Disappointment aversion and income risk: implications for portfolio allocation

Abstract

I solve the life-cycle portfolio allocation problem of a disappointment-averse (DA) agent. Unlike expected utility investors, for DA investors, the decline in income risk also drives stock allocations lower in late working life, and risky allocations are positively related to income risk over this phase of the investor’s life. The changing co-movement between returns and the disappointment/elation realization drives the effect. The DA aggregator in a heterogeneous Epstein-Zin model with market entry cost enhances the match with the empirical pattern on conditional portfolio shares. Sufficiently disappointment-averse agents abstain from investing in stocks after retirement.
1 Introduction

All households confront the portfolio choice problem, while holding an important non-traded asset: human capital. The problem also extends to employers because the Pension Protection Act of 2006 provides employers the authority to enroll workers in defined contribution plans into default investment plans (Purcell (2006)). Personal financial advisors confront the same problem.¹ The solution to this problem (normative study) or a positive study of portfolio choice requires an accurate model of a household’s risk-reward attitude in the presence of non-tradable risky income. Although the implications of expected utility models have been thoroughly studied, studies of non-expected utility theories and in particular those involving non-tradable income are limited in number.² Starmer (2000) and Rabin and Thaler (2001) summarize some of the observed patterns of choice that violate expected utility theory, particularly the Allais paradox. The interpretation of this paradox in the context of background risk suggests counterintuitive patterns in the risk-reward attitude. Accordingly, I show that a disappointment-aversion (DA) model of preferences that accommodates Allais paradox-type behavior generates a unique pattern in risky asset share with labor income risk. I further expand and present the life-cycle implications of this well-known violation of expected utility theory.

The standard life-cycle models yield a negative relation between income risk and risky asset share (Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2005), Polkovnichenko (2007)). I find that the empirical evidence suggests this relationship may depend on age. The evidence supports a negative relationship in early working life and at very high levels of income risk irrespective of age. However, the evidence suggests a positive relation between income risk and portfolio shares in late working life. The DA preferences, proposed by Gul

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¹The Bureau of Labor Statistics projects that personal financial advising as an occupation will have an above average employment growth in the next decade (Royster (2014)).

²The preference model from Epstein and Zin (1989) is an exception. It is often used to separate risk aversion from elasticity of intertemporal substitution. See Peijnenburg (2011) for a study of life-cycle model of ambiguity-averse agents. See Polkovnichenko (2005) for explorations in rank-dependent utility with non-tradable income.
(1991), which incorporate expected utility as a special case are consistent with this pattern. The standard life-cycle models also indicate declining human capital as the motivation for lowering portfolio shares with age. The DA preferences indicate declining income risk as an additional motivation to lower portfolio shares in late working life.

The estimates of the age effects on equity investment are susceptible to assumptions about the cohort and time effects (Ameriks and Zeldes (2002)). I show that use of the fraction of households with defined contribution plans or Individual Retirement Accounts as the time effect proxy yields hump-shaped age effects for the participation rate and declining age effects for the conditional equity shares. The Epstein-Zin (EZ) model with a one-time participation cost and preference heterogeneity by Gomes and Michaelides (2005) matches the participation rates in early and late working life. However, the agents with low elasticity of intertemporal substitution (EIS) and low risk aversion counterfactually invest all the savings in equities once they pay the participation cost. I show that the EZ model with the DA aggregator substantially lowers the conditional equity shares of these agents. In addition, the standard model does not generate non-participation for any level of risk aversion in retirement. By contrast the DA model obtains non-participation for sufficiently disappointment averse agents over this phase.

The DA preferences include a curvature component similar to that in expected utility models and an asymmetric weighting scheme that overweighs the disappointing outcomes. I prove that if income is independent of asset returns, a sufficiently disappointment-averse agent increases risky asset holdings as the riskless income turns risky. This pattern holds even if the expected income declines. Further, I characterize the investment behavior if the curvature component is linear and the agent is not sufficiently disappointment-averse. I prove that an agent with such preference parameters invests all his savings in the risky asset at all levels of income risk. This behavior is independent of the relation between income and asset returns. In addition, I prove that for DA preferences, welfare is concave in wealth if the curvature parameter is positive and the disappointment-aversion parameter is non-negative. I
also introduce the bequest motive in a manner that retains the concavity of welfare in wealth.

The source of the counterintuitive investment pattern under the DA model is the extra lopsided weighting scheme. With the rise in income risk, the unfavorable asset returns increasingly dissociate and the favorable asset returns increasingly associate with the disappointing or low wealth outcomes. The emphasis on these returns through the inbuilt weighting scheme raises the agent’s risk appetite and hence the increase in the risky portfolio share. In other words, the magnitude of comovement between returns and the asymmetric weighting scheme declines. However, the curvature component built into the DA preferences lowers the risk appetite and paves the way for the standard or intuitive decline in risky investment once the income risk is high enough. The result is a hump-shaped pattern in risky allocation with income risk. This pattern interacts with permanent risk in the life-cycle setting in a peculiar way. The permanent shocks in early working life affect the entire life’s earnings, which creates a high level of uninsurable risk and hence the negative relation between risky investment and income risk. By contrast the uninsurable risk declines in late working life with only a few spells of income left. The result is a positive effect of income risk on risky investment.

Another way to understand the DA agent’s counterintuitive behavior is to interpret the Allais paradox in terms of background risk. The choices under the Allais paradox can be summarized as an affinity for a riskless alternative if it is one of the options, and a preference for a risky choice if the alternative choice is risky as well. The income risk can limit the alternate choices available to the agent. In other words, income risk creates the baseline gamble. If the non-tradable income is riskless, then a sure or riskless gamble is one of the options if the agent does not invest in the risky assets. Consequently, the inbuilt affinity for sure outcomes lowers the risky allocations of the agent and possibly leads to non-participation in the equity market. However, if the income is risky, the agent is still left with a risky gamble.

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3Consider the following version of the paradox. The first set of choices is Lottery 1A: guaranteed $200, and Lottery 1B: 80% chance of $300 and 20% chance of $0. The second set of choices is Lottery 2A: equal chances of winning $200 and $0, and Lottery 2B: 40% chance of $300 and 60% chance of $0. Most people choose Lottery 1A from the first set of choices and Lottery 2B from the second set. These choices violate the independence axiom.
even if he chooses not to invest in the risky asset. The agent facing a risky alternative prefers to take advantage of the risky but attractive stock returns, which results in a higher risky share. Thus, the role of income risk as a baseline gamble helps generate the rising pattern in risky asset share with the income risk.\footnote{The curvature built into these preferences overwhelms the Allais paradox effect and leads to the standard negative relation once the income is risky enough.}

I show that a DA model with a heteroscedastic income process that has smaller shocks in late working life better matches the observed equity allocation pattern. I also study the effects of out-of-pocket medical expenses. Although these uncertain expenses lower the risky investments under the standard expected utility model (Pang and Warshawsky (2010)), they result in a mild increase in risky portfolio share under the DA model.\footnote{The increase is tempered because of the disastrous impact of large expense shocks that increase the curvature component of the marginal utility.} Such behavior does not constitute a case against the DA model as the net impact of an unfavorable health shock can lead to an increase or a decrease in risky share under the standard model (Edwards (2008, 2010), Love and Perozek (2007)). In addition, the empirical evidence about the causal impact of such shocks is mixed (Love and Smith (2010)). I also match the wealth-accumulation pattern, participation rate, and the equity allocation conditional on participation implied by the EZ model embedded with the DA aggregator to that from the Survey of Consumer Finances.

The literature on portfolio allocation is extensive. Polkovnichenko (2007), Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2005), Gomes and Michaelides (2003), Bertaut and Haliassos (1997), and Heaton and Lucas (1997, 2000b), among others, explore the effects of background risk on portfolio allocation.\footnote{See Yao and Zhang (2005), Flavin and Yamashita (2002), and Cocco (2005) for the effects of house price risk on portfolio choice. See Benzoni, Collin-Dufresne, and Goldstein (2007) for models that generate low equity allocation among young agents.} The studies most closely related to mine are Ang, Bekaert, and Liu (2005), Cocco, Gomes, and Maenhout (2005), and Gomes and Michaelides (2005). Ang, Bekaert, and Liu (2005) show that stock market non-participation is a possible optimal outcome when agents are sufficiently disappointment-averse. They consider the terminal utility problem and focus on the participation decision and com-
parison with loss-aversion preferences. By contrast, I study life-cycle asset-allocation decisions of DA agents who earn non-tradable labor income and also derive utility from intermediate consumption. In addition, I prove that the non-participation decision of sufficiently disappointment-averse agents does not hold if the income is independent and risky. Cocco, Gomes, and Maenhout (2005) and Gomes and Michaelides (2005) study the life-cycle problem in the presence of background risk for CRRA and EZ preferences. I focus on unique predictions under the DA model and the EZ model with the embedded DA aggregator.

The decline in labor income uncertainty provides an additional impetus to lower risky allocations in late working life and retirement under the DA preferences. The increasing risk aversion with age can also act as an additional driver to lower the risky share. Albert and Duffy (2012) provide experimental evidence indicating such patterns in risk aversion with age, and Barsky, Juster, Kimball, and Shapiro (1997) provide survey evidence indicating variation in risk tolerance with age. The habits model is another appealing model of preferences (Gomes and Michaelides (2003), Polkovnichenko (2007)). The additive habit model generates conservative investments in old age to ensure the habit level is sustained, but the accompanying wealth accumulation is also high. The agents may also exhibit aversion to risky investments after retirement, because of inflexibility in the labor supply. Bodie, Merton, and Samuelson (1992) study flexibility in the labor supply and find additional labor supply flexibility induces the agent to take on greater financial risk. However, the relation between income risk and risky allocation is negative in such models, unlike the more complex pattern under the DA preferences.

Gill and Prowse (2012), Choi, Fisman, Gale, and Kariv (2007) and others find experimental evidence for DA preferences. These preferences have also been used in equilibrium consumption asset pricing. However, asset pricing studies with DA models and uninsurable risk

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remain unexplored.

The rest of the article is organized as follows. In section 2, I present empirical evidence on the pattern in equity investment and its relation with income risk. In section 3, I show the effects of background risk on risky allocation under the DA model in a one-period setting. In sections 4 and 5 I set up and calibrate the life-cycle problem. In section 6, I present the results for the benchmark case and contrast them with the results from the CRRA model. In section 7, I follow up with various extensions. In section 8, I match the conditional portfolio shares, participation rates, and wealth-accumulation pattern implied by the EZ model with embedded DA aggregator to that from the Survey of Consumer Finances. I conclude in section 9.

2 Patterns in portfolio shares

2.1 Age effects

The age, time, and cohort are linearly dependent, and thus estimating their effects on portfolio shares is difficult. The estimates of age effects are sensitive to assumptions about the cohort and time effects. Ameriks and Zeldes (2002) use the Survey of Consumer Finances (SCF) data to measure age effects on stock market participation and the equity fraction of financial assets among equity owners. They show that the market participation rates increase during working life irrespective of whether the cohort or the time controls are used. However, the pattern in participation starting around the age of 60 depends on the type of control used. They find the time controls yield a declining age pattern. However, as the authors note, the pattern could be attributed to the missing cohort controls. Similarly, the missing time effects, such as the growing popularity of defined contribution (DC) plans and Individual Retirement Accounts (IRAs) or other time trends, could be the reason for the rising age pattern observed when only the cohort controls are used (Weisbenner (2002), Campbell and Viceira endogenous withdrawal from the market on the risk premium and the risk-free rate.
Figure 1: The implied age effects for the stock market participation rate are from a probit regression and that for the fraction of financial wealth invested in equity are from an OLS regression. The specifications include age dummies, cohort dummies, and a proxy for the time effect. I use the fraction of households with DC plans or IRA accounts in a given year as the time proxy. The data are from all available samples of the SCF from 1989 to 2010 and include home equity in computing financial wealth. See the Internet Appendix for more details.

In a similar vein, the time controls yield a declining\(^9\) age pattern, whereas the cohort controls produce increasing effects of age on equity shares of market participants’ financial assets. Each of these patterns could possibly be due to the other missing control.

I use a proxy for the time effects that is linearly independent of age and cohort to better ascertain the effect of age on equity investment. In particular, I use the fraction of households with the DC retirement plan and/or IRA account in each survey year as a proxy for the time effect. The implied age effects indicate a hump-shaped pattern in stock market participation rate and a declining pattern in equity investment among the participants (Fig. 1). Further, I obtain similar patterns with other alternative proxies for the time effect.\(^{10}\)

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\(^9\)The declining age-effect pattern on the equity fraction among the participants is stronger in the full 1989-2010 SCF sample compared to the 1989-1998 sample in Ameriks and Zeldes (2002).

\(^{10}\)Specifically I consider (1) the proportion of households participating in the equity market, (2) the proportion of households among the stock market participants with a DC retirement plan and/or IRA account in each survey year, and (3) the average proportion of financial assets held in a retirement account in the form of equity across the participating households over time. The second and third proxies are conditional on equity investment.
Table 1: The coefficients in column (i) are from a probit regression of stock market participation, and in column (ii), the coefficients are from an OLS regression of the equity fraction of financial assets for market participants. The independent variables include dummy variables “near-retirement”, “non-public sector” and their interaction “near-retirement × non-public sector” along with other controls. The data are for heads of households (HoH) from all SCF samples from 1989 to 2010 with ages between 55 to 60 and 30 to 34. near-retirement = 1 if 55 ≤ HoH age ≤ 60 and near-retirement = 0 if 30 ≤ HoH age ≤ 34. non-public sector = 0 if the HoH work in the public sector and non-public sector = 1 otherwise. The controls are for income, liquid assets, household characteristics, expected returns, along with time and cohort dummies. The Internet Appendix describes the controls. Observations are weighted with SCF sample weights. The t-statistics in parentheses are robust and adjusted for multiple imputation. ** significant at 5%; * significant at 10%.

These results indicate that the age effects, after fully accounting for time and cohort effects, are likely to generate declining equity allocation and a hump-shaped pattern in the propensity to participate in the equity market. The policy of declining allocation to equity with age at target-date funds provides another estimate of age effects. These funds accounted for $503 billion at the end of March 2013 (Charlson and Lutton (2013)) and are one of the mandated default investment options in retirement accounts.

### 2.2 Relation with non-financial income risk

The income risk has a substantial influence on portfolio choice. Some studies indicate a negative relationship between risky investment and income risk, whereas the results in other studies suggest a relation that depends on the investor’s age and wealth. Angerer and Lam (2009), Betermier, Jansson, Parlour and Walden (2012) and Heaton and Lucas (2000a) in-
dicate a negative relationship between income risk and risky investment.\textsuperscript{11} Whereas Massa and Simonov (2006) find a positive relation between the two among high-wealth households and a negative relationship among low-wealth households, they use Swedish household data and control for correlation between financial and non-financial income. Bonaparte, Korniotis and Kumar (2013) use Dutch household data to study the relation between equity investment and income-return correlation. The median age of the respondents in this dataset is 54. Although the equity investments decline with rising income-return correlation, the relation between equity investment and income risk (after accounting for income-return correlation) is not unambiguous.\textsuperscript{12} They also analyze data from the National Longitudinal Survey of Youth 1979 Cohort (NLSY79), which has a much younger sample, with a median age of 31. The relation between equity investment and income risk is unambiguously negative in this sample. Angerer and Lam (2009) also use NLSY79 to find a negative relationship between permanent income risk and risky investment and little or no impact of transitory income risk. These patterns point to a more complex relationship between income risk and risky investment.

Table 1 presents evidence suggesting that the equity investment increases with income risk in later stage of working life. I use data from SCF and compare equity investment near retirement between the ages of 55 to 60 to that between 30 to 34. I use the household head’s industry to proxy for income risk. I classify public sector workers as a low-income-risk group and, absent detailed industry classification, I classify the rest non-public sector workers into a high-income-risk group.\textsuperscript{13} The positive and significant interaction term "near retirement × non-public sector" suggests a higher propensity to participate in stock market among higher income-risk workers than among lower income-risk workers near retirement, compared to no such significant differences in early working life. The effect of the interaction term on equity investment is unambiguously negative if the income risk is very high. See the coefficients in Table 5 for the dummy variable named “High Inc Risk.” It identifies whether variance for income growth is in the top quartile.

\textsuperscript{11}See also Vissing-Jorgensen (2002) and Guiso, Jappelli, and Terlizzese (1996).
\textsuperscript{12}The relation is unambiguously negative if the income risk is very high. See the coefficients in Table 5 for the dummy variable named “High Inc Risk.” It identifies whether variance for income growth is in the top quartile.
\textsuperscript{13}See Campbell, Cocco, Gomes and Maenhout (1999) for estimates of income risk by industry. The public version of the SCF dataset classifies those in non-public sector industries into six groups without regard for the riskiness of their income stream.
investment is positive but insignificant among the market participants. The investment in equities is imputed based on categorical responses in SCF. The resulting poor measurement could be one reason for these insignificant estimates. I include time dummies, cohort dummies, income controls, liquid asset controls, household characteristics, and expected returns based on experienced stock market returns as computed in Malmendier and Nagel (2011) as controls in regressions in Table 1. I exclude retirees and part or sole owners of businesses. The Internet Appendix further describes these controls.

3 One-period portfolio problem with labor income

Gollier and Pratt (1996) show that the agents with HARA class utility functions subject to an uncompensated rise in background risk reduce risky asset demand. By contrast, the DA preferences can generate rising risky asset demand with the increasing background risk. This pattern applies when the background risk is low enough and the decline in the demand ensues once the background risk crosses a threshold. I illustrate this phenomenon for DA preferences in a one-period terminal consumption problem and contrast it with the implications from a CRRA model. The features of the one-period problem extend to the life-cycle setup. The one-period setting provides a clean way to isolate and explain different drivers of the risky asset demand.

The one-period problem is as follows. The investor has savings worth $A_t$ dollars that he optimally splits between a risky asset and a risk-free asset at time $t$. The investor also receives non-tradable positive labor income $Y_{t+1}$ at time $t + 1$. The investor consumes all of the terminal wealth $W_{t+1}$ at time $t + 1$.

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Starting in 2004, SCF collects the actual percentage of account balances invested in equity for most of the accounts. However, the equity portion of the combination and other mutual funds is still imputed. The half of the value of the combination funds and the complete value of other mutual funds is assigned to equity holdings.
\[ W_{t+1} = A_t R_{p,t+1} + Y_{t+1} \]

\[ R_{p,t+1} = R_{e,t+1} x_t + R_f \quad \text{Where,} \quad R_{e,t+1} = R_{t+1} - R_f \]

\[ \ln(R_{t+1}) = \ln(R) + \epsilon_{r,t+1} \]

\[ \epsilon_{r,t+1} \sim N(-\sigma_r^2/2, \sigma_r^2). \]

The risky return \( R_{t+1} \) has a log-normal distribution with mean \( \overline{R} \). The risk-free rate is \( R_f \).

I set \( \overline{R} - R_f = 6\% \), \( R_f = 2\% \), and \( \sigma_r = 18\% \). The portfolio weight is constrained such that \( 0 \leq x_t \leq 1 \). I present the impact of the mean-preserving spread in income risk by setting the expected labor income at \( \overline{Y} \) and varying the standard deviation, \( \sigma_y \), of the log-labor income. Further, I set return and income shocks independent of each other.

\[ Y_{t+1} = \overline{Y} \exp(\epsilon_{y,t+1}) \]

\[ \epsilon_{y,t+1} \sim N(-\sigma_y^2/2, \sigma_y^2). \]

\( \mu_{da,t} \) and \( \mu_{crra,t} \) in Eqs. (1) and (2) are the certainty equivalents under the DA and CRRA preferences, respectively. The outcomes below \( \mu_{da,t} \) are disappointing and the outcomes above \( \mu_{da,t} \) are elating:

\[ \mu_{da,t}^{1-\gamma} = E_t \left[ W_{t+1}^{1-\gamma} \right] - \theta E_t \left[ \left( \mu_{da,t}^{1-\gamma} - W_{t+1}^{1-\gamma} \right) I(W_{t+1} < \mu_{da,t}) \right] \quad (1) \]

\[ \mu_{crra,t}^{1-\gamma} = E_t \left[ W_{t+1}^{1-\gamma} \right]. \quad (2) \]

The two parameters \( \gamma \) and \( \theta \) determine the risk-reward attitudes under the DA preferences. I refer to \( \gamma \) as the curvature parameter and \( \theta \) as the disappointment-aversion parameter hereafter. Further, I refer to \( W_{t+1}^{1-\gamma}/(1 - \gamma) \), the valuation of wealth in a specific state, as utility. I set \( \gamma = 5 \) and \( \theta = 1 \). I set the CRRA model’s curvature parameter equal to that of the DA.
The correlation between labor income and returns is zero.

The agent’s objective is to maximize the CE by choosing the optimal portfolio weight, $x_t$. I compute the optimal $x_t$ as a function of $\sigma_y$ and plot it in Fig. 2. The optimal risky asset demand increases with the background risk under the DA model. This trend, however, flips once the income risk crosses a threshold and the risky allocation turns conservative with the increasing background risk. The portfolio weight, on the contrary, is decreasing over the entire range for the CRRA preferences.\(^{15}\)

**Proposition 3.1.** If the DA agent is sufficiently disappointment averse such that $\theta > \theta^*$ and earns a risk-free income, he (1) does not invest in the risky asset. If the DA agent earns risky income that is independent of asset returns, the (2) optimal risky investment is positive for all $\theta \geq 0$. However, the requirement is that $\overline{R}_e,t+1 > 0$.

\[
\theta^* = -\frac{E_t [R_e,t+1]}{E_t [R_e,t+1 | R_e,t+1 < 0] \times \Phi(R_e,t+1 < 0)}
\]

\(^{15}\)The constraint $x_t \leq 1$ is binding at low income risk and high $\overline{Y}/A_t$ for both the DA and CRRA models.
Proof. See Appendix A.

Fig. 2 shows that the DA agent can increase risky investment as the income turns risky. Proposition 3.1 formally shows that such a pattern exists for all the cases in which $\theta > \theta^*$. The first part of the proposition is Proposition 2.1 from Ang, Bekaert, and Liu (2005) with the risk-free income included. However, when income is risky and independent of returns, an infinitesimal investment in risky asset improves the welfare as the marginal utility and returns are independent. The result is that the agent prefers to invest in the risky asset as he transitions from risk-free to risky income even if the expected income declines. Further, Proposition 3.2 shows that the first order risk aversion is not sufficient for the agent with $\gamma = 0$ if $0 \leq \theta < \theta^*$ and the agent invests all his savings in the risky asset irrespective of income-return correlation.

**Proposition 3.2.** If the DA preference parameters are such that $\gamma = 0$ and $0 \leq \theta < \theta^*$, and the agent invests a fraction $x_t$ of the savings in a risky asset so that $x_t \leq 1$, the agent invests all of his savings in the risky asset.

Proof. See Appendix A.

The patterns in risky allocation shown in Fig. 2 relate to the life-cycle problem and the aversion to risky investment induced by declining income risk. In the life-cycle setting, the stock of human capital is large compared to the savings in early through mid working life. The weak correlation between income and stocks implies a lack of hedging motive and makes human capital more bond-like than stock-like. Taken together, the two features imply a higher allocation of savings toward risky assets. The income risk is also large at this life stage, because the permanent shocks affect the entire remaining income stream. In addition, the spread in both the permanent and transitory shocks is bigger in early working life (Section 7.5). This case is similar to the one with high $\overline{Y}/A_t$ in Fig. 2. The high income risk implies the relevant region is the one where income risk is negatively related to the risky share. A decline in income risk and a smaller remaining quantity of human capital characterize the late working life and
retirement. This stage is similar to the one with low $\bar{Y}/A_t$ and low $\sigma_y$ in Fig. 2. Although the drop in human capital yields similar effects under both models, the drop in income risk also drives risky allocations lower under the DA model. The negative relation between risky share and income risk at this life stage is unique to DA preferences. Below, I illustrate the drivers of risky allocation in the one-period problem. Similar mechanisms also apply in the life-cycle setting.

### 3.1 Drivers of non-monotonic asset demand

The first-order conditions (FOCs) in Eqs. (3) and (4) for CRRA and DA preferences are one of the ways to understand the phenomenon in Fig. 2.\(^{16}\)

\[
0 = E_t[u'(W_{t+1})R_{e,t+1}] \quad (3)
\]

\[
0 = E_t[u'(W_{t+1})D_{t+1}R_{e,t+1}] \quad (4)
\]

Where

\[
D_{t+1} = \frac{1 + \theta I(W_{t+1} < \mu_{da,t})}{1 + \theta \Phi(W_{t+1} < \mu_{da,t})} \text{ and } u'(W_{t+1}) = W_{t+1}^{-\gamma} \quad (5)
\]

The concave utility, $u(W_{t+1})$, and convex marginal utility, $u'(W_{t+1})$, in Eq. 3 increase the weights on adverse return outcomes as the wealth outcomes disperse with the increasing income spread. The result is a negative value for the expectation in Eq. 3 unless the asset allocation is re-optimized. The agent can achieve optimality only by reducing the risky allocation and hence the pattern for CRRA preferences in Fig. 2. Hereafter, I refer to the negative impact of background risk on risky allocation as the standard effect. It is driven by the on-average association of unfavorable return outcomes with the low wealth percentiles and the nature of the marginal utility, $u'(W_{t+1})$.

Fig. 3 shows the average return outcome associated with each of the 10 wealth percentile bins. For instance, the lowest curve graphs the average excess return corresponding to wealth...\(^{16}\)I thank the anonymous referee for suggesting this approach.
Figure 3: The average excess returns, $R_{a,e,t+1}$, with an increasing mean-preserving spread in labor income, $\sigma_y$, that corresponds to $(a - 10)\%$ to $a\%$ percentile range of the terminal wealth, $W_{t+1}$. $a$ takes values $10, 20, \ldots, 100$. The portfolio weight is set to optimal value at every income risk level for the DA preferences. Similar patterns emerge if the portfolio weight is held constant or optimal weights under CRRA preferences are used. The income and return shocks are uncorrelated. The curves at $\sigma_y = 0$ are in increasing order of the value of “$a$.”

in the $0 – 10^{th}$ percentile range. Fig. 3 shows that the favorable return outcomes increasingly associate with the lower wealth percentiles until the income risk crosses a threshold. The trend, however, inverts for income risk higher than this threshold. Such patterns apply if the correlation between income and returns is zero or positive and is a characteristic of the non-tradable nature of labor income.

The association and the subsequent retraction of favorable return outcomes from the disappointing states and the extra emphasis on these states coded in $D_{t+1}$ (Eq. 5) are consistent with the non-monotonic asset demand. I refer to this effect as the DA effect. $D_{t+1}$ is the additional component of marginal utility unique to DA preferences in Eq. 5 that overweights the disappointing states. I rewrite the FOC in Eq. 4 in terms of twisted distribution in Eq. 6 that overweights disappointing outcomes and in terms of twisted return premium $\hat{R}_{e,t+1}$ in Eq. 7. $\hat{p}(W_{t+1})$ is the twisted version of the data-generating distribution $p(W_{t+1})$. The hat (ˆ) represents quantities under the twisted distribution.
0 = \hat{E}_t [u'(W_{t+1}) R_{e,t+1}] \quad \text{Where } \hat{p}(W_{t+1}) = D_{t+1} \times p(W_{t+1}). \quad (6)

\hat{R}_{e,t+1} = \frac{-Cov_t [u'(W_{t+1}), R_{e,t+1}]}{\hat{E}_t [u'(W_{t+1})]} \quad \text{Where } \hat{R}_{e,t+1} = \hat{E}_t [R_{e,t+1}]. \quad (7)

\hat{R}_{e,t+1} = \frac{\hat{R}_{e,t+1} + \theta \Phi \hat{R}_{e,d}}{1 + \theta \Phi} = Cov_t [R_{e,t+1}, D_{t+1}] + \hat{R}_{e,t+1} \quad (8)

\text{Where, } \hat{R}_{e,d} = E_t [R_{e,t+1}|W_{t+1} < \mu_{da,t}]. \quad (9)

Eqs. 6 and 7 focus on the interaction between \(D_{t+1}\) and returns. In fact, \(\hat{R}_{e,t+1}\) captures the covariance between returns and the asymmetric weighting scheme, \(D_{t+1}\). \(\hat{R}_{e,t+1}\) is also a weighted sum of \(\hat{R}_{e,t+1}\) and \(\hat{R}_{e,d}\) in 1 to \((\theta \Phi)\) proportion. Eq. 8 presents both of these interpretations. The disappointment return premium, \(\hat{R}_{e,d}\), and disappointment probability, \(\Phi\), the two drivers of \(\hat{R}_{e,t+1}\) represent two separate components of the DA effect. The variation in \(\hat{R}_{e,d}\) even if \(\Phi\) and the asset’s return distribution are fixed is unique to the setup that includes background risk. In absence of such non-insurable risk, the wealth and the return percentiles align and \(\hat{R}_{e,d}\) is fixed if \(\Phi\) and the return distribution are fixed.

The log-linear approximation of Eq. 6 in Eq. 10 separates the terms that drive the standard and the DA effects. The term \(\rho\) approximates the standard effect and represents the elasticity or sensitivity of terminal consumption with respect to financial wealth. The twisted log return premium, \(\hat{r}_{t+1} - r_f + \hat{s}_{r,t}^2 / 2\), summarizes the DA effect. The replacement of the quantities under the twisted distribution in Eq. 10 with those under the data-generating distribution yields the approximate formula for CRRA preferences. The result is a constant return premium, and only the standard effect applies as the income risk varies. Campbell and Viceira (2002) use such approximation for CRRA preferences to show the condition \(\gamma > 1 / \rho\) is sufficient for a negative impact of the rising mean-preserving income spread on risky investment sourced.

\(^{17}\text{The appropriate notation, in line with the rest of the article, for the premium in disappointing states for a given } \Phi \text{ is } \hat{R}_{e,d,\Phi,t+1}. \text{ However, I use } \hat{R}_{e,d} \text{ for easy reading. When necessary, I emphasize the dependence on } \Phi \text{ and do not stress that } \hat{R}_{e,d} \text{ is a time } t \text{ measurable statistic of a time } t+1 \text{ measurable variable.}\)
through the rising $\rho$.\footnote{This condition is a sufficient condition and is based on the approximate formula for risky asset demand under CRRA preferences. Gollier and Pratt (1996) show that a mean-preserving increase in background risk yields a decline in risky asset demand for all HARA class utility models.} The same condition also approximately ensures a similar trend for $\rho$ and the standard effect for DA preferences (Fig. 4). I include the derivation of Eq. 10 and provide details in the Internet Appendix.

\[
    x_t \approx \frac{1}{\rho} \left( \frac{\hat{r}_{t+1} - r_f + \hat{\sigma}_{r,t}^2/2}{\gamma \hat{\sigma}_{r,t}^2} \right) \quad \text{if } \hat{\sigma}_{y,t} \approx 0. \tag{10}
\]

\[
    \frac{1}{\rho} = 1 + \frac{\exp \left( E_t \left[ \ln(Y_{t+1}) \right] \right)}{\exp \left( E_t \left[ \ln(A_t R_{p,t+1}) \right] \right)} \quad 0 < \rho < 1. \tag{11}
\]

I graph the DA effect and its components in Fig. 4. The hump-shaped pattern in $\hat{R}_{e,t+1}$ is consistent with the pattern in the risky portfolio weight. In addition to the dominant standard effect, the DA effect also helps drive the decline in portfolio weight at high income risk. The patterns in $R_{e,d}$ and $\Phi$, the DA effect’s components, are also consistent with the variation in $\hat{R}_{e,t+1}$.

The pattern in $R_{e,d}$ is hump shaped (Fig. 4) and similar to the pattern in $\hat{R}_{e,t+1}$. The two are positively related for a fixed $\Phi$. When the income is certain, returns in disappointing states are strictly lower than those in elating states as the rankings of wealth and return align when the background risk is absent. This pattern, in fact, generates the lowest possible value of $R_{e,d}$ for a given $\Phi$. Once the income turns risky, it drives a wedge between the rankings of wealth and return outcomes. The effect is that not only the low(high) returns but also the high(low) returns increasingly associate with the lower(higher) wealth percentiles. Fig. 3 reflects this pattern because all curves are drawn toward the median return outcome. The increasing mix of higher returns in disappointing states or low wealth percentiles increases $R_{e,d}$. The pattern, however, flips at high income risk. The retraction of advantageous returns from the low wealth percentiles at high income risk is an artifact of the lower bound on income set at zero.\footnote{The bulk of income distribution increasingly concentrates near zero for high $\sigma_y$, an artifact of a mean-} I illustrate the latter effect and provide details in the Internet Appendix.
Figure 4: The elasticity of consumption with respect to financial wealth (top-left panel), the twisted return premium (bottom-left panel), the disappointment return premium (top-right panel) and the disappointment probability (bottom-right panel), as the income risk increases. These graphs are for the case where the expected labor income and savings are equal. The pattern in the twisted log return premium (not included) \( \hat{r}_{t+1} - r_f + \frac{\sigma^2_{r,t}}{2} \) is similar to the pattern in \( \hat{R}_{e,t+1} - R_f \) (bottom-left panel).

The \( \Phi \), the other component of the DA effect, modifies the weight on \( \bar{R}_{e,d} \) in Eq. 8. The \( \Phi \) has a U-shaped variation (Fig. 4) and has two opposing effects on \( \hat{R}_{e,t+1} \). On one hand it attenuates, the hump-shaped variation induced in \( \hat{R}_{e,t+1} \) due to a similar variation in \( \bar{R}_{e,d} \), but on the other hand, it also aids in the generation of the hump-shaped variation in \( \hat{R}_{e,t+1} \) as \( \bar{R}_{e,d} < \bar{R}_{e,t+1} \). The second effect is the negative relation between \( \hat{R}_{e,t+1} \) and \( \Phi \) for a fixed \( \bar{R}_{e,d} \), and it holds if the income and return shocks are independent or positively correlated. The variation in \( \bar{R}_{e,d} \) is, however, larger compared to the variation in \( \Phi \) and is the driving force behind the variation in \( \hat{R}_{e,t+1} \). The same applies in the life-cycle setting where the dominant DA effect is the variation in \( \bar{R}_{e,d} \). In effect, the appetite for risky investment rises with income risk because of the association of advantageous returns with low-wealth states and also the emphasis on such states built into the DA model. The usual pattern reverts as the standard effect soon dominates, and the DA effect also contributes in the same direction at high-enough preserving spread on income that is bounded below at zero. Thus, the return and wealth rankings increasingly align at low wealth percentiles, producing a retraction of advantageous returns from the low wealth percentiles.
income risk.

\[ \Lambda_t = E_t [\Omega_{t+1}] - \Phi \times \theta [\Lambda_t - E_t [\Omega_{t+1} | \Omega_{t+1} < \Lambda_t]] \tag{12} \]

Where \( \Lambda_t = u(\mu_{da,t}) \) and \( \Omega_{t+1} = u(W_{t+1}) \). \( \tag{13} \)

I rewrite Eq. 1, the formula for the DA welfare, in Eq. 12. It shows that besides \( \theta \), the value of \( \Phi \) depends only on the distribution of \( u(W_{t+1}) \). In particular, \( \Phi \) is negatively related to the left skewness of \( u(W_{t+1}) \) because of the asymmetric emphasis on adverse outcomes. The inclusion of risky income that is independent of returns lowers the skewness of wealth and turns \( u(W_{t+1}) \) more left-skewed. The result is a decline in \( \Phi \) until the income risk crosses a threshold. Thereafter, the right skewness of income distribution ultimately dominates as the income risk increases and results in the rising portion of the pattern in \( \Phi \). I provide more details about the effect of income risk on the skewness of \( u(W_{t+1}) \) and the effect of skewness of \( u(W_{t+1}) \) on \( \Phi \) in the Internet Appendix.

The decline in wealth skewness with the introduction of risky income and the concurrent rise in a DA investor’s risky investment are suggestive of a diversifying impact of the income risk. However, the welfare, \( \mu_{da,t} \), strictly declines with the rising mean-preserving spread in income. The rising riskiness of income does not have any benefit for the DA agent, and he is strictly worse off. I include the proof in Appendix A.

In the following sections, I set up and solve the full-fledged life-cycle problem for the DA and CRRA preferences and use the twisted premium as a summary of the DA effect to explain contrasting features.

4 The life-cycle problem and solution method

The life-cycle problem is as follows. The agent at time \( t \) has savings \( A_t \) that are transformed into future tradable wealth according to Eq. (14). The benefit of saving is the portfolio
return $R_{p,t+1}$, which depends on the chosen portfolio weight $x_t$ and the stochastic excess return $R_{e,t+1}$ on the risky asset, and a known risk-free return $R_f$. The term $W_t$ that I refer to as wealth or tradable wealth is also known as cash-on-hand in the life-cycle literature (following Deaton (1991)). $C_t$ is consumption at time $t$. $A_t$ is the difference between wealth, $W_t$, and consumption, $C_t$. In addition to the return on savings, the agent also receives exogenous non-tradable labor income $Y_{t+1}$ and is left $(1 - h_{t+1})Y_{t+1}$ after covering housing-related expenditures:

$$W_{t+1} = A_t R_{p,t+1} + (1 - h_{t+1})Y_{t+1}, \text{ where, } A_t = W_t - C_t \quad (14)$$

$$R_{p,t+1} = R_{e,t+1}x_t + R_f$$

$$R_{e,t+1} = R_{t+1} - R_f.$$

The agent cannot borrow at the risk-free rate or short the risky asset. Thus, the portfolio weight lies between 0 and 1. In addition, current consumption cannot exceed current wealth.

$$C_t \leq W_t, \quad x_t \geq 0, \quad x_t \leq 1. \quad (15)$$

The risky asset at the agent’s disposal is a value-weighted market index (henceforth also referred to as stock). The raw returns on the market index follow a log-normal distribution with an expected rate of return $\overline{R}$ per period (Eq. (16)). The standard deviation of log returns is $\sigma_r$. I assume the investor faces a constant investment opportunity set:

$$\ln(R_t) = \ln(\overline{R}) + \eta_t, \quad \text{where, } \eta_t \sim N(-\frac{\sigma_r^2}{2}, \sigma_r^2). \quad (16)$$

The real labor income has a deterministic component $l_t \equiv l(t, Z_t)$ that depends on age and
other personal characteristics $Z_t$. Eq. (17) describes the labor income process until retirement:

$$Y_t = \exp(l_t + \nu_t + \epsilon_t) \quad \forall t \leq K$$

$$\nu_t = \nu_{t-1} + u_t \quad \text{where} \quad u_t \sim N(0, \sigma_u^2), \quad \epsilon_t \sim N(0, \sigma^2_\epsilon).$$

In addition to the deterministic trend $l_t$, labor income is also determined by a permanent component $\nu_t$ driven by shocks $u_t$ and an idiosyncratic component $\epsilon_t$. The two shocks distributed as $N(0, \sigma_u^2)$ and $N(0, \sigma^2_\epsilon)$ are uncorrelated. The transitory shock $\epsilon_t$ is also uncorrelated with the stock return shock $\eta_t$. I allow for correlation between $u_t$ and $\eta_t$ in the extension of the benchmark model.

The retirement income is a fixed fraction of permanent income in the last working period before retirement:

$$\ln(Y_t) = \ln(\lambda) + l_K + \nu_K; \quad \forall K + 1 \leq t \leq T$$

$$\Rightarrow Y_t = \lambda \exp(l_K + \nu_K), \quad K = \text{the last period agent works}.$$

The preference specification follows Epstein and Zin (2001). I set the reciprocal of the curvature parameter, $\gamma$, equal to the elasticity of intertemporal substitution (EIS), $\psi$. In section 8 I consider the case where the two differ. The value function $J_t$ in Eq. 27a summarizes the preference over gambles and static choices. $\beta$ captures the time rate of preference, and $p_t$ represents the probability of surviving to the next period, $t+1$, conditional on having survived.
The welfare $J_t(W_t, \nu_t)_{1-\gamma}/(1 - \gamma)$ is concave in wealth, $W_t$, if $\gamma > 0$ and $\theta \geq 0$. The proof is in the Internet Appendix and applies to the general case involving a bequest motive. The proof involves Proposition 4.1, a non-trivial component. The complexity arises because of the fact that the disappointing and elating outcomes of two gambles need not occur in the same states of the world. This article is the first to show concavity of welfare under the DA model in the presence of non-tradable income.

**Proposition 4.1.** If $g_{t+1}(W_{t+1}, \nu_{t+1})_{1-\gamma}/(1 - \gamma)$ is concave (quasi-concave) in wealth $W_{t+1}$ and $\theta \geq 0$, the agent’s welfare, $\mu_t(\cdot)_{1-\gamma}/(1 - \gamma)$, satisfies the following inequality for any two distinct wealth gambles, $W_{1,t+1}$ and $W_{2,t+1}$, and for all $0 < \lambda < 1$, where $W_{\lambda,t+1} = \lambda W_{1,t+1} + (1 - \lambda) W_{2,t+1}$:

$$
\lambda \frac{\mu_t(g_{t+1}(W_{1,t+1}, \nu_{t+1}))_{1-\gamma}}{1 - \gamma} + (1 - \lambda) \frac{\mu_t(g_{t+1}(W_{2,t+1}, \nu_{t+1}))_{1-\gamma}}{1 - \gamma} < (\leq) \frac{\mu_t(g_{t+1}(W_{\lambda,t+1}, \nu_{t+1}))_{1-\gamma}}{1 - \gamma}.
$$

**Proof.** See Appendix A for a sketch of the proof and the Internet Appendix for the unabridged version of the proof. 

The optimization program ends at age 100 as $p_{100} = 0$. I numerically solve for the optimal policy rules for all other periods by backward iteration. The value function scales with the permanent income $e^{\nu_t}$. The normalized value function $J_t^{\nu}(W_t^{\nu}) \equiv J_t e^{-\nu_t}$ depends only on one state variable, $W_t^{\nu} \equiv W_t e^{-\nu_t}$, the normalized wealth. I use simulated draws of shocks to
compute integrals in Eq. 19b. This step along with the fixed point problem in Eq. 19b severely slow the solution speed. I describe the numerical method in more detail in Appendix B.

5 Calibration

I follow Cocco, Gomes, and Maenhout (2005) and set the relative risk aversion $\gamma_{crra}$ for the CRRA preference model at 10. The DA and CRRA preferences are not equivalent, because the CRRA model implies little risk aversion over small gambles unless $\gamma_{crra}$ is extremely high. However, a high $\gamma_{crra}$ implies unrealistic risk aversion over large gambles. Consequently I match the DA preference parameters at only one age. I choose age 64, just before retirement, to perform the match. The fact that the portfolio implications of the two preferences are in stark contrast right after retirement motivates this choice. I choose $\gamma_{DA} = 6$ and $\theta = 0.65^{20}$ so that the average savings and risk allocations are almost the same under the two models at age 64. Thus, the ratio of human capital to financial wealth and the risk-reward attitudes are similar prior to retirement under the two preferences.$^{21}$

The agent faces mortality risk from age 66 until age 100, and dies with probability 1 at age 100. I obtain estimates for $p_t$ from Arias (2006). The adult age starts at 20 for agents without college degrees and at age 22 for those with college degrees. All agents retire at age 65 irrespective of their educational attainment.

I follow Cocco, Gomes, and Maenhout (2005) and estimate the labor income process in Eqs. (17) and (18) using Panel Study of Income Dynamics (PSID) data from 1970 to 2009. I provide details of the estimation procedure in the Internet Appendix. I fit a third-order polynomial in age to the labor income profile implied by the age dummies for each of the three education groups. The polynomial coefficients and the estimates of the other parameters of the income process are in Table C.1 in Appendix C. I set the correlation between income

$^{20}$The estimates by Choi, Fisman, Gale, and Kariv (2007) for $\theta$ range from 0 to 1.9. These values are the 5th and the 95th percentiles in Table 1 for the symmetric case.

$^{21}$I thank an anonymous referee for suggesting this simpler method of matching the CRRA and DA preference parameters.
and returns to zero for the benchmark case.\footnote{The estimates of the correlation between permanent income and returns are positive but small.} In section 7.1 I consider the case with positive correlation between permanent income and returns.

I follow Gomes and Michaelides (2005) and estimate the fraction of annual income used for mortgage and rent payments, $h_t$, as a function of age, $t$, using the updated PSID sample from 1970 to 2009. I use a truncated version of this polynomial in Eq. 20 for $h_t$. The function truncates to zero for all ages 81 and above. The details about estimation are in the Internet Appendix.

$$h_t = \max \left( 0.534 - 2.384 \times 10^{-2} \cdot t + 4.539 \times 10^{-4} \cdot t^2 - 2.990 \times 10^{-6} \cdot t^3, 0 \right) \quad (20)$$

I follow the life-cycle portfolio allocation literature and set the equity premium at $4\%$.\footnote{The real returns on the assets are lower after considering holding costs and taxes. Jagannathan, McGrattan, and Scherbina (2000) argue that the equity premium is much lower than 6\% due to diversification costs, taxes, and liquidity premium for bills. Claus and Thomas (2001) and Fama and French (2002) also argue for a low value of the equity premium.} I set the risk-free rate at $2\%$, and the standard deviation of stock returns at $18\%$.

## 6 Benchmark case

In the following subsections, I analyze the case for high school graduates. Similar results follow for the college and no-high-school groups. Table 2 lists all parameters for this case.
6.1 Policy rules

The retirement policy rules for the optimal risky asset weight are shown in Fig. 5. These are functions of (tradable) wealth, $W_t$, scaled by the permanent component of income, $e^{\nu t}$. They feature conservative allocation with (1) rising tradable wealth and (2) age. The drastic conservative attitude, the third feature of the DA preference model, beginning right at retirement, differentiates the DA model from the CRRA model. This conservative attitude overlays the rest of the retirement period under the DA preference model.

The retirement income is risk free and is a multiple of the permanent income $e^{\nu t}$ in the last working period. These two aspects are responsible for both the common as well as the differentiating features of the investment rules. The risk-free retirement income creates a risk-free but non-tradable bond holding in the agent’s portfolio and makes the investment rules for the tradable wealth, $W_t$, more aggressive. In effect, the aggressive tilt attempts to offset the non-tradable bond holding. However, in the case of a large value for the tradable wealth, the relative importance of the non-tradable bond is diminished and yields risky allocations that are more conservative. The result is a declining pattern in investment rules for a given age. Further, the stock of the riskless asset created by non-random retirement income also declines with age. This decline generates the second pattern, an increasingly conservative allocation with age for a given level of scaled wealth, $W'_t$. In the next subsection, I illustrate the determinants of the third feature, unique to the DA preferences, with the help of simulations.

The risky-allocation rules for the DA and CRRA preferences share similar features over the agent’s working life. The policy rules are decreasing in scaled wealth and become conservative as the middle-aged agent grows older. In addition, the policy rules for the DA model grow aggressive with age in early life, similar to that for the CRRA preference model. This trend, as Cocco, Gomes, and Maenhout (2005) note, is due to the rise in human capital stock in early life, because income is lower but income growth is steeper over this phase of life.

The consumption policy rules in terms of scaled consumption as a function of scaled wealth are similar for the DA and CRRA preferences. The consumption rules are concave in tradable
wealth and also indicate that the agent decumulates savings in retirement. This pattern is due to increasing mortality-induced impatience. The chances of surviving to the next year decline with increasing age and the mortality is certain at age 100. Thus, the benefits to savings depreciate with age and consequently accelerate the decumulation. However, the decumulation motive is stronger for the DA than for the CRRA agent. The difference is driven by the more conservative investments of the DA agent through retirement that yield lower average benefits to savings. The rules are similar over working life under both preferences, except that the DA agent accumulates savings at a more aggressive rate than does the CRRA agent.

6.2 Simulations

I simulate 10,000 independent income paths and obtain consumption and investment decisions along these paths using optimal policy rules at every age. I contrast the DA and CRRA agents’ behavior using average stock allocation, consumption, and savings over life.

The risky allocations decline with age for both the DA and CRRA agents, and the DA
agent’s drastic conservative approach to investment in retirement is noticeable starting at age 65 (Fig. 6, top-left panel). This attitude is due to the resolution of permanent income uncertainty in the last working period that sets the retirement income for remaining life periods. The result is a resolution of uncertainty in a large stock of human capital around retirement as observed in the drop in $\sigma_t(\ln(J_{t+1}))$. In addition to the sizeable stock of retirement income, the agent’s savings are also the highest around the retirement period. Thus, the phenomenon around retirement is similar to that associated with the passage from modest income risk to no income risk in the one-period problem with moderate values of the income-to-savings ratio.

Fig. 6 shows that the twisted premium, $\tilde{R}_{e,t+1}$, the summary of the DA effect, drops by 2.2% as income transitions from uncertainty to certainty with the drop of 8.6% in disappointing return premium, $\overline{R}_{e,d}$, being the dominant driver for the change. The rise in disappointment probability, $\Phi$, is small (increment of 0.08) and contributes to the decline in $\tilde{R}_{e,t+1}$ to a limited extent. The $\Phi$ is related to the skewness of utility $u(J_{t+1}) \equiv J_{t+1}^{1-\gamma}/(1-\gamma)$.

In effect, the extra emphasis on disappointing states built into the DA model coupled with the increasing association of adverse returns with the low wealth realizations (reflected in $\overline{R}_{e,d}$) dampens the appetite for the risky asset. The drastic decline in background risk creates sharp changes in the return and wealth associations and consequently the sharp change in risky allocation. The decline also highlights the positive relationship between risky investment and uninsurable risk in late working life that is unique to DA preferences. These preferences can generate such a drop into retirement as long as the total uninsurable or background risk through income and other possible sources declines into retirement. The post-retirement phase is associated with many risks and most notably the health risk. I explore its impact on stock allocation in section 7.6. Further, the risky allocations rise in retirement in the absence of a bequest motive. In section 7.4 I incorporate bequest in the DA preference model.

The DA and CRRA agents save little over early working life (Fig. 7, left panel) and hold

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24 Similar to the one-period problem, the right-skewness of $u(J_{t+1})$ is positively related to $\Phi$. See the Internet Appendix and section 3 for a detailed treatment of the one-period problem.

25 The changes in family composition can also act as additional sources of risk (Love (2010)).
Figure 6: The cross-sectional average of the fraction of savings invested in risky assets (top-left panel) and the standard deviation of the log of the gamble over the next period value function (bottom-left panel) at all ages over the life of DA and CRRA agents. These averages are conditioned on positive savings. The cross-sectional average of the twisted return premium and disappointment return premium conditional on positive savings for the DA agent are in the top-right panel. The cross-sectional averages of the skewness of utility and the disappointment probability for the DA agent are in the bottom-right panel.
Figure 7: The cross-sectional mean savings (left panel) and consumption (right panel) at every age over the life of the CRRA and DA agents.

a large stock of human capital they are yet to encash. Consequently, the stock allocations are high but limited by the borrowing constraint over this phase of life. The risky investment behavior is as if the human capital is more bond-like than stock-like. Both agents accumulate savings throughout working life and reduce risky investment as human capital declines with age.

The overall consumption patterns are similar for both the agents. However, the relatively uneven consumption profile for the DA model is a result of over-dependence, especially in retirement periods, on risk-free assets for consumption smoothing. The aversion to attractive but risky stock imposes a higher cost to achieve a given level of smoothing, and hence the more uneven pattern.

7 Model extensions

7.1 Labor income correlated with stock market returns

I consider the effect of correlation on the DA agent’s risky allocations. Benzoni, Collin-Dufresne, and Goldstein (2007) use cointegration between income and dividends to obtain
aversion to stocks in early working life. Cocco, Gomes, and Maenhout (2005) find that correlated labor income helps explain lower stock investments in the early stages of working life. I follow Gomes and Michaelides (2005) and set the correlation between the permanent component of labor income and stock returns at 0.15 and idiosyncratic income shocks uncorrelated with the returns.

The correlation dilutes the semblance of human capital to a bond and makes it more stock-like. The correlation yields hedging demands that diminish stock investment through working life for both the DA and CRRA preferences (Fig. 8, left panel). The retirement income, however, is riskless, which removes any hedging motive in retirement. The rising stock investments as the CRRA agent transitions into retirement reflect these changes. This pattern also applies in the presence of the bequest motive. Such a pattern is contrary to the observed reduction in risky allocation with age. By contrast, the DA model continues to generate a drop even in the presence of correlation. Similar to the benchmark case, the DA agent’s stock allocation drops with the fall in the twisted premium (Fig. 8, right panel). However, hedging demands prior to retirement reduce the size of the drop at the onset of retirement (Fig. 8, left panel).
Figure 9: Average stock allocation (left panel) and twisted return premium (right panel) for DA preferences with different $\theta$. The twisted premium is not defined for $\theta = 1.3$ for age 65 and onward.

### 7.2 Heterogeneity in disappointment aversion

Households exhibit considerable heterogeneity in their portfolio allocation (Curcuru, Heaton, Lucas, and Moore, 2004). The most notable variation is in their choice to participate in the stock market. The various samples of the SCF find that only approximately half of the US households participate in the stock market either directly or indirectly. Further, the age effects also indicate a decline in the propensity to participate in the equity market in retirement (section 2). The international evidence also points to a significant heterogeneity in portfolio choice (Guiso, Haliassos, and Jappelli (2002)). I show that heterogeneity in disappointment aversion can generate substantial variation in portfolio choice and in particular the participation choice in retirement. I graph risky allocations and twisted premium for three different values of disappointment aversion parameter $\theta = \{0.3, 0.65, 1.3\}$ in Fig. 9.\(^{26}\) The rest of the parameters and the setup are the same as that in the benchmark case.

The increment in the weight on disappointing returns with the increase in $\theta$ lowers the twisted premium (Fig. 9, right panel). The increase in $\theta$ from 0.3 to 1.3 lowers the stock investments by about a half over mid working life through retirement (Fig. 9, left panel). Thus,

\(^{26}\)These values are well within the experimental estimates of disappointment aversion parameter provided in Choi, Fisman, Gale, and Kariv (2007).
the variation in $\theta$ alone generates substantial variation in investment behavior. In addition, if $\theta$ is high enough, the agent does not invest in the risky stock at all in retirement. By contrast, the CRRA agent invests at least some amount in the risky stock in retirement for all levels of risk aversion.\(^{27}\) The approximate value of the threshold disappointment aversion, $\theta^*$, depends on the distribution of returns alone (Eq. 21).\(^{28}\) I include the proof in the Internet Appendix. The formula also applies in the presence of the bequest motive described in section 7.4.

Ang, Bekaert, and Liu (2005) show that Eq. 21 provides the exact value of the threshold in the case of the one-period problem without any non-tradable income. The twisted premium is non-positive, and the welfare declines with an infinitesimal investment in the risky asset if $\theta > \theta^*$. In addition, the agent optimally chooses not to short the stock. Consequently, non-participation is the optimal policy.

\[
\hat{\theta}^* = -\frac{E_t[R_{e,t+1}]}{E_t[R_{e,t+1}R_{e,t+1} < 0] \times \Phi(R_{e,t+1} < 0)}
\]  

(21)

Similar to the influence of income risk on risky asset demand in Proposition 3.1, the uninsurable risks in retirement can drive the agent toward risky investment for the cases with $\theta > \theta^*$. The uninsurable risks, such as out-of-pocket medical expenses in section 7.6, are either unrelated or positively related to asset returns and can engender expenses in excess of retirement income. Thus, these risks increase the dependence on financial wealth and may imply only negligible risky allocations. In addition, the loss in welfare from complete non-participation in retirement under the DA model may as well be limited given the nature of these risks. Further, the universal health plans in some countries limit the exposure to out-of-pocket medical expense risks, an important uninsurable risk in retirement (Morgan, Mueller, and Diener (2013), Peterson and Burton (2007)). The DA model can help explain non-participation in retirement in such environments.

The changes in $\theta$ also affect consumption-savings decisions. The average consumption

\(^{27}\)If the agent has CRRA preferences then an infinitesimal investment in risky assets leaves marginal utility independent of returns. Thus, the welfare improves if the agent invests at least some amount in risky assets.

\(^{28}\) $\theta^* = 0.72$ for the asset return distribution in the benchmark case.
through life is lower for higher $\theta$, and the reliance on less rewarding risk-free asset implies that more savings are needed to last through the post-retirement period. Thus, the savings for retirement increase with $\theta$. Further, the agent with higher $\theta$ also runs through the savings faster in retirement.

### 7.3 Defined contribution plan

Employers are increasingly moving away from defined benefit (DB) to defined contribution (DC) plans. I consider a simplified model of DC benefits that excludes all retirement income including social security transfers, and treat DC investment plans as a part of the complete portfolio.\(^{29}\) I use the income process from the benchmark case and set retirement income to zero at age 66 and onwards.

The uninsurable risk dwindles throughout working life without a sizeable drop at any age given the DC plan. This pattern in uninsurable risk generates risky allocations and twisted

\(^{29}\)The agents can borrow against the savings in DC accounts or liquidate their savings from these accounts after paying a penalty.
return premium devoid of a large drop (Fig. 10), unlike that in the benchmark case. The small drop in stock investment at retirement is the result of a transition from uncertain income in the last working period to no income thereafter. The absence of non-financial income also avoids the increasing pattern in risky allocations in retirement for both DA and CRRA preferences. However, the gradual declining pattern in Fig. 10 for the DA model is not unique to DC retirement plans. I obtain a gradual decline in risky allocations with DB retirement plans if the agent is subject to the heteroscedastic income process described in section 7.5.

7.4 Bequest

I add the bequest motive\(^{30}\) to the benchmark case in Eqs. (22a) and (22b). \(\mu_t\) is the certainty equivalent over the future welfare as defined earlier, whereas \(\mu_{b,t}\) is the certainty equivalent of the gamble over bequeathed wealth modified by parameter \(b\). The welfare, \(J_{t}^{1-\gamma}/(1-\gamma)\), is concave in wealth if \(b > 0, \gamma > 0, \) and \(\theta \geq 0\). The proof is in the Internet Appendix. The disappointing and elating states conditional on being alive to \(t + 1\) need not overlap with those in the event of demise in Eq. (22a). However, alternate formulations may not be concave. This article is the first to formulate a bequest motive with DA preferences and show that the proposed formulation yields concave preferences:

\[
\frac{J_t(W_t, \nu_t)^{1-\gamma}}{1-\gamma} = \max_{C_t, x_t} \frac{C_t^{1-\gamma} + p_t \beta \mu_t(J_{t+1}(W_{t+1}, \nu_{t+1})^{1-\gamma}}{1-\gamma} \\
+ (1-p_t) \beta b \mu_{b,t} (W_{t+1}/b)^{1-\gamma} \frac{1-\gamma}{1-\gamma} \\
\mu_{b,t}^T = E_t \left[(W_{t+1}/b)^{1-\gamma} \right] \\
- \theta E_t \left[ (\mu_{b,t}^{1-\gamma} - (W_{t+1}/b)^{1-\gamma} ) I (W_{t+1} < b \mu_{b,t}) \right].
\]

The agent lives until age \(T = 100\), and the terminal valuation, \(J_{T+1}^{1-\gamma}/(1-\gamma)\), due solely to

the bequest is encoded as \( b(W_{t+1}/b)^{1-\gamma}/(1-\gamma) \). The parameter \( b \) captures the intensity of the bequest motive. I follow Gomes and Michaelides (2005) and set \( b = 2.5 \) for the DA model and retain all other parameter values from the benchmark case.\(^{31}\) The twisted premium in the first-order condition\(^{32}\) in Eq. (23) is a weighted sum of that pertaining to the gamble over future welfare and the gamble over bequeathed wealth in the event of demise\(^{33}\):

\[
\hat{E}_t [R_{e,t+1}] = \frac{p_t}{a_t} \text{Cov}_t \left[ C_{t+1}^{-\gamma} R_{e,t+1} \chi_{1,t+1} \right] - \frac{1}{a_t} p_t \text{Cov}_t \left[ (W_{t+1}/b)^{-\gamma} R_{e,t+1} \chi_{2,t+1} \right] \quad (23)
\]

\[
\hat{E}_t [R_{e,t+1}] = \left[ a_{1,t} \hat{E}_{1,t} [R_{e,t+1}] + a_{2,t} \hat{E}_{2,t} [R_{e,t+1}] \right] / a_t
\]

\[
\chi_{1,t+1} = \frac{1 + \theta I(J_{t+1} < \mu_t)}{1 + \theta \Phi(J_{t+1} < \mu_t)}; \quad \chi_{2,t+1} = \frac{1 + \theta I(W_{t+1} < b\mu_{b,t})}{1 + \theta \Phi(W_{t+1} < b\mu_{b,t})}
\]

\[
a_{1,t} = p_t E_t \left[ C_{t+1}^{-\gamma} \right]; \quad a_{2,t} = (1 - p_t) E_t \left[ (W_{t+1}/b)^{-\gamma} \right]; \quad a_t = a_{1,t} + a_{2,t}
\]

\[
\hat{E}_{i,t} [\cdot] = E_t [\cdot] \times \chi_{i,t+1} \text{ for } i = 1, 2.
\]

The average stock investment in retirement drops with age in the presence of the bequest motive (Fig. 11, left panel). However, the twisted premium in the right panel of Fig. 11 is only slightly changed compared to the benchmark case. Similar to the benchmark case, the decline with age in the capitalized value of retirement income generates increasingly conservative investment rules. The motive to bequeath wealth slows the fast pace of savings decumulation observed in the benchmark case. The slower decumulation implies that the average allocation reflects age pattern in investment rules. Thus, the bequest motive drives the increasingly conservative allocations in retirement. However, the twisted premium decline around retirement sets a far more conservative baseline during retirement for the DA model relative to that for the CRRA model. The bequest motive adds to this baseline and makes it more conservative.

\(^{31}\)I use a higher value \( b = 4 \) to avoid an increasing risky-allocation pattern in retirement for the CRRA model.

\(^{32}\)See the Internet Appendix for derivation.

\(^{33}\)The first-order condition can also be written as \( \hat{E}_t [R_{e,t+1}] = - (p_t/k_t) \text{Cov}_t \left[ C_{t+1}^{-\gamma} R_{e,t+1} \right] - ((1 - p_t)/k_t) \text{Cov}_t \left[ W_{t+1}^{-\gamma}/b^{-\gamma}, R_{e,t+1} \right] \). The twisted premium \( \hat{E}_t [R_{e,t+1}] \) is a weighted sum of \( \hat{E}_{1,t} [R_{e,t+1}] \) and \( \hat{E}_{2,t} [R_{e,t+1}] \) with weights \( k_{1,t} \) and \( k_{2,t} \) respectively, where \( k_{1,t} = p_t \hat{E}_{1,t} [C_{t+1}^{-\gamma}], \quad k_{2,t} = (1 - p_t) \hat{E}_{2,t} [W_{t+1}^{-\gamma}/b^{-\gamma}] \) and \( k_t = k_{1,t} + k_{2,t} \).
as the mortality risk increases. In addition, the intent to bequeath wealth also provides an added motive to save, which yields higher savings over working life.

7.5 Heteroscedastic income risk

The labor income is homoscedastic in the benchmark case and in related extensions. Both the permanent, $u_t$, and the idiosyncratic, $\epsilon_t$, shocks add to the uncertainty in log-labor income at age $t$. Fig. 12 graphs the estimates of the spread in log-labor income due to these two shocks, $\sqrt{\sigma^2_u + \sigma^2_\epsilon}$, with age $t$.\(^{34}\) The shocks are heteroscedastic and their size varies with age. The per-period income is more uncertain over young working life than when the agent is closer to retirement.\(^{35}\) Hence, the homoscedastic model is a simplified representation of income uncertainty. This model also implies a large decline in uninsurable risk around retirement and results in an abrupt drop in risky allocation. However, the data indicate a gradual drop in the portfolio share of stocks (section 2). The differences in retirement age across agents or a progressive decline in per-period income risk or the two together can help match the observed

\(^{34}\)See the Internet Appendix for the estimation procedure.

\(^{35}\)See Baker and Solon (2003) for a study of Canadian income data; they find an age-related U-shaped pattern in idiosyncratic income uncertainty.
profile. I show the effects of a tapered decline in per-period income risk that highlights the positive relation between risky investment and income risk in the later phase of working life. This feature is unique to DA preferences.

I use a parsimonious heteroscedastic income process, Eqs. (24a)-(24c), that accounts for both riskless income after retirement at age 65 and a declining per-period income uncertainty. \( l_t \) in Eq. (24a) is the same deterministic component of income described in the benchmark case. I set \( t_{stop} = 66 \) and \( t_{start} = 20 \). This sets \( h_\epsilon = h_u = 1 \) at age 20 and \( h_\epsilon = h_u = 0 \) right after retirement at age 66. The per-period spread in log-income, \( \sqrt{\sigma_\epsilon^2 h_\epsilon^2(t) + \sigma_u^2 h_u^2(t)} \), drops linearly over the working life. The retirement income process after age \( K = 65 \) and other parameters are the same as in the benchmark case. The bequest intensity, \( b \), is set at 2.5, as in section 7.4 for the DA model.\(^{36}\) The estimates of \( \sigma_\epsilon \) and \( \sigma_u \) for this heteroscedastic process are in Table C.2 in Appendix C. I provide details of the estimation procedure in the

\(^{36}\)The CRRA model requires a higher value \( b = 4 \) to avoid the increasing risky-allocation pattern in retirement.
Figure 13: Cross-sectional average stock allocation for the DA and CRRA agents and the twisted return premium under the DA preference model conditional on positive savings. The graphs are for the high school-educated group with the heteroscedastic income process.

Internet Appendix.

\[
\ln(Y_t) = l_t + \nu_t + h(\epsilon) \epsilon \
\forall t \leq K \tag{24a}
\]

\[
\nu_t = \nu_{t-1} + h_u(t) u_t \quad \text{Where } u_t \sim N(0, \sigma_u^2) \quad \epsilon_t \sim N(0, \sigma_\epsilon^2) \tag{24b}
\]

\[
h(\epsilon)(t) = h_u(t) = (t_{\text{stop}} - t)/(t_{\text{stop}} - t_{\text{start}}) \tag{24c}
\]

The gradual decline in total uninsurable risk implied by the heteroscedastic income process generates a phased drop in stock investment along with the steadily dwindling twisted return premium (Fig. 13). The changes in the disappointing return premium, \( \overline{R}_{e,d} \), and the disappointment probability, \( \Phi \), the two components of the DA effect, are also graded and not abrupt (not included). The stock allocations are similar in early working life under the DA and CRRA preferences, but only the DA model implies lower allocations during late working life and in retirement on account of the decline in income risk.
7.6 Health risk

The declining human capital is the dominant force that makes investments conservative under the CRRA model. In addition, the drop in income risk in late working life also contributes to this pattern under the DA model. However, the late phase of life also coincides with the rise in a notable source of risk, the health risk. Its impact through uncertain out-of-pocket medical expenses is a mild rise in risky allocation in retirement under the DA model. This pattern is neither a case in favor nor against the DA model as the empirical evidence is mixed. In addition, the net effect of health risk under the standard expected utility model is also ambiguous.

Edwards (2008, 2010) notes that health shocks can increase or decrease the marginal utility of consumption. The sign of the effect on portfolio choice depends on the complementarity of health with consumption and leisure. Some studies find that the decline in health increases the marginal utility (Lillard and Weiss (1997), Edwards (2008)), whereas others conclude the opposite (Finkelstein, Luttmer, and Notowidigdo (2013), Viscusi and Evans (1990)).\(^{37}\) Further, Love and Perozek (2007) show that if the survival risk and the expected survival horizon depend on health shocks, then, all else equal, an increase in survival risk increases the risky share of the portfolio in some cases. The health shocks are also associated with significant out-of-pocket medical expenses (De Nardi, French, and Jones (2006)). Pang and Warshawsky (2010) show that such uncertain expenses act as a background risk and imply safer investment portfolios.\(^{38}\) Thus, as a net effect, an adverse health shock may yield a rise or a drop in risky share under the standard model.

The empirical evidence on the relation between health status and risky allocation is also mixed. Rosen and Wu (2004) and Berkowitz and Qiu (2006) find a positive relation between poor health and lower risky allocation.\(^{39}\) Edwards (2008) finds that the self-perceived health

---


\(^{38}\)Yogo (2009) notes that some of the medical spending is in fact an endogenous investment in health, which can lower health-related background risk.

\(^{39}\)Fan and Zhao (2009) and Coile and Milligan (2009) also report a similar relation between health and risky allocation.
risk may explain 20% of the age-related decline in risky investment after retirement. However, Love and Smith (2010) find that an agent’s health status accounts for only a small causal effect on portfolio choice. They find that after they control for unobserved heterogeneity, health does not significantly affect portfolio choice among single households, and report only a small (two to three percentage points) effect for married households and for those in the lowest health categories.

I consider the impact of out-of-pocket (OOP) medical expenses under the DA model. The parsimonious process for OOP expenses, $M_t$, in Eq. 25 avoids additional state variables and contains the computational cost. I limit the maximum value of $M_t$ at $\varphi Y_t$. I subtract this censored medical expense, $\tilde{M}_t$, from available resources in each retirement period in Eq. 26. I estimate the process in Eq. 25 separately for each of the three education groups, using households from the 1992-2010 waves of the Health and Retirement Study (HRS) and restrict the data to those households whose head is older than 65. I provide details in the Internet Appendix. $q_t$ in Eq. 25 captures the deterministic age effect. Table C.3 in Appendix C lists coefficients for the second-order polynomials used to fit the regression estimates of the age effects and also the spread of $\iota_t$ shocks. I set $\varphi = 1.24$ and find only 3% of the observations with the ratio $M_t/Y_t$ above this value in the uncensored raw data. I retain the heteroscedastic income process from section 7.5.

\[
M_t = Y_t \exp(q_t + \iota_t) \quad \forall t > 65, \text{ where } \iota_t \sim N(0, \sigma^2) \quad (25)
\]

\[
W_t = A_{t-1} R_{p,t} + (1 - h_t)Y_t - \tilde{M}_t \quad \forall t > 65, \text{ where } \tilde{M}_t = \min\{\varphi Y_t, M_t\} \quad (26)
\]

The medical expense shocks can be severely disastrous, because they can be higher than the retirement income. The effect is to increase the dependence on financial wealth for consumption. The result is a portfolio-shares pattern rising in wealth at low values of wealth (Fig. 14, top-right panel). In addition, the agent saves and does not invest the present value
Figure 14: The graphs are for DA model subject to uncertain OOP medical expenses. Left panel: cross-sectional average stock allocation and twisted return premium conditional on positive savings. Top-right panel: the investment policy rules as a function of wealth normalized by permanent income in retirement. Bottom-right panel: the estimates of the average effect of age on \( \ln(M_t/Y_t) \) and the second-order polynomial fit to these estimates.

The increment in portfolio shares in retirement until the agent is in his early 80s is due to the rising twisted premium over this phase (Fig. 14). The rise in the variance of \( M_t/Y_t \) due to the rise in the mean value of \( \ln(M_t/Y_t) \) with age is the source behind this pattern. The increase in the expected value of OOP expenses with age also makes expense shocks more severe and raises the curvature component of the marginal utility, the standard effect. It tempers the impact of a rising twisted premium. The twisted premium, however, drops later in retirement and both effects induce safer portfolio choices over the terminal phase.

The average risky allocation in retirement is 6.7% in Fig. 14, up from 3.4% in the absence of uncertain OOP expenses. The current setup helps segregate the impact of uncertain OOP expenses. The current setup helps segregate the impact of uncertain OOP expenses.

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This drop is akin to a similar drop in one period model at high income risk.
expenses. However, the agent is likely to be subject to significant uncertain OOP expenses while still earning uncertain income. The delay in retirement might be one reason for such a pattern. The overlap between the two uncertainties can avoid the mildly increasing age pattern in stock allocation in retirement in Fig. 14.

8 Epstein-Zin preferences with DA aggregator

Gomes and Michaelides (2005) show that separating risk aversion from the elasticity of intertemporal substitution (EIS) and adding a one-time entry cost to participate in the equity market creates a recipe to match the equity participation rates. The agents with high risk aversion and high EIS turn out to be the dominant participants in the equity market, whereas the agents with low risk aversion and low EIS mostly abstain from the equity market. The low risk-averse and low EIS agents save a small buffer stock and lack a stronger drive to save for the retirement. The result is that such agents are unwilling to pay the small one-time entry cost. The small fraction of these agents that do pay the entry cost invest almost all their savings in the risky asset. I show that adding disappointment aversion helps lower the risky investments of such agents to some extent. Further the rise in aversion to disappointment increases wealth accumulation and lowers risky investment over the entire lifetime (section 7.2). These trends have opposing effects on the participation decision. The higher savings imply the agent is more inclined to pay the market entry cost, whereas holding the wealth accumulation constant, the agents with lower optimal investment in stocks are less willing to pay the participation cost. However, the net effect is an increase in the participation rate with the increase in the disappointment aversion parameter.

I follow Gomes and Michaelides (2005) and consider a model with preference heterogeneity to match participation rate, conditional equity allocation, and the distribution of the financial wealth-to-income ratio. I consider a population of investors equally split between two sets of preference parameters. Fifty percent of investors have low EIS, a low value for the curvature
Figure 15: The participation rates and conditional equity allocations over life for two different sets of preference parameters. The left panel corresponds to an agent with low EIS, a low curvature component of risk aversion, and low disappointment aversion. The right panel corresponds to an agent with moderate values for the same set of parameters.
\begin{equation}
J_t = \max_{C_t, x_t} \left[ (1 - \beta) C_t^{1 - 1/\psi} + \beta \left[ p_t \mu_t^{1 - \gamma} + (1 - p_t) b \mu_{b,t}^{1 - \gamma} \right] ^{\frac{1 - 1/\psi}{1 - 1/\psi}} \right]^{1/1 - \psi} \tag{27a}
\end{equation}

\begin{equation}
\mu_t^{1 - \gamma} = E_t \left[ J_{t+1}^{1 - \gamma} \right] - \theta E_t \left[ \mu_t^{1 - \gamma} - J_{t+1}^{1 - \gamma} I (J_{t+1} < \mu_t) \right] \tag{27b}
\end{equation}

\begin{equation}
J_T = \max_{C_T, x_T} \left[ (1 - \beta) C_T^{1 - 1/\psi} + \beta \left[ b \mu_{b,T}^{1 - \gamma} \right] ^{\frac{1 - 1/\psi}{1 - 1/\psi}} \right]^{1/1 - \psi} \tag{27c}
\end{equation}

\begin{equation}
W_{t+1} = (W_t - C_t) R_f + (1 - h_{t+1}) Y_{t+1} - F I_p Y_{p,t+1}, \text{ where } Y_{p,t+1} = e^{\eta t + 1 + \nu t}. \tag{27d}
\end{equation}

\begin{equation}
W_{t+1} = (W_t - C_t) R_{p,t+1} + (1 - h_{t+1}) Y_{t+1} \tag{27e}
\end{equation}

The low EIS and low risk-aversion (low curvature and disappointment aversion component) group saves little, and only a small fraction of the agents from this group enter the equity market (Fig. 15 left panel). In contrast to the agents with the CRRA aggregator and similar parameters (but \( \theta = 0 \)), the equity market participants from this group reduce their allocation to stocks as they progress through their working life. The declining twisted premium lowers the appetite for risky assets. The higher savings accompanying the agents with higher \( \gamma \) and \( \theta \) imply that they eagerly pay the entry cost despite the lower risky weight over the lifetime.

I compare the model-implied average participation rates and average conditional allocations for different age groups with the age effects implied by the SCF data in Fig. 16. I use the age effects from Fig. 1 that control for cohort effects and use the fraction of households with DC and/or IRA accounts in each survey year as the time control.\(^4\) I simulate the model with initial wealth distribution calibrated using SCF data. The model and the data differ in participation rates in the latter phase of life. One way to improve the match over this phase is to incorporate a small group of investors with disappointment aversion parameter above the critical threshold. Further, the conditional allocations have sizeable differences in the early

\(^4\) The SCF data includes cohorts born in 1910 through 1985 and the samples are drawn from triennial surveys from 1989 to 2010 with an additional survey in 2009. I use cohort effect for those born in 1950 and time effect for the survey year 2001, the respective midpoints, in Fig. 16.
Figure 16: The participation rates (left panel) and conditional equity allocations (right panel) implied by the model and SCF data for different age groups. The model involves 50% of investors with low EIS and low risk aversion (curvature component and disappointment aversion), and the other 50% with moderate values for the same preference parameters. The data points from SCF are the age effects in Fig. 1 that control for the cohort effect, and use a fraction of households with DC/IRA accounts in a survey year as the time control. I fix the time effect to that of year 2001 and fix the cohort effect to that of 1950.

I compare the wealth-accumulation pattern implied by the model with that from SCF in Table 3. I follow Gomes and Michaelides (2005) and compare the distributions over three age groups: buffer stock savers (20-35), retirement savers (36-65), and retirees (66 and higher). I use all survey samples to compute the distribution in the data. Similar to the model in Gomes and Michaelides (2005), the DA aggregator model also comes close to matching the wealth-accumulation pattern of poorer households, except in the retirement phase. However, the extent to which the EZ-DA model overshoots the wealth-to-income ratio of the median household in the 36-65 age group is slightly higher compared to that for the model in Gomes and Michaelides (2005). The DA aggregator model also yields larger wealth accumulation at the extreme, but the quantity is not as high as that in the retirement phase in the data.

Gomes and Michaelides (2005) obtain a median value of 3.116 for households in the 36-65 age group.
Table 3: The distribution of wealth-to-labor income ratios for different age groups. Top panel: the distribution from SCF data using all survey samples from 1989 to 2010. Bottom panel: the distribution generated from the model using initial wealth distribution from SCF data.

## Conclusion

I study the life-cycle portfolio problem of a disappointment averse (DA) agent and find that the declining income risk in late working life provides an additional motivation to reduce risky allocation. The DA model generates an age-dependent relationship between income risk and risky portfolio shares in congruence with the suggestive empirical evidence. I prove that the welfare under DA preferences with a positive curvature parameter and non-negative disappointment aversion parameter is concave in wealth. The addition of the DA aggregator to a heterogeneous Epstein-Zin model with a one-time equity market participation cost helps lower the conditional equity allocations of agents with the low elasticity of intertemporal substitution and low risk aversion. The DA model also suggests that a heteroscedastic income process better matches the observed equity allocations. The DA preferences also generate non-participation in equity markets in retirement if the agent’s aversion to disappointment is strong enough. Such non-participation occurs despite the fact that the equity premium is positive and no other frictions are present. I also relate the life-cycle results to a one-period investment problem with uncertain labor income. I show that the DA preferences generate a hump-shaped pattern in risky portfolio share with income risk. I prove that the risky portfolio share must increase for a sufficiently disappointment averse agent if the income and returns
are independent.

This article suggests that background risk is a useful lens through which to probe unique features of competing preference models and a useful tool to limit the set of preferences to those that are consistent with the observations.
A Proofs

Proposition A.1. The optimal welfare of a DA agent, $\mu_{da,t}(\sigma_Y)$, endowed with savings, $A_t$, and facing a one-period investment problem while earning a non-tradable income, $Y_{t+1}$, with standard deviation $\sigma_Y$ declines with the mean-preserving increase in the income risk if $\theta \geq 0$ and $\gamma > 0$. $\mu_{da,t}(\sigma_Y)$ represents the maximum welfare corresponding to income with a standard deviation of $\sigma_Y$ at the optimal portfolio weights:

$$\mu_{da,t}(\sigma_Y) \equiv \mu_{da,t}(\sigma_Y, x^*_t) \equiv \max_{x_t} \mu_{da,t}(\sigma_Y, x_t)$$

Where, $x^*_t = \arg \max_{x_t} \mu_{da,t}(\sigma_Y, x_t)$.

Proof. Let the income with a standard deviation of $\sigma_{Y,2}$ represent a mean-preserving spread over income with the standard deviation equal to $\sigma_{Y,1}$ that is; $\sigma_{Y,1} < \sigma_{Y,2}$. Let $x^*_{t,1}$ and $x^*_{t,2}$ be the optimal portfolio weights corresponding to standard deviation of income $\sigma_{Y,1}$ and $\sigma_{Y,2}$, respectively. Thus, $\mu_{da,t}(\sigma_{Y,1}) = \mu_{da,t}(\sigma_{Y,1}, x^*_{t,1})$ and $\mu_{da,t}(\sigma_{Y,2}) = \mu_{da,t}(\sigma_{Y,2}, x^*_{t,2})$.

Note that the definition of optimal welfare $\mu_{da,t}(\sigma_{Y,1})$ implies the following:

$$\mu_{da,t}(\sigma_{Y,1}, x^*_{t,1}) \geq \mu_{da,t}(\sigma_{Y,1}, x^*_{t,2})$$

$\Rightarrow \mu_{da,t}(\sigma_{Y,1}) \geq \mu_{da,t}(\sigma_{Y,1}, x^*_{t,2}).$ \hspace{1cm} (A.1)

Theorem 3 in Gul (1991) implies the conditions $\gamma > 0$ and $\theta \geq 0$ are sufficient for the second-order stochastic dominance for DA preferences. This property yields the first inequality below, and combining this inequality with Eq. A.1 yields the final result:

$$\Rightarrow \mu_{da,t}(\sigma_{Y,1}, x^*_{t,2}) > \mu_{da,t}(\sigma_{Y,2}, x^*_{t,2})$$

$\Rightarrow \mu_{da,t}(\sigma_{Y,1}, x^*_{t,2}) > \mu_{da,t}(\sigma_{Y,2}) \Rightarrow \mu_{da,t}(\sigma_{Y,2}) = \mu_{da,t}(\sigma_{Y,2}, x^*_{t,2})$ \hspace{1cm} (A.1)

Thus, the optimal welfare declines with the increasing mean-preserving spread in income.

Proof of proposition 3.1. The DA agent’s welfare $\mu_t$ and terminal wealth $W_{t+1}$ given the savings $A_t$ and portfolio weight $x_t$ in presence of income $Y_{t+1}$ are as follows:
\[ W_{t+1} = A_t [R_{e,t+1} x_t + R_f] + Y_{t+1} \]
\[ \mu_{da,t}^{1-\gamma} = E_t \left[ W_{t+1}^{1-\gamma} \right] - \theta E_t \left[ \left( \mu_{da,t}^{1-\gamma} - W_{t+1}^{1-\gamma} \right) I(W_{t+1} < \mu_{da,t}) \right]. \]

\( R_{e,t+1} \) is the risky excess return and \( R_f \) is the risk-free rate. The wealth and the welfare limits as the portfolio weight \( x_t \) tends to zero are \( W_{t+1}^0 \) and \( \mu_{0,da,t} \), respectively.

\[
\begin{align*}
\lim_{x_t \to 0} W_{t+1} &= W_{t+1}^0 = A_t R_f + Y_{t+1} \\
\lim_{x_t \to 0} \mu_{da,t} &= \mu_{0,da,t}.
\end{align*}
\]

Thus, if income is risk free, \( \mu_{0,da,t} = W_{t+1}^0 = A_t R_f + Y_{t+1} \). Further, the differential of \( \mu_{da,t} \) with respect to the portfolio weight \( x_t \) is

\[
\frac{d \mu_{da,t}}{dx_t} = \frac{E_t \left[ W_{t+1}^{\gamma} \left( R_{e,t+1} + (1 + \theta I(R_{e,t+1} < 0)) \right) \right]}{1 + \theta \Phi(R_{e,t+1} \leq \mu_{da,t})}.
\]

**Case 1: Risk-free income**

I consider the case of risk-free income and simplify the inequality \( W_{t+1} < \mu_{da,t} \) as \( x_t \to 0^+ \) and as \( x_t \to 0^- \).

\[
\begin{align*}
\lim_{x_t \to 0^+} I(W_{t+1} < \mu_{da,t}) &= \lim_{x_t \to 0^+} I(\mu_{da,t} R_{e,t+1} x_t < 0) \\
\Rightarrow \lim_{x_t \to 0^+} I(W_{t+1} < \mu_{da,t}) &= I(R_{e,t+1} < 0) \\
\text{Similarly, } \lim_{x_t \to 0^-} I(W_{t+1} < \mu_{da,t}) &= I(R_{e,t+1} > 0).
\end{align*}
\]

I apply the above results to compute the differential of \( \mu_{da,t} \) with respect to \( x_t \) as \( x_t \to 0 \):

\[
\lim_{x_t \to 0^-} \mu_{da,t} d\mu_{da,t} \bigg| \frac{d}{dx_t} = (W_{t+1}^0)^{-\gamma} E_t \left[ R_{e,t+1} (1 + \theta I(R_{e,t+1} > 0)) \right] \]
\[
\Rightarrow \lim_{x_t \to 0^-} \mu_{da,t} d\mu_{da,t} \bigg| \frac{d}{dx_t} > 0 \quad \forall \theta > 0 \\ \\
\text{But, } \lim_{x_t \to 0^+} \mu_{da,t} d\mu_{da,t} \bigg| \frac{d}{dx_t} = (W_{t+1}^0)^{-\gamma} E_t \left[ R_{e,t+1} (1 + \theta I(R_{e,t+1} < 0)) \right] \]
\[
\Rightarrow \lim_{x_t \to 0^+} \mu_{da,t} d\mu_{da,t} \bigg| \frac{d}{dx_t} < 0 \quad \forall \theta > \theta^*.
\]
The above results imply that the welfare of the agent improves if he chooses not to short the risky asset, and his welfare declines if he goes long the asset. Thus, the agent optimally chooses not to invest in the risky asset when income is risk free and \( \theta > \theta^* \).

**Case 2: Risky income**

I again simplify the inequality \( W_{t+1} < \mu_{da,t} \) as \( x_t \to 0 \) when income is risky:

\[
\lim_{x_t \to 0} I(W_{t+1} < \mu_{da,t}) = I(A_t R_f + Y_{t+1} < \mu^0_{da,t})
\]

I use the above inequality to compute the differential of \( \mu_{da,t} \) with respect to \( x_t \) as \( x_t \to 0 \):

\[
\lim_{x_t \to 0} \mu_{da,t}^{-\gamma} \frac{d\mu_{da,t}}{dx_t} = E_t[f(Y_{t+1})g(Y_{t+1}) R_{e,t+1}]
\]

Where, \( f(Y_{t+1}) = (A_t R_f + Y_{t+1})^{-\gamma} \),

and \( g(Y_{t+1}) = \frac{1 + \theta I(A_t R_f + Y_{t+1} < \mu^0_{da,t})}{1 + \theta \Phi(A_t R_f + Y_{t+1} < \mu^0_{da,t})} \).

Note, however, that \( Y_{t+1} \) and \( R_{e,t+1} \) are independent. This relation between the returns and the income implies the following:

\[
\lim_{x_t \to 0} \mu_{da,t}^{-\gamma} \frac{d\mu_{da,t}}{dx_t} = E_t[f(Y_{t+1})g(Y_{t+1}) R_{e,t+1}]
\]

\[
\Rightarrow \lim_{x_t \to 0} \mu_{da,t}^{-\gamma} \frac{d\mu_{da,t}}{dx_t} > 0 \quad \because f > 0, \; g > 0, \; R_{e,t+1} > 0.
\]

Thus, the DA agent’s welfare increases if he invests some positive amount of the savings in the risky asset. This result applies for all \( \theta > 0 \).

**Proof of proposition 3.2.** Let \( K_{\theta,t} = E_t[R_{e,t+1} + \theta E_t[R_{e,t+1} I(R_{e,t+1} < 0)] \). The inequality \( \theta < \theta^* \) implies \( K_{\theta,t} > 0 \):

\[
\Rightarrow -\frac{E_t[R_{e,t+1}]}{E_t[R_{e,t+1} | R_{e,t+1} < 0] \times \Phi(R_{e,t+1} < 0)} > \theta
\]

\[
\Rightarrow -\theta E_t[R_{e,t+1} I(R_{e,t+1} < 0)] < E_t[R_{e,t+1}]
\]

\[
\Rightarrow K_{\theta,t} > 0. \quad (A.2)
\]
Let $\mu_{da,t}$ be the welfare for an arbitrary portfolio weight $x_t$ and income $Y_{t+1}$ that yields a wealth gamble $W_{t+1}$. This set-up implies, $E_t[R_{e,t+1}I(W_{t+1} < \mu_{da,t})] \geq E_t[R_{e,t+1}I(R_{e,t+1} < 0)]$. The equality applies only in the case of risk-free income. This inequality implies the inequality in Eq. A.3:

$$E_t[R_{e,t+1}I(W_{t+1} < \mu_{da,t})] \geq E_t[R_{e,t+1}I(R_{e,t+1} < 0)]$$

Since $E_t[R_{e,t+1}I(W_{t+1} < \mu_{da,t})] \geq E_t[R_{e,t+1}I(R_{e,t+1} < 0)]$

$$E_t[R_{e,t+1}I(W_{t+1} < \mu_{da,t})] \geq E_t[R_{e,t+1}I(R_{e,t+1} < 0)]$$

$$K_{\theta,t} \therefore \theta \geq 0.$$ (A.3)

I combine the inequality in Eqs. A.2 and Eq. A.3 to obtain the following inequality:

$$E_t[R_{e,t+1}I(W_{t+1} < \mu_{da,t})] \geq E_t[R_{e,t+1}I(R_{e,t+1} < 0)] \geq K_{\theta,t} \therefore \theta \geq 0.$$ (A.4)

The welfare $\mu_{da,t}$ for $\gamma = 0$ is given by

$$\mu_{da,t} = E_t[W_{t+1}] - \theta E_t[(\mu_{da,t} - W_{t+1}) I(W_{t+1} < \mu_{da,t})].$$

The differential of $\mu_{da,t}$ with respect to the portfolio weight $x_t$ is

$$\frac{d\mu_{da,t}}{dx_t} = \frac{E_t[R_{e,t+1}I(W_{t+1} < \mu_{da,t})]}{1 + \theta \Phi(W_{t+1} < \mu_{da,t})}.$$ (A.5)

I rewrite the above differential and use Eq. A.4 to obtain the inequality in Eq. A.5, which implies welfare improves with the increase in portfolio weight $x_t$, where $x_t \leq 1$.

$$\frac{d\mu_{da,t}}{dx_t} = \frac{E_t[R_{e,t+1}I(W_{t+1} < \mu_{da,t})]}{1 + \theta \Phi(W_{t+1} < \mu_{da,t})} > 0 \therefore 0 \leq \theta < \theta^* \text{ and } \Phi \geq 0.$$ (A.5)

Because the welfare improves with increasing portfolio weight, the agent invests all of the savings in the risky asset without violating any constraints.

**Sketch of the proof for proposition 4.1.**

**Case:** $\theta = 0$

If $\theta = 0$, the agent’s welfare folds back to expected utility case. The properties of expected utility are sufficient to prove the proposition.

**Case:** $\theta > 0$ and both $W_{1,t+1}$ and $W_{2,t+1}$ are sure gambles
If \( W_{1,t+1} \) and \( W_{2,t+1} \) are sure gambles then the welfare again folds back to expected utility case and the properties of expected utility are sufficient to prove the proposition.

Case: \( \theta > 0 \) and at least one of \( W_{1,t+1} \) and \( W_{2,t+1} \) gambles is an uncertain gamble

I rewrite the terms in the proposition to economize on the notation.

Let \( \psi_t(W_{t+1}, \nu_{t+1}) \equiv \frac{\mu_t(g_{t+1}(W_{t+1}, \nu_{t+1}))^{1-\gamma}}{1-\gamma} \) and \( G_{t+1}(W_{t+1}, \nu_{t+1}) \equiv \frac{g_{t+1}(W_{t+1}, \nu_{t+1})^{1-\gamma}}{(1-\gamma)}. \)

The inequalities in the proposition with the new notation are:

\[
\lambda \psi_t(W_{1,t+1}, \nu_{t+1}) + (1-\lambda) \psi_t(W_{2,t+1}, \nu_{t+1}) < (\leq) \psi_t(W_{\lambda,t+1}, \nu_{t+1}) \quad \forall 0 < \lambda < 1.
\]

The objective is to prove that the strong(weak) version of the inequality holds if \( G_{t+1}(W_{1,t+1}, \nu_{t+1}) \) is concave (quasi-concave) in \( W_{t+1} \), where, \( W_{\lambda,t+1} = \lambda W_{1,t+1} + (1-\lambda)W_{2,t+1} \). The properties of concavity (quasi-concavity) of \( G_{t+1}(\cdot, \nu_{t+1}) \) imply the following inequality given the definition of welfare for a DA agent when facing the gamble \( W_{\lambda,t+1} \).

\[
0 < (\leq) \psi_{\lambda,t} - E_t[\lambda G_{t+1}(W_{1,t+1}, \nu_{t+1}) + (1-\lambda)G_{t+1}(W_{2,t+1}, \nu_{t+1})]
+ \theta E_t[(\psi_{\lambda,t} - \lambda G_t(W_{1,t+1}, \nu_{t+1}) - (1-\lambda)G_t(W_{2,t+1}, \nu_{t+1}))]
\times I(\lambda G_{t+1}(W_{1,t+1}, \nu_{t+1}) + (1-\lambda)G_{t+1}(W_{2,t+1}, \nu_{t+1}) < \psi_{\lambda,t})]
\]

I use the definition/notation \( \psi_{\lambda,t} \equiv \psi_t(W_{\lambda,t+1}, \nu_{t+1}) \) in the inequality above. I further economize on notation by defining \( G_{1,t+1} \equiv G_{t+1}(W_{1,t+1}, \nu_{t+1}), G_{2,t+1} \equiv G_{t+1}(W_{2,t+1}, \nu_{t+1}), \psi_{1,t} \equiv \psi_t(W_{1,t+1}, \nu_{t+1}) \) and \( \psi_{2,t} \equiv \psi_t(W_{2,t+1}, \nu_{t+1}) \). I use the definitions of welfares \( \psi_{1,t} \) and \( \psi_{2,t} \) along with the inequality above to obtain the following inequality written in the new notation.

\[
\Delta_{1,2,\lambda} + \lambda \theta E_t[(\psi_{1,t} - G_{1,t+1}) I(G_{1,t+1} < \psi_{1,t})]
+ (1-\lambda) \theta E_t[(\psi_{2,t} - G_{2,t+1}) I(G_{2,t+1} < \psi_{2,t})] < (\leq) \theta E_t[(\psi_{\lambda,t} - \lambda G_{1,t+1} - (1-\lambda)G_{2,t+1})]
\times I(\lambda G_{1,t+1} + (1-\lambda)G_{2,t+1} < \psi_{\lambda,t})]
\]

(\( A.6 \))

\( \Delta_{1,2,\lambda} \) is defined as \( \Delta_{1,2,\lambda} \equiv \lambda \psi_{1,t} + (1-\lambda)\psi_{2,t} - \psi_{\lambda,t} \). The above inequality is independent of the relation between \( \psi_{\lambda,t}, \psi_{1,t} \) and \( \psi_{2,t} \). I consider three possible cases. These are (1) \( \psi_{\lambda,t} < \lambda \psi_{1,t} + (1-\lambda)\psi_{2,t} \), (2) \( \psi_{\lambda,t} = \lambda \psi_{1,t} + (1-\lambda)\psi_{2,t} \) and (3) \( \psi_{\lambda,t} > \lambda \psi_{1,t} + (1-\lambda)\psi_{2,t} \). These three cases translate into (1) \( \Delta_{1,2,\lambda} > 0 \), (2) \( \Delta_{1,2,\lambda} = 0 \) and (3) \( \Delta_{1,2,\lambda} < 0 \) respectively.
Case (1): $\Delta_{1,2,\lambda} > 0$

If $\Delta_{1,2,\lambda} > 0$ then this assumption combined with Eq. A.6 yields the following inequality.

$$\Delta_{1,2,\lambda} < \theta \lambda H_1 + \theta (1 - \lambda) H_2$$  \hspace{1cm} (A.7)

Where, $H_1$ and $H_2$ are as follows.

$$H_1 = E_t [(\psi_{1,t} - G_{1,t+1}) I (\lambda G_{1,t+1} + (1 - \lambda) G_{2,t+1} < \psi_{\lambda,t})] - E_t [(\psi_{1,t} - G_{1,t+1}) I (G_{1,t+1} < \psi_{1,t})]$$

$$H_2 = E_t [(\psi_{2,t} - G_{2,t+1}) I (\lambda G_{1,t+1} + (1 - \lambda) G_{2,t+1} < \psi_{\lambda,t})] - E_t [(\psi_{2,t} - G_{2,t+1}) I (G_{2,t+1} < \psi_{2,t})]$$

Consider $H_1$. It is made up of two integrals, $K_{H_1}$ and $Q_{H_1}$ as defined below.

$$K_{H_1} = E_t [(\psi_{1,t} - G_{1,t+1}) I (G_{1,t+1} < \psi_{1,t})]$$

$$Q_{H_1} = E_t [(\psi_{1,t} - G_{1,t+1}) I (\lambda G_{1,t+1} + (1 - \lambda) G_{2,t+1} < \psi_{\lambda,t})]$$

$Q_{H_1}$ is bounded above at $K_{H_1}$, which translates to $H_1 \leq 0$. Similarly, $H_2 \leq 0$. Since, $\theta > 0$ and $0 < \lambda < 1$, I have,

$$\theta \lambda H_1 + \theta (1 - \lambda) H_2 \leq 0.$$

But, given the assumption that, $\Delta_{1,2,\lambda} > 0$, Eq. A.7 indicates,

$$0 < \theta \lambda H_1 + \theta (1 - \lambda) H_2 \quad \therefore \Delta_{1,2,\lambda} > 0.$$

Thus the assumption of $\Delta_{1,2,\lambda} > 0$ yields an absurd result indicating incorrect assumption.

Case (2): $\Delta_{1,2,\lambda} = 0$

Same arguments and steps in “Case (1)” indicate that $H_1 \leq 0$ and $H_2 \leq 0$ and unless $G_{t+1}(\cdot, \nu_{t+1})$ is quasi-concave the result is a pair of equations contradicting each other.

Combined:

Thus, if $G_{t+1}(\cdot, \nu_{t+1})$ is concave (quasi-concave) then $\lambda \psi_{1,t} + (1 - \lambda) \psi_{2,t} < (\leq) \psi_{\lambda,t}$.  

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B  Numerical method

I use endogenous grid method to save on computational cost (Carroll (2006)). The integrals involved in computing expectations are not amenable to the quadrature method. I use simulated draws of shocks to compute these integrals. I use equi-distributed Sobol sequences to generate these draws.\textsuperscript{43} The computation of integrals combined with the search for a DA fixed point impedes the solution speed. I use the golden section search with parabolic interpolation for solving for the optimal portfolio weight and optimal consumption in the benchmark case. I use the same methods in the extensions except for the case in which I model the health risk. I use a grid search method for both the portfolio and consumption optimization problems when the curvatures of intertemporal and static choice are different, and for the extension of the benchmark case with health risk.

The DA fixed-point problems reduce to a generic problem of finding \( \mu \) such that \( f(\mu) = 0 \), where \( f(\mu) \equiv \mu^{1-\gamma} + \theta E [(\mu^{1-\gamma} - G^{1-\gamma}) I(G < \mu)] - E [G^{1-\gamma}] \). The function, \( f \), is monotonic in \( \mu \) as \( f'(1-\gamma) > 0 \). Thus, the problem, \( f(\mu) = 0 \), is amenable to a bisection algorithm. The two end points of the search are \([G_{\text{min}}, G_{\text{max}}]\). Note that \( f(G_{\text{min}}) \times f(G_{\text{max}}) < 0 \); thus the solution to \( f(\mu) = 0 \) lies in \([G_{\text{min}}, G_{\text{max}}]\).

C  Income and out-of-pocket medical expense estimates

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<th>No High School</th>
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<td>( \sigma_\varepsilon )</td>
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<td>0.3102</td>
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<td>( \sigma_u )</td>
<td>0.0806</td>
<td>0.0883</td>
<td>0.1287</td>
</tr>
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Table C.1: Constant, \( Age, Age^2/100 \), and \( Age^3/100,000 \) provide coefficients for the third-order polynomial in age for the numerical implementation of the deterministic part of the labor income profile, \( l_t \), for each of the three education groups. \( \lambda, \sigma_\varepsilon, \) and \( \sigma_u \) provide the estimates for the replacement ratio, standard deviations of idiosyncratic shock, and permanent shock, respectively.

\textsuperscript{43}See Judd (1998) for the benefits of equi-distributed sequences over pseudo-random number generator sequences. I also performed optimization using pseudo-random number generator sequences and found no difference.
Table C.2: Estimates of $\sigma_u$ and $\sigma_\epsilon$ for the heteroscedastic income process in Eqs. (24a)-(24c). $t_{stop} = 66$ and $t_{start} = 20$ for all education groups except $t_{start} = 22$ for the college-educated group.

<table>
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Table C.3: Constant, $Age$ and $Age^2/100$ provide coefficients for the second-order polynomial in age for numerical implementation of the deterministic part of the OOP medical expense profile, $q_t$, for each of the three education groups. $\sigma_\epsilon$ includes the estimates for the standard deviations of the process.

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References


Peijnenburg, Kim, 2011, Life-cycle asset allocation with ambiguity aversion and learning, *Available at SSRN 1785321*.


