Abstract

We explore the implications of a theoretical asset pricing model of Hughes et al (2007), and prove that when the number of assets in the economy are finite, both idiosyncratic volatility and the proportion of informed traders should affect a firm's cost of capital in opposing ways. Further, we find that their interaction is critically important in determining the magnitude of these effects. With these effects derived theoretically, we then test them empirically. Empirically, we find that on average idiosyncratic volatility forecasts are priced, and the proportion of informed trading is not priced, in line with previous literature and theory. However, the empirics align with theory, and the interaction of these two variables is crucially important, significantly affecting a firm's cost of capital.

Keywords: Idiosyncratic volatility; Information asymmetry; Informed trading; Cost of Capital
I. Introduction

How does the information environment affect expected returns? How does idiosyncratic volatility affect expected returns? The answers to these questions have important ramifications to the determination of a firm’s cost of capital, but the answers provided by the literature continue to exhibit significant ambiguity. Building on Merton (1984) and Easley and O’Hara (2004), Hughes et al (2007, hereafter HLL) present a theoretical model that demonstrates the effects of these two variables in the determination of a firm’s cost of capital. We explore this model in greater detail and find theoretically that the proportion of informed trading and the magnitude of idiosyncratic volatility may affect expected returns when investors are limited to a finite number of assets. These effects diminish as the number of assets available to investors increases. Crucially, we find that these effects interact in important ways to affect the cost of capital – an effect unexplored thus far in the empirical literature.

Once we have determined the implications of theory for these variables, we estimate their cross-sectional relationship with expected returns. On average, a higher proportion of informed trading has little effect on expected returns, in line with Duarte and Young (2009), and the infinite-asset case of HLL. However, the actual sensitivity of expected returns to the proportion of informed trading depends importantly on forecasts of the firm’s idiosyncratic volatility. When idiosyncratic volatility is forecasted to be low, a higher proportion of informed trading can significantly decrease the cost of capital.

We find that idiosyncratic volatility forecasts are positively priced on average, consistent with the theoretical results of Merton (1984). However, the magnitude of this positivity is determined decisively by the proportion of informed trading. The intuition of this is straightforward. Increased trading by informed traders resolves a degree of informational uncertainty, as some of
the private information is discerned by the uninformed. The resolution of this uncertainty should lower expected returns; the magnitude of this reduction is an empirical question. However, idiosyncratic volatility makes the resolution of the uncertainty more difficult, so stocks with higher idiosyncratic volatility require a higher return. The effect of idiosyncratic volatility on the cost of capital is present as an interaction with the proportion of informed traders. We estimate that in the extreme case, if there are no informed traders, idiosyncratic volatility has a negligible association with expected returns. By comparison, when the proportion of informed trading is “typical,” (when our measure of informed trading is near it’s mean), a one standard deviation increase in our forecast of idiosyncratic volatility for a firm is associated with an expected return increase of about 260 basis points annually. These findings are important, as our research links two disparate literatures that have a direct bearing on the determinants of firms’ cost of capital – and the link itself is important to the understanding of the relationship.

Easley and O’Hara (2004) develop a theoretical framework to model the firm’s cost of capital in an imperfect information environment. They find that greater the fraction of traders who receive private information, the lower the cost of capital to the firm (equation 21, p. 1572). The intuition behind this general equilibrium finding is that the more traders that are trading on private information, the more of this private information that makes its way into the public arena via the resultant informative trades. As the risk to the uninformed trader is resolved by these trades, the required return necessary to compensate for the private information risk is lower.

Merton (1984) develops a theoretical framework in which limited access to market securities by some participants gives rise to a positive relationship between idiosyncratic risk and expected return. If some market participants are unable to hold the market portfolio, then idiosyncratic volatility is a risk that cannot be fully diversified. Therefore, market participants demand
compensation in the form of a higher expected return in order to bear idiosyncratic volatility (equation 31, p. 496). This theoretical finding takes on further relevance in light of recent empirical findings such as Bennett and Sias (2011) who demonstrate that investors appear unable to hold fully diversified portfolios.

However, it is logical that the Merton findings and the Easley and O’Hara findings should be interdependent. The resolution of private information from trading reduces the expected return in the Easley and O’Hara (2004) model because the market participants are able to extract information from the trades. Higher idiosyncratic volatility will introduce noise into this process and reduce the informativeness of the trades made by those acting on private information. Higher idiosyncratic risk should mitigate the reduction in the cost of capital brought about by increased private trading. We demonstrate this finding theoretically, and then show that it holds empirically as well.

Duarte and Young (2009) find that lagged AdjPIN (the “adjusted” probability of informed trading), is not related to expected return. However, they do not pursue the potential effect of idiosyncratic volatility on this finding. We find that, while AdjPIN is minimally related to expected returns on average, its interactions with idiosyncratic volatility forecasts are positively related to expected returns, in line with theory.

The determination of the influences of the information environment on the cost of capital is fundamental to finance. Naturally, it is an active area. Armstrong et al (2010) find that as the number of shareholders of a particular equity increase, the cost of capital decreases, attributing this to market competition for information. Akins, Ng and Verdi (2012) find that greater competition among informed investors decreases the pricing effect of asymmetric information. Barron and Qu (2014) find in an experimental framework that high quality information increases
competition between informed traders, and reduces the price discount of an asset. Bloomfield and Fischer (2011) present a theoretical model in which they model disagreement among investors about information and how this may affect the cost of capital. Our research fills an important void in this literature, by demonstrating both theoretically and empirically that the proportion of informed traders and idiosyncratic volatility interact in order to affect the cost of capital.

This paper proceeds as follows. In section II, we discuss the interaction between the proportion of informed trading and idiosyncratic volatility through the theoretical model of HLL and some of its previously unexplored implications. We further discuss the HLL empirical implications in section III. In sections IV through VI, we present our sample and test empirically the implications of the HLL model. We find that the effects of informed trading and idiosyncratic volatility align closely with that predicted by theory. In section VII we conclude.

II. The Hughes et al (2007) Model of Asset Prices and its Implications

HLL derive a coherent theoretical framework in which the effects of “asymmetric information” on expected returns may be examined. The definition of asymmetric information in the literature has been nebulous at times, but the HLL model has the merit of a very specific definition. In their model, following Easley and O’Hara (2004), there are two classes of investors: informed and uninformed. Informed investors receive a private signal \( s \) about the price of the asset. Uninformed investors receive no signal, but can infer the signal with error from the equilibrium price. Proportion \( \mu \) of traders receive the private signal. This parameter \( \mu \) governs the degree of dispersion of the private information.
Because of its tractable analytics and valuable insight, we construct our theoretical analysis around the HLL model. The HLL model is a two-period model. There are $N$ risky assets and $M$ investors. Today investors choose portfolios and have initial wealth of $W_0$, and tomorrow assets in these portfolios payoff. It is assumed that asset payoffs ($v$) and random supply of assets ($x$) are generated by factor structures as

$$v = \bar{v} + \beta F + \Sigma^{1/2} \varepsilon$$  \hspace{1cm} (1)$$

$$x = \bar{x} + \beta_x F_x + \Sigma_x^{1/2} \eta_x$$ \hspace{1cm} (2)

where $\bar{v}$ and $\bar{x}$ are $N \times 1$ constant vectors, the factor $F$ is $K \times 1$ vector of mean zero normal random variables with covariance matrix $\Sigma_F$. $F_x$ is a $K_x \times 1$ vector of mean zero random variables that are not necessarily the same as $F$, with a covariance matrix $\Sigma_{F_x}$. $\beta$ and $\beta_x$ are respectively $N \times K$ and $N \times K_x$ constant factor loadings. The idiosyncratic component $\varepsilon$ is a vector of standard normal random variables, and $\Sigma$ is an $N \times N$ diagonal matrix of idiosyncratic asset payoff variances. There is an idiosyncratic supply component $\eta_x$ in the random supply, which is a $N \times 1$ standard normal random vector. $\Sigma_x$ is an $N \times N$ diagonal matrix of idiosyncratic supply shock variances.

Following HLL, and denoting time 0 and time 1 wealth as $W_0$ and $W_1$ respectively, assume all investors have the following CARA utility function:

$$U = -E[\exp(-AW_1)].$$ \hspace{1cm} (3)

Let $R_f$ be the risk free rate and $p$ be the vector of security prices. Let $D$ be an $N \times 1$ vector containing the number of shares invested in risky assets and define $\bar{M}$ as the investment in the risk free asset. Noting that $W_0 = pD + \bar{M}$ and $W_1 = R_f \bar{M} + Dv$, the agent will maximize utility subject to the following budget constraint:

$$W_1 = W_0 R_f + D (v - R_f p).$$ \hspace{1cm} (4)
With this standard set of preferences, investors’ objective function has a typical mean-variance expression, and investors choose $D$ given information set $J$ to solve

$$\max_D E(W_1 | J) - \frac{A}{2} \text{var}(W_1 | J)$$

which implies the optimal demand from the first order condition of the following form:

$$D_j^* = \frac{1}{A} \Sigma^{-1}_{v|J} E(v - R_f p | J)$$ (5)

Informed investors receive private signal $s$, which takes the form

$$s = (v - \bar{v} - \beta F) + bF + \Sigma_s^{1/2}\eta = \Sigma^{1/2}\varepsilon + bF + \Sigma_s^{1/2}\eta$$ (6)

where $F, \varepsilon, \eta, \eta_x$ are jointly normal and independent.\(^1\) The covariance matrix of $s$, $\Sigma_s$ is diagonal. Informed investors form expectation of asset payoffs conditional on $s$ as:\(^2\)

$$E(v | s) = \bar{v} + \Sigma_{v|s,F}^{-1}s + (\beta - \Sigma_{v|s,F}^{-1} b) \Sigma_{F|s}^{-1} b'(\Sigma + \Sigma_s)'s.$$ (7)

And the expectation of the variance of payoff $v$ conditional on $s$ is:

$$\Sigma_{v|s} = \Sigma_{v|s,F} + (\beta - \Sigma_{v|s,F}^{-1} b) \Sigma_{F|s}^{-1} (\beta - \Sigma_{v|s,F}^{-1} b)'.$$ (8)

Uninformed investors do not observe signal $s$ but can imperfectly infer $s$ from the equilibrium price. This noisy signal is $\theta$.

The equilibrium price is conjectured to be the following form:

$$p = C + B(s - \lambda(x - \bar{x}))$$ (9)

where $C$ is an $N \times 1$ vector and $B$ and $\lambda$ are $N \times N$ matrices. Uninformed investors observe $p$ and can calculate $\theta$ as

$$\theta = B^{-1}(p - C) = s - \lambda(x - \bar{x}).$$ (10)

The inferred signal $\theta$ is less informative than signal $s$, as we can see by its conditional covariance matrix, $\Sigma_\theta$:

\(^1\) Note that these are the realizations of $\varepsilon$ and $F$. therefore $V(s) = \Sigma_s$

\(^2\) Derivations for equations (7) and (8) can be found in Hughes et al (2007).
\[ \Sigma_\theta = \Sigma_S + \lambda (\beta_x \Sigma_{F|x} \beta_x' + \Sigma_x) \lambda'. \] (11)

After forming the moment conditions with joint density functions of \( \theta, \nu, F \), we get the expectations of \( \nu \) by uninformed traders conditional on \( \theta \) and its conditional variance, as

\[ E(\nu|\theta) = \bar{\nu} + \Sigma_{\nu|\theta,F} \Sigma_\theta^{-1} \theta + (\beta - \Sigma_{\nu|\theta,F} \Sigma_\theta^{-1} b) \Sigma_{F|\theta} b' (\Sigma + \Sigma_\theta) \theta \] (12)

\[ \Sigma_{\nu|\theta} = \Sigma_{\nu|\theta,F} + (\beta - \Sigma_{\nu|\theta,F} \Sigma_\theta^{-1} b) \Sigma_{F|\theta}(\beta - \Sigma_{\nu|\theta,F} \Sigma_\theta^{-1} b)'. \] (13)

Substituting equations 7 and 8 into equation 5 yields informed investors demand function (DI). Similarly, substituting equations 12 and 13 into equation 5 yields the uninformed investors demand function (DU). \( \mu \) is the proportion of informed investors and impose the market clearing condition that total demand from the informed and the uninformed investors equals the supply gives

\[ x = M[\mu \times DI + (1 - \mu) \times DU]. \] (14)

For a large economy in the HLL model, the number of investors expand at the same rate as the number of assets \((N/M = 1)\) as they increase to infinity. Under these conditions, they derive the expected risk premium of an asset to be

\[ E(\bar{\nu} - R_F p) = \beta A (\mu \Sigma_{F|s}^{-1} + (1 - \mu) \Sigma_{F|\theta}^{-1})^{-1} \frac{\beta x}{N} \] (15)

where conditional covariance of \( F \) conditional on \( s \) or \( \theta \) has been proven in HLL to be

\[ \Sigma_{F|s}^{-1} = \Sigma_F^{-1} + b' (\Sigma + \Sigma_s)^{-1} b \] (16)

\[ \Sigma_{F|\theta}^{-1} = \Sigma_F^{-1} + b' (\Sigma + \Sigma_\theta)^{-1} b \] (17)

Equation 15 relates the expected return of an asset to the proportion of informed traders \((\mu)\) and idiosyncratic volatility \((\Sigma)\), through the return premium.

**Proposition 1:**
Proof: See Appendix A.1.

The risk premium for assets, and thus expected returns, are decreasing in $\mu$. The logic of this is straightforward. Uninformed investors face information risk for holding assets, and demand return compensation to bear this risk. However, they are able to infer additional information by the trading activity of informed investors, reducing their information risk. The reduction of information risk provided by increased $\mu$ corresponds to a reduced required return on the part of uninformed investors. A very similar result may be seen in Easley and O’Hara (2004).\(^3\)

The risk premium for assets is increasing in idiosyncratic volatility, $\Sigma$. Again, the logic is straightforward. With a finite number of assets, perfect diversification cannot be achieved. Therefore, investors must bear idiosyncratic risk, and require increased expected return to compensate them for this.

In the case of either proposition, an increasing number of assets reduce the magnitude of their effect. As $N \rightarrow \infty$, both derivatives converge to zero. Are there enough assets to render these derivatives equal to zero in applied situations? This is an empirical question, and we explore that possibility in detail in section III. However, empirical investigations of both of these claims exist in the literature. For example, Easley et al (2002) find that $PIN$ (the probability of informed trading) is positively priced. However, Duarte and Young (2009) introduce $AdjPIN$, which is $PIN$ adjusted to eliminate liquidity effects, and find that it is unrelated to expected returns.\(^4\)

\(^4\) This is but two of many studies that look at the empirical association between expected returns and some measure of asymmetric information.
Proposition 2: \[
\frac{\partial E(u - R_f p)}{\partial \mu \partial \Sigma} > 0
\]

Proof: See Appendix A.2.

This proposition is yet to be explored in the literature either theoretically or empirically. The logic behind the finding is compelling. Proposition 1a holds because increased trading in private information allows uninformed investors to discern the private signals of the informed with error. Since they are able to resolve some of the information risk they face, the risk premium they require for holding the asset is reduced. However, idiosyncratic volatility creates noise for the uninformed investors’ attempts to discern the informed traders’ signal. Proposition 2 arises because investors demand a greater risk premium than otherwise to compensate them for the increased information risk, since the informed traders’ trades are less informative as idiosyncratic volatility rises. This cross-partial derivative may have notable empirical importance. For example, for some parameterizations of the model, idiosyncratic volatility may be only priced to a slight degree (Proposition 1b), but may affect the relationship between expected returns and the proportion of informed traders greatly.

This cross-partial derivative also gives rise to a subtler effect that may be driving the positive relationship between expected returns and idiosyncratic volatility. Idiosyncratic volatility interferes with the quality of the signal received by the informed investors, and deduced by the uninformed. This can be seen, for example, in equations 16 and 17, in which we see that the covariance matrices of the systematic factors are increasing in \( \Sigma \) for both the informed and uninformed. It is quite possible that there exist a sufficiently large number of assets in the economy such that imperfect diversification is “good enough” that idiosyncratic volatility would hardly increase the expected return to assets. It seems likely that the United States stock market would provide enough assets that this would be the case. However, when the proportion of
informed trading is high, this distortion in the perception of the signal may give rise to positively priced idiosyncratic volatility unrelated to diversification difficulties. We will revisit this possibility shortly when we discuss our empirical findings.

III. Empirical Implications

Although classical finance theory suggests that idiosyncratic volatility should not have an association with expected returns, empirical results to the contrary abound in the literature, though with often conflicting results. Tinic and West (1986) find a positive relationship between market returns and idiosyncratic volatility. Malkiel and Xu (1997) find evidence of a positive expected return-idiosyncratic volatility relationship. Spiegel and Wang (2005) and Fu (2009) also both find a positive association between expected returns and idiosyncratic volatility. Ang et al (2006) finds that lagged idiosyncratic volatility is negatively related to expected returns, while Huang et al (2010) and Han and Lesmond (2011) find that this unusual negativity may be explained by either return reversals or liquidity factors, respectively.

The exploration of the association between information and expected returns has yielded varying results as well. Easley et al (2002) show that stocks with higher PINs have higher stock returns. Duarte and Young (2009) argue that this result arises primarily from liquidity factors with which PIN is positively correlated.\(^5\) They construct AdjPIN to eliminate the effects of liquidity on PIN and find that lagged AdjPIN is unrelated to expected returns. Mohanram and Rajgopal (2009) similarly find that PIN is not a reliable indicator of future returns. Easley et al (2010) find that trading strategies involving PIN can yield abnormal profits for traders. Kelly

\(^5\) Several papers find liquidity to be inversely related to asset returns. See for example Amihud (2002) and Pastor and Stambaugh (2003).
and Ljungqvist (2012) find using natural experiments that information asymmetry is positively
priced, and linked to expected returns through the channel of liquidity.

Much of the research into the relationship between expected return and idiosyncratic volatility
has focused on the behavior of lagged volatility and subsequent returns (see, notably Ang et al
(2006, 2009)). While these relationships are of interest, our focus here is different. In the
Hughes et al (2007) model that we extend, the agents in the model are fully aware of all model
parameters, similar to Merton (1987) and Easley and O’Hara (2002). It is therefore appropriate
to use the best estimate of idiosyncratic volatility in the construction of our relationships.\(^6\) We
therefore construct idiosyncratic volatility forecasts to be our measures of idiosyncratic volatility.

Ang et al. (2006, 2009) propose an estimate for firm-level monthly idiosyncratic volatility from the daily return observations in that month. Specifically, they estimate a linear
factor model of the form

\[
R_{it} - r_{f\tau} = \alpha_i + \sum_{j=1}^{M} \beta_{ij} F_{j\tau} + \epsilon_{i\tau}
\]

for asset \(i\) is estimated for month \(t\). Trading days in month \(t\) are indexed by \(\tau\), \(\beta_{ij}\) are the
sensitivities of firm \(i\) to the \(M\) factors indexed by \(j\). \(R_{i\tau}\) is the return to asset \(i\) during day \(\tau\). \(r_{f\tau}\) is
the daily risk free rate. The idiosyncratic volatility for month \(t\) is then defined as \(\hat{\sigma}_{it}\), the sample
standard deviation of the residuals in the month. Ang et al. (2006, 2009) use the lagged estimates as the volatility forecasts. Since there exists evidence that volatility exhibits some
form of autocorrelation, inefficient estimates result. Alternatively, if we assume that the
volatility series does not follow a random walk, we may model \(\sigma_{it}\) using an autoregressive

\(^6\) In this sense, our study is in the vein of Fink et al (2012), who claim that “Use of the full information set may be
appropriate when testing theoretical models such as Merton (1987) and Malkiel and Xu (2002), as their models
assume that all parameters in the model are known.”
process to create out-of-sample forecasts. If we construct a sample of estimates of \( \hat{\sigma}_{it} \), then we may fit an ARMA model of the form

\[
\hat{\sigma}_{it} = a_0 + \sum_{m=1}^{p} a_m \hat{\sigma}_{i,t-m} + \sum_{n=1}^{q} b_n e_{i,t-n} + e_{it}
\]  

(19)

where \( e_{it} \) is the time series error for the idiosyncratic volatility of asset \( i \) in month \( t \). We may form a time \( t-1 \) forecast of idiosyncratic volatility for one month ahead as

\[
\hat{\sigma}_{i,t} | \Phi_{t-1} = \hat{a}_0 + \sum_{m=1}^{p} \hat{a}_m \hat{\sigma}_{i,t-m} + \sum_{n=1}^{q} \hat{b}_n \hat{e}_{i,t-n}.
\]  

(20)

Finally, we also estimate firm-level monthly idiosyncratic volatility using a GARCH-type model over an expanding time window using monthly data. Consider an EGARCH (p,q) model of the following form:

\[
R_{it} - r_{it} = \alpha_i + \sum_{k=1}^{K} \beta_{ik} F_{kt} + \varepsilon_{it}
\]

\[
\varepsilon_{it} \sim N(0, \sigma_{it}^2)
\]  

(21)

\[
\ln(\sigma_{it}^2) = a_i + \sum_{m=1}^{p} b_{i,m} \ln(\sigma_{i,t-m}^2) + \sum_{n=1}^{q} c_{i,n} \left[ \theta \left( \frac{\varepsilon_{i,t-n}}{\sigma_{i,t-n}} \right) + \gamma \left[ \frac{\varepsilon_{i,t-n}}{\sigma_{i,t-n}} - \left( \frac{2}{\pi} \right)^{\frac{1}{2}} \right] \right] + \varepsilon_{i,t-n}.
\]

(22)

\( R_{it} \) is the return to asset \( i \) in month \( t \in \{1, \ldots, T\} \), \( r_{it} \) is the risk free rate in month \( t \). \( \beta_{ik} \) are the sensitivities of firm \( i \) to the \( K \) monthly factors indexed by \( k \). For a particular choice of \( p \) and \( q \), we may estimate a sequence of in-sample monthly idiosyncratic volatility forecasts, or use the estimated parameters to forecast out-of-sample.

Expected idiosyncratic volatility may be estimated by a forecast from the EGARCH (p,q) model in Eq. (21) using

\[
\hat{\sigma}_{i,t}^2 | \Phi_{t-1} = \exp \left[ \hat{a}_i + \sum_{m=1}^{p} \hat{b}_{i,m} \ln(\hat{\sigma}_{i,t-m}^2) + \sum_{n=1}^{q} \hat{c}_{i,n} \left[ \theta \left( \frac{\hat{\varepsilon}_{i,t-n}}{\hat{\sigma}_{i,t-n}} \right) + \gamma \left[ \frac{\hat{\varepsilon}_{i,t-n}}{\hat{\sigma}_{i,t-n}} - \left( \frac{2}{\pi} \right)^{\frac{1}{2}} \right] \right] \right].
\]  

(22)
where $\Phi$ is the information set up to $t-1$.

IV. Data

Our proxy for percentage of informed investors is $\text{AdjPIN}$ introduced by Duarte and Young (2009). $\text{AdjPIN}$ is the estimated probability of informed trading excluding the liquidity effect. The series includes NYSE and AMEX firms traded from 1983 to 2004.$^7$ In an effort to minimize the known microstructural issues with smaller stocks, we limit our data set to NYSE stocks, and further exclude equities with share prices less than $5$. We collect the return data from CRSP, and the firm accounting variables from Compustat. We winsorize asset returns at the .5% level. Assuming stable probability of informed trading over the year, we map annual $\text{AdjPIN}$ to monthly idiosyncratic volatilities and obtain a sample with 269,552 firm-month observations.

We incorporate several control variables that have been demonstrated to correlate with expected returns in previous literature. We include the systematic risk betas from the Fama-French model and denote the market beta, small-minus-big beta and high-minus-low beta as $\text{MKTBETA}$, $\text{SMBBETA}$, and $\text{HMLBETA}$, respectively. We measure a firm’s size by the lagged market value of equity ($\text{ME}$) at the end of the fiscal year as in Fama and French (1992), which equals monthly price per share at closing multiplied by the shares outstanding in June. We construct Book to market ratio ($\text{BEME}$) as the fiscal year-end book value of common equity over the calendar year end market equity (December). Book equity equals the total common equity plus investment tax credit and deferred taxes, minus preferred stock liquidation value (or redemption value). The liquidity measure we use is the historical liquidity beta as in Pástor and Stambaugh (2003), $\text{LIQBETA}$. We also include variables to control for momentum and asset

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$^7$ We thank Jefferson Duarte for making the $\text{AdjPIN}$ series available online. We restrict ourselves up to 2004 due to the data constraint about the availability of $\text{AdjPIN}$.
growth effects. Momentum factor $\textit{CRETURN}$ is represented by the cumulative returns from month (t-7) to (t-2). Firm asset growth ($\textit{AG}$) was shown by Cooper et al (2008) to be strongly related to equity returns. We define this variable following that work as the percentage change of total assets. $\textit{ME}$, $\textit{BEME}$ and $\textit{AG}$ are all winsorized at .5% and 99.5% level, and reported as natural logs. We denote these log transformations as $\textit{LNME}$, $\textit{LNBEME}$ and $\textit{LNAG}$. Table I, Panel A provides descriptive statistics for the stock returns, $\textit{AdjPIN}$, and all control variables for our data period of January 1983 to December 2004.

To construct our idiosyncratic volatility measures, we first estimate the monthly realized idiosyncratic volatility for individual stocks from daily returns as in Eq. (18) choosing the factors to be the Fama-French three factor model. We then multiply the standard deviation by the square root of the number of trading days in the month. We denote the simple lagged monthly idiosyncratic volatility measure, used in Ang et al (2006), $\textit{IVOL}_{\textit{LAG}}$. Our autoregressive forecasts are estimated and formed as in Eq. (20). For the parameter estimation, we fit an ARMA (p,q) model to an expanding window of realized volatilities. For each firm i and each month t, we use all previous observations to estimate the parameters of all ARMA (p,q) models for p,q $\in \{0,1,2,3\}$. We then use the AIC criterion to choose the best model, and forecast next month’s idiosyncratic volatility. For these ARMA(p,q) forecasts, $\hat{\sigma}_{t+1} | \Phi_{t-1}$ is defined $\textit{IVOL}_{\textit{ARMA}}$.

We construct our final idiosyncratic volatility forecast using EGARCH(p,q) models consistent with Eq. (22). We use return series through $t-1$ and specify our factor model as in Fama and French (1992). Following Fu (2009) we evaluate nine combinations of p,q $\in \{1,2,3\}$ and select the best fitting EGARCH (p,q) model for each stock using the AIC criterion. We then forecast expected idiosyncratic volatility one month ahead. We denote this volatility forecast $\textit{IVOL}_{\textit{EGARCH}}$. 

\[14\]
We report the descriptive statistics of the lagged volatility $IVOL_{LAG}$ and the other two forecasts, $IVOL_{ARMA}$ and $IVOL_{EGARCH}$ in Table 1 Panel B. Interestingly, $IVOL_{EGARCH}$ has higher mean and higher standard deviation than either $IVOL_{LAG}$ or $IVOL_{ARMA}$.

To further compare the forecasting errors of these volatility measures, we compute their in-sample Root Mean Square Errors (RMSE) and report them in Table 2. Because we do not observe the true volatilities, we use ex-post volatility estimates as our benchmark “true” volatilities. When we use the computed monthly volatility from the same month daily returns (Eq. 18) as the benchmark, $IVOL_{ARMA}$ naturally has the lowest RMSE of 3.903. By comparison, $IVOL_{LAG}$ has a RMSE of 4.361 and $IVOL_{EGARCH}$ has the highest RMSE, with a value of 6.296. We next use the in-sample monthly EGARCH forecast $\hat{\sigma}_{i,t}$ as our “true” volatility (Eq. 21). Surprisingly, the ARMA forecast still has the lowest RMSE, with a value of 4.067. $IVOL_{EGARCH}$ underperforms this result with a RMSE of 4.234. $IVOL_{LAG}$ has the highest RMSE with a value of 4.948. Regardless of our estimation method for the “true” measure of idiosyncratic volatility, $IVOL_{ARMA}$ forecasts produce the least forecasting error. As a result, $IVOL_{ARMA}$ will be our idiosyncratic volatility forecast variable of choice for our empirical tests.

V. Cross Sectional Results

Our analysis here will take place at the monthly frequency, and we will estimate cross-sectional regressions following Fama and Macbeth (1973). For each month $t=1,\ldots,T$, we estimate the following cross-sectional regressions:

$$R_{it} = \beta_0 + \gamma_1 IVOL_{it} + \gamma_2 AdjPIN_{it} + \gamma_3 AdjPIN_{it} * IVOL_{it} + \sum_{j=1}^{J} \beta_j X_{jit} + e_{it}, \quad (18)$$

where $i=1,\ldots,N_t$ and $N_t$ is the number of firms in the sample at time $t$. $IVOL$ represents the idiosyncratic volatility forecast. By default, our idiosyncratic volatility forecast will be
IVOL\textsubscript{ARMA}, as we saw in Section III that this provides the best forecast of realized idiosyncratic volatility. AdjPIN is the lagged adjusted probability of informed trading measure of Duarte and Young (2009). The set of variables given by the vector X includes all of our control variables. We average the T estimated cross sectional coefficients, and we construct Newey-West t-statistics (with 4 lags) to establish confidence intervals for the resultant estimates.

Our estimates of these Fama-Macbeth regressions are presented in Table III. We can see in Model 1 the association of AdjPIN with expected returns. AdjPIN has a negligible effect, in line with the findings of Duarte and Young (2009).\textsuperscript{8} In model 2, we see that IVOL\textsubscript{ARMA} is both economically and statistically significant. Both of these models omit any interaction between the variables, and provide results consistent with previous literature. Model 3 provides a basis for comparison by including both effects in our regressions. In the estimations provided in Model 4 however, we see that the interactions of these two variables are important. The coefficient on AdjPIN becomes negative and significant, while the coefficient on IVOL\textsubscript{ARMA} loses significance. Crucially, their interaction is positive and significant at all conventional levels.

We can see the importance of the interaction term through the marginal effects of the variables. We see from Model 4 that

\[
\frac{\partial R_n}{\partial \text{AdjPIN}} = -0.083 + 0.011(\text{IVOL}\textsubscript{ARMA})
\] (19)

and

\[
\frac{\partial R_n}{\partial \text{IVOL}\textsubscript{ARMA}} = -0.052 + 0.011(\text{AdjPIN})
\] (20)

\textsuperscript{8} All models include a host of control variables. Cooper et al (2008) find that firms’ asset growth (measured by LNAG) is negatively associated with expected returns. Huang et al (2010) find that lagged returns (LAGRET) are associated with expected returns and resolve the idiosyncratic volatility pricing paradox of Ang et al (2006). Fu (2009) includes CRETURN as a control variable in estimating the association between expected idiosyncratic volatility and expected return.
For a firm with the mean level of $IVOL_{ARMA}$ (8.195), we see from equation 19 that $\frac{\partial R_{it}}{\partial AdjPIN} = 0.007$, a modest effect similar to the insignificant coefficients we see in models 1 and 3 on $AdjPIN$. For a firm with the mean level of $AdjPIN$ (15.568), from equation 20 we see that $\frac{\partial R_{it}}{\partial IVOL_{ARMA}} = 0.119$, an economically significant value similar to models 2 and 3.

It is from these baseline results that we can see clearly the importance of the interaction terms. For example, if $IVOL_{ARMA}$ is one standard deviation below the mean (which would yield an $IVOL_{ARMA}$ of 4.746), then $\frac{\partial R_{it}}{\partial AdjPIN} = -0.031$. This implies that a one standard deviation increase in $AdjPIN$ (which is 5.916) is associated with a monthly expected return decrease of 0.183%. This is an important effect, implying a reduction in the annual cost of capital of 219 basis points.

The association between $IVOL$ and expected returns is similarly dependent upon the level of the proportion of informed trading. Consider two cases. In the first case, $AdjPIN$ is at the 25th (11.430) percentile, and in the second case, it is at the 75th percentile (18.563). In the first case $\frac{\partial R_{it}}{\partial IVOL_{ARMA}} = 0.073$, indicating that a one standard deviation increase in $IVOL_{ARMA}$ (3.449) would be associated with a 0.251% increase in monthly expected returns. Stated differently, this $IVOL_{ARMA}$ increase would add 302 basis points annually to the cost of capital. However, if $AdjPIN$ is in the 75th percentile, $\frac{\partial R_{it}}{\partial IVOL_{ARMA}} = 0.152$, about twice as large as case one. In this case, a one standard deviation increase in $IVOL$ is associated with a 627 basis point increase in the cost of capital.

Finally, if we consider the corner situation where $AdjPIN$ is zero, we see in this instance that we are not able to statistically distinguish the association between $IVOL_{ARMA}$ and expected returns from zero. These relationships provide an indication that the pricing of idiosyncratic volatility may have less to do with difficulty investors have with diversification, and more to do
with the interference idiosyncratic volatility creates when the agents in the model discern their respective signals. If there is no signal to discern, $IVOL_{ARMA}$ has a trivial association with expected returns.

These findings align well with the theoretical model of Hughes et al (2007), and our Propositions 1 and 2 seem well supported. When we ignore the second order effects, we see that $IVOL_{ARMA}$ is positively associated with expected returns (models 1 and 3), in accordance with Proposition 1b. In models 1 and 3, $AdjPIN$ appears to have negligible association with expected returns. This is consistent with Proposition 1a (as proposition 1a has a weak inequality) as well as the findings of Duarte and Young (2009). In model 4 we see strong support for Proposition 2, as the interaction of $AdjPIN$ and $IVOL_{ARMA}$ is positive and significant at all conventional levels. Further, it appears that this second-order effect is economically important in describing a firm’s cost of capital.

As a basis of comparison, we include in Table IV, which presents results of regressions similar to those of Table III, save that the idiosyncratic volatility variable is now $IVOL_{EGARCH}$. The results illustrated in Table IV are similar to those in Table III, though slightly weaker. This is to be expected, as $IVOL_{EGARCH}$ is less effective in forecasting idiosyncratic volatility than $IVOL_{ARMA}$. Even still, the results are quite similar. The association between expected return and idiosyncratic volatility is positive for the mean level of $AdjPIN$, in this case $\frac{\partial R_{it}}{\partial IVOL_{EGARCH}} = 0.034$. We see similar increases in this relationship when $AdjPIN$ is high. For example, when $AdjPIN$ is one standard deviation above the mean, $\frac{\partial R_{it}}{\partial IVOL_{EGARCH}} = 0.064$. This implies that a one standard deviation increase in $IVOL_{EGARCH} (5.889)$ will be associated with a monthly expected return increase of 0.379%. This is slightly weaker than, but closely in line with, the results we see in Table III.
VI. Robustness Check – Competition for Private Information

Akins et al (2012) find that the pricing of asymmetric information decreases when there is more competition for private information. This naturally has important effects on the cost of capital, and omission of this effect could potentially affect our results, if idiosyncratic volatility is correlated with competition for private information.

To control for competition for private information, we construct two measures, following Akins et al (2012) who assume that institutional investors are informed investors.\(^9\) The first measure of competition is the number of institutional investors holding a firm’s stock in a given year, \(Investor_i\).\(^{10}\) For the second measure of competition among informed investors, we construct a Herfindahl measure of the distribution of institutional investors among firms. The rationale for this approach is that an even distribution of institutional investors implies greater competition for private information, and a more rapid dissemination of those private signals. Again following Akins et al (2012), we construct

\[
HerfInst_i = -1 \times \sum_{j=1}^{N} \left( \frac{Investor_{i,j}}{Investor_i} \right)^2 \tag{21}
\]

Where \(Investor_{i,j}\) is the number of shares held by investor \(j\) in stock \(i\). \(N\) is the number of institutional investors in stock \(i\). This index is computed annually. A higher value of \(HerfInst\) indicates more competition.

In Table V, we first replicate Akins et al (2012) and then add the \(IVOL\) variable and the interaction term between \(IVOL_{ARMA}\) and \(AdjPIN\). \(Competition\) in all models is the Herfindahl index from equation 21. In models 1 and 2, the interaction term \(AdjPIN*Competition\) is negative.

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\(^{10}\) Data is available through Thomson-Reuters Institutional Holdings database.
and significant at the 1% level, confirming the findings of Akins et al (2012). Further, $AdjPIN*IVOL_{ARMA}$ is positive and significant in model 3, confirming that the interaction of informed trading and idiosyncratic volatility is an important determinant of the cost of capital, even when we consider the effects of competition for private information. Indeed, the coefficient on this variable of interest in model 3 is not much different from the corresponding coefficient in model 4 of Table III, indicating a relatively low level of correlation between our variable of interest and the competition variable (as well as its interaction with $AdjPIN$).

VII. Conclusion

The role of the information environment on the cost of capital is an important topic in both the finance and accounting literature. We expand the conversation to include the interaction between the proportion of informed trading in a stock and its idiosyncratic volatility. We examine existing theory in greater detail, and find that the pricing of idiosyncratic volatility may be importantly affected by the proportion of informed trading.

These theoretical findings are then confirmed by empirical tests. While idiosyncratic volatility forecasts in our sample are positively priced, the magnitude of investor reward for idiosyncratic risk is directly tied to the proportion of informed trading, as measured by $AdjPIN$ of Duarte an Young (2009). In the extreme case of zero informed trading, idiosyncratic volatility forecasts are unrelated to expected returns. However, when the proportion of informed trading is high, expected returns exhibit substantial sensitivity to idiosyncratic volatility forecasts. These results are robust to the inclusion in the model of competition for private information, an important consideration put forth in Akins et al (2012).
These results shed light on an aspect of a firm’s cost of capital that has yet to be fully explored. There have been many explorations of both the information environment and the effect of idiosyncratic volatility separately on a firm’s cost of capital. We believe this is both the first theoretical and empirical exploration of the ways in which the interactions of these two variables are associated with different costs of capital.

Future research into the information environment and the cost of capital could build on these findings. For example, there may be an optimal idiosyncratic volatility – proportion of informed trading solution over which firms may be able to exert some control. In the HLL model from which we develop our hypotheses, such interactions are not explored. This may be an interesting theoretical extension to our findings.
References


Table I. Summary Statistics.

The sample period is from January 1983 to December 2004 for firms traded on the NYSE and Amex exchanges, when available. \( RET \) is the percentage return to the firm over the one-month holding period. \( IVOL_{LAG} \) is the lagged idiosyncratic volatility estimate using daily data as in Ang et al. (2006). \( IVOL_{ARMA} \) are expected idiosyncratic volatilities formed using monthly volatility information through time \( t-1 \) to forecast time \( t \) values. The forecasts are made using ARMA (p,q) model where the AIC criterion is used to choose the optimal number of lags for each stock at each time period. EGARCH are EGARCH(p,q) expected idiosyncratic volatilities formed using information from 1963 through time \( t-1 \) to forecast time \( t \) values. The AIC criterion is used to choose the optimal number of lags for each stock at each time period. \( AdjPIN \) is the adjusted PIN measure in Duarte and Young (2009). \( MKTBETA, SMBBETA, HMLBETA \) and \( LIQBETA \) are beta estimates from time series regressions with Fama-French three factors and the liquidity series from Pástor and Stambaugh (2003) using the previous 60 month data on a rolling basis. \( LNME \) is the natural log of a firm’s market equity at the end of the fiscal year. \( LNBEME \) is the log of the fiscal year-end book value of common equity over the calendar year-end market equity. \( CRETURN \) is represented by the cumulative returns from month \( (t-7) \) to \( (t-2) \) and is expressed as a percentage return. \( LNAG \) is the log of the percentage change of total assets of the firm.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Median</th>
<th>Std Dev</th>
<th>Lower Quartile</th>
<th>Upper Quartile</th>
<th>n</th>
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</thead>
<tbody>
<tr>
<td>( RET ) (%)</td>
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<td>10.650</td>
<td>-4.412</td>
<td>6.767</td>
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</tr>
<tr>
<td>( AdjPIN ) (%)</td>
<td>15.568</td>
<td>14.782</td>
<td>5.913</td>
<td>11.430</td>
<td>18.563</td>
<td>269,552</td>
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<tr>
<td>( MKTBETA )</td>
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<td>0.687</td>
<td>1.304</td>
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<td>( SMBBETA )</td>
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<td>0.870</td>
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<tr>
<td>( HMLBETA )</td>
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<td>-0.202</td>
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<tr>
<td>( LIQBETA )</td>
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<td>-0.153</td>
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<tr>
<td>( LNME )</td>
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<td>1.609</td>
<td>5.543</td>
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<td>( LNBEME )</td>
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<td>-0.978</td>
<td>-0.090</td>
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<td>( CRETURN )</td>
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<td>21.221</td>
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<td>( LNAG )</td>
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<tr>
<th>Panel B – Idiosyncratic Volatility Forecasts</th>
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</table>

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<tr>
<th>Variable</th>
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<th>Std Dev</th>
<th>Lower Quartile</th>
<th>Upper Quartile</th>
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<tr>
<td>( IVOL_{LAG} )</td>
<td>8.027</td>
<td>6.936</td>
<td>4.786</td>
<td>4.931</td>
<td>9.817</td>
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<tr>
<td>( IVOL_{ARMA} )</td>
<td>8.195</td>
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<td>3.449</td>
<td>5.805</td>
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<tr>
<td>( IVOL_{EGARCH} )</td>
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<td>7.796</td>
<td>5.889</td>
<td>5.769</td>
<td>10.715</td>
<td>269,552</td>
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Table II: Idiosyncratic Volatility Forecast Comparison

The sample period is from January 1983 to December 2004 for firms traded on the NYSE exchange, when available. *IVOL*$_{LAG}$ is the lagged idiosyncratic volatility estimate using daily data as in Ang et al. (2006). *IVOL*$_{ARMA}$ are expected idiosyncratic volatilities formed using DSQRET information through time t-1 to forecast time t values. The forecasts are made using ARMA (p,q) model where the AIC criterion is used to choose the optimal number of lags for each stock at each time period. *EGARCH* are EGARCH(p,q) expected idiosyncratic volatilities formed using information from 1963 through time t-1 to forecast time t values. The AIC criterion is used to choose the optimal number of lags for each stock at each time period. *EGARCH* are EGARCH(p,q) expected idiosyncratic volatilities formed using information through time t to forecast time t values.

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<th>Forecast Variable</th>
<th>RMSE</th>
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<tr>
<td><em>IVOL</em>$_{ARMA}$</td>
<td>3.903</td>
</tr>
<tr>
<td><em>IVOL</em>$_{EGARCH}$</td>
<td>6.296</td>
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<td><em>IVOL</em>$_{LAG}$</td>
<td>4.361</td>
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</table>

<table>
<thead>
<tr>
<th>Forecast Variable</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>IVOL</em>$_{ARMA}$</td>
<td>4.067</td>
</tr>
<tr>
<td><em>IVOL</em>$_{EGARCH}$</td>
<td>4.234</td>
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<tr>
<td><em>IVOL</em>$_{LAG}$</td>
<td>4.948</td>
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Table III: Fama-MacBeth regressions with using $IVOL_{ARMA}$

The sample covers firms traded on the NYSE from January 1983 to December 2004. Market data is collected from CRSP and accounting variables are collected from Compustat. RET is the percentage return to the firm over the one month holding period. LAGRET is the percentage return to the firm over the preceding one month holding period. MKTBETA, SMLBETA, HMLBETA and LIQBETA are beta estimates from time series regressions with Fama-French three factors and the liquidity series from Pástor and Stambaugh (2003) using the previous 60 month data on a rolling basis. LNME is the natural log of a firm’s market equity at the end of the fiscal year. LNBEME is the log of the fiscal year-end book value of common equity over the calendar year end market equity. CRETURN is represented by the cumulative returns from month (t-7) to (t-2) and is expressed as a percentage return. LNAG is the log of the percentage change of total assets of the firm. $IVOL_{ARMA}$ are expected idiosyncratic volatilities formed using idiosyncratic volatility information through time t-1 to forecast time t values. The forecasts are made using ARMA (p,q) model where the AIC criterion is used to choose the optimal number of lags for each stock at each time period.

<table>
<thead>
<tr>
<th>AdjPIN</th>
<th>IVOL_{ARMA}</th>
<th>AdjPIN * IVOL_{ARMA}</th>
<th>MKTBETA</th>
<th>SMBBETA</th>
<th>HMLBETA</th>
<th>LIQBETA</th>
<th>LNME</th>
<th>LNBEME</th>
<th>CRETURN</th>
<th>LNAG</th>
<th>LAGRET</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.003</td>
<td>0.120***</td>
<td>0.011***</td>
<td>0.159</td>
<td>0.007</td>
<td>0.069</td>
<td>0.141</td>
<td>-0.136***</td>
<td>0.133**</td>
<td>0.002</td>
<td>-0.490***</td>
<td>-0.042***</td>
<td>0.088</td>
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<tr>
<td>(0.0376)</td>
<td>(5.035)</td>
<td>(5.221)</td>
<td>(0.747)</td>
<td>(0.052)</td>
<td>(0.521)</td>
<td>(0.667)</td>
<td>(-3.166)</td>
<td>(1.975)</td>
<td>(0.692)</td>
<td>(-2.997)</td>
<td>(-7.346)</td>
<td>0.095</td>
</tr>
<tr>
<td></td>
<td>-0.083***</td>
<td></td>
<td>-0.144</td>
<td>-0.157</td>
<td>0.199*</td>
<td>0.076</td>
<td>-0.066*</td>
<td>0.182***</td>
<td>0.002</td>
<td>-0.581***</td>
<td>-0.044***</td>
<td>0.097</td>
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<tr>
<td></td>
<td>(0.561)</td>
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<td>(-0.796)</td>
<td>(-1.312)</td>
<td>1.644</td>
<td>(0.359)</td>
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<tr>
<td></td>
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<td></td>
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<td>(-1.064)</td>
<td>1.631</td>
<td>(0.317)</td>
<td>(-1.423)</td>
<td>(2.633)</td>
<td>(0.692)</td>
<td>(-3.430)</td>
<td>(-7.777)</td>
<td></td>
</tr>
</tbody>
</table>

Note: * indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.
Table IV: Fama-MacBeth regressions using $IVOL_{EGARCH}$

The sample covers firms traded on the NYSE from January 1983 to December 2004. Market data is collected from CRSP and accounting variables are collected from Compustat. $RET$ is the percentage return to the firm over the one month holding period. $LAGRET$ is the percentage return to the firm over the preceding one month holding period. $MKTBETA$, $SMBBETA$, $HMLBETA$ and $LIQBETA$ are beta estimates from time series regressions with Fama-French three factors and the liquidity series from Pástor and Stambaugh (2003) using the previous 60 month data on a rolling basis. $LNME$ is the natural log of a firm’s market equity at the end of the fiscal year. $LNBEME$ is the log of the fiscal year-end book value of common equity over the calendar year end market equity. $CRETURN$ is represented by the cumulative returns from month (t-7) to (t-2) and is expressed as a percentage return. $LNAG$ is the log of the percentage change of total assets of the firm. $IVOL_{EGARCH}$ idiosyncratic volatility forecasts are EGARCH(p,q) expected idiosyncratic volatilities for time $t$, formed using information from 1963 through time $t-1$ to forecast time $t$ values. The AIC criterion is used to choose the optimal number of lags for each stock at each time period.

<table>
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<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AdjPIN$</td>
<td>0.002</td>
<td>-0.040</td>
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<td></td>
<td>(0.230)</td>
<td>(-7.321)</td>
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<tr>
<td>$IVOL_{EGARCH}$</td>
<td>0.027***</td>
<td>0.027***</td>
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<tr>
<td></td>
<td>(2.832)</td>
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<td>$AdjPIN*IVOL_{EGARCH}$</td>
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<td></td>
<td>0.005***</td>
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<td></td>
<td></td>
<td>(3.814)</td>
</tr>
<tr>
<td>$MKTBETA$</td>
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<td></td>
<td>(0.462)</td>
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<td>-0.035</td>
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<td>(-0.277)</td>
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<tr>
<td>$HMLBETA$</td>
<td>0.100</td>
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<td>0.085</td>
</tr>
<tr>
<td></td>
<td>(0.785)</td>
<td>(0.775)</td>
<td>(0.666)</td>
</tr>
<tr>
<td>$LIQBETA$</td>
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<td>0.138</td>
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<tr>
<td></td>
<td>(0.701)</td>
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<td>(0.630)</td>
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<tr>
<td>$LNME$</td>
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<td>-0.124***</td>
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<tr>
<td></td>
<td>(-3.050)</td>
<td>(-2.814)</td>
<td>(-2.960)</td>
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<tr>
<td>$LNBEME$</td>
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<td>0.149</td>
<td>0.148**</td>
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<tr>
<td></td>
<td>(2.211)</td>
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<td>(2.151)</td>
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<tr>
<td>$CRETURN$</td>
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<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.512)</td>
<td>(0.479)</td>
<td>(0.354)</td>
</tr>
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<td>-0.548</td>
<td>-0.530***</td>
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<tr>
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<td>(-3.302)</td>
<td>(-3.200)</td>
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<tr>
<td>$LAGRET$</td>
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<td>-0.041</td>
<td>-0.041***</td>
</tr>
<tr>
<td></td>
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<td>(-7.285)</td>
<td>(-7.321)</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.092</td>
<td>0.095</td>
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</table>
Table V: Robustness check – Competition Variables

The sample covers firms traded on the NYSE from January 1983 to December 2004. Market data is collected from CRSP and accounting variables from Compustat. IVOL-ARMA idiosyncratic volatility forecast formed using information through time t-1 to forecast time t values. The forecasts are made using ARMA model where the AIC criterion is used to choose the optimal number of lags for each stock, each time period. RET is the percentage return to the firm over the one month holding period. AdjPIN is the adjusted PIN measure related to asymmetric information in Duarte and Young (2009). PSOS is the symmetric order-flow shocks, the PIN component that related to illiquidity in Duarte and Young (2009). LAGRET is the percentage return to the firm over the preceding one month holding period. MKTBETA, SMLBETA, HMLBETA and LIQBETA are beta estimates from time series regressions with Fama-French three factors and the liquidity series from Pástor and Stambaugh (2003) using the previous 60 month data on a rolling basis. LNME is the natural log of a firm’s market equity at the end of the fiscal year. LNBEME is the log of the fiscal year-end book value of common equity over the calendar year end market equity. CRETURN is represented by the cumulative returns from month (t-7) to (t-2), expressed as a percentage return. LNAG is the log of the percentage change of total assets of the firm. Competition is proxied by Herfinst(t-1) as in equation 21. As in Akins et al (2012), we convert competition variables into quintile ranks each quarter, and quintile ranks are scaled from 0 to 1, with the lowest quintile mapping to zero.

<table>
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<th>(2)</th>
<th>(3)</th>
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<tr>
<td>AdjPIN</td>
<td>0.039*</td>
<td>0.046**</td>
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<tr>
<td></td>
<td>(1.918)</td>
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<td>IVOL-ARMA</td>
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<td>0.102</td>
<td>0.059</td>
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<td>(0.648)</td>
<td>(1.224)</td>
<td>(0.673)</td>
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<tr>
<td>AdjPIN * IVOL-ARMA</td>
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<td></td>
<td></td>
<td></td>
<td>(5.322)</td>
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<tr>
<td>MKTBETA</td>
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<tr>
<td></td>
<td>(0.788)</td>
<td>(-0.773)</td>
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<td>SMBBETA</td>
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<td>(0.094)</td>
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<td>(0.7868)</td>
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<td>HMLBETA</td>
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<td>0.202</td>
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<tr>
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<td>(0.556)</td>
<td>(1.682)</td>
<td>(1.326)</td>
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<tr>
<td>LIQBETA</td>
<td>0.138</td>
<td>0.062</td>
<td>0.057</td>
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<td>(0.649)</td>
<td>(0.293)</td>
<td>(0.270)</td>
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<tr>
<td>LNME</td>
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<td>-0.058</td>
<td>-0.073</td>
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<td>(-2.731)</td>
<td>(-1.273)</td>
<td>(-1.567)</td>
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<tr>
<td>LNBEME</td>
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<td>0.174***</td>
<td>0.168**</td>
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<tr>
<td></td>
<td>(1.933)</td>
<td>(2.575)</td>
<td>(2.475)</td>
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<td>CRETURN</td>
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<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.551)</td>
<td>(0.692)</td>
<td>(0.463)</td>
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<tr>
<td>LNAG</td>
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<td>-0.539***</td>
<td>-0.500***</td>
</tr>
<tr>
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<td>(-2.785)</td>
<td>(-3.242)</td>
<td>(-3.003)</td>
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<tr>
<td>LAGRET</td>
<td>-0.042***</td>
<td>-0.044***</td>
<td>-0.044***</td>
</tr>
<tr>
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<td>(-7.811)</td>
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<td>R²</td>
<td>0.093</td>
<td>0.101</td>
<td>0.104</td>
</tr>
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</table>
Appendix A: Comparative Statics from the Hughes et al (2007) Model

Proposition 1. $a$ \[ \frac{\partial E(\bar{v} - \bar{R}_f p)}{\partial \mu} \leq 0 \]

\[ \frac{\partial E(\bar{v} - \bar{R}_f p)}{\partial \mu} = -\frac{\beta A(b'(\Sigma + \Sigma_\theta)^{-1}b - b'(\Sigma + \Sigma_\theta)^{-1}b)\beta' \bar{x}}{N[(\Sigma^{-1} + b'(\Sigma + \Sigma_\theta)^{-1}b) + (1 - \mu)(\Sigma^{-1} + b'(\Sigma + \Sigma_\theta)^{-1}b)]^2} \quad \text{(A1)} \]

The numerator of the right hand side expression is weakly negative. It must be, since $\Sigma_\theta > \Sigma_s$ from equation 11. It is equal to zero in trivial cases. The denominator is always positive. Therefore,

\[ \frac{\partial E(\bar{v} - \bar{R}_f p)}{\partial \mu} \leq 0 \quad \text{(A2)} \]

Proposition 1. $b$. \[ \frac{\partial E(\bar{v} - \bar{R}_f p)}{\partial \Sigma} \geq 0 \]

\[ \frac{\partial E(\bar{v} - \bar{R}_f p)}{\partial \Sigma} = -\frac{\beta A(-b'(1 - \mu)(\Sigma + \Sigma_\theta)^{-2}b - b'(\Sigma + \Sigma_\theta)^{-2}b)\beta' \bar{x}}{N[(\Sigma^{-1} + b'(\Sigma + \Sigma_\theta)^{-1}b) + (1 - \mu)(\Sigma^{-1} + b'(\Sigma + \Sigma_\theta)^{-1}b)]^2} \]

\[ = \frac{\beta A(b'(1 - \mu)(\Sigma + \Sigma_\theta)^{-2}b + b'(\Sigma + \Sigma_\theta)^{-2}b)\beta' \bar{x}}{N[(\Sigma^{-1} + b'(\Sigma + \Sigma_\theta)^{-1}b) + (1 - \mu)(\Sigma^{-1} + b'(\Sigma + \Sigma_\theta)^{-1}b)]^2} \]

The denominator of this expression is the same as the denominator in A1, and positive. Further, assuming $0 < \mu < 1$, the numerator is always positive. Therefore,

\[ \frac{\partial E(\bar{v} - \bar{R}_f p)}{\partial \Sigma} > 0 \quad \text{(A3)} \]

Proposition 2: \[ \frac{\partial E(\bar{v} - \bar{R}_f p)}{\partial \mu \partial \Sigma} > 0 \]

\[ \frac{\partial E(\bar{v} - \bar{R}_f p)}{\partial \mu \partial \Sigma} = 2\beta A(b'(\Sigma + \Sigma_\theta)^{-1}b - b'(\Sigma + \Sigma_\theta)^{-1}b)(-b'(1 - \mu)(\Sigma + \Sigma_\theta)^{-2}b - b'(\Sigma + \Sigma_\theta)^{-2}b)\beta' \bar{x} \]

\[ = \frac{\beta A(-b'(\Sigma + \Sigma_\theta)^{-2}b + b'(\Sigma + \Sigma_\theta)^{-2}b)\beta' \bar{x}}{N[(\Sigma^{-1} + b'(\Sigma + \Sigma_\theta)^{-1}b) + (1 - \mu)(\Sigma^{-1} + b'(\Sigma + \Sigma_\theta)^{-1}b)]^2} \]

\[ - \frac{\beta A(-b'(\Sigma + \Sigma_\theta)^{-2}b + b'(\Sigma + \Sigma_\theta)^{-2}b)\beta' \bar{x}}{N[(\Sigma^{-1} + b'(\Sigma + \Sigma_\theta)^{-1}b) + (1 - \mu)(\Sigma^{-1} + b'(\Sigma + \Sigma_\theta)^{-1}b)]^2} \]
\[
\frac{\beta A (b' (\Sigma + \Sigma_\sigma)^{-2} - b' (\Sigma + \Sigma_\theta)^{-2} b) \beta \bar{x}}{N[\mu (\Sigma_F^{-1} + b' (\Sigma + \Sigma_\sigma)^{-1} b) + (1 - \mu) (\Sigma_F^{-1} + b' (\Sigma + \Sigma_\theta)^{-1} b)]^2} - \frac{2 \beta A (b' (\Sigma + \Sigma_\sigma)^{-1} - b' (\Sigma + \Sigma_\theta)^{-1} b) (b' (1 - \mu) (\Sigma + \Sigma_\theta)^{-2} b + b' \mu (\Sigma + \Sigma_\theta)^{-2} b) \beta \bar{x}}{N[\mu (\Sigma_F^{-1} + b' (\Sigma + \Sigma_\sigma)^{-1} b) + (1 - \mu) (\Sigma_F^{-1} + b' (\Sigma + \Sigma_\theta)^{-1} b)]^2}
\]

Given that \[\mu (\Sigma_F^{-1} + b' (\Sigma + \Sigma_\sigma)^{-2} b) + (1 - \mu) (\Sigma_F^{-1} + b' (\Sigma + \Sigma_\theta)^{-2} b)\] > 0, the denominator is always positive. To determine whether the sign of the numerator is positive, it is sufficient to show that

\[
\beta A b' \left[ ((\Sigma + \Sigma_\sigma)^{-2} - (\Sigma + \Sigma_\theta)^{-2}) \ast \left[ \mu (\Sigma_F^{-1} + b' (\Sigma + \Sigma_\sigma)^{-1} b) + (1 - \mu) (\Sigma_F^{-1} + b' (\Sigma + \Sigma_\theta)^{-1} b) \right] - 2((\Sigma + \Sigma_\sigma)^{-1} - (\Sigma + \Sigma_\theta)^{-1}) (b' (1 - \mu) (\Sigma + \Sigma_\theta)^{-2} b + b' \mu (\Sigma + \Sigma_\theta)^{-2} b) \right] \beta \bar{x} > 0.
\]

Examining this term more closely, we see that

\[
((\Sigma + \Sigma_\sigma)^{-2} - (\Sigma + \Sigma_\theta)^{-2}) \ast \left[ \mu (\Sigma_F^{-1} + b' (\Sigma + \Sigma_\sigma)^{-1} b) + (1 - \mu) (\Sigma_F^{-1} + b' (\Sigma + \Sigma_\theta)^{-1} b) \right] - 2((\Sigma + \Sigma_\sigma)^{-1} - (\Sigma + \Sigma_\theta)^{-1}) (b' (1 - \mu) (\Sigma + \Sigma_\theta)^{-2} b + b' \mu (\Sigma + \Sigma_\theta)^{-2} b)
\]

\[
= ((\Sigma + \Sigma_\sigma)^{-2} - (\Sigma + \Sigma_\theta)^{-2}) (b' (\Sigma + \Sigma_\sigma)^{-1} b \mu + \Sigma_F^{-1} + b' (\Sigma + \Sigma_\theta)^{-1} b (1 - \mu)) - 2((\Sigma + \Sigma_\sigma)^{-1} - (\Sigma + \Sigma_\theta)^{-1}) (b' (1 - \mu) (\Sigma + \Sigma_\theta)^{-2} b + b' \mu (\Sigma + \Sigma_\theta)^{-2} b)
\]

\[
= b' (\Sigma + \Sigma_\sigma)^{-3} b \mu + (\Sigma + \Sigma_\sigma)^{-2} \Sigma_F^{-1} + b' (\Sigma + \Sigma_\theta)^{-1} (\Sigma + \Sigma_\sigma)^{-2} b (1 - \mu) - b' (\Sigma + \Sigma_\sigma)^{-1} (\Sigma + \Sigma_\theta)^{-2} b \mu - (\Sigma + \Sigma_\theta)^{-2} \Sigma_F^{-1} - b' (1 - \mu) (\Sigma + \Sigma_\theta)^{-3} b - 2b' (1 - \mu) (\Sigma + \Sigma_\theta)^{-2} (\Sigma + \Sigma_\sigma)^{-1} b - 2b' \mu (\Sigma + \Sigma_\sigma)^{-3} b + 2b' (1 - \mu) (\Sigma + \Sigma_\theta)^{-3} b + 2b' \mu (\Sigma + \Sigma_\sigma)^{-2} (\Sigma + \Sigma_\theta)^{-1} b
\]

\[
= (\Sigma + \Sigma_\sigma)^{-2} \Sigma_F^{-1} - (\Sigma + \Sigma_\theta)^{-2} \Sigma_F^{-1} + b' (\Sigma + \Sigma_\sigma)^{-1} (\Sigma + \Sigma_\sigma)^{-2} b (1 + \mu) - b' (2 - \mu) (\Sigma + \Sigma_\sigma)^{-2} b - b' (\Sigma + \Sigma_\sigma)^{-3} b \mu + b' (1 - \mu) (\Sigma + \Sigma_\theta)^{-3} b
\]

\[
= (\Sigma + \Sigma_\sigma)^{-2} \Sigma_F^{-1} - (\Sigma + \Sigma_\theta)^{-2} \Sigma_F^{-1} - b' (2 - \mu) (\Sigma + \Sigma_\sigma)^{-2} (\Sigma + \Sigma_\sigma)^{-1} b - b' (\Sigma + \Sigma_\sigma)^{-3} b \mu + [b' (\Sigma + \Sigma_\sigma)^{-3} b (1 + \mu) + b' (1 - \mu) (\Sigma + \Sigma_\sigma)^{-3} b] \text{ (given that } \Sigma_\theta > \Sigma_s, \text{ hence } b' (\Sigma + \Sigma_\sigma)^{-3} b (1 + \mu) )
\]
\[
\begin{align*}
&= (\Sigma + \Sigma_s)^{-2} \Sigma_f^{-1} - (\Sigma + \Sigma_\theta)^{-2} \Sigma_f^{-1} - \left[ b'(2 - \mu)(\Sigma + \Sigma_\theta)^{-2} (\Sigma + \Sigma_s)^{-1} b + b'(\Sigma + \Sigma_s)^{-3} b \mu \right] + 2b(\Sigma + \Sigma_\theta)^{-3} b \\
&> (\Sigma + \Sigma_s)^{-2} \Sigma_f^{-1} - (\Sigma + \Sigma_\theta)^{-2} \Sigma_f^{-1} - \left[ b'(2 - \mu)(\Sigma + \Sigma_\theta)^{-3} b + b'(\Sigma + \Sigma_s)^{-3} b \mu \right] + 2b(\Sigma + \Sigma_\theta)^{-3} b \\
\end{align*}
\]

(given that $\Sigma_\theta > \Sigma_s$, hence $-\left[ b'(2 - \mu)(\Sigma + \Sigma_\theta)^{-2} (\Sigma + \Sigma_s)^{-1} b \right] > -\left[ b'(2 - \mu)(\Sigma + \Sigma_s)^{-2} (\Sigma + \Sigma_s)^{-1} b \right]$)

\[
\begin{align*}
&= [(\Sigma + \Sigma_s)^{-2} - (\Sigma + \Sigma_\theta)^{-2}] \Sigma_f^{-1} - 2b'[((\Sigma + \Sigma_s)^{-3} - (\Sigma + \Sigma_\theta)^{-3})b \\
&= [(\Sigma + \Sigma_s)^{-1} - (\Sigma + \Sigma_\theta)^{-1}][(\Sigma + \Sigma_s)^{-1} + (\Sigma + \Sigma_\theta)^{-1}] \Sigma_f^{-1} - 2b'[(\Sigma + \Sigma_s)^{-1} - (\Sigma + \Sigma_\theta)^{-1}][(\Sigma + \Sigma_s)^{-2} + (\Sigma + \Sigma_\theta)^{-1} + (\Sigma + \Sigma_\theta)^{-2}]b \\
\end{align*}
\]

$\Sigma_\theta > \Sigma_s$, and hence $\frac{1}{\Sigma + \Sigma_s} - \frac{1}{\Sigma + \Sigma_\theta} > 0$, so we need to know boundary for

\[
\begin{align*}
&= [(\Sigma + \Sigma_s)^{-1} + (\Sigma + \Sigma_\theta)^{-1}] \Sigma_f^{-1} - 2b'[(\Sigma + \Sigma_s)^{-2} + (\Sigma + \Sigma_s)^{-1}(\Sigma + \Sigma_\theta)^{-1} + (\Sigma + \Sigma_\theta)^{-2}]b \\
&= [(\Sigma + \Sigma_s)^{-1} + (\Sigma + \Sigma_\theta)^{-1}] \Sigma_f^{-1} - 2b'[(\Sigma + \Sigma_s)^{-2} + 2(\Sigma + \Sigma_s)^{-1}(\Sigma + \Sigma_\theta)^{-1} + (\Sigma + \Sigma_\theta)^{-2}]b + 2b'(\Sigma + \Sigma_s)^{-1}(\Sigma + \Sigma_\theta)^{-1} b \\
&= [(\Sigma + \Sigma_s)^{-1} + (\Sigma + \Sigma_\theta)^{-1}] [\Sigma_f^{-1} - 2b'[(\Sigma + \Sigma_s)^{-1} + (\Sigma + \Sigma_\theta)^{-1}] b] + 2b'(\Sigma + \Sigma_s)^{-1}(\Sigma + \Sigma_\theta)^{-1} b \\
&> [(\Sigma + \Sigma_s)^{-1} + (\Sigma + \Sigma_\theta)^{-1}] [\Sigma_f^{-1} - 2b'[(\Sigma + \Sigma_s)^{-1} + (\Sigma + \Sigma_\theta)^{-1}] b] \\
\end{align*}
\]

(given that $2b' \frac{1}{(\Sigma + \Sigma_s)(\Sigma + \Sigma_\theta)} b > 0$)
$(\Sigma + \Sigma_{\delta})^{-1} + (\Sigma + \Sigma_{\theta})^{-1} > 0$, and HLL model illustrates that systematic information intensity $b$ is related to the convergence of the investor’s posterior precision, so assume identical distribution for risky asset payoffs with one factor, they specify $b = \frac{k}{\sqrt{N}}$ where $k$ is a non-zero constant. As $N$ becomes large, we have

$$\Sigma_F^{-1} - 2b [(\Sigma + \Sigma_{\delta})^{-1} + (\Sigma + \Sigma_{\theta})^{-1}] b = \Sigma_F^{-1} - \frac{2k^2}{N} [(\Sigma + \Sigma_{\delta})^{-1} + (\Sigma + \Sigma_{\theta})^{-1}] > 0$$

Therefore, we have

$$\frac{\partial E(\bar{y} - R_f p)}{\partial \mu \sigma} > 0 \quad \text{(A4)}$$