partial \varepsilon}
- m \frac{\beta_1'}{\sigma_1} f(q) \, dq
- m \frac{\beta_2'}{\sigma_2} f(q) \, dq
= -m \frac{r_{x,2} + \delta_2 - c}{\sigma_2^2(R_2 - c)} \left( 1 - \sigma_2 \right) + \delta_2 \frac{f_2}{\sigma_2} < 0
$$

The sign $\frac{\partial \pi_1}{\partial \varepsilon} < 0$ shows that, investors’ situation will be deteriorated, with more distinct securitization originators, and this effect tends to enhance with higher $m$.

Banks’ profits are respectively:

$$
\pi_1 = (N - m) \left( \int \frac{\rho_1'}{\sigma_1} [r_{x,1} + \delta_1 - r] f(q) \, dq + \left( \frac{\delta_1}{\rho_{1,\min}} + \left( \frac{1}{\delta_1} \right) q R_1 + (1 - q)c - r \right) f(q) \, dq \right)
= (N - m) \left( \int \frac{\rho_1'}{\sigma_1} [r_{x,1} + \delta_1 - q R_1 - (1 - q)c] f(q) \, dq + \left( \frac{\delta_1}{\rho_{1,\min}} q R_1 + (1 - q)c - r \right) f(q) \, dq \right)
$$
\[
\pi_2 = m \left\{ \int_{\delta_2}^{1/\sigma_2} [r_{s,2} + \delta_2 - r] f(q) dq + \left( f_{\bar{q}_s}^{\beta_1} \right) \int_{\beta_{2,\min}/\sigma_2}^{1} [q R_2 + (1 - q)c - r] f(q) dq \right\}
\]

\[
= m \left\{ \int_{\delta_2}^{1/\sigma_2} [r_{s,2} + \delta_2 - q R_2 - (1 - q)c] f(q) dq + \left( f_{\bar{q}_s}^{\beta_1} \right) \int_{\beta_{2,\min}/\sigma_2}^{1} [q R_2 + (1 - q)c - r] f(q) dq \right\}
\]

Then

\[
\frac{1}{N - m} \frac{\partial \pi_1}{\partial \epsilon} = \left[ r_{s,1} + \delta_1 - \frac{\beta_1^{'}\epsilon_1}{\sigma_1} R_1 - \left( 1 - \frac{\beta_1^{'}\epsilon_1}{\sigma_1} \right) c \right] f \left( \frac{\beta_1^{'}\epsilon_1}{\sigma_1} \right) \frac{\partial \beta_1^{'}\epsilon_1}{\partial \epsilon} + \frac{\partial r_{s,1}}{\partial \epsilon} \int_{\delta_2}^{1/\sigma_2} f(q) dq - \left[ r_{s,1} + \delta_1 - \bar{q}_s R_1 - (1 - \frac{\beta_1^{'}\epsilon_1}{\sigma_1} \right. \]

\[
\frac{1}{N - m} \frac{\partial \pi_2}{\partial \epsilon} = \left[ r_{s,2} + \delta_2 - \frac{\beta_2^{'}\epsilon_2}{\sigma_2} R_2 - \left( 1 - \frac{\beta_2^{'}\epsilon_2}{\sigma_2} \right) c \right] f \left( \frac{\beta_2^{'}\epsilon_2}{\sigma_2} \right) \frac{\partial \beta_2^{'}\epsilon_2}{\partial \epsilon} + \frac{\partial r_{s,2}}{\partial \epsilon} \int_{\delta_2}^{1/\sigma_2} f(q) dq - \left[ r_{s,2} + \delta_2 - \bar{q}_s R_2 - (1 - \frac{\beta_2^{'}\epsilon_2}{\sigma_2} \right. \]

And

\[
\frac{1}{N - m} \left( \frac{\partial \pi_1}{\partial \epsilon} - \frac{\partial \pi_2}{\partial \epsilon} \right) = \delta_1 f \left( \frac{\beta_1^{'}\epsilon_1}{\sigma_1} \right) \frac{1}{\sigma_1 (R_1 - c)} \frac{\partial r_{s,1}}{\partial \epsilon} \left[ r_{s,1} + \delta_1 - c - \bar{q}_s (R_1 - c) \right] f(\bar{q}_s) \frac{\partial \bar{q}_s}{\partial \epsilon}
\]

\[
- \delta_2 f \left( \frac{\beta_2^{'}\epsilon_2}{\sigma_2} \right) \frac{1}{\sigma_2 (R_2 - c)} \frac{\partial r_{s,2}}{\partial \epsilon} \left[ R_2 - c \right] \left( \beta_2^{'}\epsilon_2 - \bar{q}_s \right) f(\bar{q}_s) \frac{\partial \bar{q}_s}{\partial \epsilon}
\]

\[- \left( r_{s,2} + \delta_2 - c \right) \left( 1 - \frac{1}{\sigma_2} \right) f \left( \frac{\beta_2^{'}\epsilon_2}{\sigma_2} \right) \frac{1}{\sigma_2 (R_2 - c)} \frac{\partial r_{s,2}}{\partial \epsilon}
\]

The impact on average quality of securitization pool is

\[
Q = (N - m) \int_{\delta_2}^{1/\sigma_2} q f(q) dq + m \int_{\delta_2}^{1/\sigma_2} q f(q) dq,
\]

then

\[
\frac{\partial Q}{\partial \epsilon} = (N - m) \frac{\beta_1^{'}\epsilon_1}{\sigma_1} f \left( \frac{\beta_1^{'}\epsilon_1}{\sigma_1} \right) \frac{1}{\sigma_1 (R_1 - c)} \frac{\partial r_{s,1}}{\partial \epsilon} (N - m) \bar{q}_s f(\bar{q}_s) \frac{\partial \bar{q}_s}{\partial \epsilon} + m \frac{\beta_2^{'}\epsilon_2}{\sigma_2} \frac{1}{\sigma_2 (R_2 - c)} \frac{\partial r_{s,2}}{\partial \epsilon}
\]

\[
(N - m) \bar{q}_s f(\bar{q}_s) \frac{\partial \bar{q}_s}{\partial \epsilon}
\]
\[
+ m \frac{\beta'_z}{\sigma_z} f \left( \frac{\beta'_z}{\sigma_z} \right) \left( \frac{1}{\sigma_z^2} \frac{\partial r_{z,2}}{\partial \epsilon} + \frac{1}{\sigma_z} \frac{\partial r_{z,2} - \epsilon}{\partial \epsilon} \right) - m \tilde{q}_z f(\tilde{q}_z) 
= \frac{(N-m)f_1 \beta'_z}{\sigma_1(\theta_1 - c)} \frac{\partial r_{z,2}}{\partial \epsilon} + m f_2 \frac{\beta'_z}{\sigma_2} \frac{\partial r_{z,2}}{\partial \epsilon} - m f_2 \frac{1}{\sigma_2^2} \beta'_z \left( \frac{k_1 + k_2}{\sigma_1(\theta_2 - c)} \right) \tilde{q}_z \left( \frac{r_{z,2} - \epsilon}{\theta_2 - c} \right) f' 
+ \left( \frac{1}{\sigma_1} - 2 \right) f_1 \frac{\partial r_{z,2}}{\partial \epsilon}
\]

**Difference in Liquidity Premium: impacts on security prices, threshold and volume**

Differentiate on first order condition equations (A3.2) and (A3.3) with respect to \( \delta \) gives:

\[
\frac{(r_{z,1} - c)(1 - \sigma_1) + \delta_1}{\sigma_1^2(\theta_1 - c)} f' \left( \frac{\beta'_1}{\sigma_1} \right) \frac{1}{\sigma_1(\theta_1 - c)} \frac{\partial r_{z,1}}{\partial \Delta} + \frac{(1 - \sigma_1)}{\sigma_1^2(\theta_1 - c)} f \left( \frac{\beta'_1}{\sigma_1} \right) 
= f \left( \frac{\beta'_1}{\sigma_1} \right) \frac{1}{\sigma_1(\theta_1 - c)} \frac{\partial r_{z,1}}{\partial \Delta} - f(\tilde{q}_z) \frac{(N-m)}{(N-m)(\theta_1 - c) + m(\theta_2 - c)} \frac{\partial r_{z,2}}{\partial \Delta}
\]

and

\[
\frac{(r_{z,2} - c)(1 - \sigma_2) + \delta_2}{\sigma_2^2(\theta_2 - c)} f' \left( \frac{\beta'_2}{\sigma_2} \right) \frac{1}{\sigma_2(\theta_2 - c)} \left( \frac{\partial r_{z,2}}{\partial \Delta} + 1 \right) + \frac{(1 - \sigma_2)}{\sigma_2^2(\theta_1 - c)} f \left( \frac{\beta'_2}{\sigma_2} \right) 
= f \left( \frac{\beta'_2}{\sigma_2} \right) \frac{1}{\sigma_2(\theta_2 - c)} \left( \frac{\partial r_{z,2}}{\partial \Delta} + 1 \right) - f(\tilde{q}_z) \frac{(N-m)}{(N-m)(\theta_1 - c) + m(\theta_2 - c)} \frac{\partial r_{z,2}}{\partial \Delta}
\]

then with the denotations of \( \Psi_z, K_1,K_2 \), we have

\[
\ddot{\tilde{G}}_1 \frac{\partial r_{z,1}}{\partial \Delta} + K_2 \frac{\partial r_{z,2}}{\partial \Delta} = 0
\]

\[
\ddot{\tilde{G}}_2 \frac{\partial r_{z,2}}{\partial \Delta} + K_1 \frac{\partial r_{z,1}}{\partial \Delta} = - \frac{1}{\sigma_2(\theta_2 - c)} \Psi_z
\]

That is,

\[
\frac{\partial r_{z,2}}{\partial \Delta} = - \frac{\ddot{\tilde{G}}_1}{\ddot{\tilde{G}}_1 \ddot{\tilde{G}}_2 - K_1 K_2} \Psi_z
\]

\[
\frac{\partial r_{z,1}}{\partial \Delta} = - \frac{K_2 \ddot{\tilde{G}}_2}{\ddot{\tilde{G}}_1 \ddot{\tilde{G}}_2 - K_1 K_2} \Psi_z
\]

Then

\[
\frac{\partial (r_{z,2} - r_{z,1})}{\partial \Delta} = - \frac{\ddot{\tilde{G}}_1 + K_2}{\ddot{\tilde{G}}_1 \ddot{\tilde{G}}_2 - K_1 K_2} \Psi_z
\]
\[
\frac{\partial \bar{q}_t}{\partial \Delta} = \frac{K_1 \partial r_{s,1}}{f^R} \frac{\partial \bar{r}_{s,2}}{\partial \Delta} + \frac{K_2 \partial r_{s,2}}{f^R} \frac{\partial \bar{r}_{s,1}}{\partial \Delta} = \frac{K_2}{f^R} \frac{\Psi_2}{\bar{g}_1 \bar{g}_2 - K_1 K_2} \left[ \frac{(r_{s,1} - c)(1 - \sigma_1) + \delta_1}{\sigma_1^2 (R_1 - c)} f'_1 + \left( \frac{1}{\sigma_1} - 2 \right) f_1 \right]
\]

And banks' retain levels:

\[
\frac{\partial \beta'_t}{\partial \Delta} = \frac{1}{\bar{g}_1 - c} \frac{\partial r_{s,1}}{\partial \Delta}, \quad \text{which differs from} \quad \frac{\partial r_{s,1}}{\partial \Delta} \quad \text{by a multiplier.}
\]

For the impact on securitization volume,

\[
\frac{\partial V}{\partial \Delta} = (N - m) f \left( \frac{\beta'_1}{\sigma_1} \right) \frac{1}{\sigma_1 (R_1 - c)} \frac{\partial r_{s,2}}{\partial \Delta} + mf \left( \frac{\beta'_2}{\sigma_2} \right) \frac{1}{\sigma_2 (R_2 - c)} \left( \frac{\partial r_{s,1}}{\partial \Delta} + 1 \right)
\]

\[
+ N \frac{K_2}{f^R} \frac{\Psi_2}{\bar{g}_1 \bar{g}_2 - K_1 K_2} \frac{1}{\sigma_1 (R_1 - c)} \left[ \frac{(r_{s,1} - c)(1 - \sigma_1) + \delta_1}{\sigma_1^2 (R_1 - c)} f'_1 + \left( \frac{1}{\sigma_1} - 2 \right) f_1 \right]
\]

\[
= (N - m) f \left( \frac{\beta'_1}{\sigma_1} \right) \frac{1}{\sigma_1 (R_1 - c)} \frac{K_2}{\bar{g}_1 \bar{g}_2 - K_1 K_2} \psi_2 - mf \left( \frac{\beta'_2}{\sigma_2} \right) \frac{1}{\sigma_2 (R_2 - c)} \frac{\delta_1}{\bar{g}_1 \bar{g}_2 - K_1 K_2} \psi_2 + N \frac{K_2}{f^R} \frac{\psi \psi_1}{\bar{g}_1 \bar{g}_2 - K_1 K_2}
\]

\[
+ mf \left( \frac{\beta'_2}{\sigma_2} \right) \frac{1}{\sigma_2 (R_2 - c)} \frac{K_2}{\bar{g}_1 \bar{g}_2 - K_1 K_2}
\]

\[
= \frac{\psi_2}{\bar{g}_1 \bar{g}_2 - K_1 K_2} \left[ (N - m) f_1 \frac{K_2}{\sigma_1 (R_1 - c)} - mf_2 \frac{\delta_1}{\sigma_2 (R_2 - c)} \right] + N \frac{K_2}{f^R} \frac{\psi \psi_1}{\bar{g}_1 \bar{g}_2 - K_1 K_2} + \frac{mf_2}{\sigma_2 (R_2 - c)}
\]

where \( \psi_1 = \frac{1}{\sigma_1 (R_1 - c)} \left[ (r_{s,1} - c)(1 - \sigma_1) + \delta_1 \right] \sigma_1^2 (R_1 - c) f'_1 + \left( \frac{1}{\sigma_1} - 2 \right) f_1 \).
Figure 1: Probability Partitioning of Bank Decision

Figure 2: Shifted Loan Supply Function with Securitization
Figure 3: Equilibrium Effects of Information Indicator $\sigma$ (Base Model)
Panel A: Liquidity premium $\delta=0.015$

Scenario 1:
$R_\sigma = 1.11; c = 0.8; r = 1.05; \delta = 0.015$;

Scenario 2:
$R_\sigma = 1.08; c = 0.9; r = 1.03; \delta = 0.015$;

Panel B: Liquidity premium $\delta=0.020$

Scenario 1:
$R_\sigma = 1.11; c = 0.8; r = 1.05; \delta = 0.020$;

Scenario 2:
$R_\sigma = 1.08; c = 0.9; r = 1.03; \delta = 0.020$;
Panel C: $H_1$ and $(f - \sigma f')$ for Above Scenarios:

Scenario 1:
$R_\sigma = 1.11; c = 0.8; r = 1.05; \delta = 0.015;$

Scenario 2:
$R_\sigma = 1.08; c = 0.9; r = 1.03; \delta = 0.015;$

Note: For the variables: $R_\sigma$ is average loan rate, $c$ is collateral rate; $r$ is a representative bank's cost of capital (or deposit rate); $\delta$ is bank's liquidity premium, $\sigma$ is bank's information indicator. For equilibrium variables: $r_s$ is security price, $\tilde{\beta}$ is security investors’ acceptable threshold of loan repayment probability, $\beta'$ is bank's hold back level, $V$ is security volume, $Q$ is average quality of the securitization pool, $\pi_{bank}$ is profit of bank, and $\pi_i$ is investors' profit.
Figure 4: Equilibrium Effects of Liquidity Premium $\delta$

Panel A: Information indicator $\sigma = 0.99$

Scenario 1:  
$R_\sigma = 1.11; c = 0.8; r = 1.05; \sigma = 0.99$;

Scenario 2:  
$R_\sigma = 1.08; c = 0.9; r = 1.03; \sigma = 0.99$;

Panel B: Information indicator $\sigma = 1.01$

Scenario 1:  
$R_\sigma = 1.11; c = 0.8; r = 1.05; \sigma = 1.01$;

Scenario 2:  
$R_\sigma = 1.08; c = 0.9; r = 1.03; \sigma = 1.01$;
Panel C: $H_1$ and $(f - \sigma f')$ for Above Scenarios:

**Scenario 1:**
\[ R_\sigma = 1.11; c = 0.8; r = 1.05; \sigma = 0.99; \]

**Scenario 2:**
\[ R_\sigma = 1.08; c = 0.9; r = 1.03; \sigma = 0.99; \]

**Scenario 1:**
\[ R_\sigma = 1.11; c = 0.8; r = 1.05; \sigma = 1.01; \]

**Scenario 2:**
\[ R_\sigma = 1.08; c = 0.9; r = 1.03; \sigma = 1.01; \]

**Note:** For the variables: $R_\sigma$ is average loan rate, $c$ is collateral rate; $r$ is a representative bank's cost of capital (or deposit rate); $\delta$ is bank's liquidity premium, $\sigma$ is bank's information indicator. For equilibrium variables: $r_s$ is security price, $\hat{\beta}$ is security investors’ acceptable threshold of loan repayment probability, $\beta'$ is bank's hold back level, $V$ is security volume, $Q$ is average quality of the securitization pool, $\pi_{\text{bank}}$ is profit of bank, and $\pi_I$ is investors' profit.
Figure 5: Equilibrium Effects of Loan Rate $R_\sigma$

Panel A: Information indicator $\sigma = 0.99$
Scenario 1: $c = 0.8; r = 1.05; \delta = 0.020; \sigma = 0.99;$
Scenario 2: $c = 0.9; r = 1.03; \delta = 0.015; \sigma = 0.99;$

Panel A: Information indicator $\sigma = 1.01$
Scenario 1: $c = 0.8; r = 1.05; \delta = 0.020; \sigma = 1.01;$
Scenario 2: $c = 0.9; r = 1.03; \delta = 0.015; \sigma = 1.01;$
Panel C: $H_1$ and $(f - \sigma f')$ for Above Scenarios:

Scenario 1:
$c = 0.8; r = 1.05; \delta = 0.020; \sigma = 0.99$;

Scenario 2:
$c = 0.9; r = 1.03; \delta = 0.015; \sigma = 0.99$;

Note: For the variables: $R_o$ is average loan rate, $c$ is collateral rate; $r$ is a representative bank's cost of capital (or deposit rate); $\delta$ is bank's liquidity premium, $\sigma$ is bank's information indicator. For equilibrium variables: $r_2$ is security price, $\bar{F}$ is security investors’ acceptable threshold of loan repayment probability, $\beta'$ is bank's hold back level, $V$ is security volume, $Q$ is average quality of the securitization pool, $\pi_{\text{bank}}$ is profit of bank, and $\pi_i$ is investors' profit.
Figure 6: Equilibrium Effects of Information Gap $\varepsilon$

Panel A: Information indicator of Bank 1 $\sigma_1 = 1$

Scenario 1:

$R_1 = R_2 = 1.11; \delta_1 = \delta_2 = 0.020; c = 0.8; r = 1.05; N = 10000; m = 0.5*N;$

Scenario 2:

$R_1 = R_2 = 1.08; \delta_1 = \delta_2 = 0.015; c = 0.9; r = 1.03; N = 10000; m = 0.5*N;$
Panel B: Information indicator of Bank 1 $\sigma_1 = 0.97$

Scenario 1:

$R_1 = R_2 = 1.11; \delta_1 = \delta_2 = 0.020; c = 0.8; r = 1.05; N = 10000; m = 0.5*N$

Scenario 2:

$R_1 = R_2 = 1.08; \delta_1 = \delta_2 = 0.015; c = 0.9; r = 1.03; N = 10000; m = 0.5*N$
Panel C: Information indicator of Bank 1 $\sigma_1 = 0.95$

Scenario 1:

$R_1 = R_2 = 1.11; \delta_1 = \delta_2 = 0.020;$
$c = 0.8; r = 1.05; N = 10000; m = 0.5*N;$

Scenario 2:

$R_1 = R_2 = 1.08; \delta_1 = \delta_2 = 0.015;$
$c = 0.9; r = 1.03; N = 10000; m = 0.5*N;$
Panel D: Information indicator of Bank 1 $\sigma_1 = 1.02$

Scenario 1:

$R_1 = R_2 = 1.11; \delta_1 = \delta_2 = 0.020; c = 0.8; r = 1.05; N = 10000; m = 0.5*N;$

Scenario 2:

$R_1 = R_2 = 1.08; \delta_1 = \delta_2 = 0.015; c = 0.9; r = 1.03; N = 10000; m = 0.5*N;$

Note: For the variables: $R_1, R_2$ are average loan rates of Bank 1 and 2 respectively, $\delta_1, \delta_2$ are liquidity premiums, $c$ is collateral rate, $r$ is banks’ cost of capital (or deposit rate), $N$ is the loan market size, and $m$ is the part of loan market that occupied by Bank 2. For equilibrium variables: $r_{s,1}$ and $r_{s,2}$ are security prices of Bank 1 and 2 respectively, $\tilde{q}_s$ is intrinsic securitization threshold, $\beta'_1, \beta'_2$ are banks’ hold back levels, $V$ is security volume, $V_2$ is security volume of Bank 2, $Q_1, Q_2$ are average quality of banks’ securitization pool, $\pi_1, \pi_2$ are profits of banks, and $\pi_I$ is investors’ profit.
Figure 7: Equilibrium Effects of Loan Market Share (compare $m = 0.8 \cdot N$ and $m = 0.5 \cdot N$ for impacts of information gap $\varepsilon$)

Scenario 1: $R_1 = R_2 = 1.11; \delta_1 = \delta_2 = 0.020; c = 0.8; 
\quad r = 1.05; N = 10000; m = 0.8*N or 0.5*N;

Scenario 2: $R_1 = R_2 = 1.08; \delta_1 = \delta_2 = 0.015; c = 0.9; 
\quad r = 1.03; N = 10000; m = 0.8*N or 0.5*N;

Note: For the variables: $R_1, R_2$ are average loan rates of Bank 1 and 2 respectively, $\delta_1, \delta_2$ are liquidity premiums, $c$ is collateral rate, $r$ is banks’ cost of capital (or deposit rate), $N$ is the loan market size, and $m$ is the part of loan market that occupied by Bank 2. For equilibrium variables: $\pi_{1,0.8}, \pi_{1,0.5}$ are investors’ profit for cases for $m = 0.8 \cdot N$ and $m = 0.5 \cdot N, (r_{s,2} - r_{s,1})_{0.8}, (r_{s,2} - r_{s,1})_{0.5}$ are security price between Bank 2 and Bank 1 for $m = 0.8 \cdot N$ and $m = 0.5 \cdot N, \hat{q}_{s,0.8}, \hat{q}_{s,0.5}$ are intrinsic securitization thresholds for the two cases, $(Q_1 - Q_2)_{0.8}, (Q_1 - Q_2)_{0.5}$ are quality difference between Bank 1 and Bank 2 for the two cases, $(\pi_{2,per} - \pi_{1,per})_{0.8}, (\pi_{2,per} - \pi_{1,per})_{0.5}$ are banks’ profit differences (per loan) for the two cases, $(V_2/V)_{0.8}, (V_2/V)_{0.5}$ are securitization market share of Bank 2 for the two cases.
Figure 8: Equilibrium Effects of Liquidity Premium Gap $\Delta$ (Liquidity premium of Bank 1 $\epsilon_1 = 0.015$)

Panel A: Information indicator less than 1

Scenario 1:
$R_1 = R_2 = 1.11; \sigma_1 = \sigma_2 = 0.99;$
$c = 0.8; r = 1.05; N = 10000; m = 0.5*N;$

Scenario 2:
$R_1 = R_2 = 1.08; \sigma_1 = \sigma_2 = 0.99;$
$c = 0.9; r = 1.03; N = 10000; m = 0.5*N;$
Panel B: Information indicator greater than 1 (= 1.01)

Scenario 1:
\[ R_1 = R_2 = 1.11; \sigma_1 = \sigma_2 = 1.01; \]
\[ c = 0.8; r = 1.05; N = 10000; m = 0.5 \times N; \]

Scenario 2:
\[ R_1 = R_2 = 1.08; \sigma_1 = \sigma_2 = 1.01; \]
\[ c = 0.9; r = 1.03; N = 10000; m = 0.5 \times N; \]
Figure 9: Equilibrium of Loan Market with Different Demand Level and Availability Constraint

\[ R_{max} = R - \frac{1 - q}{q} r_c \]
Appendix Figure 1: Robustness Check for Information Gap ε (Differ in both liquidity premium and information)

Scenario 1:
\[ R_1 = R_2 = 1.11; \delta_1 = 0.015; \delta_2 = 0.020; \]
\[ c = 0.8; r = 1.05; N = 10000; m = 0.5*N; \sigma_1 = 1; \]

Scenario 2:
\[ R_1 = R_2 = 1.08; \delta_1 = 0.014; \delta_2 = 0.015; \]
\[ c = 0.9; r = 1.03; N = 10000; m = 0.5*N; \sigma_1 = 1; \]